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AINSLEY'S
ENGINEERS MANUAL
OF THE
LOCAL MARINE BOARD
EXAMINATIONS

7/6

THE

ENGINEERS' MANUAL

OF THE

LOCAL MARINE BOARD

EXAMINATIONS,

FOR CERTIFICATES OF COMPETENCY, AS SECOND AND FIRST-CLASS ENGINEERS,

(WITH UPWARDS OF ONE HUNDRED AND FIFTY
ILLUSTRATIVE DIAGRAMS, &c.)

BY THOMAS L. AINSLEY,

AUTHOR OF "A GUIDE BOOK TO THE LOCAL MARINE BOARD
EXAMINATION," ETC., ETC.

Seventeenth Edition.

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PREFACE.

THIS Work is intended for the use of Engineers desirous of obtaining Certificates of Competency, either as Second-Class or First-Class Engineer. The Work is drawn up with a view to *self-instruction*, or for the use of Schools.

It contains rules for working out the several Problems that are presented to candidates, with model solutions of, and numerous exercises to each Problem. A number of Examination Papers are also given. The chapter on the "Indicator" is illustrated by a large number of choice Diagrams, taken from Engines by the most eminent makers—including some from the Screw and Paddle Engines of the *Great Eastern;* H.M.S.S. *Nile;* the Iron-clad *Research;* the R.M.S.S. *China;* the *Sea King* (Confederate Cruiser); the S.S. *Hansa*, &c., &c.; also some rare and valuable ones from a *Compound Marine Engine* by Messrs. ROWAN & Co., &c. This species of engine combines the various improvements of a high pressure of steam with superheating and surface condensation, and is reported by Professor RANKINE to have acted with a smaller consumption of fuel than has been hitherto attained in any engine whatever.

The mode of reading Indicator Diagrams is fully treated of, and also practical hints are given as to the important information to be obtained from them. It also contains chapters on Surface Condensers, Superheated Steam, Proportions of Engines, and such other related subjects as are required for the *viva voce* Examinations.

The Answers are given in full at the end of the book, with copious details of the intermediate parts of the solution.

T. L. A.

SOUTH SHIELDS, *July 15th*, 1865.

ADVERTISEMENT TO SEVENTEENTH EDITION.

In the present Edition of the Engineers' Manual an attempt has been made to adapt it to the present requirements of the Board of Trade Examinations. The Examination Papers are a selection from some of those which have been given to Candidates at the Examinations.

T. L. A.

March 4th, 1880.

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EXAMINATION OF ENGINEERS FOR CERTIFICATES OF COMPETENCY,

Under the Merchant Shipping Acts, &c., Amendment Act, 1862.

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1. Under the provisions of the "Merchant Shipping Acts, &c., Amendment Act, 1862," no "Foreign-going Steam Ship," or "Home Trade Passenger Steam Ship, can obtain a clearance or transire, or legally proceed to sea, from any port in the United Kingdom, unless in the case of a Foreign-going Steam Ship of 100 nominal horse-power or upwards, the First and Second Engineers, and in the case of a Foreign-going Steam Ship of less than 100 nominal horse-power, or a Home Trade Passenger Steam Ship, the First or Only Engineer (as the case may be), have obtained and possess valid Certificates either of competency or service appropriate to their several stations in such Steam Ship, or of a higher grade.

2. Every person who, having been engaged to serve as First or Second Engineer in a Foreign-going Steam Ship of 100 nominal horse-power and upwards, or as First or Only Engineer in a Foreign-going Steam Ship of less than 100 nominal horse-power, or in a Home Trade Passenger Steam Ship, goes to sea as such First, Second, or Only Engineer, without being at the time entitled to and possessed of such a Certificate as the Act requires, or who employs any person as First or Second Engineer in a Foreign-going Steam Ship of 100 nominal horse-power and upwards, or as First or only Engineer in a Foreign-going Steam Ship of less than 100 nominal horse-power, or a Home Trade Passenger Steam Ship, without ascertaining that he is at the time entitled to and possessed of such Certificate, for each offence incurs a penalty not exceeding fifty pounds.

3. The Certificates of Engineers are of two descriptions, viz., Certificates of Competency, and Certificates of Service; and for each description of Certificate there are two grades, viz., "First-class Engineer's Certificates," and "Second-class Engineer's Certificates."

4. Certificates of Competency will be granted to those persons who pass the requisite examinations, and otherwise comply with the requisite conditions. For this purpose Examiners have been appointed by the Board of Trade, and arrangements have been made for holding the examinations in the places and at the times named in the Table marked A on page 4. The Examiners are selected generally from the Engineer Surveyors of the port; but no Engineer Surveyor is to undertake the duty unless he receives special instructions from the Board of Trade.

5. The application for examination is to be made on Form Exn. 3. The same Rules are to be observed by Engineers in making application to be examined, in paying the fees, and in forwarding testimonials, as in the case of applications by Masters and Mates.

QUALIFICATIONS FOR CERTIFICATES OF COMPETENCY.

6. **Second-class Engineer.**—A candidate for a Second-class Engineer's Certificate must be 21 years of age;

(*a*) He must have served an apprenticeship to an Engineer, and prove that during the period of his apprenticeship he has been employed on the making and repairing of engines: Or if he has not served an apprenticeship, he must prove that for not less than three years he has been employed in some factory or workshop on the making or repairing of engines. In either case he must also have served one year at sea in the Engine Room; or

(*b*) He must have served at least four years at sea in the Engine Room.

(*c*) He must be able to give a description of boilers, and the methods of staying them, together with the use and management of the different valves, cocks, pipes, and connexions.

(*d*) He must understand how to correct defects from accident, decay, &c., and the means of repairing such defects.

(*e*) He must understand the use of the barometer, thermometer, hydrometer, and salinometer.

(*f*) He must state the causes, effects, and usual remedies for incrustation and corrosion.

(*g*) He must be able to state how a temporary or permanent repair could be effected in case of derangement of a part of the machinery, or total break down.

(*h*) He must write a legible hand, and understand the first five rules of arithmetic, and decimals.

(*i*) He must be able to pass a creditable examination as to the various constructions of paddle and screw engines in general use; as to the details of the different working parts, external and internal, with the use of each part.

7. **First-class Engineer.**—A candidate for a First-class Engineer's Certificate must be 22 years of age.

In addition to the qualification required for a Second-class Engineer,

(*a*) He must either possess, or be entitled to a First-class Engineer's Certificate of Service; or in the event of his not being so possessed or entitled he must have served for one year with a Second-class Engineer's Certificate of competency.

By this it is intended that the same rule shall be observed in the examination of engineers who are not in possession of a First-class Certificate of Service, as is observed in the examination of masters and mates; viz.,—that before the certificate of a higher grade is granted, certain service in the lower grade must be performed.

The Examiner should therefore be satisfied that applicants for the First-class Engineer's Certificate have not only been *in possession* of a Second-class Certificate for twelve months, but that they have actually *served* for a period of not less than twelve months, in the engine-room, with a Second-class Certificate, in the capacity of a second engineer, and that their names have been entered in the articles of agreement accordingly.

(*b*) He must be able to make rough working drawings of the different parts of the engines and boilers.

(*c*) He must also be able to take off and calculate indicator diagrams.

(*d*) He must be able to calculate safety-valve pressures, and the strength of the boiler.

(*e*) He must be able to state the general proportions borne by the principal parts of the machinery to each other.

(*f*) He must be able to explain the method of testing and altering the setting of the slide valves, and of testing the fairness of the paddle and screw shafts and of adjusting them.

(*g*) He must be conversant with surface condensation, superheating, and the working of steam expansively.

(*h*) His knowledge of arithmetic must include the mensuration of superficies and solids and the extraction of the square root.

8. **An Extra First-class Engineer's Examination** is voluntary, and is intended for such persons as wish to prove their superior qualifications, and are desirous of having Certificates for the highest grade granted by the Board of Trade.

The candidate must be entitled to or possessed of a First-class Engineer's Certificate of Competency, and in addition to the qualifications required for a First-class Engineer:

(*a*) He must possess a thorough knowledge of the construction and working of marine engines and boilers in all their parts, and be so far acquainted with the elements of theoretical mechanics as to comprehend the general principles on which the machine works.

(*b*) He must understand how to apply the indicator and draw the proper conclusions from the diagrams.

(*c*) He must be acquainted with the principles of expansion, and able to prove, or at least to illustrate, the use of the expansion gear.

(*d*) He must be able to draw rough sketches of any part of the machinery, with figured dimensions, fit to work from.

GENERAL RULES AS TO EXAMINATIONS AND FEES.

9. The examination will be partly *viva voce*, and partly by examination papers. It will be directed specially to the above points, and to the duties and business of an Engineer generally.

If the candidate passes the *viva voce* examination creditably, a set of questions will be given to work out.

He will be allowed to work out these questions according to the methods he is accustomed to use, and will be allowed five hours to perform the work: and

(*a*) If at the expiration of the time allowed he has worked out correctly the whole of the questions set him, he will be declared to have passed.

(*b*) If at the expiration of the time allowed he has not worked out the whole of the questions set him, but if the result of the *viva voce* examination taken in connexion with the answers to such of the questions as he has worked out are sufficient to satisfy the examiner that the applicant is competent to take charge of engines of 100 nominal horse-power or upwards, he will be declared to have passed.

(*c*) In other cases he will be declared to have failed.

A report of the Examination, and the Examination Papers, will be forwarded to the Board of Trade on the Form (Exn. 15).

10. The fee for examination must be paid to the Superintendent of the Mercantile Marine Office. If a candidate fail in his examination, half the fee he has paid will be returned to him by the Superintendent on his producing the Form Exn. 17, which will be given him by the Examiner. The fees are as follows:—

	£	s.	d.
First-class Engineer's Certificate (whether extra or ordinary)	2	0	0
Second-class Engineer's Certificate	1	0	0
For First-class Engineer's Certificate, if already in possession of a Second-class Certificate	1	0	0

11. If the applicant passes he will receive a Form Exn. 16 from the Examiner, which will entitle him to receive his Certificate of Competency from the Superintendent of the Mercantile Marine Office at the port to which he has directed it to be forwarded. If his testimonials have been sent to the Registrar General of Seamen to be verified, they will be returned with his Certificate.

12. If an applicant is examined for the higher grade, and fails, but passes an examination of the lower grade, he may receive a Certificate accordingly, but no part of the fee will be returned.

13. If the applicant fails in working out the examination papers, he may present himself for re-examination whenever he thinks he has acquired sufficient knowledge to enable him to pass. But if he fails in the *viva voce* or practical part of the examination, he may not present himself for re-examination until the expiration of three months from the date of failure.

CERTIFICATES OF SERVICE.

14. Every person who, before the 1st April, 1862, had served as either First Engineer in a British Foreign-going Steam Ship of 100 nominal horse-power and upwards, or who has attained or attains the rank of Engineer in the service of Her Majesty or of the East India Company, is entitled to a First-class Engineer's Certificate of Service; and every person who, before the above-mentioned date, has served as Second Engineer in any British Foreign-going Steam Ship of 100 nominal horse-power or upwards, or as First or only Engineer in any other Steam Ship, or who has attained or attains the rank of First-class Assistant Engineer in the Service of Her Majesty, is entitled to a Second-class Engineer's Certificate of Service.

15. Applications for Certificates of Service must be made on the printed Form Exn. 22, to be obtained *free of charge* of the Registrar-General of Seamen, Adelaide Place, London Bridge, London, or of the Superintendent of any Mercantile Marine Office.

TABLE A.

EXAMINATION DAYS
AT

Places. 1.	Days. For Engineers. 2.
BRISTOL	Second and Fourth Tuesday in each month.
DUNDEE	Thursday in each week.
GLASGOW* GREENOCK*	Thursday; held alternately at each place.
HULL	First and Third Tuesday in each month.
LIVERPOOL*	Thursday in each week.
LONDON*	Wednesday in each week.
SHIELDS, NORTH	First and Third Wednesday in each month.
SOUTHAMPTON	First and Third Tuesday in each Month.
SUNDERLAND	The last Monday in each month.

* At these places Engineers' Extra Examinations are held.

NOTICE OF ALTERATION IN EXAMINATION PAPERS.

From the 1st day of February, 1874, all candidates presenting themselves for examination will be required to give written answers to eight out of a list of ten questions selected from Form Exn. 15 *a*, "Elementary Questions for the first examinations of Engineers for certificates of competency, and of Masters Mates for certificates in steam." These questions are intended to furnish a record to some extent of the candidate's knowledge at the time of his examination, and also to induce the candidates to pay more attention to their handwriting and spelling.

The Form Exn. 15 *b*, on which these answers will be written, contains also some questions as to the experience of the candidate, to be answered by him in writing. A copy of Exn. 15 *b* is contained in Form Exn. 15 *a*.

The *viva voce* questions on the practical management of steam engines and boilers will remain on the same footing as at present. Examiners may add to their *viva voce* questions any of those contained in Exn. 15 *a*.

The arithmetical questions on Forms Exn. 10 and Exn. 11 will be cancelled, and Forms Exn. 10 *a* and Exn. 11 *a*, or other similar forms, to be from time to time issued, will be used instead. No further notice will be given of the changing of these forms. The questions will be promiscuous exercises on the arithmetic of marine engineering, and the candidate will not be expected to work all the questions, but only so many of them as he may think sufficient to satisfy the examiner that he possesses the required knowledge of arithmetic, and that he can apply it. The arithmetical standard for candidates is not altered; it will continue as at present to include decimal fractions for second-class, and to include square root for first-class candidates.

First-class candidates will be required to make, from a copy, an intelligible hand sketch, or a working drawing of some one or more of the principal parts of a steam-engine, and to mark in, without a copy, all the necessary dimensions in figures, so that the sketch or drawing could be worked from. This is considered an important qualification for a first-class engineer, and as his sending home an unworkable sketch to replace a break down would be a practical failure in competency, so the omission of one necessary dimension, or a disagreement between any two dimensions, will fail the candidate.

The examination questions are designed to test the degree of knowledge of this kind possessed by the candidate, and thereby to ascertain how much his heart has been given to his work.

The examination papers for a second-class certificate will consist of questions relating to the candidate's experience in the construction and management of engines, of questions in elementary knowledge, and of a set of engineering arithmetical questions.

In addition to these examination papers a candidate for a first-class certificate will have also another set of arithmetical questions, and an examination paper on indicator diagrams and working drawings.

The first set of new questions for first-class require a knowledge of the strength of a shaft against twisting and against transverse breaking, the strength of an iron lever, the loss by friction, and the cut off by a given amount of lap.

The new questions for second-class candidates contain nothing beyond what is contained in the old set of questions, except that one of the questions requires a knowledge of the weight of wrought iron.

In addition to these examination papers, the *viva voce* examinations will be continued as heretofore. These alterations are not intended in any way to remove from the individual examiner the entire responsibility of passing or of failing any candidate. The examination papers are planned to assist the examiner, and to afford a written evidence, to some extent, of the thoroughness of the examination, and to promote uniformity at the various examining ports.

TIME AND MARKS.

All candidates will be allowed six hours for the arithmetical paper and elementary questions, Form Examination 15 *b*—viz., from 10 A.M. to 4 P.M.

First-class candidates are directed to leave the diagram question, No. 9 on Examination 11 *a*, until the second day; the diagram will not be shown to the candidate on the first day.

The *viva voce* examination and drawing will be on the second day; but they may be continued on subsequent days if requisite.

To secure uniformity at the various examining ports, a system of marks has been arranged, and the candidates are not required to work all the questions on any of the forms, but only as many as they may consider will entitle them to the required number of marks.

The number of marks allowed for each question correctly worked is stated in the first column of data table laid before the candidate.

If the method adopted for any question be in the main correct, for each slip or omission in the method one mark will be forfeited.

If the numeration in the working of any question be in the main correct, for each error in the numeration one mark will be forfeited.

Marks gained for one question will not be forfeited on account of errors in any of the other questions.

The maximum number of marks on the second-class arithmetical paper is 33; the minimum required to pass is 22.

The maximum number of marks on the first-class arithmetical paper is 24; the minimum required to pass is 16.

For each full and correct answer to elementary questions, 2 marks will be allowed. For answers to the two questions on defects and repairs, on page 3 of Examination 15 *b*, from 0 to 4 marks will be allowed for each. The total marks on Examination 15 *b* will, therefore be 24, and the minimum required is 16.

Marks will be allowed for only ten questions on Form Examination 15 *b*, that is, for eight elementary questions, and for the two questions marked F and G. If either F or G be omitted, the number of elementary questions may be increased to make up ten in all.

Marks beyond the minimum for any Form may be applied to make up a defect of marks for any other Form at the rate of two marks overplus, to make up for one mark deficient.

ELEMENTARY QUESTIONS FOR THE FIRST EXAMINATIONS OF ENGINEERS FOR CERTIFICATES OF COMPETENCY; AND OF MASTERS AND MATES FOR CERTIFICATES IN STEAM.

Exn. 15 b.

NAVAL DEPARTMENT

Port	Class for which examined
Date	Candidate's name

Issued by the Board of Trade, April, 1878.

A.—Where and how long did you serve in works at the making or at the repairing of engines?

B.—How long have you served in the stokehole?

C.—How long have you served in the engine-room at sea, and in what capacities?

D.—With what descriptions of engines have you served at sea—paddle or screw, or both, jet condensing, surface condensing, or non-condensing engines, compounds, trunks, inverted cylinders, or horizontal engines? What size were the engines?

E.—With what descriptions of boilers have you served at sea—rectangular or cylindrical, wet bottomed or dry bottomed, multitubular, sectional, or flue boilers?

F.—What engine defects have come under your notice at sea, what caused these defects, and how were they remedied? Give the names of the steamers for verification.

G.—What boiler defects have come under your notice at sea, what caused these defects, and how were they remedied? Give the names of the steamers for verification.

For the questions to be answered on the following pages see the book of Elementary questions. The questions need not be written, only the answers to them.

Question No.

Question No.

Question No.

Question No.

Question No.

Question No.

Question No.

Question No.

EXPLANATION OF SIGNS USED.

The following signs are made use of both in arithmetic and algebra:—

SIGNS OF OPERATION.

1. The sign +, called *plus* (which is the Latin for *more*), signifies *additive*, or to be *added*, and shows that the number before which it stands is to be *added;* thus $3 + 4$ (read *three plus four*) means that 4 is to be added to 3, making 7.

2. The sign —, called *minus* (which is the Latin for *less*), signifies *subtractive*, or to be *subtracted*, and shows that the number before which it stands is to be *subtracted;* thus $13 - 5$ (read as *thirteen minus five*) means that 5 is to be subtracted from 13, leaving 8.

3. The sign × (into), signifies *multiplied by*, and shows that the numbers between which it stands are to be *multiplied;* thus, 3×4 (read *three into four*, or *three multiplied by four*), means that 3 is to be multiplied into 4, making 12. Sometimes a full stop at the bottom of the figure is used for this; thus, 2×7, or 2.7, are both used to express twice seven.

4. The sign ÷, signifies *divided by*, and means that the number which stands before it is to be divided by the one which follows it. Thus, $14 \div 2$, (read *fourteen by two*), means that 14 is to be divided by 2, making 7. The operation of division is also indicated by writing the divisor under the dividend with a line between them; thus, 14 by 2 is also frequently denoted thus $\frac{14}{2}$.

5. The sign =, signifying *equal to* (or amounting to), means that the numbers between which it stands are equal to each other; that is, have the same arithmetical value, each taken as a whole.

Examples of the preceding, with the results in each case, will stand thus:—

1. 14 and 3 equals 17, or $14 + 3 = 17$. 2. $10 - 3 = 7$.
3. $7 \times 5 = 35$. 4. $14 \div 2 = 7$, or $\frac{14}{2} = 7$.

6. The signs : *is to*, : : *so is*, are the signs of *proportion:* as 2 : 4 : : 8 : 16; that is, as 2 is to 4 so is 8 to 16.

7. The signs $\overline{14 - 4} + 10 = 20$ show that the difference between 4 and 14 added to 10 is equal to 20. The line drawn over 14 and 4 is called a *vinculum.*

8. The signs $10 - \overline{2 + 5} = 3$ signify that the sum of 2 and 5 taken from 10 is equal to 3.

9. 8^2 is read 8 *squared*, and means that the 8 is to be multiplied by itself; thus, $8 \times 8 = 64$; hence 64 is called the *square* of 8.

10. The sign √ prefixed to any number, signifies that the *Square Root* of that number is required; thus,

$\sqrt{64}$ is read *the square root of* 64, and means that number which when multiplied by itself gives 64; hence 64 is called the square of 8.

11. H.P. is read *horse-power.*
12. N.H.P. is read *nominal horse-power.*
13. I.H.P. is read *indicated horse-power.*
14. U.W. is read *unit of work.*

THE PRINCIPLES AND PRACTICE OF ARITHMETIC.

CHAPTER I.

DEFINITIONS, PRELIMINARY NOTIONS, NUMERATION, AND FUNDAMENTAL OPERATIONS.

ARTICLE I.—DEFINITION I.

1. **Arithmetic** is the science which treats of *numbers*—of the mode of expressing them—of the manner of computing by them—and of the various uses to which they are applied in the practical business of life.

2. A **Unit** or **Unity** is the name given to that quantity which is to be reckoned as *one* when other quantities of the same kind are to be measured. Thus, each of the terms, *a* man, *a* house, *a* pound, &c., denotes one individual of its kind, being the same as *one* man, *one* house, *one* pound, &c., respectively: and these are the basis or elements by means of which *several* men, *several* houses, *several* pounds, &c., may be computed.

3. Number is the relation of a quantity to its unit, the notion of number being suggested by successive repetitions of the individual unit; or number is the name by which we signify how many objects or things are considered whether *one* or *more*. When, for instance, we speak of one ship, two steamers, three masts, or four yards, the number of things referred to will be one, two, three, or four, according to the case; and so one, two, three, four, and the rest, are call numbers. Numbers thus viewed are termed *whole numbers* or *integers*; and for the sake of uniformity, the unit is considered the first or least integer.

4. Numbers used to express one or more individuals of *specified* kinds are called *concrete* numbers; whereas *two*, *three*, *four*, &c., by themselves, not particularizing the *kinds* of individuals, are termed *abstract* numbers.

NOTATION.

5. **Def. 1.**—*Notation* is the method of expressing any proposed number by certain symbols or characters appropriated for that purpose.

6. **Def. 2.**—The *symbol* or *representative* of unit or unity is 1; but instead of other numbers being expressed by assemblages or multitudes of units *placed* together, which would soon become embarrassing, other characters or symbols have been invented, by means of which every number, however great, may be expressed; and instead of a different symbol being adopted for every different number, which would soon become equally inconvenient, *all* numbers are expressed by means of the following ten symbols, or as they

are usually termed *figures*, and sometimes *digits*, which have their names respectively annexed:

1	2	3	4	5	6	7	8	9	0
one	two	three	four	five	six	seven	eight	nine	zero

the first *nine* of which are called by their names; and the *last*, which is variously denominated *nought*, *cypher*, or *zero*, when standing by itself has no signification, or at most, denotes the absence of number, and is to be regarded merely as an *auxiliary* digit, for the purposes to be hereafter explained.

7. **Def. 3.**—Whenever a figure is placed on the *right* of the same or any figure, it has, by *universal agreement*, the effect of increasing the value of the last-mentioned figure *tenfold*, at the same time that it retains its own value.

Thus beginning with the auxiliary digit 0, we have the following numbers and their representations:

10	11	12	13	14	&c.
ten	eleven	twelve	thirteen	fourteen	&c.

20	21	22	&c.
twenty	twenty-one	twenty-two	&c.

and it is obvious that by means of *two* figures, this kind of notation may be continued till we arrive at ninety-nine whose symbol is 99.

8. **Def. 4.**—Beyond this number, the use of *two*, either of the *same* or *different* figures, will not enable us to go, but a repetition of the contrivance in the last Article will, by means of *more* figures, supply the defect.

Thus, supposing the effect of any figures being placed on the right of symbols formed as above, to be to increase all their values *tenfold*, we shall have

100	101	102	&c.
one hundred	one hundred and one	one hundred and two	&c.

so likewise of succeeding numbers; thus, we have

234	567
two hundred and thirty-four	five hundred and sixty-seven

and again 999 will be *nine hundred and ninety-nine*, which is the largest number capable of being expressed by *three* figures.

Here, the *first* figure on the right hand is said to occupy the *unit's place*, the *second* the place of *tens*, and the *third* that of *hundreds*.

Of the auxiliary digit 0, the sole use is in the effect specified in the last two articles; and all figures to the *right* of it, will therefore be unaffected by it.*

9. **Def. 5.**—In estimating numerical magnitudes, we proceed in order from *hundreds*, to *thousands*, *tens of thousands*, and *hundreds of thousands*, *millions*,

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* The word *cypher* is from the Arabic word *Tsaphara*, blank or void. At the end or in the middle of any number the cypher is of use to keep the significant digits in their proper rank, when the units or the hundreds or any other denomination may be wanting, *e.g.*, 60 means 6 tens followed by no units, 606 means 6 hundreds with no tens, but 6 units. At the beginning of a number cyphers would be useless: if so placed they could only indicate the absence of any higher class, *e.g.*, 096 means only 9 tens and 6 units, the cypher showing that there are no hundreds, which is equally intelligible if the cypher be omitted.

tens of millions, and *hundreds of millions*, in precisely the same manner as we have done above from *units* to *tens*, and from *tens* to *hundreds*.

10. Agreeable to the principle of article 7 it is *assumed* that "any figure placed on the right of one or more others, has the effect of increasing every one of them tenfold, without altering its own value," and this enables us to express with facility any number whatever.

Thus,

1. 1000 will represent one *thousand*.
2. 5493 will represent five *thousands*, four *hundreds*, and ninety-three.
3. 23456 will represent twenty-three *thousands*, four *hundreds*, and fifty-six.
4. 729054 will represent seven hundred and twenty-nine *thousands* and fifty-four.
5. 1803205 will represent one *millon*, eight hundred and three *thousands*, two *hundreds*, and five.
6. 32754081 will represent thirty-two *millions*, seven hundred and fifty-four *thousands*, and eighty-one.
7. 473025004 will represent four hundred and seventy-three *millions*, twenty-five *thousands*, and four.

11. If the first three figures, beginning from the right hand, be denominated so many *units*, tens of *units*, and hundreds of *units*, it follows that the next three figures taken the same way will be *thousands*, tens of *thousands*, and hundreds of *thousands*; the next three in order will be *millions*, tens of *millions*, and hundreds of *millions*, and so on.

Whence to express in figures any number proposed, we have only to consider in which of these divisions each part of it ought to be found, observing that *three* figures from the right must be taken to make each division complete, before we proceed to the next.

EXAMPLES.

Ex. 1. Express, by means of figures, thirty-five thousand, eight hundred and nineteen.

Here, eight hundred and nineteen belongs to the *first* division on the right, and is written 819; also, thirty-five thousand must be found in the *second* division from the right, and is 35, whence the proposed number may be expressed by 35819.

Ex. 2. Express in figures the number five million, twenty-five thousand, six hundred and seven.

In this case the first division on the right will be 607; the second will be 025, the digit 0 being affixed to the left of the others without altering their value, to make up the required number of *three;* and the third is 5, so that the expression will be 5025607.

Ex. 3. Express by figures the following number, five hundred and seventy million, two hundred and six thousand and fifty-four.

Here, the first division is 054, the 0 altering only the value of the figures in the *subsequent* divisions; the second division is 206; and the third is 570; whence the number proposed is correctly expressed by 570206054.

EXAMPLES FOR PRACTICE.

1. One hundred. 2. One hundred and one. 3. One hundred and ten. 4. Nine thousand and nine. 5. Nine thousand and ninety. 6. Nine thousand nine hundred and nine. 7. Five thousand and seventy-four. 8. Ten thousand seven hundred. 9. Ninety thousand and ninety. 10. Three hundred and five thousand. 11. Nine hundred thousand nine hundred. 12. Five hundred and five thousand five hundred and fifty. 13. One million.

three thousand and eight. 14. Five million thirty thousand and forty-nine. 15. Nine million nine hundred thousand and six. 16. Fifty-eight million and nine. 17. Seventy million three hundred and two thousand four hundred and forty-one. 18. Two hundred and twenty-two millions and thirty-five. 19. Six hundred and four million sixty thousand and five. 20. Eight hundred million three thousand and thirty-three. 21. Nine hundred million nine hundred thousand and nine hundred. 22. Seven hundred million and seven. 23. One hundred and eighty million. 24. Five hundred million. 25. Five hundred and eighty million two hundred and forty-five thousand one hundred and ninety-two. 26. Seven hundred and seven million seven thousand and seventy-seven.

12. This method of notation can never present any difficulty, provided it be carefully remembered that every division of figures as we proceed from the right hand towards the left must be *completed*, as far as it is possible, and, by a little practice, we shall be enabled to write down any number by beginning at the *left* hand.

EXAMPLE.

Ex. 1. To write down six hundred and thirteen millions five hundred and nineteen, we observe that the division of millions will be 613; that of thousands will be 000, and that of units 519; so that the number is expressed in arithmetical symbols by 613000519.

13. It will be observed, from what has been said, that each of the nine figures or digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, has an *absolute* value of itself, whereas the auxiliary digit 0 has no such value; and on this account the former are termed *significant* figures, in contra-distinction to the last. It will, moreover, have occurred to the reader that every one of these significant digits, in addition to its *absolute* value, which is fixed and certain, possesses also a *local* value dependent upon the situation in which it is placed; thus, in the expression of the number four thousand three hundred and twenty-one, which will be 4321, the 1 in the first place on the right hand retains its absolute value; the second figure 2, in its situation, denotes two *tens* or *twenty;* the third is three *hundred*, and the fourth is four *thousand;* so that the local values of 2, 3, and 4, are respectively, *ten* times, a *hundred* times, and a *thousand* times, as great as their absolute values; and it is the circumstance of assigning to each of the significant figures a *local* as well as an absolute value, which confers upon the system the immense power which it possesses.

NUMERATION.

14. **Def. 6.**—*Numeration* is the art of reading or estimating the value of a number expressed by figures, and is, therefore, the *reverse* of Notation.

15. From the circumstance of every figure possessing a local as well as an absolute value, it follows that the value of each figure must be estimated by the place which it occupies; hence, a figure standing by itself expresses so many *units;* a figure in the second place so many *tens;* a figure in the third place so many *hundreds*, and so on; consequently, if we suppose any numerical expression to be divided into *periods* or portions, each consisting of three figures as far as they go, the figures of the period on the right will be

units and tens and hundreds of *units;* those of the next will be units, tens and hundreds of *thousands;* those of the third will be units, tens and hundreds of *millions*, and so on.

Thus,

1. 25 is twenty-five.
2. 304 is three hundred and four.
3. 5287 is five thousand two hundred and eighty-seven.
4. 70639 is seventy thousand six hundred and thirty-nine.
5. 306583 is three hundred and six thousand five hundred and eighty-three.
6. 1648305 is one million six hundred and forty-eight thousand three hundred and five.
7. 53024367 is fifty three million twenty-four thousand three hundred and sixty-seven.
8. 257008005 is two hundred and fifty-seven million eight thousand and five.

In each of these instances we conceive the expression to be separated into periods of three figures each as far as they go, beginning at the right hand, as in 257008005 we observe that 005 is the first period, 008 the second, and the third period is 257, denotes two hundred and fifty-seven millions 008 eight thousands and 005 five units.

16. The last article will be rendered more clear by the following scheme, called the *Numeration Table:*—

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
9	8	7	6	5	4	3	2	1
	9	8	7	6	5	4	3	2
		9	8	7	6	5	4	3
			9	8	7	6	5	4
				9	8	7	6	5
					9	8	7	6
						9	8	7
							9	8
								9

wherein the local value of every figure in each of the horizontal rows is pointed out by the name written *upwards* at the top of the whole; thus, in the *third* horizontal line from the bottom the figures will be read, *nine hundred and eighty-seven*, and in the second line from the top, *ninety-eight millions, seven hundred and sixty-five thousand, four hundred and thirty-two.*

EXAMPLES FOR PRACTICE.

Express in words:—

1. 43
2. 60
3. 12
4. 21
5. 100
6. 101
7. 110
8. 500
9. 505
10. 550
11. 1000
12. 2020
13. 3303
14. 4004
15. 7707
16. 8880
17. 87054
18. 70707
19. 60880
20. 99404
21. 903756
22. 202202
23. 400400
24. 550550
25. 1000001
26. 8047328
27. 4090300
28. 5210007
29. 6030405
30. 9009900
31. 41041014
32. 3000006
33. 20084216
34. 5001860
35. 8080808
36. 55700005
37. 76014059
38. 6006606
39. 56700505
40. 120015015
41. 202202200
42. 100100101
43. 275008005
44. 100010001
45. 79030184
46. 408076032
47. 401400056
48. 908500060

ADDITION.

17. Addition is the collecting together of two or more numbers, and the amount of all of them is termed their *sum.* The sign + *(plus)* is employed to indicate addition, as 7 + 2 signifies that 2 is to be added to 7. Also, the sign = *(equal)* signifies that the numbers between which it is placed are equal: thus, 8 + 1 = 9.

The process of addition depends upon the principle that the sum of two numbers is equal to the sum of their respective parts. Thus,

Let it be required to find the sum of two numbers, 1724 and 4638, and explain the process.

1724 = 1 thousand + 7 hundreds + 2 tens + 4 units,
4638 = 4 thousands + 6 hundreds + 3 tens + 8 units,

and as the sum of these two numbers is equal to the sums of their respective parts, that sum is

5 thousands + 13 hundreds + 5 tens + 12 units.

To each of the four parts into which the first number is separated add the part of the second which is under it, beginning at the units. Thus, 8 units and 4 units are 12 units, that is, 1 ten and 2 units; again, 3 tens and 2 tens are 5 tens; 6 hundreds and 7 hundreds are 13 hundreds, or 1 thousand and 3 hundreds. Lastly, 4 thousands and 1 thousand are 5 thousands, hence the sum is either 5 thousands 13 hundreds 5 tens and 12 units, or 6 thousands 3 hundreds 6 tens and 2 units = 6362.

Hence,

18. The rule for simple addition is as follows:—

RULE I.

Write the numbers *to be added together in vertical columns so that the units of all the numbers may be in one column, the tens in the second, the hundreds in the third, and so on. Draw a line under the last number, and, beginning with the column of units, add successively the* numbers *contained in each column; if the* sum *does not* exceed nine, *write it down under the line, but if it contains* tens *reserve them to be* added *to the* next *column, writing down only the* units *of each column, and under the* last *column put the entire* sum, *whatever it may be. If the* sum *of any column be an* exact number *of* tens, *write* 0 *for the units and* carry *the tens to the next column.*

Example.

Ex. 1. Let it be required to find the sum of 26389, 38127, 2815, 6497, 835, and 3745.

Write the numbers as at the side, so that the figures of the same class shall be in the same vertical column; then taking the sum of each class, we find there are 38 units, 27 tens, 31 hundreds, 25 thousands, and 5 tens of thousands. Now 38 units are 3 tens and 8 units, then writing 8 below the units column, carry the 3 tens to the 27 tens, which together make 30 tens, or 3 hundreds and 0 tens. Write 0 below the column of tens and reserve the 3 hundreds to be added to the 31 hundreds; this gives 34 hundreds, or 3 thousands and 4 hundreds, and writing 4 below the column of hundreds, carry the 3 thousands to the 25 thousands, and we get 28 thousands, or 2 tens of thousands and 8 thousands. Writing the 8 below the column of thousands, carry the 2 tens of thousands, making the entire sum = 78408.

$$\begin{array}{r} 26389 \\ 38127 \\ 2815 \\ 6497 \\ 835 \\ 3745 \\ \hline 78408 \end{array}$$

19. **Verification of Addition.**—The usual verification is to add both upwards and downwards and see if the sums agree. This is generally sufficient. If more is required, or if the student cannot get a long column to cast the same way both up and down, he can cut it up and add each portion separately; then add the sums.

EXERCISES IN SIMPLE ADDITION.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
321413	543123	536123	123456	761284	657890	692387	876578
452734	234512	453215	234561	612874	278679	4956	495
130421	713145	1234	345612	8719	5798	87658	54939
3718	104234	4231	456223	46759	67843	769378	8797
24561	36142	51234	561234	587999	488567	5790	358428
341323	3451	613254	612345	987678	37429	87958	768453

(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
662593	846914	516398	425396	567453	169964	145673	197794
395266	415327	854627	674958	654359	435434	366535	543543
841923	723456	735829	827694	531769	744315	679654	765976
356627	674216	916358	731045	765453	476757	341345	415161
725983	328427	827146	556677	147954	496059	569765	954131
346783	736259	633289	889900	645679	695969	694313	643167

(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
987825	916427	695024	986257	985626	372519	586372	148537
736349	625736	538426	427385	796842	463726	477754	697296
856925	346831	827836	514986	915638	298534	638831	526438
734316	857936	735985	726326	809274	851372	951490	723649
827842	735784	216515	915827	444444	319628	479291	859698
936736	426467	859827	734482	913258	738543	863748	852619
842625	849753	910756	386912	872364	497791	376546	419648
759519	358358	683625	219863	410698	345345	356633	777777
846325	647846	745841	391285	742367	679567	459681	999999
987846	386921	526606	842163	946208	161514	453148	555555
333445	666777	888999	615827	807609	131549	567963	724483
335445	666777	888999	736846	915827	761346	313499	952637

25. Add together the addends (1) under exercises (1), (9), and (17); (2) under (2), (10), and (18); (3) under (3), (11), and (19); (4) under (4), (12), and (20); (5) under (5), (13), and (21); (6) under (6), (14), and (22); (7) under (7), (15), and (23); and (8) under (8), (16), and (24).

26. Add together three hundred and nine million, four hundred and seventeen thousand, and eighty-seven; six hundred and seventy-five thousand, and forty-nine; seven thousand and ninety-seven million, eight hundred and fourteen thousand, three hundred and five; seventy-nine million, five hundred and four thousand, and forty-nine; six thousand and seventy-eight million, four hundred and thirty-nine thousand, six hundred and forty-seven; seven thousand million, eight hundred and seventy-six thousand, four hundred and twenty-nine.

SUBTRACTION.

20. The process of finding a number which shall be equal to the difference of two numbers is called *subtraction*. It is customary to call the quantity from which the subtraction is made, the **minuend**; the quantity to be subtracted, the **subtrahend**; and the result of the subtraction, the **difference**. Thus, then, we have, minuend — subtrahend = difference.

We may also write this as,

minuend = subtrahend + difference,

which shows the connection between subtraction and addition.

The operation of SUBTRACTION is indicated or expressed by the sign —, which is read *minus* or *less by*, with the use of the sign =; thus, the excess of 7 above 3 will be expressed in the *form* $7 - 3 = 4$, which is *read* 7 minus 3 equals 4; where the sign — between 7 and 3 denotes the subtraction of the latter from the former, and the sign = between 3 and 4 shows the *equality* of the excess to 4.

21. The process of subtraction involves two principles; the one is the equal augmentation or diminution of the numbers. In either way, the difference of the two numbers will not be altered; for if the greater number be either increased or diminished by 7 for example, and the less be increased by 7, the numbers themselves will be altered.

The other principle is this: since 12 exceeds 7 by 5, and 8 exceeds 6 by 2, then 12 and 8 together, or 20 exceeds 7 and 6 together, or 12 by 5 and 2 together or 7.

Let it be required to take 231 from 574.

Write the numbers as in the margin, units under units, tens under tens, and hundreds under hundreds, then 4 units exceed 1 unit by 3 units, 7 tens exceed 3 tens by 4 tens, 5 hundreds exceed 2 hundreds by 3 hundreds. Therefore, by the second principle, all the first column together exceeds all the second column by all the third column together, that is, by 3 hundreds 4 tens 3 units, or 343 which is the difference between 574 and 231.

Hundreds.	Tens.	Units.
5	7	4
2	3	1
3	4	3

Again, let it be required to subtract 23957 from 802126.

	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
802126 =	8	0	2	1	2	6
23957 =		2	3	9	5	7

Now here a difficulty immediately arises since 7 is greater than 6, and cannot be taken from it, neither can 5 be taken from 2, 9 from 1, 3 from 2, nor 2 from 0. To obviate this we must have recourse to the first principle, and add the same quantity to both these numbers, which will not alter their difference. Add ten to the first number, making 16 units, and add ten also to the second number, but, instead of adding ten to the number of units, add one to the number of tens, making 6 tens. Again, add ten tens to the first number, and one hundred to the second, then add ten hundred to the first, and one thousand to the second, and so on, adding equal quantities to each. In this way the numbers will be changed into the following:—

Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
8	10	12	11	12	16
1	3	4	10	6	7
—	—	—	—	—	—
7	7	8	1	6	9

and the difference 778169 is obtained in the usual manner.

Hence, *when the upper figure is the less we must augment it by ten, and retain one to be added to the lower figure immediately to the left.*

22. The rule for simple subtraction is as follows:—

RULE II.

1°. *Put the smaller number under the greater, taking care, as in addition, that units shall be under units, tens under tens, hundreds under hundreds, and so on.*

2°. *Beginning at the* units, *take each figure in the* lower *line from the figure* above *it, if the* lower *figure be not the greater of the* two, *setting down the remainder below it.* (See the operation in Ex. 1, page 17).

3°. *But if any figure in the* lower *line be* greater *than that* above *it, add* 10 *to the* upper *one, and then take the* lower *figure from that* sum, *setting down the remainder, and carrying one (i.e. adding* 1*) to the* next lower *figure, and with which proceed as before, and so on, till the whole is finished.* (See Ex. 2, page 17).

The following examples will illustrate this rule.

EXAMPLES.

Ex. 1. Let it be required to subtract 42572 from 76594.

In the units column, 2 from 4 leaves 2, set down 2.
In the tens column, 7 from 9 leaves 2, set down 2.
In the hundreds column, 5 from 5 leaves 0, set down 0.
In the thousands column, 2 from 6 leaves 4, set down 4.
In the ten thousands column, 4 from 7 leaves 3, set down 3.

From 76594 *minuend.*
Subt. 42572 *subtrahend.*
Rem. 34022 *difference.*

Ex. 2. Subtract 7495 from 9263.

In the process adopted in practice the figures in the minuend are not, as in the second example No. 21, page 16, actually altered; and perhaps we might more simply explain the practical process as follows:

9263
7495
1768

To subtract 5 from 3 is impossible, so separate 1 ten from the 6 tens, and adding it to the 3 units, say 5 from 13 leaves 8. Now we are supposed to have separated 1 ten from the 6 tens, but as the figure really remains 6, we still have to take 1 from it; also we have to take from it the 9 in the lower line, so instead of taking away first 1 and then 9 more, take away 10 at once; but 10 from 6 being impossible, separate 1 from the place of hundreds, and adding it as 10 tens to the 6 tens, say 10 from 16 leaves 6. As we have not really taken 1 from the 2 hundreds, we have still 1 to take from it; also we have to take the 4 in the lower line; instead of taking first 1 and then 4, take away 5 at once; but 5 from 2 being impossible separate 1 from the place of thousands and add it as 10 hundreds to the 2 hundreds, and say 5 from 12 leaves 7. As we have not really diminished the figure 9 in the place of thousands, we have still 1 to take from it, and likewise we have to take away the 7 in the lower line, so taking away 8 at once from 9 we have 1 left in the place of thousands, and the entire difference is 1768.

Ex. 3. Let it be required to subtract 27385 from 64927.

Then placing the former number under the latter (as in the margin), we proceed thus:

64927 *minuend.*
27385 *subtrahend.*
37542 *difference.*

In the units column, 5 from 7 leaves 2, set down 2.
In the tens column, 8 from (not 2) but 12 leaves 4, set down 4 and carry 1.
In the hundreds column, (3 + 1, *i.e.* 3 + 1 carried) from 9 = 4 from 9 leaves 5, set down 5.
In the thousands column, 7 from (not 4) but 14 leaves 7, set down 7 and carry 1.
In the ten thousands column, (2 + 1, *i.e.* 2 + 1 carried) from 6 = 3 from 6 leaves 3, set down 3.

Ex. 4. As another example, let 86025704 be subtracted from 130741392.

Then, having arranged the numbers as in the margin, we proceed thus: 4 from 12, 8, carry 1; 1 and 0, 1, 1 from 9, 8; 7 from 13, 6, carry 1; 1 and 5, 6, 6 from 11, 5, carry 1; 1 and 2, 3, 3 from 4, 1; 0 from 7, 7; 6 from 10, 4, carry 1; 1 and 8, 9, 9 from 13, 4, carry 1; and 1 carried being taken from 1, leaves 0, therefore, the remainder is 44715688.

From 130741392 *minuend.*
Subt. 86025704 *subtrahend.*
Rem. 44715688 *difference.*

23. **Verification of Subtraction.**—The best verification is to add the subtrahend and difference. This ought to give back the minuend, or original quantity from which the subtraction was made.

EXERCISES IN SIMPLE SUBTRACTION.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
706205	804601	980001	600501	702001	601002	501001	602004
84694	265061	980000	600492	26000	46003	20106	11906

(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
701628	508000	403000	393436	321288	345876	206011	8500000
20449	129	26901	219050	213788	123457	48605	90909

(17)	(18)	(19)	(20)	(21)
10000001000	1010001110101 1	10101160330	101100110110	479863217896
7077070077	1011100110110	80209337	10011101011	241826424862

Take each subtrahend 12 times from its minuend in the following examples:—

(22)	(23)	(24)	(25)	(26)	(27)
7432326	6677298	7213545	4362579	6002109	8100630
157689	67527	57636	9873	45108	6156

28. What number remains when one million four hundred thousand six hundred and nineteen has been taken from one hundred millions and two?
29. What is the difference between sixteen thousand and eighteen, and one million?
30. If eighty-nine be added to fourteen thousand six hundred and forty-three, how many must be added to the sum to make it ten millions?
31. How many must be added to the sum of 99, 416305, and 2108, that it may exceed the difference between 19104 and 605 by 1143200?
32. What must be added to 7543 to make it 16000?
33. The sum of two numbers is 84207, the less is 12327, what is the greater?

24. By help of the plus (+) and minus (—) signs, we can easily connect together in a single row a set of numbers, of which some are to be added and others subtracted; thus $4 + 6 - 3 - 2$ means that 4 and 6 are to be *added*, and 3 and 2 are to be *subtracted*, so that $4 + 6 - 3 - 2 = 5$. Instead of subtracting first 3 and then 2, we may, of course, subtract 5 at once; so that the above is the same as $10 - 5 = 5$. In some cases where additive and subtractive quantities are indicated, there is a difficulty which may be stated in a simple form by means of an example.

$$2 - 7 + 8 - 1$$

If we begin from the left-hand end, our first operation is to subtract 7 from 2, which cannot be done directly; we have recourse, therefore, to a different arrangement, namely, to take all the additions first, and all the subtractions last. We write it, therefore, as

$$2 + 8 - 7 - 1$$

After adding the 2 and the 8, we may either subtract the 7 and the 1 in succession, or we may add the 7 to the 1 and subtract their sum from the sum of the 2 and the 8. The simplest way, therefore, of getting at the result of a set of mixed additions and subtractions is to add the additive quantities, and then (separately) add the subtractive quantities, and subtract the sums. Thus, if we have

$$125 + 427 - 684 + 237 - 15$$

125	684	789	
427	15	699	
237	—	—	
—	699	90	result.
789			

Examples for Practice.

1. $361 + 483 - 246 - 179 = 419$.
2. $573 - 184 + 602 - 67 = 924$.
3. $12064 + 700628 - 109641 + 637 - 2604 = 601086$.
4. $23596 - 625 + 72311075 - 13758 - 3506285 + 6879 = 68820880$.
5. $1 - 2 + 4 - 8 + 16 - 32 + 64 - 128 + 256 - 512 + 1024 = 2048 + 4096 - 8192 + 16384 - 32768 + 65536 - 131072 + 262144 - 524288 + 1048576 - 2097152 + 4194304 = 2796203$.

MULTIPLICATION.

25. **Multiplication** is a short method of finding the amount of a number repeated any number of times; thus, when 3 is multiplied by 4, the number produced by the multiplication is the sum of 3 repeated 4 times, which sum is equal to $3 + 3 + 3 + 3$ or 12.

26. The number which is repeated, or, in other words, is to be multiplied, is called the **multiplicand**; the number denoting the repetitions—*i.e.*, the number by which it is to be multiplied—is called the **multiplier**; and the amount, or the number which is found by multiplying the former by the latter is called the **product**.

The operation of Multiplication is expressed by the sign ×, which is read *into*, or *multiplied by*; thus, $5 \times 7 = 35$. Sometimes a full stop at the bottom of the figure is used for this; thus, 2×7 or 2.7, are both used to express twice seven. So again, $4 \times 5 \times 13 = 260$, expresses the continued product of 4, 5, and 13.

Multiplier × Multiplicand = Product. Multiplicand × Multiplier = Product.

27. If more than two numbers are multiplied together, the result is called the **continued product**.

28. The multiplicand and multiplier are termed **factors** of the product, because they are factors or makers of the product.

29. To perform the operation of multiplication readily, a *table*, called the *multiplication table*, must first be learnt, and the result which arises from multiplying one number by another, provided neither be greater than 12, must be committed to memory; it is one of the few operations in arithmetic where the memory of rules is indispensible.

THE MULTIPLICATION TABLE.

1	**2**	3	4	5	6	7	8	9	10	11	12
2	4	**6**	8	10	12	14	16	18	20	22	24
3	**6**	9	**12**	15	18	21	24	27	30	33	36
4	8	**12**	16	**20**	24	28	32	36	40	44	48
5	10	15	**20**	25	**30**	35	40	45	50	55	60
6	12	18	24	**30**	36	**42**	48	54	60	66	72
7	14	21	28	35	**42**	49	**56**	63	70	77	84
8	16	24	32	40	48	**56**	64	**72**	80	88	96
9	18	27	36	45	54	63	**72**	81	**90**	99	108
10	20	30	40	50	60	70	80	**90**	100	**110**	120
11	22	33	44	55	66	77	88	99	**110**	121	**132**
12	24	36	48	60	72	84	96	108	120	**132**	144

In this table the first horizontal line consists of the first twelve numbers in order; the second consists of the products when multiplied by 2; the third contains their products when by 3; the fourth when multiplied by 4, and so on; and the table is repeated in the following manner.

Thus, to make use of the *second* line of figures, we say—

twice 1 are 2
twice 2 are 4
twice 3 are 6
twice 4 are 8
twice 5 are 10
twice 6 are 12
twice 7 are 14
twice 8 are 16
twice 9 are 18
twice 10 are 20
twice 11 are 22
twice 12 are 24

NOTE.—It should be noticed that the product of any two numbers is the same whichever of them is taken as the multiplier—*e.g.*, 7 times 8 = 8 times 7. This is illustrated in the above Table by a double row of dark figures forming the diagonal of the square.

30. When two numbers are to be multiplied together, it is a matter of indifference, so far as the product is concerned, which of them be taken as the multiplicand or multiplier; in other words, the product of the first multiplied by the second, will be the same as the product of the second multiplied

by the first. Thus, 4 × 5 and 5 × 4 express the same thing, namely, 20. This is best seen as follows:—

```
* * * * *
* * * * *
* * * * *
* * * * *
```

Here there are 20 stars, or asterisks. It is evident that the total number —20—is not altered by the way in which we choose to group them, in order to count them. Thus we may either say there are 5 in each line, and 4 lines, or that there are 4 in each column, and 5 columns. Now one of these ways of counting takes the 20 as 4 × 5, the other as 5 × 4. Hence these are simply two ways of arriving at the same result, or product.

32. The fundamental principles upon which the process of multiplication depends are these:—

(1) If we separate any multiplicand into any number of parts, and multiply each part severally by any number and add the results, the *whole* multiplicand is thus multiplied, *e.g.*, 15, which may be separated into 8 and 7 is multiplied by 9, if 8 and 7 be each multiplied by 9, and the results added together.

(2) If the multiplier be separated into any number of parts, and the multiplicand be multiplied severally by each of these parts and the results added together, this is equivalent to multiplication by the *whole* multiplier, *e.g.*, if it be required to multiply 17 by 12, and we multiply 17 by 4, and 17 by 8, and add these results, we have then taken 17 exactly, 4 + 8 times or 12 times.

Let it be required to multiply 6739 by the single figure 8.

Since the product 6739 by 8 is evidently equal to the sum of the products of all its parts, and

$$6739 = 6000 + 700 + 30 + 9$$

we must, therefore, multiply each of these parts by 8, and add together the results; we have, therefore, the following operation:—

Ten thousands.	Thousands.	Hundreds.	Tens.	Units.		
	6	7	3	9	6739 8	
	—	—	—	—	——	
			7	2	72 =	product of 9 by 8 (*a*)
		2	4		240 =	,, 30 ,, 8 (*b*)
	5	6			5600 =	,, 700 ,, 8 (*c*)
4	8				48000 =	,, 6000 ,, 8 (*d*)
—	—	—	—	—	——	——
5	3	9	1	2	53912 =	,, 6739 ,, 8

Now 9 units multiplied by 8 gives 72 units (*a*), 3 tens multiplied by 8 gives 24 tens (*b*), 7 hundreds multiplied by 8 gives 56 hundreds (*c*), and 6 thousands multiplied by 8 gives 48 thousands (*d*). Writing these results in the ordinary way, and adding them together, as above, the sum, which is the required *product*, is 53912.

In practice the partial products 72, 240, 5600, and 48000, are not written down, but combined mentally into one sum; thus, we say 8 times 9 units are 72 units, that is to say, 7 tens and 2 units, we accordingly write down 2 in the units' place, and carry 7 to the place of tens; 8 times 3 tens are 24 tens, and the 7 tens carried make 31 tens, or 3 hundreds and 1 ten. Put the 1 in the tens' place and carry the 3 hundreds; 8 times 7 hundreds are 56 hundreds, and the 3 hundreds carried make 59 hundreds, or 5 thousands 9 hundreds; write the 9 in the place of hundreds and carry 5 to the thousands. We have next, 8 times 6 thousands are 48 thousands, and the 5 thousands carried are 53 thousands, which is the entire number of thousands, the whole product being 53912.

$$\begin{array}{r}6739\\8\\\hline 53912\end{array}$$

The reason of the following rule will now be evident.

When the multiplier is not greater than 12.

RULE III.

1°. *Place the multiplier under the multiplicand, units under units, and draw a line under the multiplier to separate it from the product.*

2°. *Commencing at the* unit's figure *multiply each figure of the* multiplicand *by the* multiplier. *If the* product *of the* multiplier *and* any figure *of the* multiplicand *is* less *than* ten, *set it down under that digit; but set down the* right-hand figure only *of the* product *when it is a number of* more *than* one figure, *and carry, as in addition. Set down the last product in full.*

EXAMPLES.

Ex. 1. Multiply 6209748 by 7.

To work out this sum, commence by placing the multiplier under the multiplicand, units under units. Beginning at the right hand, and multiplying each figure in the multiplicand separately, the work is as follows:—

7 times 8, 56, write down the 6 and carry 5; 7 times 4, 28, which, with 5 carried, make 33, set down 3 and carry 3; 7 times 7, 49, and 3 carried, are 52, set down 2 and carry 5; 7 times 9, 63, and 5 are 68, set down 8 and carry 6; 7 times 0, 0 and 6 are 6, set down 6; 7 times 2 are 14, set down 4 and carry 1; 7 times 6 are 42, and 1 43, set down 43.

$$\begin{array}{r}6209748\\7\\\hline 43468236\end{array}$$

The following, worked like the above example, require no further explanation.

Multiply	73826	9073142	460052	531462
By	8	9	11	12
Product	590608	81658278	5060572	6377544

EXAMPLES FOR PRACTICE.

1. 34264896 × 2
2. 654321987 × 3
3. 376543198 × 4
4. 379865782 × 5
5. 91823740526 × 6
6. 6521734782 × 7
7. 485868788 × 8
8. 573241789 × 9
9. 987654321 × 10
10. 891237654 × 11
11. 647853291 × 12
12. 919273654 × 12
13. Multiply 7452341 six times successively by the following multipliers, 2, 3, 4, 5, 6, 7, 8, 9, 11, and 12.

32. To multiply any number by 10, we have only to remove each of the figures of the multiplicand one place to the left and their value will be increased ten times, or, in other words, any number is multiplied by 10 by annexing *one* cypher, by 100 by annexing *two* cyphers, by 1000 by annexing *three* cyphers, &c.; *e.g.*, 85 × 10 = 850, for, by annexing the cypher, the 5

units have become 5 *tens*, and the 8 *tens* have become 8 *hundreds*, *i.e.*, *the several parts* of the multiplicand have each received a tenfold increase, and, therefore, the whole number has been multiplied by 10. Again, 2376 × 100 = 237600, where the value of each figure is increased a hundred times by writing to the right of the multiplicand as many cyphers as there are in the multiplier.

(a) When the significant figure of the multiplier is not a unit, as for example 30, 400, or 700. Since these multipliers are the same, as 10 times 3, 100 times 4, or 1000 times 7; the multiplicand is first multiplied by the significant figure 3, 4, or 7, (by Rule III), afterwards the product is multiplied by 10, 100, or 1000 as in Art. 32) by writing one, two, or three cyphers to the right of the product. Thus, to multiply 468 by 700, we have the operation in the margin.

$$\begin{array}{r} 468 \\ \underline{700} \\ 327600 \end{array}$$

When the multiplier consists of many figures, the process is as follows:—

EXAMPLE.

Ex. 1. Multiply 4786 by 2783. That is, to take 4786 2783 times and add them all together; or, to take it 2000 times, 700 times, 80 times, and 3 times, and the sums together; or, to multiply it by 2000, by 700, by 80, and by 3, and add the products together.

(A)

$$\begin{array}{rr} 4786 \times 3 = & 14358 \\ 4786 \times 80 = & 382880 \\ 4786 \times 700 = & 3350200 \\ 4786 \times 2000 = & \underline{9572000} \\ \text{And the sum is} & \underline{13319438} \\ \therefore 4786 \times 2783 = & 13319438 \end{array}$$

Ordinary form (B)

Having placed the multiplier under the multiplicand, units under units, tens under tens, &c., proceed thus:—

$$\begin{array}{lc|cccccccc} & & & & & & \text{Thousands.} & \text{Hundreds.} & \text{Tens.} & \text{Units.} \\ & & & & & & 4 & 7 & 8 & 6 \\ & & & & & & 2 & 7 & 8 & 3 \\ \hline (1)\ 4786 \times 3 = 14358 & = & & & & 1 & 4 & 3 & 5 & 8 \\ (2)\ 4786 \times 8 = 38288 & & & & & & & & & \\ \therefore 4786 \times 8 \textit{ tens} = 38288 \textit{ tens} & = & & & 3 & 8 & 2 & 8 & 8 & \\ (3)\ 4786 \times 7 = 33502 & & & & & & & & & \\ \therefore 4786 \times 7 \textit{ hunds.} = 33502 \textit{ hunds.} & = & & 3 & 3 & 5 & 0 & 2 & & \\ (4)\ 4786 \times 2 = 9572 & & & & & & & & & \\ \therefore 4786 \times 2 \textit{ thous.} = 9572 \textit{ thous.} & = & & 9 & 5 & 7 & 2 & & & \\ \hline \text{Now, by simple addition, the sum of these products} & = & 1 & 3 & 3 & 1 & 9 & 4 & 3 & 8 \\ \hline \end{array}$$

$$\therefore 4786 \times 2783 = 13319438$$

If the ordinary method of performing the operation (see under B above) be compared with the detailed form (under A), it will be observed that by arranging the figures in the second line of multiplication one place to the left of those in the first, those in the third one place to the left of those in the second, and so on, we retain the figures in each line in their proper places without the addition of the cyphers at the end of each line.

The reason of the following rule will now be evident.

33. **When the multiplier is greater than 12,** we proceed as follows:—

RULE IV.

1° *Place the multiplier under the multiplicand so that the units of the former may be under those of the latter, the tens under tens, &c., and draw a line under the whole.*

2°. *Write down the* product *of the* whole multiplicand *by the* unit's digit *of the* multiplier, as in rule. *In like manner write down the* product *of the* multiplicand *by* each *of the remaining figures of the* multiplier, observing to place the right-hand figure of each line *in the column* under *the* figure *of the* multiplier *from which it came* (see Ex. 1).

(a) *If the* multiplicand *contains a* cypher, *treat it as if it were a* number, recollecting that 0 × 1 = 0, 0 × 2 = 0, *and so on.*

(b) *If* one *or* more *of the* figures (not final figures) *of the* multiplier *be* 0, *the corresponding* partial product, *or* products, *will be* 0 *or cyphers, and the lines may be* entirely omitted, *recollecting to give its* proper value *to the* product, *arising from* multiplying *by the* next figure (see Ex. 2).

3°. *Then* add *all these partial products together, and their* sum *will be the entire* product *of the* factors.

EXAMPLES.

Ex. 1. Multiply 821436 by 672576.

```
      821436
      672576
     -------
     4928616
    5750052
   4107180
  1642872
 5750052
4928616
------------
552478139136
```

Here the first figure (6) of the first partial product is set below the figure 6 in the multiplier, the first figure (2) of the second partial product is set below 7, the multiplying figure, the first figure (0) of the third partial product is set below 5, the figure multiplied by, and so on; the first or right-hand figure on each row standing directly under the multiplying figure producing it.

Ex. 2. Multiply 32407 by 6005.

```
    32407
     6005
 --------
   162035
194442
---------
194604035
```

Here the first figure (5) of the first partial product is placed immediately below 5, the figure multiplied by, and since the next two figures of the multiplier being cyphers, the corresponding products will be 0 or cyphers, and are entirely omitted, and the multiplicand is then multiplied by 6 and the right-hand figure of the product is set in the column directly below the multiplier 6.

Ex. 3. Multiply 729 by 63817 = 63817 × 729.

```
   63817
     729
 -------
  574353
 127634
446719
--------
46522593
```

Ex. 4. 700013 × 54836 = 54836 × 700013

```
      54836
     700013
     ------
     164508
    54836
383852
-----------
38385912868
```

The following example will illustrate a point of some utility connected with the position of the first figure obtained by multiplication.

Ex. 5. Multiply 3421975 by 11912.

In this example it will be observed that though there are five figures to multiply by, we have only three separate lines of products. The reason is that in the first place we multiply by 12, next by 9, and then by 11, which shortens the sum by two whole lines—a considerable saving. It is necessary, however, to be careful to *place the first figure* in each case *obtained by multiplication directly* under the *figure by which you are multiplying.*

```
    3421975
      11912
   --------
   40063700
  30797775
37641725
-----------
40762566200
```

EXAMPLES FOR PRACTICE.

1. 58726341 × 23
2. 78954236 × 34
3. 98765240 × 57
4. 93876129 × 95
5. 43679854 × 806
6. 67869578 × 903
7. 23589647 × 678
8. 86483279 × 567
9. 1001001 × 999
10. 5832764985 × 4689
11. 815085 × 20048
12. 5906408 × 90064
13. 6437063 × 5006701
14. 958866 × 804002
15. 31622777 × 6324553
16. 378421896 × 5928578
17. 28814412 × 12345678
18. 586371829 × 673857
19. 95400621 × 7003041
20. 948375628 × 8764853
21. 678924 × 678924
22. 987654321 × 1234567890
23. 4771213 × 602059999
24. 46358148222 × 34817061
25. 7733889 × 6866883
26. 57298492692 × 700809050321
27. 38015732 × 400700065
28. 427282 × 744615 × 15905.
29. 461762 × 930937 × 972744 × 708421
30. 9090909 × 1111111 × 999999 × 1010101
31. 9999 × 99980001 × 999700029999
32. 9999 × 9998 × 1000 × 10001 × 10002
33. 9999 × 9999 × 9999 × 9999 × 9999 × 9999
34. In a book of tables there are, on one page, 15 columns, on the average each column contains 157 figures; how many figures are there on a page, and how many on 167 pages.

34. If the multiplier to any proposed multiplicand consists of any one or more of the nine digits, followed by a cypher, or any number of cyphers, then multiply according to the following

RULE V.

1°. *Place the* multiplier *under the* multiplicand, *so that the* significant *figure of the* multiplier *shall stand* under *the* unit's figure *of the* multiplicand, *and* multiply *the* successive *figures of the* multiplicand *by the* significant figure *of the* multiplier, according to Rule IV.

2°. *Then, to the* product *thus obtained, place to the* right *the* same number *of cyphers as are* contained *in the* multiplier.

EXAMPLE.

Multiply 123456789 by 80 and 800000.

Multiplicand	123456789	Multiplicand	123456789
Multiplier	80	Multiplier	800000
Product	9876543120	Product	98765431200000

In the first of these examples we multiply first by 8, according to Rule III, then annex (*i.e., join to*) to the product *one* cypher, because the multiplier contains *one* cypher, in order to preserve the product in its proper place, as the product of 8 *tens*. In the second example the same rule is followed, but five cyphers are annexed, because the multiplier contains *five* cyphers, in order to preserve the product in its proper place as the product of 8 *hundred of thousands*.

35. If the multiplier and multiplicand both end with cyphers, we may omit them in the working, and proceed according to the following

RULE VI.

Multiply the significant *figures of the factors, as directed in* Rule IV. *Then, to the* product, affix *as many* cyphers *as have been* omitted from the end *of both the* multiplier and multiplicand:

The principle of this annexation has been already explained (No. 32).

EXAMPLES.

Thus, if 570 be multiplied by 3200, and 4076800 by 307000.

```
   (1)                 (2)
   570             4076800
  3200              307000
  ----           ---------
   114              285376
  171              122304
  -------        -------------
  1824000        1251577600000
```

In the first case, the 7 multiplied by 2 is the same as 70 multiplied by 200; and 70 multiplied by 200 gives 14000. In the second the product of the significant figures is 40768 × 307 = 12515776, to this *five* cyphers must be annexed, because 100 × 1000 = 100000; and 12515776 × 100000 = 1251577600000.

EXAMPLES FOR PRACTICE.

1. 80437050 × 90
2. 98311950 × 110
3. 50014060 × 270
4. 4381792800 × 5600
5. 41508473 × 7002800
6. 42456008000 × 5400
7. 5432149 × 8705040950
8. 765321 × 760
9. 325758 × 72000
10. 504603 × 26600
11. 567425 × 4080
12. 240318 × 317000
13. 49864223 × 70860470
14. 3000100300 × 40010000000

36. It is sometimes advantageous to split up a multiplier which is the product of two or more numbers, and multiply by its factors; thus, if we have to multiply by 36, it is easier to multiply in this case by 6 and 6 (6 × 6 = 36), or by 4 and 9 (4 × 9 = 36), than to multiply by long multiplication, that is, by 3 tens and 6. In any case we have two rows of multiplication, but in the last case we have an addition into the bargain.

EXAMPLE.

Multiply 57894362 by 48.

Since 48 = 6 × 8; or, 4 × 12 = 48, then the answer is found as below.

```
                              57894362                                      57894362
                                     6                                             4
                             ---------                                     ---------
6 times the multiplicand = 347366172       4 times the multiplicand = 231577448
                                     8                                            12
                            ----------                                    ----------
                            2778929376 Ans.                               2778929376 Ans.
∴ (6 times 8 times =) 48 times the         ∴ (4 times 12 times =) 48 times the
        multiplicand.                               multiplicand.
```

EXAMPLES FOR PRACTICE.

1. 685732 × 15
2. 903421 × 18
3. 356628 × 36
4. 838777 × 48
5. 777838 × 49
6. 434560 × 56
7. 735846 × 64
8. 279819 × 72
9. 38926392 × 77
10. 356718 × 81
11. 29362983 × 84
12. 817938 × 96
13. 108143145 × 121
14. 117974340 × 132
15. 128699280 × 144

37. **Verification of Multiplication.—I.** By casting out nines.—Add together the figures of the multiplicand, multiplier, and product separately, not counting any 9 that may occur, rejecting also 9 whenever, in adding up, the sum amounts to 9 or more; note each result. Multiply the first two

remainders, *i.e.*, the remainder arising from casting out nines in the multiplicand and multiplier, retaining, as before, only what is left after the rejection of all the nines from this product, if the sum of the digits exceed nine; then, if the remainder which thus arises is the same as that from the product of the two factors, the operation is likely to be correct, unless there be some compensation of errors, or some figure misplaced.*

$$\begin{array}{rl} \text{Multiply } 90376 & \ldots\ldots 7 \\ 2083 & \ldots\ldots 4 \\ \hline 271128 & \\ 723008 & \\ 1807520 & \\ \hline 188253208 & \ldots\ldots 1 \end{array}$$

Thus, in the annexed example, we say (omitting the 9) 3 and 7 are 10; then 1 and 6 are 7, which write down. Again, 2 and 8 are 10; then 1 and 3 are 4, which is also put down near the multiplier. Lastly, the product of 4 and 7 are 28, and 2 and 8 are 10, which is one above 9. Write, then, 1 near the product, and cast the nines out of the product thus, 1 and 8 are 9; 8 and 2 are 10; 1 and 5 are 6 and 3 are 9; 2 and 8 are 10; which being 1 above 9 shows that the operation most probably is correct.

The usual way of noting the four results is to make a cross, to put the first in the left-hand opening, the second in the opposite opening, the third above, and the fourth below. If the upper and lower results are the same, the work, as just observed, is ***most likely*** correct, but otherwise it is wrong.

$$\begin{array}{r} 21800475 \\ 1089 \\ \hline 196204275 \end{array} \qquad (b)\ \begin{array}{ccc} & 0 & \\ 0 & \times & 0 \\ & 0 & \end{array}$$

In this example both the multiplier and multiplicand leave no remainder when the nines are cast out, *i.e.*, both numbers are divisible by 9, without a remainder.

Then 0 × 0 = 0, place this 0 at *b*; next, casting out the nines from the product, there is no remainder. Hence, the operation, most probably, is correct.

The method may even be used in detail to verify each step of the sum, as follows:—

349751	remainder	2
28637	,,	8
2448257	,,	5
1049253	,,	6
2098506	,,	3
2798008	,,	7
699502	,,	4
10015819387	,,	7

Thus checking every separate multiplication and the addition as well as the complete result.

II. The truth of all results in multiplication may be proved by using the multiplicand as multiplier, and the multiplier as multiplicand; if the product thus obtained be the same as the product found at first, the results are in all probability true.

---

* It is plain, that if any of the figures in the product were made to exchange places, the agreement of the third and fourth results would remain, though the product would be wrong; as would also be the case if one figure were increased and another diminished by the same number; all, therefore, that we can safely infer is, that the agreement spoken of must have place if the work be correct, so that if it fail, the work is wrong. Suppose, for instance, that we had made 73084163 × 7584 = 554270392192, then, applying the test, we get from the first factor the result 5, for the second the result 6, and from the product of these the result 3; but, from the above-stated product of the two numbers the result is 4. This product, therefore, is incorrect, and, upon revising the multiplier we find that the 3, after the nought, should have been 2.

$$\begin{array}{ccc} & 3 & \\ 5 & \times & 6 \\ & 4 & \end{array}$$

DIVISION.

38. The object of **division** is to find how many times one number is contained in another. The quantity to be divided is called the **dividend,** the quantity by which we divide is the **divisor,** the *number of times* is the **quotient,** and what remains over (if any such there be) is called the **remainder.** **Dividend = divisor × quotient + remainder.**

The operation of DIVISION is expressed by the sign ÷, which is read *by* or *divide by*; thus, $42 \div 7 = 6$, implies that the result of the division of 42 by 7 is 6. The number 42 which is divided is called the dividend, that which divides, *i.e.*, 7 the divisor and the result 6, the quotient.

39. The first idea of obtaining the result is to use subtraction and count the times we have to use it.

Thus, to find how many times 8 is contained in 34.

$$\begin{array}{rl} 34 & \\ 8 & (1) \\ \hline 26 & \\ 8 & (2) \\ \hline 18 & \\ 8 & (3) \\ \hline 10 & \\ 8 & (4) \\ \hline 2 & \end{array}$$

We see we can take 8 away from 34 *four* times in succession, and then we leave 2. But if we had helped ourselves by the multiplication table (of *eight* times) we might have done it more shortly. For since $5 \times 8 = 40$, 8 will go 5 times into 40, exactly; therefore, 8 will not go 5 times into 34. Again, $4 \times 8 = 32$, and thus 8 will go 4 times into 34 and leave something over. This "something over" is evidently $34 - 32$, or 2.

So long as the quotient is a small number the process of continued subtraction may be employed, but when the dividend contains the divisor a large number of times, it would be necessary to abridge the operation by taking away as many times the divisor at once as we please, provided the number of times is marked at each step. For example, to divide 115 by 12 we may take away 8 times 12 at once, and afterwards take away 12; therefore, 12 may be subtracted 9 times, and the remainder is 7.

$$\begin{array}{rl} 115 & \\ 96 & = \text{8 times 12} \\ \hline 19 & \\ 12 & = \text{once 12} \\ \hline 7 & = \text{remainder.} \end{array}$$

In order to avoid the labour of repeated subtractions, we may lay down the following principle, viz., that if we separate any dividend into any number of parts, and find how often the divisor may be subtracted from these parts (or how often the divisor is *contained*, as it is called, in each of these), we shall, by adding these results, obtain the correct quotient of the whole dividend divided by that divisor, because it is evident that the *whole* dividend will contain the divisor as many times as its *several parts together* contain it.

Let it be required to divide 3168 by 27. Here the quotient will consist of three digits, and therefore there will be at least 3 separate subtractions. Now the figure in the hundreds' place cannot be more than 1, and if the product 27 hundreds, or 2700, be subtracted from the total product 3168, the remainder 468 must contain the products of the tens and units of the quotient multiplied by the divisor 27. We now enquire how often 27 is contained ten times in 468, and this is found to be only once ten times; then subtracting the partial product 27 tens, or 270 from 468, the remainder is 198. Lastly, we have to divide 198 by 27, which gives 7 for a quotient and a remainder 9; and, therefore, 3168 contains 27, 100 + 10 + 7, or 117 times, leaving 9 for the remainder.

3168		
2700	=	100 times 27
468		
270	=	10 times 27
198		
189	=	7 times 27
9		

It will be seen that as often as 27 is contained in 31, so many hundred times it will be contained in 3100, or in 3168; and as often as 27 is contained in 46, so many ten times it will be contained in 460, or 468, and in this manner any quotient figure is just as readily obtained as the last or unit's figure of it.

40. The preceding articles contain the principles of division, and all that remains is to apply them in the most economical manner.

EXAMPLE.

Suppose we have to divide 2987618 by 3605.

Operation with cyphers in full.	Operation without annexing cyphers.
3605)2987618(800 + 20 + 8	3605)2987618(828
. 2884000 or 828	28840
103618	10361
72100	7210
31518	31518
28840	28840
2678	2678

41. **If the divisor be not greater than 12.**

RULE VII.

1°. ***Set down the dividend with a line under it to separate it from the future quotient; and write the divisor on the left of the dividend with a curve line between them.***

2°. *By the multiplication table find the* greatest *number of times the* divisor *is* contained *in the* first *figure of the* dividend, *or, if necessary, the* first two *or* first three *figures of the dividend, and place the figure denoting the times* directly *under the* figure *divided, or under the* last figure *if* more *than* one *have been taken, and* carry *what is over, that is, regard the figure which is* over *to be* prefixed *to the* following *figure of the dividend.*

3°. *Divide this number by the divisor, set down the result as the next figure of the quotient, carry the remainder to the next figure of the dividend; and if the divisor is* not *contained in any figure of the dividend, place a* cypher *in the* quotient *and* prefix *this figure to the next one of the dividend as if it were a remainder, and proceed in the same manner till all the figures of the dividend are exhausted.* The number thus found is the quotient.

EXAMPLES.

Ex. 1. Divide 25602 by 3.

Placing the dividend and divisor (3) as in the margin, we proceed thus:— 3 is contained in 2, *no* times, so that nothing is to be placed under the 2; 3 is contained in 25, *i.e.*, 25 thousands, 8 times (*i.e.*, 8 thousands) and 1 over, 8 and carry 1; this 1 regarded as *prefixed* to the 6, gives the number 16, *i.e.*, 16 hundreds, 3 is in 16, 5 times and 1 over; 3 is in 10 (*i.e.*, 1 hundred) 3 times and 1 over; 3 is in 12, 4 times. Therefore, the quotient is 8534; and this is the complete quotient, as there is no *remainder*.

$$\begin{array}{r} 3)\underline{25602} \\ 8534 \end{array}$$

Ex. 2. Divide 7804623 by 5.

We say, 5 in 7, 1 and 2 over; 5 in 28, 5 and 3 over; 5 in 30, 6; 5 in 4, 0 and 4 over; 5 in 46, 9 and 1 over; 5 in 12, 2 and 2 over; 5 in 23, 4 and 3 over. As there is here a remainder, we annex it, with the divisor 5 under it, to the figures of the quotient, and call $1560924\frac{3}{5}$ the complete quotient.

$$\begin{array}{r} 5)\underline{7804623} \\ 1560924\frac{3}{5} \end{array}$$

Ex. 3. Divide 84111648 by 12.

We say, 12 in 8, *no* times, so that nothing is to be placed under the 8; 12 in 84 goes 7 times and 0 over; 12 in 1, *no* times and 1 over, put down 0 under the 1; 12 in 11, 0 times and 11 over; this 11 regarded as prefixed to the next figure of the dividend gives the number 111; therefore say, 12 in 111, 9 times and 3 over; 12 in 36, 3 times and 0 over; 12 in 4, 0 times and 4 over; 12 in 48, 4 times.

$$\begin{array}{r} 12)\underline{84111648} \\ 7009304 \end{array}$$

EXAMPLES FOR PRACTICE.

1. 135792695 ÷ 2
2. 584697386 ÷ 3
3. 399345884 ÷ 4
4. 298244760 ÷ 5
5. 400678493 ÷ 6
6. 276586437 ÷ 7
7. 6947421006 ÷ 8
8. 2470263075 ÷ 9
9. 254096146 ÷ 10
10. 1101182267 ÷ 11
11. 1095137170 ÷ 12
12. 59437055312 ÷ 12

Divide each of the following dividends six times successively by each of the following divisors—2, 3, 4, 5, 6, 7, 8, 9, 11, 12.

1. 6154778
2. 7000000
3. 4086791
4. 8821000
5. 2000707
6. 9370005
7. 9914060
8. 4706009

42. **If the divisor be greater than 12.**

RULE VIII.

1°. *Place the divisor and dividend in the same line, separated by a small curve line, and on the right of the dividend draw another line of the same kind*; thus,—

divisor) dividend (quotient

2°. *Mark off from the* left-hand *side of the* dividend *a number of figures* equal *in number to those of the* divisor, *or* one more, *if necessary, and find the* greatest *number of times the* divisor *is contained in this number; place the figure which denotes this number on the* right *as the first figure of the quotient.*

3°. *Multiply the divisor by this number and place the product under the number marked off at the left of the dividend.* Subtract *the said* product *from that part of the* dividend *under which it stands.*

4°. *Bring down the* next *figure of the* dividend *and place it to the* right *of the remainder, and if the* number *thus formed be* greater *than the* divisor, *find the* greatest *number of times the* divisor *is contained in it, and write this number as the* second *figure of the quotient; but if this number be* less *than the* divisor *bring down the* next *figure of the dividend, or* more, *until* a number *not* less *than the* divisor, *is formed,* remembering *to place* a cypher *in the* quotient *for* every figure *brought down,* except the last; *find how often the divisor* is contained in this number; *then multiply, subtract, and bring down, &c., as before, till all the figures of the dividend are exhausted.*

The number thus obtained is the quotient required.

NOTE.—When the divisor is large, the learner will find assistance in determining the quotient figure, by finding how many times the first one or two figures on the left hand of the divisor is contained in the first one or more of those of the dividend. This will give pretty nearly the right figure. Some allowance must, however, be made for carrying from the product of the other figures of the divisor, to the product of the first into the quotient figure. If any product be greater than the number which stands above it, the last figure in the quotient must be changed for one of smaller value; but if any remainder be greater than the divisor, or equal to it, the last figure of the quotient must be changed for a greater.

EXAMPLES.

Ex. 1. Let it be required to divide 256434 by 346.

```
346)256434(741 quotient.
    2422
    ----
     1423
     1384
     ----
       394
       346
       ---
        48
```

Looking at the leading figure of the divisor, and also at that of the dividend, with the view of seeing whether the latter contains the former, which it does not, 3 being greater than 2; we therefore commence with the number 25, formed by the first *two* figures of the dividend, and seeing that 3 is contained in 25 8 times, we should put 8 for the first quotient figure; but bearing in mind that when the *whole* divisor is multiplied by this 8 we must attend to the *carryings;* we perceive that 8 is too great, we therefore try 7, and find 7 times 346 to be 2422, a number less than 2564 above it, so that we can *subtract;* the remainder is 142, which, when the next figure of the dividend is brought down, becomes 1423. We now take this as a dividend, and looking at the *leading figures* in this new dividend and the divisor, we see that the latter *will go* 4 times, we therefore put 4 for the second quotient figure, and multiplying and subtracting we get 39 for the second remainder, and, by bringing down another figure we get 394 for a new dividend; the divisor goes into this *once*, so that the quotient is 741, and the final remainder 48; this remainder must be annexed with the divisor underneath to the quotient figures, so that the complete quotient is 741 $\frac{48}{346}$, which is the 346th part of 256434.

Ex 2. Divide 108419716214 by 5783.

5783) 108419716214 (18748005 quotient.
Quotient figure (1)...... 5783
50589 .... 1st remainder with next figure.
(8)...... 46264
43257 .... 2nd " "
(7)...... 40481
27761 .... 3rd " "
(4)........ 23132
46296 .... 4th " "
(8)........ 46264
322 .... 5th " "
☞ (0)..........
3221 .... 6th " "
☞ (0)............
32214 .... 7th " "
(5)............ 28915
3299 final remainder.

It must be noticed that if any *dividend* formed by a remainder and a figure brought down should be less than the divisor, that the divisor will go *no times* in that dividend; so that a 0 will be the corresponding quotient figure; and that, then, a second figure must be brought down as in the operation annexed. The steps marked ☞ are inserted merely to show the principle. In practice we simply put down the two noughts in the quotient, and go at once to 32214 for the divisor.

Ex. 3. Divide 6421284 by 642.

642)6421284(10002
642
1284
1284

642 goes once into 642, and leaves no remainder. Bring down the next fignre (1) of the dividend, then 642 is in 1 no times, put 0 in the quotient after the 1. The next figure of the dividend (2) being brought down to the right of the 1 forms the dividend 12, then 642 is in 12 no times, put a cypher as the next quotient figure; bring down the next figure of the dividend (8), then 642 in 128 goes no times, write 0 as the corresponding quotient figure; but the next figure brought down (4) makes the quantity brought down 1284, which contains the divisor twice, and gives no remainder.

EXAMPLES FOR PRACTICE.

1. 983296 ÷ 13
2. 9220290 ÷ 17
3. 800062 ÷ 23
4. 20067690 ÷ 37
5. 9500864 ÷ 43
6. 7846723 ÷ 52
7. 9943000 ÷ 78
8. 8904030 ÷ 89
9. 56703264 ÷ 123
10. 367347398 ÷ 506
11. 571414204 ÷ 809
12. 94010610 ÷ 987
13. 147109629898869 ÷ 1709
14. 2107791630 ÷ 3654
15. 1395243584 ÷ 5678
16. 3855999705 ÷ 6789
17. 940870015 ÷ 8764
18. 1000000000 ÷ 9999
19. 1000000000 ÷ 1001
20. 100000000000000 ÷ 11111
21. 484231571378 2 ÷ 570634
22. 815240906170 ÷ 763054
23. 490002800004 ÷ 900702
24. 6680943744279021 ÷ 95400621
25. 7219326311126635269 ÷ 987654321
26. 2000000018760631 ÷ 31622777
27. 79222833228805843200 ÷ 879510067
28. 2962296118145871914806 56 ÷ 972744

29. Divide 1000........) with as many noughts added as may be necessary to give ten places of figures in the quotient) by 2302585093.

43. When the divisor is a composite number, that is, can be separated into two or more factors, the division can be effected by the following rule:—

RULE IX.

1°. *Divide by one factor, setting down the quotient and remainder.*

2°. *Divide the quotient thus obtained by another factor, setting down the quotient and remainder, and so on, till all the factors are employed.* The last quotient will be the answer required.

3°. *The proper remainder is found, when the divisor is resolved into but two factors, by multiplying the second remainder by the first divisor, and to the product adding the first remainder,* but when more than two factors are employed *by multiplying the remainders after every line by all the divisors except their own, and adding the results.*

EXAMPLES.

Ex. 1. Divide 569736869 by 15.

$$15\left\{\begin{array}{c}3\\ 5\end{array}\right.\left|\begin{array}{l}569736869\\ \hline 189912289\ldots\ldots 2\\ \hline 37982457\ldots\ldots 4\end{array}\right.$$

Here, since 15 is the product of 3 and 5, it is obvious that the quotient may be obtained from successive division by 3 and 5.

Here, the remainder 2 in the first quotient is 2 units of the upper line; but the remainder 4 in the second line consists of 4 units of the second line; and as each unit in the second line is *three times* as great as each unit in the upper line, the remainder 4 is equal to 3 × 4 units of the upper line, *i.e.*, is equal to 12 ordinary units, hence the whole remainder is 2 + 12, or is 14.

Ex. 2. Divide 8327965 by 72.

$$72\left\{\begin{array}{c}8\\ 9\end{array}\right.\left|\begin{array}{l}8327965\\ \hline 1040995\ldots\ldots 5\\ \hline 115666\ldots\ldots 1\end{array}\right.$$

Ex. 3. Divide 8327965 by 99.

$$99\left\{\begin{array}{c}9\\ 11\end{array}\right.\left|\begin{array}{l}8327965\\ \hline 925329\ldots\ldots 4\\ \hline 84120\ldots\ldots 9\end{array}\right.$$

To deduce the remainders which would have been left had the divisions been performed by 72 and 99 in the usual way, we may observe that the first partial remainder 5 (Ex. 2) must be *units*; but the second remainder must be regarded as so many collections of 8 units each, and that the first partial remainder 4 (Ex. 3) must be *units*; but the second dividend being as so many collections of 9 units each, the second remainder must, in this case, be regarded as so many collections of 9 units each; hence the true remainders in these examples are respectively

$$1 \times 8 + 5 = 13, \text{ and } 9 \times 9 + 4 = 85.$$

Ex. 4. Divide 2671998 by 192.

The factors for 192 are $4 \times 6 \times 8 = 192$.

$$192\left\{\begin{array}{c}4\\ 6\\ 8\end{array}\right.\left|\begin{array}{l}2671998\\ \hline 667999\ldots\ldots 2\\ \hline 111333\ldots\ldots 1 \times 4 + 2 = 6\\ \hline 13916\ldots\ldots 5 \times 6 \times 4 + 6 = 126\end{array}\right.$$

Dividing by 4 the remainder is 2, dividing by 6 the remainder is 1, dividing by 8 the remainder is 5, which gives a total remainder 126, found thus: the remainder 2 is two units, the second remainder is one 4 and 2, making 6, the third remainder 5 is five times 24 (4 × 6) = 120, because having divided by 4 × 6 = 24, the quotient 111333 are twenty-fours, so that any remainder must consist of twenty-fours; 120 added to the previous remainder 6, gives 126. Thus the answer is—quotient 13916, remainder 126.

Ex. 5. Divide 24533279 by 432.

Since $6 \times 8 \times 9 = 432$, we may divide successively by these numbers.

$$432\left\{\begin{array}{c}6\\ 8\\ 9\end{array}\right.\left|\begin{array}{l}24533279\\ \hline 4088879\ldots\ldots 5\\ \hline 511109\ldots\ldots 7 \times 6 + 5 = 47\\ \hline 56789\ldots\ldots 8 \times 8 \times 6 + 47\\ \qquad\qquad = \text{remainder } 431\end{array}\right.$$

The first remainder is 5, the second remainder is 7, then 7 times (the first divisor =) 6 + 5 (the first remainder) = 47. The third remainder is 8, then 8 times 8 (the second divisor) = 64, and 64 times the first divisor 6, = 384 + remainder 47, gives the final remainder 431.

EXAMPLES FOR PRACTICE.

1. $6489275432689467 \div 14$
2. $5984327896483207 58 \div 22$
3. $5698347568 9268 \div 36$
4. $5986432685946896 \div 63$
5. $987654321012345 \div 66$
6. $543210123456789 \div 121$
7. $359132139162 1911 \div 132$
8. $4902550716552769 \div 144$

44. **Division may also be abridged where the divisor is terminated by a cypher or cyphers; we proceed as follows:—**

RULE X.

1°. ***Cut off the cyphers from the divisor, and as many figures from the right-hand of the dividend as there are cyphers so cut off at the right-hand end of the divisor, then proceed with the remaining figures in the usual manner* (Rule VII or VIII), *and if there are anything remaining after the division annex those figures which are cut off from the dividend; otherwise, the figures cut off will be the remainder.***

NOTE.—The same rule applies when the divisor and dividend both terminate with cyphers.

EXAMPLES.

Ex. 1. Divide 3704196 by 20.

2,0)370419,6

185209 $\frac{16}{20}$

In the first of these examples you mark off with a comma the cypher or 0 in the divisor, and the first figure 6 to the right in the dividend; this is equivalent to dividing both divisor and dividend by 10. You next divide the remaining figures 370419, to the left in the dividend, by the divisor 2, according to Rule VII; thus is obtained the quotient 185209, and remainder 1; to this remainder you annex the figure 6, which was cut off, and you have the complete remainder 16. The quotient may now be correctly represented thus, 185209$\frac{16}{20}$.

Ex. 2. Divide 31086901 by 7100.

```
71,00)310869,01(4378 3101/7100
      284
      ---
       268
       213
       ---
        556
        497
        ---
         599
         568
         ---
          31
```

In the second example you follow the same rule; that is, you cut off two cyphers in the divisor and two figures in the dividend, and obtain the quotient in the usual way, which is 4378, and remainder 31; to this 31 annex the two figures cut off from the dividend, and you have the complete remainder 3101.

Ex. 3. Divide 271830 by 30.

3,0)27183,0

9061 *Ans.*

1st. Cut off a cypher from the divisor, and also one from the dividend.

2nd. Divide 27183, the remainder of the dividend, by 3, the remainder of the divisor: the quotient 9061 is the answer.

EXAMPLES FOR PRACTICE.

1. $9357864837986496 \div 50$
2. $674008694738425 \div 700$
3. $987654321670 \div 3000$
4. $17932810740000 \div 2600$
5. $82354605678 00 \div 1440$
6. $84106000 \div 81200$
7. $4837958649732067 89 \div 120$
8. $6550000280034 \div 6300$
9. $267995346 87 \div 7890000$
10. $634394567 \div 164600$

45. **Verification of Division.**—(1.) Multiply the quotient by the divisor, or the divisor by the quotient, and to the product add the remainder, if there be one. The result ought to be the same as the dividend; because we are only adding the divisor the same number of times, as it was subtracted in the operation of division.

(2.) *Subtract the remainder, if any,* from *the dividend, and divide the* difference *so obtained by the quotient.* The result should be equal to the divisor, if the working be correct.

ON THE USE OF BRACKETS.

46. **Brackets,** which are of several kinds—*e.g.*, ( ), { }, [ ]—are used to denote that all numbers included within any pair of them are to be considered as forming but one number, and are, therefore, to be equally affected by any number not included within the same pair of brackets, thus—

$$(6 + 3) \times 8 = 9 \times 8 = 72$$

This is read, the sum of 6 and 3 multiplied by 8 = 72.

47. **A vinculum** is a sign sometimes used instead of brackets. It consists of *a line* drawn over the numbers to be considered as forming one number; thus, in $\overline{4+7} = 11$, $4 + 7$ is supposed to form but a single term, and is read, the sum of 4 and 7; and $2 \times \overline{8+3} = 22 = 2 \times 11 = 22$.

48. Putting an additive quantity—that is, one which has the sign of addition prefixed or understood—within a bracket, or with a vinculum over it, does not in any way alter its value, neither does the removing a bracket or a vinculum affect the value of such a quantity.

49. A vinculum placed over, or a bracket enclosing, a subtractive quantity, or one having the negative sign prefixed, alters its value, unless we change all the signs but the first; thus, $5 - 3 + 2$, and $5 - \overline{3+2}$ are not the same thing; for, $5 - 3 + 2 = 4$, but $5 - \overline{3+2}$, $(3 + 2)$ being considered now as but one quantity, $= 0$; for $3 + 2 = 5$, therefore $5 - \overline{3+2}$ is the same as $5 - 5$, which leaves nothing, or, in other words, it is equal to 0. If, however, we change all the signs except the first, the value of the quantity is not altered by the vinculum (or bracket); thus, $5 - 3 + 2 = 4$, and $5 - \overline{3-2}$, or $5 - (3 - 2)$, also, are equal to 4.

Again, $27 - 4 + 7 - 3 = 27$.
$27 - (4 + 7 - 3) = 19$.
But, $27 - 4 - 7 + 3$ changing all the signs of the original quantities but the first $= 27$.

The following example will show how the bracket or the vinculum affects numbers, according as we make it include an additive or a subtractive quantity:—

$48 + 7 - 3 - 8 + 7 - 2 = 49$.
$48 + (7 - 3 - 8 + 7 - 2) = 49$; what is within the bracket being additive, it is not necessary to change any signs.
$48 + 7 - (3 + 8 - 7 + 2) = 49$; it is now necessary to change all the signs *within* the bracket.
$48 + 7 - 3 - (8 - 7 + 2) = 49$; it is necessary in this case, also, to change the signs.
$48 + 7 - 3 - 8 + (7 - 2) = 49$; it is not necessary in this case.

In the above we have sometimes put an additive and sometimes a subtractive quantity within the brackets; in the former case we were obliged to change the signs of all the terms connected by the bracket, except the first—that is, to change all the signs *within* the bracket; in the latter, to preserve the original value of the quantity, it was not necessary to change any sign.

In reality, when a subtractive quantity is put within brackets, the signs of *all* the terms in the bracket are changed, since the negative sign (—) refers to the whole quantity within the bracket, and not to the first term, which has become positive (+). Hence—

When an expression in brackets *is* preceded *by the* sign —, *the brackets may be struck out, if the* signs *of all the terms within the brackets be changed, namely,* + into — *and* — into +.

50. When a quantity in brackets, or under the vinculum, is to be multiplied by any number, each of its parts must be multiplied, for, to multiply the whole, we must multiply *each* of its parts. Thus, $3 \times (7 + 8 - 3) = 3 \times 7 + 3 \times 8 - 3 \times 3$; and $\overline{4+5} \times \overline{8+3-6}$ means that each of the terms under the *latter* vinculum is to be multiplied by each of these under the *former*, or by their sum.

51. When a quantity in brackets, or under the vinculum, is to be divided, we must, on removing the brackets or vinculum, put the divisor under each of the terms connected by the sign of addition, or subtraction, otherwise the value of what was to be divided will be changed; thus, $(5 + 6 - 7) \div 3 = \frac{5}{3} + \frac{6}{3} - \frac{7}{3}$, for we do not divide the whole unless we divide *all* its parts.

The line placed between the dividend and divisor occasionally assumes the place of a vinculum; and, therefore, when the quantity to be divided is subtraction, it will sometimes be necessary to change the signs—as already directed—thus, $\frac{6}{2} + \frac{13-3}{2} = \frac{6+13-3}{2}$, but $\frac{27}{3} - \frac{15-6+9}{3} = \frac{27-15+6-9}{3}$. For when, as in these cases, *all* the terms are put under the vinculum, the effect—as far as the subtraction signs are concerned—is the same as if the vinculum were removed altogether, and then the signs should be changed *back again* to what they must be considered to have been *before* the vinculum was affixed.

Sometimes one set of brackets is used within another set, in which case the inner set is to be first removed, then the outer.

Ex. Simplify $36 - \{18 - (18 - 12)\}$

First removing the inner brackets the expression becomes

$$36 - \{18 - 18 + 12\}$$

The remaining brackets being removed it becomes

$$36 - 18 + 18 - 12$$

$$\text{or } 36 - 12 = 24.$$

NOTE.—Observe that the inner brackets being preceded by the sign —, on striking the brackets out the sign of the terms within them are changed, — to +. Similarly, the signs are changed in removing the outer brackets.

Similarly, if more than one set of brackets is used within another set, we begin by first removing the inmost brackets, then the next outer set, and so on till all are removed.

Ex. Simplify $12 - \left[8 + \left\{6 - \left(4 - \overline{3 - 2}\right)\right\}\right]$

Ex. Find the value of $12 - \left[8 + \left\{6 - \left(4 - \overline{3 - 2}\right)\right\}\right]$

Removing the vinculum we have the expression

$$12 - \left[8 + \left\{6 - \left(4 - 3 + 2\right)\right\}\right]$$

Again, by removing the inmost brackets we have

$$12 = \left[8 + \left\{6 - 4 + 3 - 2\right\}\right]$$

By removing the inner brackets the expression becomes

$$12 = \left[8 + 6 - 4 + 3 - 2\right]$$

Lastly, removing the outermost brackets we have

$$\begin{aligned} &12 - 8 - 6 + 4 - 3 + 2 \\ &= 1 \end{aligned}$$

Find the value of the following expressions :—

$$\begin{aligned} &\left[8 + 2\left\{9 + 3\left(4 + 3\right) + 17\right\} + 21\right] \times 9 \\ &= \left[8 + 2\left\{9 + 21 + 17\right\} + 21\right] \times 9 \\ &= \left[8 + 94 + 21\right] \times 9 \\ &= 123 \times 9 \\ &= 1107 \end{aligned}$$

EXAMPLES FOR PRACTICE.

Find the value of the following expressions :—

1. $43 + \overline{7 - 3 - 14}$; $8 \times (17 \times 1)$; $(16 - 3) \times 2$.
2. $3 \times (2 \times 4)$; $(61 + 29) \div 3$; $\left\{198 \div 9\right\} \times 6$.
3. $\left\{(28 \div 7) + 9\right\} \times 2$; $\left\{(16 + 18 - 2) \div 8 + 4\right\} \times 3$.
4. $15 \times 37153 - 73474 - 67152 \div 4 + 40734 \times 2$.
5. $494871 - 94853 + (45079 - 3177) - (54312 - 3987) - (1763 + 231) + 379 \times 379$.
6. $(7854 - 4913) \times 3 - (20374 - 12530) \div 53 - 6 + (395456 - 2364) \div 556$.
7. $\frac{37004 \times 1607 + 157}{5} - \frac{5048 \times 206}{8} + \frac{371700}{59}$
8. $(503407 \times 82) - (3792 \times 48) + (94185 \div 273)$.
9. $\left[\left\{(19 + 17) \div 6 + 4\right\} \times 10 - 91\right] \div 3$; $\left[\left\{27 + 14 + 6(3 + 15 - 13) - 11\right\} \div 5\right] \times 7$

CHAPTER II.

APPLICATION OF ARITHMETIC TO NUMERICAL MAGNITUDES OF VARIOUS DENOMINATIONS NOT CONNECTED BY THE COMMON BASE OF NOTATION.

52. The numbers which have been hitherto introduced into our calculations have been independent of any particular unit, that is, *abstract* numbers; but these numbers can only afford a clear conception of the magnitude of objects when the unit is defined and known. By the number 24 we specify that the number to be measured is composed of 24 times the unit supposed to be known; but when we say that the day is composed of 24 hours, we mean that the unit of time is the duration of *one hour*, and that 24 of these hours are equal to the duration of one day. Numbers of this kind, composed of a particular unit, which is repeated as many times as are indicated by an abstract number, are termed *concrete numbers*, and are consequently products of which the multiplicand is the unit, and the multiplier an abstract number. Hence, 24 hours means 24 times one hour, and 36 miles signifies 36 times one mile.

53. A compound quantity is one consisting of several others, expressed in different units, as 17 miles 57 yards 2 feet, £3 17s. 6d., and the different divisions into which money, time, weight, length, or distance, &c., are divided, constitute so many scales or systems of numeration, by means of which operations on concrete and compound numbers are assimulated to those on abstract and simple numbers.

The processes employed in cases of this nature are *Reduction*, and the fundamental operations then called *Compound Addition*, *Compound Subtraction*, *Compound Multiplication*, and *Compound Division*, each of which will be exemplified in order, and the usual tables of the different divisions of money, weights and measures, by means of which these processes are conducted, will be found at the end of the work.

REDUCTION.

54. **Reduction** is the name given to the methods by which quantities are changed to others of the same values but of different denominations; as, for instance, the changing, or *reducing* pounds to shillings, pence to farthings, yards to miles, hours, &c., to years, and so on.

EXAMPLES.—£2 = 40s.; where we say that two pounds has been reduced to shillings.
110s. = £5 10s.; where we say one hundred and ten shillings has been reduced to pounds and shillings.

NOTE I.—It must be carefully noticed that the values of these quantities are not at all altered by the reduction: the values remain the same, the names only are changed.

NOTE II.—The term *reduction* will be found to be of very wide application, thus—when we wish to ask "*What numbers multiplied together will give* 77?" We express the question by saying "*Reduce* 77 *to its prime factors.*"

Reduction is of two kinds, viz.:—

1. When the quantity is to be converted from a higher to a lower denomination; *e.g.*, *reduce* £5 to shillings.

2. When the change has to be from a lower denomination to a higher; *e.g.*, *reduce* 106 shillings to pounds.

1. To reduce a quantity to a lower denomination.

Ex. Reduce £82 to pence.

Since £1 = 20s. £82 will be equal to 82 times 20s., *i.e.* (20s. × 82) or 1640 shillings.

Also, since 1 shilling = 12 pence, 1640 shillings will be equal to 1640 times 12 pence, or 19680 pence.

The preceding example enables us to lay down the following rule for reducing quantities to a lower denomination.

RULE XI.

Multiply the* highest *denomination by the* number *which expresses how many of the* next lower *denomination make one of the higher, and* add *to the product such number of the lower denomination (if any) as may be included in the given sum. Repeat this operation till the given quantity is expressed in the required denomination.

EXAMPLES.

Ex. 1. Reduce £163 6s. 8¾d. to farthings. Thus, as 20s. make £1, we multiply the number 163 by the number 20, adding in the number 6; the product is 3266, the number of shillings contained in £163 6s. Again, since 12 pence make 1s., we multiply the last number by 12, taking in the 8; the product is 39200, the number of pence in £163 6s. 8d. And lastly, because 4 farthings make 1d., multiply by 4, and taking in the 3 farthings, we get 156803, the number of farthings required.

$$\begin{array}{rrr} £ & s. & d. \\ 163 & 6 & 8\frac{3}{4} \\ 20 & & \\ \hline 3266 & & \\ 12 & & \\ \hline 39200 & & \\ 4 & & \\ \hline 156803 & & \end{array}$$

Ex. 2. Reduce £28 17s. 6½d. to farthings.

In this example we have several terms of different denominations.

$$\begin{array}{lll} £ & s. & d. \\ 28 & 17 & 6\frac{1}{2} \\ 20 & & \\ \hline 577 \text{ shillings.} & & \\ 12 & & \\ \hline 6930 \text{ pence.} & & \\ 4 & & \\ \hline 27722 \text{ farthings.} & & \end{array}$$

1st. Multiply 28 by 20, and add 17 to the product

2nd. Multiply 577 by 12, and add 6 to the product.

3rd. Multiply 6930 by 4, and add 2 to the product.

Ex. 3. Let it be required to reduce 317 days 14 hours and 36 minutes to minutes. Since 24 hours make one day, we multiply by 24; and, in adding in the 14, we include the *units* in the units' amount of the product—that is, in the first result of the first partial product—and the tens (1) in the first result of the second partial product. We thus get 7622, the number of hours; this number we multiply by 60, because 60 minutes make 1 hour, and we add in the 36—units with units and tens with tens, as before—and we thus find the number of minutes to be 457356.

$$\begin{array}{llll} \text{D.} & \text{H.} & \text{M.} & \\ 317 & 14 & 36 & \\ 24 & & & \\ \hline 1272 & & & \\ 635 & & & \\ \hline 7622 \text{ hrs.} & = 317^d\ 14^h & & \\ 60 & & & \\ \hline 457356 \text{ mins.} & = 317^d\ 14^h\ 36^m & & \end{array}$$

Ex. 4. **Reduce 17 tons 16 cwts. 1 qr. 19 lbs. to ounces.**

tons. cwts. qr. lbs.
17 16 1 19
20
356 cwts.
4
1425 qrs.
28
11409
2851
39919 lbs.
16
239514
39919
638704 oz. *Answer.*

In multiplying by 28, in adding in the 19, we include the units in the units' amount of the product, that is, in the first result of the first partial product—and the tens (1) in the first result of the second partial product.

Ex. 5. **Reduce 27 furlongs 3 yards to inches (*Long Measure*).**

furs. poles. yds.
27 0 3
40
2)1080 poles.
$5\frac{1}{2}$
5400 = 5 times 1080
540 = $\frac{1}{2}$ of 1080
5940 = $5\frac{1}{2}$ times 1080
+ 3
5943 yards.
3
17829 feet.
12
213948 inches.

Having reduced the furlongs to poles by multiplying the number 27 by 40 = 1080, we have next to reduce the poles to yards. Since $5\frac{1}{2}$ yards make a pole, or perch, we have to multiply by $5\frac{1}{2}$. Now to *multiply* by $\frac{1}{2}$ means simply to take *half* the multiplicand, that is, to *divide* it by 2 In this example we have $1080 \times 5 = 5400$, and $1080 \div 2 = 540$; then $5400 + 540 = 5940 = 5\frac{1}{2}$ times 1080.

Ex. 6. **Reduce 14 acres 2 roods 17 poles to square feet (*Square Measure*).**

acres. roods. poles.
14 2 17
4
58 roods.
40
4)2337 poles.
$30\frac{1}{4}$
70110 = 30 times 2337
$584\frac{1}{4}$ = $\frac{1}{4}$ of 2337
$70694\frac{1}{4}$ = $30\frac{1}{4}$ times 2337
9
$636248\frac{1}{4}$ *Answer.*

In this example we have to reduce square poles to square yards, which is done by multiplying by $30\frac{1}{4}$, the number of square yards in 1 square pole. Now, to multiply by $\frac{1}{4}$ is to take a *fourth part*, or to divide the multiplicand by 4. In multiplying by 9 we say, 9 times $\frac{1}{4}$ is 2 and $\frac{1}{4}$, write down $\frac{1}{4}$ (or quarter) and carry 2.

2. To reduce a quantity to a higher denomination.

RULE XII.

1°. *Divide the given quantity by the number which shows how many of the lower denomination make one of the* next *higher.*

2°. *Repeat this operation till the given quantity is expressed in the required denomination.*

3°. *If there should be a* remainder *after any divisor, we must set it down as a term of the same denomination as the dividend from which it came.*

EXAMPLES.

Ex. 1. Reduce 337587 farthings to pounds.

4)337587
12) 84396d. 3f.
2,0) 703,3s. 0d.
£351 13s. 0¾d.

Dividing the number of farthings by 4, the quotient is 84396 pence, and the remainder is 3 farthings, that is, a term of the same denomination as the dividend from which it came. Dividing the number of pence by 12 gives 7033 shillings, and no remainder, and dividing the number of shillings by 20 gives 351 pounds and a remainder of 13 shillings; hence, 337587 farthings are equal to 84396 pence and 3 farthings, or 7033 shillings and no pence, or equal to £351 13s. 0¾d.

N.B.—We divide by 20 by the usual short method, cutting off the last figure of the dividend and divisor.

Ex. 2. Reduce 15234 farthings to £ s. d.

4)15234	1st. Divide by 4; the remainder, 2, is 2 farthings, or ½d.
12) 3808d. 2f.	2nd. Divide by 12; the remainder, 4, is 4 pence, or 4d.
2,0) 31,7s. 4d.	3rd. Divide by 20; the remainder, 17, is 17 shillings, or 17s.
£15 17s. 4½d.	*Answer* £15 17s. 4½d.

Ex. 3. Let it be required to convert 591241 lbs. into tons, cwts, &c.

28 { 7)591241
28 { 4)84463
4)21115--21 lbs.
2,0)527,8--3 qrs.
263 tons 18 cwt. 3 qrs. 21 lbs.

As 28 lbs. make 1 qr., the next higher denomination to pounds, we divide first by 28, or by 7 and by 4, the two *factors* of 28, as it is better to use *short* division; we thus get 21115 qrs., with 21 lbs. over. This number divided by 4, gives the number of cwts.—namely, 5278 and 3 qrs. over; and lastly, dividing by 20, the number of cwts. in 1 ton, we get, finally, 263, the number of tons; so that there is 263 tons 18 cwt. 3 qrs. 21 lbs. in 591241 lbs.

Ex. 4. Reduce 3192783 square feet to acres, roods, &c., &c.

9)3192783 sq. feet.
354753 sq. yds. 6 sq. feet.

We want now to reduce 354753 square yards to acres, roods, &c. The first thing to be done is to divide 354753 by 30¼ to bring the quantity to perches.

```
30¼)354753
 4       4
----------
121)1419012(11727
    121
    ---
     209
     121
     ---
      880
      847
      ---
       331
       242
       ---
        892
        847
        ---
         45
```

Now, *if the dividend and the divisor be both multiplied by the same term, the quotient will not be thereby* affected; therefore, we multiply 30¼ by 4, thus making 121, and 354753 by 4, thus making 1419012, and dividing 1419012 by 121 the quotient is 11727, *i.e.*, 11727 perches and 45 quarters of a square yard remains, *i.e.*, 11¼ square yards remain.

It now remains to reduce 11727 perches to roods, &c.

```
4,0)1172,7
    ------
  4)293 roods 7 perches.
    ---
     73 acres 1 rood 7 perches.
```

Therefore, the whole *answer* is 73 acres 1 rood 7 poles 11¼ square yards 6 square feet.

Ex. 5. Reduce 22870062 square inches to acres, &c.

```
    ⎧ 12)22870062
144 ⎨    --------
    ⎩ 12)1905838-- 6 ⎫
         --------    ⎬ 126 inches.
       9) 158819--10 ⎭
          -------
30¼   )   17646-- 5 feet.
 4            4
---       -----
    ⎧ 11)70584 quarter yards.
121 ⎨    -----
    ⎩ 11) 6416--8 ⎫
          ----    ⎬ 41 qr. yds. = 10 yds. 2 ft. 36 in.
     4,0)58,3--3  ⎭                        5  126
         ----
        4)14--23 poles.
          --                              ---------
          3 ac. 2 ro. 23 pol. 10 yd.     8 ft. 18 in.
```

Ex. 6. How many pounds avoirdupoise are equal to 2275 pounds troy?

2275 lbs. troy = 2275 × 5760 grains = 13104000 grains

1 lb. avoirdupoise = 7000 grains

∴ 2275 lb. troy = (13104000 ÷ 7000) lbs. avoirdupoise = 1872 lbs. avoirdupoise *Answer.*

```
            2275 lbs. troy.
            5760
           -----
           13650
          15925
         11375
         --------
7,000)13104,000 grains.
      ---------
         1872 lbs. avoirdupoise.
```

Since each of the above rules (Rules XI and XII) is the converse of the other, the accuracy of any result obtained by either of them may be tested by working the result back again by by the other rule.

EXAMPLES FOR PRACTICE.

1. Reduce (proving each result) £91 10s. 3½d. to farthings; £71 13s. 6½d. to farthings; £137 9s. 11d. to half-pence; £71 18s. 3d. to three pences; £19 10s. 9d. to threepenny pieces; 180565 guineas to half-crowns.

2. Reduce (proving each result) 71834 pence to £ s. d.; 9463 farthings to £ s. d.; 712920 pence to half-crowns; 444912 farthings to guineas; 5673542 farthings to pounds.

3. Reduce (proving each result) 12 tons 15 cwt. 3 qrs. 17 lbs. 13 ozs. (avoirdupoise) to ounces; 125 tons 3 qrs. 14 ozs. 10 drs. to drams; 5838297 ozs. to tons; 83 lbs. 10 ozs. 3 drs. (av.) to drams; 17 tons 4 cwt. 2 qrs. 19 lbs. 13 ozs. to ounces (av.); 823 cwt. 2 lbs. 12 ozs. to drams (av.); 9641827 ozs. (av.) to tons, cwt. &c.; 249382 grains to lbs. troy, and also to lbs. av.; 17 ozs. 14 dwt. 12 grs. to grains troy; 1000 million ounces to tons, &c., av.; 685440 grains to stones av.

4. Reduce (proving each result) 41 miles 6 furlongs to poles; 213 leagues 6 furlongs 5 inches to inches; 200 miles 1 furlong 9 poles 2½ yards to feet and inches; 10 million inches to miles, furlongs, &c.; 713 million feet to furlongs; 720 acres 100 sq. inches to sq. inches; 8134926 sq. feet to acres, roods, &c; 20 million cubic inches to cubic yards; 8374859 sq. yards to acres.

5. Reduce (proving each result) 563 gallons to pints; 365843 gills to gallons; 2 bushels 3 pecks 2 quarts to pints; 1 load to pecks; 141 gallons 1 pint to pints; 4000000 pints to hogsheads; 1916382 pecks to quarts; 81927368 gills to gallons, &c.

6. Reduce (proving each result) 36 weeks 5 days 17 hours to seconds; 1 month of 30 days 23 hours 59 seconds to seconds; 10 million minutes to years (allowing 365 days 6 hours to a year); 365 days 5 hours 48 minutes 48 seconds to seconds; 10000000 seconds to years.

7. How many pounds, avoirdupoise, are there in 21000 pounds troy (reduce to grains and back).

8. Supposing a cubic foot of water to weigh 1000 ounces avoirdupoise, how many troy grains are there in a cubic inch?

9. A gallon of water measures 277 cubic inches very nearly, and weighs 10 lbs. avoirdupoise. How many cubic feet and inches are there in a ton of water?

COMPOUND ADDITION.

55. **Compound Addition is the collecting into one sum two or more numbers expressed in different denominations; and the process is precisely similar to that for the addition of simple numbers, with this difference, that the numbers connecting the different denominations must be employed instead of ten.**

Ex. Find the sum of £73 12s. 9½d., £25 8s. 4¾d., £68 3s. 11½d., £28 11s. 7¾d., and £17 14s. 11¾d.

£	s.	d.
73	12	9½
25	8	4¾
68	3	11½
28	11	7¾
17	14	11¾
213	11	9¼

Here the numbers are arranged so that those of the same denomination are in the same vertical column; then, beginning at the lowest denomination, viz., farthings, the sum is 13 farthings, which are equivalent to 3 pence and 1 farthing, the ¼ is therefore put down, and then 3 pence are carried to the next column and added thereto, making the entire number of pence 45. But 45 pence are 3 shillings and 9 pence, and writing 9 pence under the column of pence, the 3 shillings are added with the numbers in the column of shillings, making 51 shillings, which are equal to 2 pounds 11 shillings; writing 11 under the column of shillings, and carrying the 2 pounds to the left column, the entire number of pounds is found to be 213; consequently, the sum of the whole is £213 11s. 9¼d.

It may be noticed that, in adding up the shillings column, the best way is to disregard the *tens* in that column till all the units have been added: then, having reached the top unit figure, to proceed downwards, taking in every *ten* that appears. Thus, in the present example, the sum of the units' column of shillings is 21; so that proceeding downwards, taking in each *ten* as we meet with it, we say 31. 41. 51. so that the sum is 51 shillings, or £2 11s.

Two other examples are here annexed: the one in avoirdupoise weight and the other in time. In the first the sum of the drams is found to be 56 drams; by reduction we find that in these drams there are 3 ozs. 8 drs. (56 divided by 16 goes 3 times and 8 over), we therefore put down 8 drs. and carry 3 ozs. to the next column, which gives 46 ozs., or 2 lbs. 14 ozs., (46 divided by 16 goes 2 times and 14 over), writing down the 14 ozs. and carrying the 2 lbs. to the column of lbs., we set down 41 lbs. for the sum of this column; therefore, the whole sum is 41 lbs. 14 ozs. 8 drs.

lbs.	ozs.	drs.
7	3	8
4	6	9
11	11	15
8	13	11
9	10	13
41	14	8

In the next example the column of seconds amounts to 110 seconds, that is, to 1 minute 50 seconds; the 50 seconds is put down, and the 1 carried to the next column, the amount of which is 79 minutes, that is, 1 hour 19 minutes, put down 19 and carry 1: the hour column amounts to 83 hours, or 3 days 11 hours, 11 and carry 3 to the days column, the amount of which is 119 days; therefore, the whole amounts to 119 days 11 hrs. 56 secs.

d.	h.	m.	s.
18	9	0	37
27	21	11	19
34	13	9	15
13	14	21	19
10	7	14	16
14	18	23	4
119	11	19	50

To add together a set of concrete quantities of different denominations, the rule is as follows:—

RULE XIII.

1°. *Arrange the quantities to be added one under another, so that all in the same vertical column may be of the same denomination.*

2°. Add *up the quantities of lowest denomination; find how many of the* next *denomination are contained in the sum; put the* remainder *under the column and* carry quotient *to the next column.*

3°. *Proceed in this way from column to column till all have been added up.*

NOTE.—The principle of this rule is too obvious to require any explanation, the carryings merely transfer the quantities of advanced denominations to the columns in which these denominations are arranged, just as in the addition of abstract numbers.

EXAMPLES FOR PRACTICE.

(1)

£	s.	d.
23674	10	6[illegible]
36720	15	10[illegible]
9837	12	2[illegible]
304	5	9[illegible]
7640	19	8[illegible]
47000	16	10[illegible]
36239	14	6[illegible]
4762	9	7[illegible]
76830	15	6[illegible]
84525	12	7[illegible]

(2)

£	s.	d.
36798	5	7¼
7690	12	6[illegible]
49379	3	9¼
3987	12	5[illegible]
329	15	11¼
59842	10	6
7604	19	7¾
68432	8	9[illegible]
5000	12	6¼
93245	16	7¾

(3)

£	s.	d.
56042	13	6[illegible]
7421	9	0[illegible]
69840	0	7[illegible]
827	19	11[illegible]
7642	12	6[illegible]
98329	16	5[illegible]
946	5	8[illegible]
7600	19	11[illegible]
89217	12	6[illegible]
74235	16	6[illegible]

Add together 1 and 2, and 2 and 3.

(4)

£	s.	d.
78946218	17	9
68346236	12	4¼
7134682	14	4½
94365423	18	6¾
948796	16	5¼
46349745	19	2¼
9932436	11	3½
54390325	10	2¼
9143690	12	4¼
96245731	1	1¼
9946740	7	5
47210040	2	1½
9732102	1	2
42468403	9	3½
9054650	10	4
34240354	2	3
9024053	1	2¾
94326000	0	0
99453005	0	6
94600504	0	7½
40504050	3	0
94305067	4	0

(5)

£	s.	d.
49679677	19	9½
98798768	18	11½
98765432	16	10¾
49678989	10	8¾
12345678	19	10¼
42697687	16	8¼
49944994	17	11½
49666783	18	10½
98765439	19	11¼
76579999	17	10¼
99999976	16	9½
27863000	17	11¾
97877999	19	11¾
12345678	11	10¼
49678919	10	9½
66778899	11	10¼
97667899	19	10¼
91678910	18	9½
76543210	16	6¼
76543210	17	10½
87654321	16	9¼
88987654	13	9¼

(6)

£	s.	d.
49849698	19	11¾
6813984	18	10¼
98564932	17	8¼
498698	16	9¾
98648975	15	7½
89768493	14	6¼
491876	13	5½
41993219	12	4¼
486719	11	3¼
99483619	10	2½
2978413	19	11¼
419382	18	10½
98941389	17	9¼
27863000	17	11¼
984978	16	10¼
81924198	15	9½
61324918	14	8¼
483296	13	7½
998694	12	6¼
49132869	11	5¾
91989768	10	4¼
892300	19	11¼

(7)

£	s.	d.
98461795	19	10¼
97358912	18	9½
78924587	16	11¾
84612698	4	4
10798546	18	9½
68924679	1	1¼
54679284	19	11¼
27863000	17	11¾
49643210	18	10¼
58643516	19	8½
98456891	17	2¼
12345678	17	10¼
45678910	14	2½
24681012	14	3½
51437891	13	2¾
42316789	10	11
34689101	12	2½
23456789	10	10
34567891	11	11½
45432109	12	10
14567891	11	0¾
23456789	4	6

(8) **Avoirdupoise.**

tons.	cwt.	qrs.	lbs.	ozs.	drs.
3114	10	2	10	10	0
2120	16	0	14	0	12
5240	10	3	0	10	8
1716	10	3	10	7	10
3289	15	2	0	5	6
4174	10	0	15	0	14
3256	12	1	13	14	12

(9) **Troy Weight.**

lbs.	ozs.	dwts.	grs.
827	3	15	16
174	10	0	11
801	0	13	0
527	11	12	13
725	5	4	7
275	6	11	14
327	4	5	11

(10) **Apothecaries' Weight**

lbs.	ozs.	drs.	sc.	grs.
27	3	2	1	5
64	1	3	0	3
76	2	4	2	8
63	10	1	1	2
87	5	2	0	1
86	2	7	1	9

(11) **Long Measure.**

miles.	fur.	po.	yds.	ft.	in.
3217	3	20	0	0	10
4149	0	27	4	2	0
1320	0	26	3	1	6
2100	0	0	5	0	11
730	7	0	0	2	4
1257	3	20	1	1	2
3101	0	7	1	2	11

(12) **Square Measure.**

ac.	ro.	po.	yds.	ft.	in.
2320	1	15	13	2	110
1276	3	26	10	1	70
3104	2	17	26	3	126
4764	1	25	2	7	87
7476	2	13	20	1	136
2542	1	12	15	2	132
1876	2	16	10	1	120

(13) **Cubic Measure.**

c. yds.	ft.	in.
372	15	768
420	13	1076
325	19	376
823	20	1045
598	12	376
432	16	1567

(14)

ft.	in.	10ths
1	9	8
0	11	3
2	4	5
3	3	9
3	8	0
3	0	5

COMPOUND SUBTRACTION.

56. **Compound Subtraction is the taking a less number from a greater, when both numbers are composed of different denominations.**

EXAMPLES.

Ex. 1. Let it be required to find the difference between £586 16s. 2d., and £232 18s. 11d.

£	s.	d.
586	16	2
232	18	11
353	17	3

or

£	s.	d.
586	36	14
233	19	11
353	17	3.

We cannot take 11d. from 2d., therefore separate the 16s. in the minuend into 15s. and 12d., and add the 12d. to the 2d., making 14d.: from 14d. take 11d., and write 3d. in the remainder. Now, from the 16s. (as the figure remains unaltered) we have still to take 1s., besides the 18s. in the subtrahend, *i.e.*, we have to take away 19s.; but as we cannot take 19s. from 16s., separate the £6 in the minuend into £5 and 20s., and add the 20s. to the 16s., making 36s.; take 19s. from 36s., and write 17s. in the remainder. From the £6 remaining unaltered we have still to take away £1 besides the £2 in the subtrahend, *i.e.*, we have to take away altogether £3; subtract £3 from £6, and write £3 in the remainder. Next, subtract 3 from 8, and write down remainder 5; and lastly, take 2 from 5 gives remainder 3; whence the entire *difference* is £353 17s. 3d.

Ex. 2. Find the difference between 35 yards 2 feet 8 inches and 52 yards 1 foot 4 inches.

yds.	ft.	in.		yds.	ft.	in.
52	1	4	or	52	4	16
35	2	8		36	3	8
16	1	8		16	1	8

Since the difference of two quantities is not altered by adding the same quantity to both, we must first add 12 inches to the upper line, and 12 inches, or 1 foot, to the lower line, making 16 inches in the one and 3 feet in the other. Again, as 3 feet cannot be subtracted from 1 foot, we must add 3 feet to the upper line, and 3 feet, or 1 yard, to the lower, making 4 feet in the former and 36 yards in the latter. The subtraction can now be effected, the difference being 16 yds. and 1 ft. 8 in. Whence we have the following:—

RULE XIV.

1°. *Place the* less *of the two quantities* under *the* greater, *arranging the denominations as in addition.*

2°. *Commence with the* lowest *denomination, and* subtract, *if the upper number be sufficiently great; if not,* increase *it by as many as will make* 1 *of the* next *denomination, and then* subtract, *taking care afterwards to carry* 1, *as in subtraction of abstract numbers, and proceed in like manner with each denomination till the subtraction is finished.*

Examples for Practice.

(1)

£	s.	d.
45	14	7½
12	7	5¾

(2)

£	s.	d.
125	6	2¼
4	15	5¾

(3)

£	s.	d.
147	8	5¾
29	13	8¼

(4)

£	s.	d.
2975	17	3
897	19	7¼

(5)

£	s.	d.
4683	13	5½
2075	17	9¾

(6)

tons.	cwt.	qrs.	lbs.	ozs.
386	15	3	24	10
185	16	3	25	11

(7)

lbs.	ozs.	dwt.	grs.
6	5	11	21
3	10	10	23

(8)

mls.	fur.	yds.	ft.	in.
7	3	105	2	6
3	7	216	1	9

(9)

ac.	rds.	pols.
1798	2	17
489	3	28

(10)

sq. yds.	sq. ft.	sq. ins.
27	8	116
19	6	135

(11)

wks.	dys.	hrs.	min.	sec.
5	6	17	33	17
5	6	14	57	43

(12)

cub. yds.	cub. ft.	cub. in.
17	17	17
15	25	252

(13)

gals.	qts.	pts.
36	2	0
33	3	1

(14)

mos.	wks.	dys.
12	2	5
8	3	6

(15)

yrs.	wks.	dys.
27	35	4
18	47	6

(16)

yrs.	dys.	hrs.
26	213	11
19	231	21

COMPOUND MULTIPLICATION.

57. By **Compound Multiplication** we find what quantity is equal to the result obtained by repeating a compound quantity (*i.e.*, a quantity composed of denominations, which, though differing among themselves, are all of the same kind), any given number of times.

The following is the rule for multiplying a compound quantity:—

1. When the multiplier is not greater than 12.

RULE XV.

Place the multiplier under *the quantity of least denomination of the* multiplicand, *multiply this term by it, and divide the product by the number that expresses how many of such quantities make* 1 *of the* next *denomination; put down the* remainder *(if any)* under *the* right-hand term *and* carry *the* quotient *to the product arising from the multiplication of the next term, and so on, till all the terms have been multiplied.*

Examples.

Ex. 1. Required to multiply £17 18s. 9d. by 8, and explain the process.

£	s.	d.
17	18	9
		8
136	144	72

£	s.	d.
17	18	9
		8
143	10	0

2,0)15,0
£7 10s.

12)72(6
72
0

Putting the multiplier (8) under the pence, and multiplying them by the 8, the product is 72 pence, and dividing them by 12, the number of pence in a shilling, we get 6 and 0 over, so that in 72 pence there are 6 shillings and 0 pence; put down the 0 under the pence, and carry the 6 shillings to 144, the shillings' product, which gives 150 shillings, dividing by 20, the shillings in a pound, we get 7 and 10 over, so that in a 150 shillings there are 7 pounds 10 shillings; putting down the 10 under the shillings we carry the 7 to the pounds' product. The complete product is £143 10s. 0d.

Ex. 2. Multiply £16 11s. 2¼d. by 7.

1. ¼ × 7 = 7 farthings = 1¾d; set down ¾d. and carry 1.
2. 2d. × 7 = 14d., which, with the 1d. carried, = 15d. = 1s. 3d., set down 3 and carry 1.
3. 11s. × 7 = 77, which, with the 1s. carried, = 78s. = £3 18s.; set down 18s. and carry 3.
4. £16 × 7 = 112, which, with 3 carried, = £115; set down £115.

£	s.	d.
16	11	2¼
		7
115	18	3¾

Ex. 3. Multiply 2634 tons 13 cwt. 3 qrs. 25 lbs. by 9.

tons.	(20) cwt.	(4) qrs.	(28) lbs.
2634	13	3	25
			9
23712	5	3	1

20)125(6
120
5

4)35(8
32
3

28)225(8
224
1

2. When the multiplier is greater than 12, and is such as to admit of being formed from factors.

RULE XVI.

Multiply by each factor in succession, as in short multiplication.

NOTE.—When the multiplicand contains farthings, if one of the factors of the multiplier be even, it will often be advantageous to use it first, as the farthings may disappear.

EXAMPLES.

Ex. 1. Multiply £23 7s. 11¾d. by 63.

Since 7 times 9 are 63; if we multiply the given quantity by 7 and by 9 successively, *i.e.*, multiply £23 7s. 11¾d. by 7, and then multiply that result by 9, we shall by that means effect the multiplication by 63; hence—

	£	s.	d.
	23	7	11¾
			7
7 times the multiplicand =	163	15	10¼
			9
(9 times 7 times, *i.e.*) 63 times the multiplicand =	1474	2	8¼

Ex. 2. Multiply £32 14s. 4¾d. by 36.

Since 36 = 6 × 6, or 4 × 9, or 3 × 12, the sum may stand thus:—

£	s.	d.	£	s.	d.	£	s.	d.
32	14	4¾	32	14	4¾	32	14	4¾
		6			3			4
196	6	4½	98	3	2¼	130	17	7
		6			12			9
1177	18	3	1177	18	3	1177	18	3

Ex. 3. Multiply £27 4s. 0½d. by 240.

Since 240 = 4 × 6 × 10, the sum may stand thus:—

	£	s.	d.
	27	4	0½
			4
4 times the multiplicand =	108	16	2
			6
(6 times 4 times) *i.e.*, 24 times multiplicand =	652	17	0
			10
(4 times 6 times 10 times) *i.e.*, 240 times multiplicand =	6528	10	0

3. When, however, the multiplier exceeds 12, and cannot be broken up into factors.

RULE XVII.

Take the nearest* number *to it, which is composed of factors, multiply by these factors successively, and correct the result by adding or subtracting from the product so many times the multiplicand as the assumed composite number is less or greater than the given multiplier.

EXAMPLES.

Ex. 1. Multiply £105 10s. 2d. by 39.

Since 6 × 6 + 3 = 39.

If we multiply the given sum of money by 6, and then that result by 6, and to that continued product add 3 times the original sum of money, we shall obtain the product of £105 10s. 2d. and 39.

Or, since 4 × 10 − 1 = 39, we may multiply £105 10s. 2d. by 4, and the product by 10, and from this last product subtract £105 10s. 2d.

	£	s.	d.	
Multiplicand =	105	10	2	
			6	
6 times the multiplicand =	633	1	0	
			6	
(6 times 6 times, *i.e.*) 36 times the multiplicand =	3798	6	0	
3 times the multiplicand =	316	10	6	
∴ by addition, 39 times the multiplicand =	4114	16	6	***Answer.***

Ex. 2. Multiply £24 17s. 8¾d. by 23.

Since 23 = 4 × 6 − 1, we may multiply £24 17s. 8¾d. first by 4, then the product by 6, and from this last product subtract £24 17s. 8¾d. Or, since 2 × 11 + 1 = 23, we may first multiply by 2, then this product by 11, and add to the last product £24 17s. 8¾d.

	£	s.	d.
	24	17	8¾
			4
	99	10	11
			6
	597	5	6
Subtract	24	17	8¾
	572	7	9¼

	£	s.	d.
	24	17	8¾
			2
	49	15	5½
			11
	547	10	0½
Add	24	17	8¾
	572	7	9¼

Ex. 3. Multiply 37 yds. 2 ft. 11 in. by 53.

Since 6 × 9 − 1 = 53 the work will stand thus:—

	yds.	ft.	in.
	37	2	11
			6
	227	2	6
			9
	2050	1	6
Subt.	37	2	11
	2012	1	7

Ex. 4. Multiply 3 tons 27 lbs. 13 ozs. by 76.

Here 7 × 11 − 1 = 76.

	tons.	cwt.	qrs.	lbs.	ozs.
	3	0	0	27	13
					7
	21	1	2	26	11
					11
	231	19	0	13	9
Subt.	3	0	0	27	13
	228	18	3	13	12

When the multiplier is large we may often proceed as in the following examples:—

Ex. 5. Multiply 591 tons 11 cwts. 0 qrs. 6 lbs. by 346.

	tons.	cwts.	qrs.	lbs.	
	591	11	0	6	(*a*)
				10	
10 times the multiplicand =	5915	10	2	4	(*b*)
				10	
∴ 100 times the multiplicand =	59155	5	1	12	(*c*)
				3	
∴ 300 times the multiplicand =	177465	16	0	8	
40 times the multiplicand = 4 times the line (*b*) =	23662	2	0	16	
6 times the multiplicand = 6 times the line (*a*) =	3549	6	1	8	
By addition 346 times the multiplicand =	204677	4	2	4	

Ex. 6. Multiply £83 16s. 3¼d. by 6428.

	£	s.	d.	
	83	16	3¼	(*a*)
			10	
10 times the multiplicand =	838	2	8½	(*b*)
			10	
∴ 100 times the multiplicand =	8381	7	1	(*c*)
			10	
∴ 1000 times the multiplicand =	83813	10	10	
			6	
∴ 6000 times the multiplicand =	502881	5	0	
400 times the multiplicand = 4 times the line (*c*) =	33525	8	4	
20 times the multiplicand = 2 times the line (*b*) =	1676	5	5	
8 times the multiplicand = 8 times the line (*a*) =	670	10	2	
By addition, 6428 times the multiplicand = *Answer*	538753	8	11	

N.B.—This sum might have been done as below:—

£83 16s. 3¼d. = 80461 farthings.
∴ 6428 times £83 16s. 3¼d. = 6428 times 80461 farthings.
= 517203308 farthings.
= £538753 8s. 11d.

Note.—Should the multiplier consist of *many* factors, it will be found, in that case, convenient to reduce the multiplicand to the lowest denomination contained in it, then to multiply this result by the multiplier, and then to reduce the result back again.

Examples for Practice.

1. £213 4s. 2½d. × 3, 5, 7, 9.
2. £6024 8s. 5¼d. × 4, 6, 8, 10.
3. £853 12s. 6½d. × 8, 9, 10, 11, 12.
4. £3027 10s. 3¾d. × 3, 8, 5, 7, 12.
5. 3 qrs. 6 lbs. 13 ozs. 15 drs. × 8.
6. 4 tons. 13 cwt. 17 lbs. 10 ozs. × 9.
7. 1 cubic yd. 15 ft. 1124 in. × 7.
8. 17 gal. 2 qts. 1 pt. 2 gls. × 4, 5, 6.
9. Multiply £1608 13s. 11½d. by 42, 36, 81, 84, 63, 35, and 27.
10. Multiply £9700 18s. 3¼d. by 23, 41, 26, 34, and 107.
11. Multiply 14 tons 7 cwt. 63 lbs. by 8, 17, 42, 605, 19, and 191.
12. Multiply 18 gals. 3 qts. 1 pt. by 16, 17, 19, 401, and 1500.
13. Each sailor being supposed to weigh 1 cwt. 1 qr. 9 lbs., what is the weight of a crew of 861 men?
14. What is the weight of a million of bricks at 4 lbs. 2 ozs. each, and the cost at 27s. 8d. per 1000?
15. Telegraph posts being placed at 132 feet apart, what distance will 1235 poles do for?

16. The circumference of a wheel is 16 ft. 1½ in. How many miles will it run in 125375 revolutions?
17. The pitch of an endless screw is half an inch, and it is driven at the rate of 150 turns a minute. What is the edge velocity of the wheel in feet per hour?

COMPOUND DIVISION.

58. **Compound Division** is a method of dividing a compound number, that is, a number consisting of several denominations into as many equal parts as there are units in the divisor; or, it is the finding how many times one compound number is contained in another consisting of like denominations.

1. When the divisor is an *abstract* number not greater than 12.

RULE XVIII.

Place the numbers as in Simple Division, then find how often the divisor *is contained in the* highest *denomination of the dividend; put this number down in the quotient; if there be a remainder, reduce that remainder to the next inferior denomination, adding to it the number of that denomination in the* dividend, *and repeat the divisor: carry on this process through the whole dividend.*

Examples.

Ex. 1. Divide £172 11s. 5½d. into 5 equal parts, *i.e.*, find the 5th part of £172 11s. 5½d.

$$\begin{array}{rrr} £ & s. & d. \\ 5)172 & 11 & 5\frac{1}{2} \\ \hline 34 & 10 & 3\frac{1}{2} \end{array}$$

Here, the quotient obtained by dividing £172 by 5 is £34 with a remainder £2, put the 34 under the pounds and bring the £2 into shillings, and add the 40 shillings so obtained to the 11 shillings in the dividend, making 51 shillings; divide 51 shillings by 5; the quotient is 10 shillings, with a remainder of 1 shilling, write the 10 under shillings; bring the 1 shilling over into pence, and add 12 pence to 5 pence in the dividend; divide 17 pence, so obtained, by 5, and the quotient is 3 pence, with 2 pence remainder; bring 2 pence into farthings, and add 8 farthings so obtained, to the halfpenny, or 2 farthings, in the dividend, making 10 farthings; divide 10 farthings by 5, and the quotient is 2 farthings, or a halfpenny; write ½ in the farthings' place of the quotient. Hence the entire quotient is £34 10s. 3½d.

Proof.—By multiplying the quotient by the divisor we obtain the original *dividend;* thus:—

$$\begin{array}{rrr} £ & s. & d. \\ 34 & 10 & 3\frac{1}{2} \\ & & 5 \\ \hline 172 & 11 & 5\frac{1}{2} \end{array}$$

Ex. 2. Divide 1080 tons 12 cwt. 3 qrs. 21 lbs. by 7.

$$\begin{array}{rrrr} \text{tons.} & \text{cwt.} & \text{qrs.} & \text{lbs.} \\ 7)1080 & 12 & 3 & 21 \\ \hline 154 & 7 & 2 & 7 \end{array}$$

Dividing the tons by 7 we have 154 tons, with the remainder 2. This remainder is 2 tons, or 40 cwt., which, with the odd 12 cwt., makes 52 cwt. Dividing this we get 7 cwt., and a remainder of 3 cwt., or 12 qrs. We have now 12 + 3 = 15 qrs. to divide. This gives 2 qrs. and a remainder 1 qr., or 28 lbs; with the 21 lbs. this gives 49 lbs. The division of this gives 7 lbs.

Examples for Practice.

1. £84 7s. 6d. ÷ 2; £84 7s. 6d. ÷ 3; £102 2s. 8d. ÷ 4.
2. £137 6s. 4½d. ÷ 5; £284 10s. 9d. ÷ 6; £192 4s. 10¾d. ÷ 7.
3. £1928 11s. 5½d. ÷ 7; £311 0s. 6d. ÷ 8; £5062 18s. 9d. ÷ 8.
4. £4987 13s. 0d. ÷ 9; £586 13s. 6½d. ÷ 10.; £2856 9s. 7¾d. ÷ 11.
5. £10584 5s. 4½d. ÷ 11; £5135 13s. 6d. ÷ 12; £10121 14s. 3d. ÷ 12.
6. 23 lbs. 8 ozs. 13 dwt. 1 gr. ÷ 7; 41 lbs. 11 ozs. 4 dwt. 6 grs. ÷ 6; 114 tons 5 cwt. 24 lbs. ÷ 8.
7. 4 tons 3 qrs. 8 lbs. 5 ozs. ÷ 9; 306 yds. 6 in. ÷ 11; 42 mls. 5 fur. 4 yds. ÷ 12.
8. 59 mls. 3 fur. 16 yds. ÷ 12; 307 ac. 17 pls. ÷ 11; 577 gals. 2 qts. ÷ 10.

2. When the divisor is greater than 12, and cannot be resolved into **factors,** we proceed according to Long Division.

EXAMPLES.

Ex. 1. Divide £23343 17s. 9d. by 365.

Here, say 365 is in 2334, 6 times, 6 times 365 = 2190, which place under 2334 and subtract, the remainder is 144; bring down 3 to the right of the remainder, and we have the dividend 1443, which contains 365 3 times, 3 times 365 = 1095; write this down under 1443 and subtract; the remainder is £348.

Multiply £348 by 20 to bring it to shillings, adding in the 17 shillings, which gives 6977 shillings. Then 365 into 697 is 1, and 365 × 1 = 365, which place under 697 and subtract, leaving remainder 332, bring down 7, and the dividend thus formed is 3327, into which 365 goes 9 times; 9 times 365 = 3285, which place under 3327 and subtract, the remainder is 42 shillings. Multiply 42 by 12 to bring them to pence, add the 9 pence to the product, which is 513; divide by 365, the quotient is 1d.; multiply 365 by 1 = 365; subtract 365 from 513, remainder 148d.; multiply 148d. by 4, product 592 farthings; dividing 592 by 365 gives quotient 1 farthing, and 227 farthings over.

```
        £     s.  d.  £
    365)23343 17  9 ( 63
        2190
        ----
         1443
         1095
         ----
          348
           20
          -----
          6977(19s.
          365
          ----
          3327
          3285
          ----
            42
            12
          ----
           513(1d.
           365
           ---
           148
             4
           ---
           592(1
           365
           ---
Remainder  227 farthings.
```

Ex. 2. Divide 204677 tons 4 cwt. 2 qrs. 4 lbs. by 346.

```
      tons.   cwt. qrs. lbs. tons  cwt. qrs. lbs.
  346)204677   4    2    4  (591   11    0    6
      1730
      ----
       3167
       3114
       ----
         537
         346
         ---
         191
          20
         ----
         3824 cwt.
         346
         ----
          364
          346
          ---
           18
            4
          ---
           74 qrs.
           28
          ---
          596
          148
          ----
         2076 lbs.
         2076
         ----
```

The tons are dealt with as in Simple Division.

The 191 tons *over* are converted into 3820 cwt., which, with the 4 cwt. in the dividend, make 3824.

This 3824 cwt. is divided by 346 as in Simple Division.

The 18 cwt. *over* are similarly converted into 72 qrs., which with the 2 qrs. in the dividend, make 74 qrs.

This 74 qrs. is divided by 346, as in Simple Division, and gives 0 for the quotient.

The 74 qrs. *over* are converted into lbs., and, with the 4 lbs. in the dividend, give 2076 lbs.

This 2076 lbs. divided by 346, as in Simple Division, give 6 lbs. as quotient.

EXAMPLES FOR PRACTICE.

1. £1030 13s. 11¼d. ÷ 13; £289 0s. 8½d. ÷ 17; £456 0s. 11¼d. ÷ 23.
2. £1216 3s. 9d. ÷ 244; £1866102 3s. 3½d. ÷ 257; £16647 11s. 6d. ÷ 753.
3. £371 2s. 9½d. ÷ 29; £712 18s. 7¼d. ÷ 41.
4. £7638862 10s. 3¾d. ÷ 879; £41005 2s. 6d. ÷ 984; £787163 9s. 4d. ÷ 7804.
5. £247075 2s. 9½d. ÷ 9346; £79166820 4s. 8½d. ÷ 8975; £732441473 3s. 7½d. ÷ 87031.

3. When the divisor is large, but can be broken up into two or more factors, we may divide separately by each of these.

EXAMPLE.

Ex. Divide £5762 3s. 6d. by 24.

Since 24 = 4 × 6, or = 3 × 8, or = 2 × 12, the work may stand thus:—

```
        £     s.  d.              £     s.  d.               £     s.  d.
   ( 4)5762   3   6          ( 3)5762   3   6         (  2)5762   3   6
24 {  ————————————        24 {  ————————————        24 {   ————————————
   ( 6)1440  10  10½         ( 8)1920  14   6         ( 12)2881   1   9
      ——————————————            ——————————————             ——————————————
       240    1   9¾             240    1   9¾              240    1   9¾
```

EXAMPLES FOR PRACTICE.

	£	s.	d.			£	s.	d.	
1.	187	14	11	÷ 14	7.	319	2	9	÷ 132
2.	275	15	1¾	÷ 18	8.	576	3	0	÷ 144
3.	345	10	5	÷ 25	9.	457	18	4	÷ 400
4.	200	16	9	÷ 35	10.	1137	10	0	÷ 2400
5.	400	0	9	÷ 54	11.	1556	5	0	÷ 3600
6.	363	18	2¼	÷ 81	12.	362	10	0	÷ 6000

4. Division by 10, 100, 1000, &c., is usually performed by *pointing off*, *one*, *two*, *three*, &c., figures, respectively, from the right of the dividend.

Ex. 1. Divide £3415 14s. 7d. by 100.

```
  £      s.  d.
34,15   14   7
   20
 ————
 3,14 s.
   12
 ————
 1,75 d.
    4
 ————
 3,00 f.
```

Here, dividing 3415 by 100, we have a quotient 34, with remainder 15; we may *point* off, therefore, the last *two* figures as the remainder, leaving the rest for the quotient; reducing now this remainder to shillings, and adding in the term 14s., we have to divide 314s. by 100, and since the quotient is 3, with remainder 14, we may again *point* off the last *two* figures as the remainder, and so on.

Ex. 2. Divide £57618 15s. 0d. by 1200.

Since 1200 = 12 × 100, the work may stand thus:—

```
        £     s.  d.
  12)57618   15   0
     ——————————————
100) 48,01   11   3
        20
      ————
      0,31 s.
        12
      ————
      3,75 d.
         4
      ————
      3,00 f.
```

Here there is no quotient from shillings and we have *Ans.* £48 0s. 3¾d.

N.B.—In a case where one of the factors of the divisor is 10, 100, &c., it is generally best to divide *last* by that factor.

EXAMPLES FOR PRACTICE.

	£	s.	d.	
1.	176	16	8	÷ 10
2.	329	1	3	÷ 100
3.	1511	9	2	÷ 1000
4.	645	16	8	÷ 10000
5.	262	10	0	÷ 2400
6.	1447	18	4	÷ 1000
7.	26380	4	2	÷ 25000
8.	1668	15	0	÷ 150

9. Divide 140 weeks 3 days 14 hours by 100.

5. When the divisor and the dividend are both compound numbers of the same kind.

RULE XIX.

Reduce both numbers to the same denomination, divide as in Simple Division, and the result will be the answer required.

EXAMPLE.

Ex. How often is £3 18s. 7d. contained in £31 8s. 8d., that is, divide £31 8s. 8d. by £3 18s. 7d.

We must first bring both dividend and divisor to the *same denomination*, viz., pence, before the division can be effected; therefore—

£ s. d.	£ s. d.
3 18 7	31 8 8
20	20
78	628
12	12
943	943)7544(8
	7544

Hence, the quotient is the abstract number 8, or the dividend contains the divisor exactly 8 times.

PROOF.—By multiplying the divisor by the quotient we have £3 18s. 7d. × 8 = £31 8s. 8d. by compound multiplication.

EXAMPLES FOR PRACTICE.

How many times does

	£	s.	d.		£	s.	d.
1.	20237	5	0	contain	8	5	0
2.	619	15	6	„	47	13	6
3.	7934	11	0	„	67	16	4
4.	12075	0	0	„	95	16	8
5.	417597	0	0	„	375	17	6
6.	95	6	3	„	3	16	3

7. 49 tons 7 cwt. contain 1 ton 3 cwt. 2 qrs.
8. 11 tons 4 cwt. 3 qrs. 20 lbs. contain 3 cwt. 1 qr. 12 lbs.
9. 2 miles 76 yds. contain 14 yds. 1 ft. 6 in.
10. 1221 wks. 3 dys. 12 hrs. contain 3 wks. 12 hrs.

CHAPTER III.

THE GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

DEFINITIONS.

59. When one number divides another without any remainder, or is contained an exact number of times in it, the former is said to be a **Measure** of the latter.

Thus, 4 and 7 are contained an exact number of times in 28, *i.e.*, they divide it without any remainder; therefore, 4 and 7 are each said to measure 28. Similarly, 2, 3, 4, and 6 are measures of 12.

60. When one number will divide each of two or more numbers without remainder, it is said to be a **Common Measure** of these numbers.

Thus, 4 will measure 24, 32, and 112; therefore, 4 is a *common measure* of 24, 32, and 112. Similarly, 6 is a common measure of 18, 24, and 30.

61. The *greatest* number which divides each of two or more numbers without remainder is called the **Greatest Common Measure** of these numbers.

Thus, 2, 3, and 6 are common measures of 18 and 30, but 6 is the *greatest* common measure of 18 and 30.

Greatest common measure is usually written G.C.M.

62. One number is a multiple of another number when it *contains*, *i.e.*, can be divided by that other number an exact number of times.

Thus, 6 is a multiple of 2, because 6 can be exactly divided by 2; also, 18 and 36 are multiples of 6.

63. When one number can be exactly divided by two or more numbers, it is said to be a **Common Multiple** of these numbers.

Thus, since 72 can be exactly divided by 8 and 12, it is said to be a *Common Multiple* of 8 and 12. So 420 is a common multiple of 2, 3, 4, 5, 6, 7, 10, 12, 14, 15, 20, 21, 28, 30, 35, 42, 60, 70, 84, 105, 140, and 210.

64. The *least* number which can be divided by each of two or more numbers is called the **Least Common Multiple** of these numbers.

Thus, 18 is the least common multiple of 3, 6, and 9.

The least common multiple is usually written L.C.M.

The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, &c.

The multiples of 5 are 5, 10, 15, 20, 25, 30, &c.

The common multiples of 3 and 5 are 15, 30, 45, &c.

The L.C.M. of 3 and 5 is 15.

Hence, we see that all the common multiplies of 3 and 5 are multiples of their L.C.M.

The same may be seen to be true of the common multiples of three or more numbers.

The terms Measure and Multiple are thus related: since 5 is a *measure* of 15, therefore, 15 is a *multiple* of 5; here 5 is said to *measure* 15 by the units of the quotient 3.

65. **A Composite Number** is a number which is the product of two or more numbers, *i.e.*, a number which is exactly divisible by other numbers beside itself and unity.

Thus 63 is a *composite* number, because it is the product of 7 × 3 × 3. So 4, 6, 8, 10, 12, 14, 15, 16, 18, 20, 21, 24, &c., are composite numbers.

Hence every *multiple* of two or more numbers is a composite number.

66. The **Factors of any number** are those numbers which, when multiplied together, will produce that number.

Thus, 2 and 15, 3 and 10, 5 and 6, are factors of 30.

67. **A Prime Number** is one which cannot be resolved into factors, *i.e.*, a number which is not exactly divisible by any number beside itself and unity.

Thus, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, &c., are *prime* numbers, none of them being reduceable to factors.

Every measure of a number is not necessarily a *prime number*, but every number may be reduced into *prime factors*, *i.e.*, into factors which are prime numbers. Thus, 24 is measured by 6 and by 4, but neither 6 nor 4 is a prime number, the first being the product of 3 × 2, and second of 2 × 2.

68. When two or more numbers have no common measure but unity they are said to be **Prime to one another.**

Thus, 14 and 55 are prime to each other, because neither 7 nor 2, the measures of 14, is a measure of 55.

69. **How to find the G.C.M. and L.C.M. of two or more numbers.**

Two problems are suggested by the consideration of the subject, viz.:—

1st. How to find the G.C.M. of two or more numbers.

2nd. How to find the L.C.M. of two or more numbers.

70. In investigating the methods by which each of these may be found, we shall be materially assisted by the following proposition:—

"*A Common Measure of any two numbers is also a measure of their sum, of their difference, and of any multiple of any of them.*"

Thus, 3 being a common measure of 9 and 15, will measure their sum, their difference, and also any multiple of either 9 or 15.

The sum of 9 and 15 = 9 + 15 = 24 = 3 × 8
therefore, 3 measures their sum 24.

The difference of 15 and 9 = 15 − 9 = 6 = 2 × 3
therefore, 3 measures their difference 6.

Again, 36 is a multiple of 9, and 36 = 3 × 12
therefore, 3 measures this multiple of 9; and similarly any other multiple of 9.

Again, 75 is a multiple of 15, and 75 = 3 × 25
therefore, 3 measures this multiple of 15, and similarly any other multiple of 15.

71. To find the greatest common measure of two numbers.

RULE XX.

Divide the greater *number by the* less; *if there be a* remainder, *divide the* first divisor *by this* remainder; *if there be still a remainder, divide the* second divisor *by this* remainder, *and so on, always dividing the last preceding divisor by the last remainder, until the remainder is* 0, *or* 1. The last divisor will be the greatest common measure required.

Note I.—When the remainder is 1, the given quantities have no common measure except unity, *i.e.*, they are prime to each other.

Note II.—It is not absolutely necessary that our products should be less than the dividend. All that the principle requires is that we should take the difference *between the dividend and the nearest multiple of the divisor.* The method given in the rule is that most usually employed; though, when the next higher multiple would be nearer to the dividend than the next lower, the actual work is considerably lessened by the adoption of the higher multiple. See Ex. 2.

The process will be best understood by an example.

Examples.

Ex. 1. Find the greatest common measure G.C.M. of 348 and 738.

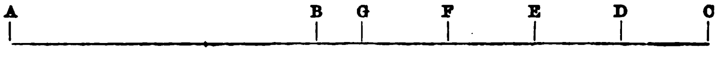
```
348)738(2
    696
    ---
     42)348(8
        336
        ---
         12)42(3
            36
            --
             6)12(2
               12
               --
```

The larger, 738, is divided by 348, leaving the remainder 42. 348 is now divided by 42, leaving remainder 12. 42 is divided by 12, leaving remainder 6. 12 is divided by 6, leaving no remainder. The last divisor 6 is the G.C.M.

Reason of the process.—Whatever number divides 348 and 738 must divide twice 348 or 696, and, therefore, 738 — 696, or 42. For, suppose it does not go an exact number of times in 42, then (since it does go an exact number of times in 696) it would not go an exact number of times in 696 + 42, or 738. but by the hypothesis it does, since it is a measure of 738. Hence, every number that divides 348 and 738 will also divide 348 and 42. And, similarly, every number which divides 348 and 42 will also divide 738. Hence, the greatest number which divides 42 and 348 will be the greatest number which will divide 348 and 738; that is, the G.C.M. of 348 and 738 is the same as the G.C.M. of 348 and 42. In the same manner it may be shown that the G.C.M. of 348 and 42 is the G.C.M. of 42 and 12, and the G.C.M. of 42 and 12 is the G.C.M. of 348 and 738.

72. The reason for this rule may be represented by lines as follows:—

To find the G.C.M. of 49 and 112.



Let AC represent a line of 112 inches, and AB 49 inches. It is required to find the line of greatest length which exactly measures AB and AC. From BC cut off BD, to equal AB. Then since the required line is contained an exact number of times in AB, it is also in BD. But it is contained an exact number of times in AC, therefore it must be in DC, that is, in 14 inches; and it must be the line of greatest length which is exactly contained in DC and DB. From DB cut off parts DE, EF, FG, each equal to DC, then the required line must be contained an exact number of times in BG, that is, in 7 inches, and be the greatest line, which is exactly contained in BG and GF. But BG is half GF, therefore BG is the required line.

Ex. 2. Find the greatest common measure of 891 and 3429.

```
891)3429(4
    3564
    ----
     135)891(7
         945
         ---
          54)135(2
             108
             ---
              27)54(2
                 54
                 --
```

In finding the G.C.M. in this example, the work has been shortened by taking the quotient figures 4 and 7, because the products 3564 and 945 differ less from the dividends 3429 and 891, than if 3 and 6 had been taken.

Here 27 is the G.C.M. of 891 and 3429.

Ex. 3. Find the greatest common measure of 3852 and 762896.

```
3852)762896(198
     3852
     -----
     37769
     34668
     -----
      31016
      30816
      -----
        200)3852(19
            3800
            ----
              52)200(4
                 208
                 ---
                   8)52(6
                     48
                     --
                      4)8(2
                        8
                        -
```

Here 4 is the G.C.M. of 3852 and 762896.

73. This process enables us to find the G.C.M. in all cases. But in the case of small numbers it can often be seen by inspection, and the student must accustom himself to do so. Thus, since 48 = 12 × 4, and 60 = 12 × 5, 12 is evidently the greatest common measure of 48 and 60.

74. To find the G.C.M. of three numbers, we first find the G.C.M. of two of the numbers, and then the greatest common measure of the result of the third number; and, similarly, if there were more numbers.

EXAMPLES FOR PRACTICE.

Find the greatest common multiple of—

1. 24 and 36; 36 and 45; 121 and 143; 87 and 145; 289 and 529.
2. 567 and 576; 1001 and 1331; 729 and 999; 175 and 2042; 1878 and 3443.
3. 2314 and 3721; 7512 and 12207; 13731 and 19366; 55328 and 22755.
4. 107358 and 107271; 814407 and 91797; 63404 and 42867; 12679235 and 1864381.
5. 2442431 and 1562009; 1718446 and 4439618; 2158654 and 12130265.
6. 252177 and 3628013; 68590142 and 85044059; 318732033187 and 32033187203.
7. 726221432301 and 432301726221; 318732033187 and 320331873203.

75. To find the Least Common Multiple of two or more numbers.

RULE XXI.

1°. *Having arranged all the numbers in a row, cancel those which* exactly *divide any of the others.*

2°. *Select from the numbers which remain that* one *which appears to contain* *the greatest number of different factors, and divide all the uncancelled numbers in succession by the greatest number which will divide each of them and the aforesaid* selected *number, putting down the quotients for a second row.*

3°. *Treat this second row as the first, that is, cancel every number which* exactly *divides any of* the others. *Then, if the uncancelled numbers in the second row be all prime to one another, the operation will be finished; for the continued product of the numbers in this line and the divisor will be the least common multiple. Should the uncancelled numbers in the second row be not all prime to one another,* then *we may conclude that an* unsuitable *number has been selected, and another number, taken from the given numbers, must be used instead,* always *taking that which offers the greatest number of different factors.*

EXAMPLES.

Ex. 1. Find the least common multiple, L.C.M., of 6, 8, 12, 14, 15, 18, 20, 21, and 24.

$$24)\underline{\cancel{6},\ \cancel{8},\ \cancel{12},\ 14,\ 15,\ 18,\ 20,\ 21,\ \text{and}\ 24}$$

$$\cancel{7},\ \cancel{5},\ 3,\ 5,\ 7$$

$$3 \times 5 \times 7 \times 24 = 2520$$

Strike out all the numbers which will divide any of the other numbers exactly, as 6, 8, 12, which divide 24. Selecting 24 as the number containing the most factors, write it outside. Divide the other numbers in succession by the greatest number which will divide each of them and 24; thus, the G.C.M. of 24 and 14 is 2, leaving 7; of 24 and 15 is 3, leaving 5; of 24 and 18 is 6, leaving 3; of 24 and 20 is 4, leaving 5; of 24 and 21 is 3, leaving 7. Then 7 cancels 7, 5 cancels 5, and the remaining figures in the row, viz., 3, 5, 7, being all prime to one another, their continued product and the divisor 24 gives the L.C.M. = 2520.

Reason of the process.—We see that 2520 = 24 × 3 × 5 × 7. Hence it contains as factors all the given numbers, for 6, 8, 12 are each contained in 24; the part of 14, viz., 7, not contained in the 24 is introduced by the factor 7; the part of 15 not contained in 24 is introduced by the factor 5, and so on for the other numbers. Again, we cannot dispense with any of the factors 3, 5, 7; for, if the L.C.M. did not contain the factor 3, beside the factor 6 in the 24, it would not be a multiple of 18; and, similarly, the other factors, 5 and 7, are necessary. Hence, 2520 is a common multiple of all the given numbers, and if we took away any of the factors composing it, it would cease to contain one or more of the given numbers. It is, therefore, the L.C.M. of the given numbers.

Ex. 2. Find the least number that can be divided by 5, 6, 9, 15, 24, 27, and 126.

The factors—

$$126)\underline{\cancel{5},\ \cancel{6},\ \cancel{9},\ 15,\ 24,\ 27,\ 126}$$

$$15,\ 4,\ \cancel{3}$$

Therefore, 126 × 15 × 4 = 7560, the number required.

Strike out 5, 6, 9, as they are contained in 15, 24 and 27 respectively; then the G.C.M. of 126 and 24 is 6, leaving 4; of 126 and 27 is 9, leaving 3; cancel the 3 as it is contained in 15. Then the continued product of 126 × 15 × 4 = 7560, the number required.

Ex. 3. Required the least common multiple of 7, 12, 15, 27, 35, 40, and 45.

$$45)\underline{\cancel{7},\ 12,\ \cancel{15},\ 27,\ 35,\ 40,\ 45}$$

$$\cancel{4},\ 3,\ 7,\ 8$$

$$\therefore 45 \times 3 \times 7 \times 8 = 7560 \ \textit{Ans.}$$

3 divides 45 and 12, leaving 4; 9 divides 45 and 27, leaving 3; 5 divides 45 and 35, leaving 7; 5 divides 45 and 40, leaving 8; cancel 4 as it is contained in 8; and since the remaining figures 3, 7, 8 have no common measure but unity; the continued product of these and 45 gives 7560 as the L.C.M. required.

Ex. 4. Required the L.C.M. of 8, 9, 10, 12, 25, 32, 75, and 80.

$$\begin{array}{l} 75)\underline{\cancel{8},\ 9,\ \cancel{10},\ 12,\ \cancel{25},\ 32,\ 75,\ 80} \\ \quad\ 3,\ \cancel{4},\ 32,\ \cancel{16} \end{array}$$

$$\therefore 75 \times 3 \times 32 = 7200$$

76. The least common multiple of two or more numbers may also be found by the following process:—

RULE XXII.

1°. "*Write the numbers in a line, and strike out any that are contained in any of the others. Divide those not struck out by any number that will exactly divide one of them; under any which it exactly measures place the corresponding quotient; under any which it partially measures containing some factor common to it, (but not being itself wholly contained in it), place the quotient obtained by dividing it by the common factor; and under any which it does not measure at all, repeat the number itself.*

2°. *Now treat the new line thus formed in the same manner as the first, and so on until all the numbers left in any line have no common measure but unity.*

3°. *Then the continued product of the numbers in this line and all the divisors is* L.C.M. *required of the given numbers.*"

NOTE.—It will generally be most convenient to take pretty large numbers, if possible, for divisors, as fewer lines will thus be necessary, especially if such be chosen as contain themselves many simple factors. Thus, 12 contains the factors 2, 3, 4, 6, and 12, and is, therefore, when possible, a very good divisor to be employed.

EXAMPLES.

Ex. 1. Find the L.C.M. of 24, 16, 6, 20, 4, 8, 10, 30, 12, 25.

$$\begin{array}{l} 12)\underline{24,\ 16,\ \cancel{6},\ 20,\ \cancel{4},\ \cancel{8},\ \cancel{10},\ 30,\ \cancel{12},\ 25} \\ \qquad \cancel{2},\ 4,\ \cancel{5},\ \cancel{5},\ 25 \end{array}$$

$$\therefore \text{L.C.M.} = 12 \times 4 \times 25 = 1200.$$

Strike out 6, 4, 8, and 12, as they are contained in 24; having placed 12 outside as divisor, divide the other numbers in succession by the greatest number which will divide each of them and 12; thus, 12 goes into 24 2; the G.C.M. of 12 and 16 is 4, leaving 4; of 12 and 20 is 4, leaving 5; of 12 and 30 is 6, leaving 5; bring down 25. Next, cancel 2 as it is contained in 4; cancel 5 and 5 because they are contained in 25; and the remaining figures, viz., 4 and 25, are prime to one another, having no common measure but unity. Then the continued product of 4 × 25 × 12 = 1200, the L.C.M. required.

Ex. 2. What is the least number that can be divided by each of 14, 16, 40, 50, 25, 8, 64.

$$\begin{array}{l} 10)\underline{14,\ \cancel{16},\ 40,\ 50,\ \cancel{25},\ \cancel{8},\ 64} \\ \qquad 7,\ 4,\ 5,\ 32 \end{array}$$

$$\therefore 7 \times 4 \times 5 \times 32 \times 10 = 11200 \text{ L.C.M.}$$

Ex. 3. Find the least common multiple of 27, 24, 6, 15, 5, 9, 126.

$$\begin{array}{l} 9)\underline{27,\ 24,\ \cancel{6},\ 15,\ \cancel{5},\ \cancel{9},\ 126} \\ 2)\underline{\ 3,\ 8,\ 5,\ 14} \\ \quad\ \ 3,\ 4,\ 5,\ 7 \end{array}$$

$$\therefore 3 \times 4 \times 5 \times 7 \times 9 \times 2 = 7560$$

Compare this with Ex. 2, page 59.

EXAMPLES FOR PRACTICE.

Find the least common multiple of—

1. 2, 3, 4, 6; 4, 5, 15, 20; 15, 21, 35, 40; 36 and 75.
2. 319 and 407; 333 and 504; 2961 and 799; 8, 9, 12, 18.
3. 6, 15, 27, 35; 3, 9, 7, 15, 18, and 42; 75, 125, and 1000; 54, 96, and 64.
4. 12, 20, 24, 54, 81, 63, and 14; 225, 255, 289, 1023, and 4095.
5. 8, 18, 28, 35, 54, 72, and 90; 30, 60, 90, 150, 180, 300, 450, and 900.
6. 12, 18, 24, 27, 36, and 42; 15, 21, 25, 36, 48, 60, 78, 9, 18, 24, 72, and 144.
7. 9, 16, 42, 63, 21, 14, and 72; 34, 51, 85, 65, 104, and 117.
8. 29, 27, 216, 261, 297, and 783; 343, 5929, 7007, 8281, and 16807.
9. 221, 117, 153, 289, 169, and 81; 529, 437, 361, 161, and 133.
10. 105, 403, 248, 221, 119, and 12648; 3528, 25725, 23625, and 432.
11. 658, 752, 1222, 1410, and 2444; 13792381, 32080621, 33452239, and 5806592401.

MISCELLANEOUS EXAMPLES IN G.C.M. AND L.C.M.

1. What multiple of 5 results from multiplying 314725 by 9?

2. What number is the same multiple of 5 that 148995 is of 5?

3. Of all the numbers beside unity that may be subtracted from 7905 an exact number of times, which is that which may be taken the *greatest*, and which the *least*, number of times?

4. What is the least number from which 34, 510, 91, and 16, may each be taken an exact number of times?

5. What is the least number by which 17595 must be multiplied, that 36 may be taken from the product an exact number of times?

6. A school was found to contain such a number of boys that whether arranged in sixes, sevens, nines, or elevens, there were always five over; how many children at *least* did the school contain?

7. Find all the numbers which will divide 396, 5184, and 6912, without a remainder, and the smallest number they will all divide without a remainder.

In practice the L.C.M. may often be written down at once. Example—8, 12, 15, 16, and 18. The L.C.M. $= 15 \times 16 \times 3 = 720$, for the numbers 15 and 16 having no common factor, must both remain in the L.C.M, 8 is included in the 16, 12 partly in the 15, partly in the 16, and 18 containing the factor 3 twice, requires the addition of the factor 3, since 15 contains it only once, and 16 not at all.

8. What is the least number of dollars at 4s. 2d. each, which is equal to an exact number of sovereigns?

9. Standard gold being coined at the rate of £3 17s. 10½d. per oz., what is the least number of ounces which can be coined into an exact number of sovereigns?

10. Eight bells begin tolling simultaneously, and they toll at intervals of 1, 2, 3, 4, 5, 6, 7, and 8 seconds, respectively; after what interval of time will they again be tolling at the same instant?

11. What is the common unit which will express 10080 minutes, and 15840 minutes, in the smallest possible integral numbers?

12. Find all the common multiples of 27, 36, and 45, which are less than 2000?

13. Two bells, which toll respectively every 4 and 6 seconds, commence tolling together; how often will they be tolling simultaneously in an hour?

14. Three boys start together to run round a ring, and they run round it in 4, 5, and 6 minutes respectively; when will they all be together again, and how many times will they each have been round?

15. A has 5040 shillings, B has 5292, and they wish each to separate their own money into equal heaps, so that A's heaps are as large as B's, and each as large as possible; how many shillings must they put into each heap, and how many heaps will they each have?

16. Two foot rules are divided, one into 120 divisions and the other into 96; if they are placed side by side, how many divisions will coincide?

17. Two cog-wheels containing 32 and 36 cogs, respectively, work together; if the large wheel has made 64 revolutions, how often have the same cogs been in contact?

CHAPTER IV.

FRACTIONS.

DEFINITIONS.

77. **A Fraction** is a part or parts of unity or one.

The nature of a fraction will be understood by supposing that a unit of any kind is divided into several equal parts. One or more of these parts is called a fraction of that unit.

A fraction in its simplest form consists of two numbers placed one over the other with a line between them.

The number under the line denotes the number of parts into which the unit is divided, and is called *The Denominator* of the fraction, *i.e.*, the "*name given*," because it gives the name to the parts.

The figure placed above the line is called *The Numerator*, *i.e.*, the "*numberer*," because it shows how many of the parts named by the denominator are to be taken.

Thus, if the unit be a yard, the fraction $\frac{3}{8}$ signifies that the yard is divided into 8 equal parts, and three of these parts are taken. It is read *three-eighths*. If the unit be divided into 2, 3, 4, 5, or 6 equal parts, the corresponding fractions with their names are as follows:—$\frac{1}{2}$, *one-half*; $\frac{1}{3}$, *one-third*; $\frac{1}{4}$, *one-fourth*; $\frac{1}{5}$, *one-fifth*; $\frac{1}{6}$, *one-sixth*.

VULGAR FRACTIONS.

78. In treating of the subject of Vulgar Fractions, it is usual to make the following distinctions:—

1. **A Proper Fraction** is one whose numerator is *less* than its denominator; thus, $\frac{3}{8}$, $\frac{9}{10}$, are proper fractions.

2. **An Improper Fraction** is one whose numerator is either *equal to*, or *greater than*, its denominator, as $\frac{7}{7}$, $\frac{11}{4}$, $\frac{10}{8}$, $\frac{101}{43}$.

3. **A Simple Fraction** has only one numerator, and one denominator, which are called the *terms* of the fraction, as $\frac{3}{4}$, $\frac{3}{5}$, $\frac{9}{8}$.

4. **A Mixed Number** consists of a whole number and a fraction; thus, $1\frac{1}{2}$, $6\frac{3}{8}$, and $15\frac{1}{6}$, which represent, respectively, 1 unit together with $\frac{1}{2}$ of a unit; 6 units together with $\frac{3}{8}$ths of a unit, and 15 units together with $\frac{1}{6}$th of a unit.

NOTE.—It is evident that $1\frac{1}{2}$, $6\frac{3}{8}$, and $15\frac{1}{6}$ is each only a shortened form of $1 + \frac{1}{2}$, $6 + \frac{3}{8}$, and $15 + \frac{1}{6}$, the sign + (*plus*) being understood.

5. **A Complex Fraction** is one which has either a fraction or a mixed number in one or both terms of the fraction; thus,—

$$\frac{\frac{3}{5}}{\frac{5}{7}} \quad \frac{2\frac{1}{2}}{3} \quad \frac{3}{4\frac{3}{5}} \quad \frac{2\frac{1}{7}}{5\frac{1}{6}} \quad \frac{\frac{3}{5} \text{ of } \frac{1}{2}}{2\frac{1}{6}} \quad \text{are complex fractions.}$$

6. **A Compound Fraction** is a fraction of a fraction, that is, two or more simple fractions connected by the word of; thus, $\frac{3}{5}$ of $\frac{3}{4}$, where $\frac{3}{4}$ is the quantity of which $\frac{3}{5}$ is to be taken, $2\frac{1}{2}$ of $\frac{5}{7}$ of $3\frac{3}{4}$.

79. It is clear, from what has been said, that every integer may be considered as a fraction whose denominator is 1; thus, $5 = \frac{5}{1}$, for the unit is divided into 1 part, comprising the whole unit, and 5 of such parts, that is, 5 units, are taken.

80. To show that $\frac{2}{3}$ of $1 = \frac{1}{3}$ of 2, or the third part of 2.

$\frac{2}{3}$ of 1 is 2 third parts of unity.

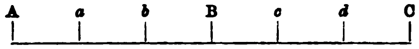
Now 1 is 3 third parts of unity.

Therefore, 2 is 6 third parts of unity.

Therefore, $\frac{1}{3}$ of 2 is $\frac{1}{3}$ of 6 third parts of unity, *i.e.*, is 2 third parts of unity.

But $\frac{2}{3}$ of $1 = \frac{1}{3}$ of 2, *i.e.*, $= 2 \div 3$.

Or, we may show that $\frac{2}{3}$ of $1 = \frac{1}{3}$ of 2 by the following illustration.



Let AB, BC each represent unity; then AC represents 2.

Divide AB, BC each into 3 equal parts in *a b*, *c d*. Then AC is divided into 3 equal parts in *b c*.

Again, since AC represents 2, A*b* represents $\frac{1}{3}$ of 2.

Hence, $\frac{2}{3} = \frac{1}{3}$ of 2.

Hence we may consider the fraction $\frac{2}{3}$ to represent either "unity divided into 3 equal parts, of which 2 parts are taken," or else "the integer 2 divided into 3 equal parts, of which one is taken."

81. The following definition of a fraction will now be easily understood.

A Fraction is a simple means of expressing the division of the numerator by the denominator, *e.g.*,—

The fraction $\frac{3}{7} =$ (1) three sevenths of one.

$=$ (2) one seventh of three, which is found by dividing 3 by 7, *i.e.*, the numerator by the denominator.

Hence we see that one of the definitions above involves the other.

82. From the notion attached to the words numerator and denominator, it is evident that—

1.—A fraction is *increased* by *increasing* the numerator.

2.—A fraction is *increased* by *decreasing* the denominator.

3.—A fraction is *diminished* by *diminishing* the numerator.

4.—A fraction is *decreased* by *increasing* the denominator.

For, by increasing or diminishing the numerator we take more or fewer of the parts of the unit; by diminishing the denominator the magnitude of the parts is increased, while the same number of parts are taken.

Also, if the denominator be increased, the magnitude of the parts is diminished and the fraction is diminished.

Thus the fraction—

$\frac{3}{5}$ is three times greater than $\frac{1}{5}$; $\frac{4}{7}$ is the double of $\frac{2}{7}$.

$\frac{1}{4}$ is three times greater than $\frac{1}{12}$; $\frac{3}{5}$ is the double of $\frac{3}{10}$.

$\frac{1}{5}$ is the third part of $\frac{3}{5}$; $\frac{5}{21}$ are the half of $\frac{10}{21}$.

$\frac{3}{5}$ is less than $\frac{4}{5}$; $\frac{4}{13}$ are less than $\frac{4}{5}$.

83. To express a whole number as a fraction with a given denominator.

RULE XXIII.

Multiply the number by the given denominator, and the result will be the numerator of the required fraction.

EXAMPLE.

Ex. Express 4 as a fraction with a denominator 7.

$$4 = \frac{4 \times 7}{7} = \frac{28}{7}.$$

For since 1 contains 7 *seventh* parts, 4 contains 4 × 7 or 28 *seventh* parts, that is, $4 = \frac{28}{7}$.

EXAMPLES FOR PRACTICE.

Reduce the following numbers to fractions having a common denominator.

1. 5; 6; 7; 8; 9; 10; 11; 12, to denominators 7 and 15.
2. 7 to denominators 11, 13, and 143; 909 to denominators 11 and 101.

84. To multiply any given fraction by any given integer.

EXAMPLE.

Ex. $\frac{5}{12} \times 3$.

Here we want to make the quotient, found by dividing 5 by 12, *three* times greater. Now, any quotient may be made *three* times greater in either of two ways, viz.:—

1st. By making the *dividend* or *numerator* (§ 82), three times *greater*; or,

2nd. By making the *divisor* or *denominator* (§ 82), three times *less*.

So that $\frac{5}{12}$ may be multiplied by 3, either—

1st. By keeping the same denominator and multiplying its numerator by 7.

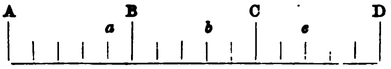
Thus, $\frac{5}{12} \times 3 = \frac{5 \times 3}{12} = \frac{15}{12} = \frac{5}{4}$, or

2nd. By keeping the same numerator, and dividing its denominator by 3.

Thus, $\frac{5}{12} \times 3 = \frac{5}{12 \div 3} = \frac{5}{4}$.

We may show how to multiply a fraction by a whole number by the following illustration.

Ex. $\frac{4}{5} \times 3 = \frac{12}{5}$.



Let equal lines AB, BC, CD, each represent unity; divide each of them into 5 equal parts. Then Aa represents $\frac{4}{5}$. Now $Aa = ab = bc$, because they each represent 4 of the smaller divisor; hence, AC represents 3 times Aa, or $\frac{4}{5} \times 3$. But Ac represents $\frac{12}{5}$ since it contains 12 smaller divisions.

Hence we deduce the following rule for multiplying any given fraction by any given integer.

RULE XXIV.

Multiply the numerator *of the fraction by the* integer, *or* divide *the* denominator *by it.*

EXAMPLES.

$\frac{3}{5} \times 9 \frac{3 \times 9}{5} = \frac{27}{5}$. $\frac{7}{15} \times 4 = \frac{7 \times 4}{15} \frac{28}{15}$.

$\frac{13}{27} \times 9 = \frac{13 \times 9}{27}$, or $\frac{13}{27 \div 9}$, that is, either $\frac{117}{27}$, or $\frac{13}{3}$.

$\frac{6}{35} \times 7 = \frac{6 \times 7}{35}$, or $\frac{6}{35 \div 7}$, that is, either $\frac{42}{35}$, or $\frac{6}{5}$.

85. It sometimes happens that when a fraction has been multiplied by a whole number, it can afterwards be reduced to lower terms; this will be the case when the denominator and the multiplier have some common factor; this factor may be rejected from both the multiplier and denominator *before* multiplication, instead of from the new numerator and denominator *after* multiplication.

Thus, $\frac{7}{12} \times 8 = \frac{56}{12} = \frac{14}{3}$ is better worked thus, $\frac{7}{\underset{3}{\cancel{12}}} \times \overset{2}{\cancel{8}} = \frac{7}{3} \times 2 = \frac{14}{3}$, striking out, or dividing by, the factor 4, which is common to 8 and 12 *before* multiplication, instead of from 56 and 12 *after* multiplication. Here the 12 and 8 are divided by 4, leaving only 2 in the multiplier and 3 in the denominator.

Again, $\frac{4}{77} \times 14$, or $7 \times 2 = \frac{4}{\underset{11}{\cancel{77}}} \times \overset{2}{\cancel{14}} = \frac{4}{11} \times 2 = \frac{8}{11}$, striking out or dividing by the factor 7, which is common to 77 and 14. 7 goes in 14, 2 times, write 2 above the 14; 7 goes in 77, 11 times, write 11 under 77.

$\frac{7}{15} \times 5 = \frac{7}{\underset{3}{\cancel{15}}} \times \cancel{5} = \frac{7}{3}$; $\frac{4}{\underset{3}{\cancel{21}}} \times \overset{4}{\cancel{28}} = \frac{16}{3}$; $\frac{12}{\underset{3}{\cancel{39}}} \times \overset{2}{\cancel{26}} = \frac{24}{3}$; $\frac{108}{\underset{7}{\cancel{35}}} \times \overset{4}{\cancel{20}} = \frac{432}{7}$;

$\frac{55}{\underset{7}{\cancel{63}}} \times \overset{5}{\cancel{45}} = \frac{275}{7}$; and $\frac{25}{\cancel{48}} \times \overset{2}{\cancel{96}} = 50$.

EXAMPLES FOR PRACTICE.

Find the value of the following fractions.

1. $\frac{4}{9} \times 3$; $\frac{17}{24} \times 7$; $\frac{41}{63} \times 9$; $\frac{4}{21} \times 28$; $\frac{13}{39} \times 26$; $\frac{29}{840} \times 231$; $\frac{5}{378} \times 42$; $\frac{13}{77} \times 1001$; $\frac{137}{10001} \times 246$; [illegible] $\times 326$; [illegible] $\times 8059$; $\frac{1001}{10001} \times 137 \times 365$; $\frac{10001}{1002001} \times 7 \times 7 \times 11 \times 11 \times 13 \times 13 \times 17 \times 17$.*

86. To divide any given fraction by any given integer.

EXAMPLE.

Ex. $\frac{14}{16} \div 7$.

Here we want to make the quotient, found by dividing 14 by 16, *seven times* less. Now, any quotient may be made seven times less in either of two ways, viz.:—

1st. By making the *dividend* seven times *smaller*; or,

2nd. By making the *divisor* seven times *greater*.

So that $\frac{14}{16}$ may be divided by 7, either—

1st. By keeping the same denominator, and dividing its numerator by 7.

Thus, $\frac{14}{16} \div 7 = \frac{14 \div 7}{16} = \frac{2}{16}$; or,

---

* Operations containing fractions with very high numbers are of little practical value, decimal fractions being preferred. But as exercises of arithmetical accuracy we shall give among the rest, a few cases of high numbers.

2nd. By keeping the same numerator, and multiplying its denominator by 7.

Thus, $\frac{14}{16} = \frac{14}{16 \times 7} = \frac{14}{112}$.

Hence we deduce the following rule for dividing any given fraction by any given integer.

RULE XXV.

Divide the numerator *of the fraction by the given* integer, *or* multiply *the* denominator *by it.*

EXAMPLES.

Ex. 1. Divide $\frac{12}{5}$ by 3.

$\frac{12}{5} \div 3 = \frac{12 \div 3}{5} = \frac{4}{5}$ (by keeping the same denominator and dividing the numerator).

Ex. 2. Divide $\frac{4}{5}$ by 3.

$\frac{4}{5} \div 3 = \frac{4}{5 \times 3} = \frac{4}{15}$ (dividing the numerator is impossible, therefore, multiply the denominator).

86. In the same manner as in multiplying a fraction by a whole number, if there is a factor common to the numerator and the divisor, this factor may be struck out before the denominator is multiplied.

Thus, $\frac{4}{5} \div 6 = \frac{4}{30} = \frac{2}{15}$ is better worked thus, $\frac{4}{5} \div 6 = \frac{\overset{2}{\cancel{4}}}{5} \div \overset{3}{\cancel{6}} = \frac{2}{5} \div 3 = \frac{2}{15}$.

Dividing numerator 4 and divisor 6 by the common factor 2.

$\frac{4}{21} \div 28 = \frac{1}{147}$; $\frac{12}{35} \div 8 = \frac{3}{70}$; $\frac{108}{35} \div 24 = \frac{9}{70}$; $\frac{55}{63} \div 110 = \frac{1}{126}$; $\frac{25}{48} \div 300 = \frac{1}{576}$.

EXAMPLES FOR PRACTICE.

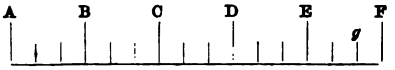
Find the value of the following fractions.

1. $\frac{3}{4} \div 6$; $\frac{5}{8} \div 5$; $\frac{4}{5} \div 10$; $\frac{24}{5} \div 8$; $\frac{24}{5} \div 16$; $\frac{45}{63} \div 18$.
2. $\frac{12}{13} \div 3$; $\frac{14}{21} \div 7$; $\frac{5}{11} \div 11$; $\frac{741}{17} \div 969$; $\frac{21}{34} \div 840$; $\frac{13}{17} \div 1001$.
3. Divide $\frac{107}{10301}$ by 146; $\frac{10001}{137}$ by 146; $\frac{11}{13}$ by 10001; $\frac{1001}{13}$ by 77; $\frac{4}{21}$ by 28; $\frac{12}{35}$ by 8.
4. Divide $\frac{108}{35}$ by 24; $\frac{55}{63}$ by 110; $\frac{25}{48}$ by 300; $\frac{2082778}{1001}$ by 5117; $\frac{10452}{11353}$ by 804.
5. $\frac{7 \times 7 \times 11 \times 13 \times 13 \times 17 \times 17}{1002001} \div 1001$; $\frac{88402}{308715} \div 413$.

87. To represent an improper fraction as a whole or a mixed number.

EXAMPLE.

Ex. $\frac{14}{3} = 4\frac{2}{3}$.



Let equal lines AB, BC, CD, DE, EF, each represent unity. Divide them each into 3 equal parts; then Ag contains 14 such parts, that is, Ag = $\frac{14}{3}$.

But AE is 4 units, and Eg is $\frac{2}{3}$, ∴ Ag = $4\frac{2}{3}$, that is, $\frac{14}{3} = 4\frac{2}{3}$.

Hence the rule—

RULE XXVI.

Divide the numerator *by the* denominator; *the* quotient *will be a whole number, and the remainder (if any) will be the numerator of the fractional part of the mixed number, and the divisor its denominator. If there be no remainder the improper fraction is equal to the whole number thus found.*

EXAMPLES.

Ex. 1. $\frac{63}{9} = 7$.

Ex. 2. $\frac{35}{35} = 1$.

Ex. 3. $\frac{116}{15}$)116($7\frac{11}{15}$

105

11

15

Ex. 4. $\frac{11975}{357}$)11975($33\frac{194}{357}$

1071

1265

1071

194

357

N.B.—All improper fractions which occur in any answer should (except the contrary be desired) be expressed as a whole or mixed number.

EXAMPLES FOR PRACTICE.

Find the mixed quantities equivalent to—

1. $\frac{7}{3}$, $\frac{19}{6}$, $\frac{88}{9}$, $\frac{149}{11}$, $\frac{199}{13}$, $\frac{1002}{7}$, $\frac{69}{17}$, $\frac{68}{15}$, and $\frac{403}{132}$.
2. $\frac{440}{13}$, $\frac{1001}{77}$, $\frac{247}{19}$, $\frac{8787}{29}$, $\frac{20471}{37}$, $\frac{7168}{1024}$, and $\frac{87402}{181}$.
3. $\frac{744812}{8088}$, $\frac{26586822}{7480237}$, $\frac{87748621}{1483}$, $\frac{18524257}{7948263}$, $\frac{101919}{8871}$, and $\frac{4220856}{1862}$.

88. To reduce a mixed number to its equivalent improper fraction.

A quantity made up of two others, one of which is an integer and the other a fraction, may be represented in the form of a fraction alone.

Let us take $3\frac{4}{5}$, which is called a mixed quantity and is intended to express the integer 3 and the fraction $\frac{4}{5}$, taken together, or $3 + \frac{4}{5}$, and is read *three* and *four-fifths*.

Now, $1 = \frac{5}{5}$

$\therefore 3 = \frac{3 \times 5}{5} = \frac{15}{5}$.

$\therefore 3 + \frac{4}{5} = \frac{3 \times 5 + 4}{5} = \frac{19}{5}$.

Hence the following rule:—

RULE XXVII.

Multiply the integer by the denominator of the fraction, and to the product add the numerator, this sum placed over the denominator will form the improper fraction required.

EXAMPLE.

Ex. Represent $121\frac{5}{11}$ and $344\frac{24}{29}$ as improper fractions.

$121\frac{5}{11} = \frac{121 \times 11 + 5}{11}$

$= \frac{1331 + 5}{11} = \frac{1336}{11}$.

or, $\frac{1336}{11}$ and $\frac{10000}{29}$ are the fractions required.

$344\frac{24}{29}$

29

3096

688

24

10000

29

Examples for Practice.

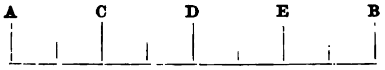
Reduce the following mixed numbers to improper fractions.

1. $2\frac{5}{7}$; $5\frac{4}{9}$; $2\frac{11}{31}$; $18\frac{7}{48}$; $32\frac{11}{15}$; $69\frac{61}{71}$; $115\frac{13}{16}$; $143\frac{6}{7}$.
2. $993\frac{443}{720}$; $100\frac{99}{100}$; $333\frac{1}{3}$; $186\frac{228}{1001}$; $89\frac{770}{1441}$; $99\frac{99}{10011}$.
3. $111\frac{1}{9}$; $1001\frac{999}{1001}$; $10101\frac{1}{99}$; $111111\frac{1}{9}$; $1638\frac{201}{54393}$.
4. $1\frac{1008}{1409}$; $218\frac{109}{60000}$; $4\frac{18158}{50728}$; $2\frac{649}{2846}$; $25\frac{471}{5265}$.

89. **The value of a fraction is not altered by multiplying or dividing both its numerator and its denominator by the same quantity.**

Example.

Ex. $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$.



Let the length AB represent unity, or one yard. Divide AB into 4 equal parts in CDE. Then AE represents $\frac{3}{4}$ of a yard. Again, divide each of these fourth parts AC, CD, DE, EB, into 2 equal parts; the whole yard AB is thus divided into 8 equal parts, of which AE would contain 6, therefore, AE is $\frac{6}{8}$ of a yard; hence, $\frac{3}{4}$ and $\frac{6}{8}$ are the same thing; the line being divided into parts *twice as small* in the latter as in the former case, but *twice as many* of those smaller parts being taken.

Conversely, sine $\frac{6}{8} = \frac{3}{4}$, we see that if the numerator and denominator of a fraction are divided by the same number, the value of the fraction is not altered, since by dividing the numerator of a fraction we *decrease* its value, but by dividing its denominator we *increase* its value in the same degree that we before decreased it.

90. **We may hence reduce any fraction to lower terms by dividing both numerator and denominator by any common factor; and a fraction is said to be reduced to its *lowest terms* when the numerator and denominator, being both divided by their greatest common measure, are prime to one another.**

The following rule for reducing a fraction to its lowest terms will now be self-evident.

RULE XXVIII.

Find the* G.C.M. *of the numerator and denominator of the fraction, divide each of them by it, and set down the quotient of the numerator as a new numerator, and the quotient of the denominator as a new denominator.

Examples.

Ex. 1. Reduce the fraction $\frac{891}{3429}$ to its lowest terms.

```
891)3429(4
    3564
    ----
     135)891(7
         945
         ---
          54)135(2
             108
             ---
              27)54(2
                 54
                 --
```

$$\frac{891}{3429} = \frac{891 \div 27}{3429 \div 27} = \frac{33}{127}$$

∴ G.C.M. is 27.

Here 27 is the G.C.M. required, and by dividing both numerator and denominator by this quantity, the fraction $\frac{891}{3429}$ is reduced to $\frac{33}{127}$; where 33 and 127 being prime to one another, the fraction is in its lowest terms.

If the given fraction be an improper one, first reduce it to a mixed number, and then reduce the remainder to its lowest terms, *e.g.*, to bring $\frac{3471}{195}$ to its lowest terms.

$$\begin{array}{r} 195)3471(17\frac{156}{195} \\ \underline{195} \\ 1521 \\ \underline{1365} \\ 156 \end{array}$$

And finding the G.C.M. of 156 and 195, viz. 39, the given fraction in its lowest terms becomes $17\frac{4}{5}$.

Ex. 2. Reduce the fraction $\frac{1847}{8209}$ to its lowest terms.

The greatest common measure of 1847 and 8209 is 1.

∴ the fraction $\frac{1847}{8209}$ is already in its *lowest terms.*

91. In many instances it is unnecessary to find the greatest common measure, the fractions being reducible by successive divisions of the numerators and denominators by common factors discovered by inspection.

Thus, $\frac{4356}{9504} = \frac{1089}{2376} = \frac{363}{792} = \frac{121}{264} = \frac{11}{24}$ by dividing by 4, 3, 3, and 11;

and $\frac{14652}{15048} = \frac{3663}{3762} = \frac{407}{418} = \frac{37}{38}$ by dividing by 4, 9, and 11;

also, $\frac{1760}{2200} = \frac{176}{220} = \frac{44}{55} = \frac{4}{5}$ by dividing by 10, 4, and 11.

Sometimes it is not possible to see any number that will divide both numerator and denominator. In this case recourse must be had to the method of finding the G.C.M.

EXAMPLES FOR PRACTICE.

Reduce to their lowest terms—

1. $\frac{4}{6}$, $\frac{10}{15}$, $\frac{24}{32}$, $\frac{45}{60}$, $\frac{108}{120}$, $\frac{15}{30}$, $\frac{28}{35}$, $\frac{144}{160}$, $\frac{12}{36}$, $\frac{63}{144}$.
2. $\frac{34}{51}$, $\frac{143}{165}$, $\frac{147}{189}$, $\frac{435}{567}$, $\frac{555}{999}$, $\frac{308}{350}$, $\frac{354}{7650}$, $\frac{689}{901}$, $\frac{969}{8721}$, $\frac{629}{943}$.
3. $\frac{120}{840}$, $\frac{336}{840}$, $\frac{600}{2520}$, $\frac{2040}{2352}$, $\frac{540}{1680}$, $\frac{3780}{27720}$, $\frac{2310}{6300}$, $\frac{138978}{360808}$.
4. $\frac{11050}{35581}$, $\frac{714285}{999999}$, $\frac{109375}{10000000}$, $\frac{185795}{222210}$, $\frac{314175}{100005}$, $\frac{150822}{190551}$, $\frac{112788}{185408}$, $\frac{2521777}{3828601}$.
5. $\frac{48249049}{48530781}$, $\frac{1761161}{17479683}$, $\frac{860149}{2006816}$, $\frac{2442431}{1882009}$, $\frac{1718446}{4430618}$, $\frac{16409021}{17221847}$, $\frac{1269671687}{7281427811}$.
6. $\frac{42614674094432}{149720237927424}$, $\frac{7241379310344827586206896551}{9999999999999999999999999999}$.

92. To bring fractions to equivalent ones having a common denominator.

Let it be required to reduce $\frac{5}{8}$, $\frac{4}{9}$, $\frac{5}{12}$ to equivalent fractions having a common denominator.

It is evident that no simple fraction whatever can be equivalent to $\frac{5}{8}$ unless its denominator be some multiple of 8, because the fraction being already in its lowest terms, we cannot *divide* its numerator and denominator by any common measure, and the only other way to obtain an equivalent fraction is to multiply both the numerator and denominator by some number, thereby obtaining a denominator which will be a multiple of 8.

In the same way we may show that no simple fraction equivalent to $\frac{4}{9}$ can be found which has not a multiple of 9 for its denominator, nor any equivalent to $\frac{5}{12}$ not having a multiple of 12 for its denominator.

We, therefore, conclude that any common denominator to the fractions $\frac{5}{8}$, $\frac{4}{9}$, $\frac{5}{12}$, must be a common multiple of their denominators, *i.e.*, must be exactly divisible by 8, 9, and 12.

The least common denominator will, therefore, be the L.C.M. of 8, 9, and 12, (§ 76) which is 72, and we take this for the common denominator of the fractions.

Taking the fractions in order, it is evidently necessary to multipiy the denominator 8 of $\frac{3}{8}$ by 9, to make it 72; and to keep the value of the fraction unaltered, we multiply 3, its numerator, also by 9,

$$\left.\begin{array}{ll} \text{and we then have} & \frac{3}{8} = \frac{3 \times 9}{8 \times 9} = \frac{27}{72} \\ \text{similarly} & \frac{4}{9} = \frac{4 \times 8}{9 \times 8} = \frac{32}{72} \\ \text{and} & \frac{5}{12} = \frac{5 \times 6}{12 \times 6} = \frac{30}{72} \end{array}\right\} \text{All these fractions have the same denominator, viz., 72.}$$

From what has now been said the following rule will be self-evident.

RULE XXIX.

Find the least common multiple **(L.C.M.)** ***of all the denominators of the given fractions; this will be the common denominator. Divide this*** **L.C.M.** ***separately by the denominators of each of the given fractions, and multiply each successive quotient by the numerator which corresponds to the denominator used to find that quotient.***

Note.—If the given fractions be in their *lowest* terms, the above rule will reduce them to others having the *least* common denominators; if the *least* common denominator be required, the given fractions should be reduced to their lowest terms before the rule is applied.

Examples.

Ex. 1. Bring $\frac{4}{5}, \frac{5}{6}, \frac{9}{10}, \frac{11}{12}, \frac{19}{20}$ to equivalent fractions having the least common denominator.

$$\begin{array}{r|ccccc} 12 & \cancel{5}, & \cancel{6}, & \cancel{10}, & 12, & 20 \\ \hline & & & & 1, & 5 \end{array}$$

$$\therefore \text{L.C.M.} = 1 \times 5 \times 12 = 60.$$

Divide the L.C.M. by all the denominators in succession.

$$\begin{array}{ll} 60 \div 5 = 12 & 60 \div 12 = 5 \\ 60 \div 6 = 10 & 60 \div 20 = 3 \\ 60 \div 10 = 6 & \end{array}$$

Now look to the original fractions; multiply every number by the result of its denominator in the preceding list, and we shall then have the numerators of the fractions required.

$$\begin{array}{ll} 4 \times 12 = 48 \therefore \frac{4}{5} = \frac{48}{60} & 11 \times 5 = 55 \therefore \frac{11}{12} = \frac{55}{60} \\ 5 \times 10 = 50 \therefore \frac{5}{6} = \frac{50}{60} & 19 \times 3 = 57 \therefore \frac{19}{20} = \frac{57}{60} \\ 9 \times 6 = 54 \therefore \frac{9}{10} = \frac{54}{60} & \end{array}$$

Ex. 2. Reduce $\frac{5}{8}$, $\frac{11}{12}$, and $\frac{7}{18}$ to their least common denominator.

The L.C.M. of 8, 2, 18 being 72, we have

$$\frac{5}{8} = \frac{5 \times 9}{8 \times 9} = \frac{45}{72}; \quad \frac{11}{12} = \frac{11 \times 6}{12 \times 6} = \frac{66}{72}; \quad \frac{7}{18} = \frac{7 \times 4}{18 \times 4} = \frac{28}{72}.$$

Here the factors 9, 6, 4 of the new numerators are obtained by dividing the common denominator (= L.C.M.) by the original denominators 8, 12, 18 respectively.

93. If the denominators have no common measure the fractions may still be transformed into other equivalent fractions having a common denominator.

Let it be required to make $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{7}$, to have a common denominator; then, since the continued product of the denominators is expressed by $2 \times 5 \times 7$, we have—

$$\frac{1}{2} = \frac{1 \times 5 \times 7}{2 \times 5 \times 7} = \frac{35}{70},$$

$$\frac{2}{5} = \frac{2 \times 2 \times 7}{5 \times 2 \times 7} = \frac{28}{70},$$

$$\frac{3}{7} = \frac{3 \times 2 \times 5}{7 \times 2 \times 5} = \frac{30}{70}.$$

So that $\frac{35}{70}$, $\frac{28}{70}$, and $\frac{30}{70}$, are the new equivalent fractions with the common denominator 70.

And hence we have the following rule.

RULE XXX.

Multiply each numerator by all the denominators **except** ***the one placed under it, for the new numerator, and multiply all the denominators together for a common denominator.***

EXAMPLE.

Ex. 1. Reduce $\frac{1}{5}$, $\frac{2}{7}$, and $\frac{1}{9}$, to equivalent fractions with a common denominator.

Then, since the continued product of the denominators is expressed by 2 × 5 × 7, we have—

$$\frac{1}{5} = \frac{1 \times 7 \times 9}{5 \times 7 \times 9} = \frac{63}{315},$$

$$\frac{2}{7} = \frac{2 \times 5 \times 9}{7 \times 5 \times 9} = \frac{90}{315},$$

$$\frac{1}{9} = \frac{1 \times 5 \times 7}{9 \times 5 \times 7} = \frac{35}{315},$$

Or the fractions with a common denominator are—

$$\frac{63}{315},\ \frac{90}{315},\ \text{and}\ \frac{35}{315}.$$

*EXAMPLES FOR PRACTICE.

Reduce to the least common denominator—

1. $\frac{1}{3}$, $\frac{2}{9}$, $\frac{5}{12}$, and $\frac{7}{18}$; $\frac{3}{7}$, $\frac{9}{14}$, $\frac{17}{21}$, and $\frac{25}{28}$; $\frac{5}{6}$, $\frac{4}{7}$, $\frac{4}{9}$, and $\frac{2}{11}$; $\frac{15}{16}$, $\frac{17}{18}$, and $\frac{19}{20}$.
2. $\frac{33}{40}$, $\frac{8}{63}$, $\frac{9}{28}$, and $\frac{19}{81}$; $\frac{13}{15}$, $\frac{17}{20}$, $\frac{99}{100}$, and $\frac{17}{18}$.
3. $\frac{7}{16}$, $\frac{11}{18}$, $\frac{17}{24}$, $\frac{19}{36}$, and $\frac{25}{48}$; $\frac{4}{7}$, $\frac{3}{10}$, $\frac{5}{12}$, $\frac{17}{35}$, $\frac{4}{63}$, and $\frac{15}{28}$.
4. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{19}{27}$, $\frac{8}{81}$, and $\frac{16}{243}$; $\frac{3}{5}$, $\frac{7}{10}$, $\frac{6}{25}$, $\frac{11}{30}$, $\frac{13}{45}$, and $\frac{23}{60}$; $\frac{131}{230}$, $\frac{16}{184}$, $\frac{111}{276}$.
5. $\frac{4}{7}$, $\frac{11}{12}$, $\frac{2}{15}$, $\frac{2}{27}$, $\frac{8}{35}$, and $\frac{17}{40}$; $\frac{1}{3}$, $\frac{4}{18}$, $\frac{14}{27}$, $\frac{6}{81}$, $\frac{12}{162}$, and $\frac{16}{243}$.
6. $\frac{2}{15}$, $\frac{3}{16}$, $\frac{4}{18}$, $\frac{6}{20}$, $\frac{7}{24}$, $\frac{8}{25}$, $\frac{10}{27}$, $\frac{11}{30}$, $\frac{12}{45}$, $\frac{13}{48}$, $\frac{14}{54}$, and $\frac{16}{90}$.
7. $\frac{13}{91}$, $\frac{14}{63}$, $\frac{101}{143}$, $\frac{38}{55}$, $\frac{16}{35}$, $\frac{115}{245}$, and $\frac{49}{77}$; $\frac{19}{34}$, $\frac{20}{119}$, $\frac{67}{105}$, $\frac{17}{30}$, $\frac{181}{357}$, and $\frac{199}{210}$.

Express with a common denominator—

8. $\frac{3}{5}$, $\frac{3}{7}$, $\frac{4}{11}$; $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{13}$; $1\frac{1}{3}$, $2\frac{1}{5}$, $3\frac{1}{7}$; 7, $\frac{5}{8}$, $10\frac{10}{11}$, and $26\frac{1}{7}$.

94. To reduce a compound fraction to a simple one.

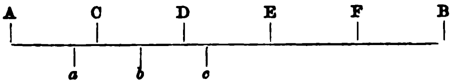
EXAMPLE.

Ex. 1. The expression $\frac{3}{4}$ of $\frac{3}{5}$ is a compound fraction, and it is required to reduce it to a simple one.

For, *one-fourth* of $\frac{3}{5}$ is $\frac{3}{20}$ (§ 86), therefore, *three-fourths*, which must be three times as great, is $\frac{9}{20}$ (§ 85).

$$\tfrac{3}{4} \text{ of } \tfrac{3}{5} = (1)\ 3 \text{ times } (\tfrac{1}{4} \text{ of } \tfrac{3}{5}) = 3 \text{ times } (\tfrac{3}{5} \div 4) = 3 \text{ times } \frac{3}{5 \times 4}$$

$$\therefore \tfrac{3}{4} \text{ of } \tfrac{3}{5} = (2)\ \frac{3}{5 \times 4} \times 3 = \frac{3 \times 3}{5 \times 4} = \frac{9}{20} = \frac{\text{the product of the numerators.}}{\text{the product of the denominators.}}$$



Let AB represent unity, and divide it into 5 equal parts in CDEF. Then AE represents $\frac{3}{5}$. Divide AE into 4 equal parts in *abc*; then A*c* is $\frac{3}{4}$ of AE, that is, A*c* is $\frac{3}{4}$ of $\frac{3}{5}$. Now if we divide each of the parts AC, CD, &c., into 4 equal parts, then AB will be divided into 20 equal parts, and AE into 12 parts of which A*c* contains 9. Hence A*c* = $\frac{9}{20}$,

$$\text{That is, } \tfrac{3}{4} \text{ of } \tfrac{3}{5} = \tfrac{9}{20} = \frac{3 \times 3}{4 \times 5}.$$

Hence, to reduce a compound fraction to a simple one we have the following rule:—

RULE XXXI.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

EXAMPLES.

Ex. 1. Reduce $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ to a simple fraction.

$\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7} = \frac{2}{3}$ of $\frac{24}{35} = \frac{48}{105}$.

Mixed numbers must be reduced to improper fractions before the rule can be applied.

Ex. 2. $2\frac{3}{4}$ of 5 of $3\frac{1}{2} = \frac{11}{4}$ of $\frac{5}{1}$ of $\frac{7}{2} = \frac{385}{8} = 48\frac{1}{8}$.

In reducing compound fractions to simple ones, we may strike out factors common to one of the numerators and one of the denominators; for this is in fact simply dividing the numerator and denominator of the fraction by the same number (§ 89).

Ex. 3. $\frac{3}{5}$ of $6\frac{1}{3}$ of $\frac{7}{8}$ of $\frac{16}{19} = \frac{\overset{1}{\cancel{3}}}{5} \times \frac{\overset{1}{\cancel{19}}}{\underset{1}{\cancel{3}}} \times \frac{7}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{1}{\cancel{19}}}$

The working is here considerably shortened by cancelling, thus:—

1. Reduce to unity the 3 and the 19 in the line of numerators, and do the same to the 3 and the 19 in the line of denominators. The value of the answer will not be altered because we have divided the numerator and denominator thereof by the same number—viz., 3 × 19, or 57 (§ 89).

2. Divide the 16 in the line of the numerators, and the 8 in the line of the denominators by 8, and put 2 above the 16, because 16 ÷ 8 = 2, and 1 beneath the 8, because 8 ÷ 8 = 1.

3. Now, applying the rule we get

$$\left.\begin{array}{l}\text{The product of the numerators} = 1 \times 1 \times 7 \times 2 = 14 \\ \text{The product of the denominators} = 5 \times 1 \times 1 \times 1 = 5\end{array}\right\} = \textit{Ans.} = \frac{14}{5} = 2\frac{4}{5}.$$

N.B.—It is not usual to put down the *ones* when cancelling.

Ex. 4. $\frac{\cancel{4}}{\cancel{7}}$ of $\frac{\overset{\cancel{2}}{\cancel{14}}}{\cancel{23}}$ of $\frac{\overset{3}{\cancel{69}}}{\underset{\underset{\cancel{2}}{\cancel{8}}}{\cancel{80}}}$ of $\frac{\cancel{10}}{11} = \frac{3}{11}$ *Ans.*

Here the 14 and 7 are divided by 7; the 23 and 69 by 23; the 10 and 80 by 10; the 4 and 8 by 4; the 2 and 2 by 2; leaving only 3 in the numerator and 11 in the denominator. This operation is called *cancelling.*

EXAMPLES FOR PRACTICE.

Simplify the following compound fractions:—

1. $\frac{1}{2}$ of $\frac{2}{3}$; $\frac{3}{5}$ of $\frac{3}{4}$; $\frac{4}{15}$ of $\frac{21}{32}$; $\frac{25}{32}$ of $\frac{24}{25}$; $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$; $2\frac{1}{3}$ of $16\frac{1}{2}$.
2. $\frac{11}{12}$ of $\frac{12}{13}$ of $\frac{21}{22}$; $\frac{5}{9}$ of $\frac{3}{7}$ of $\frac{7}{15}$; $\frac{4}{17}$ of $\frac{13}{8}$ of $\frac{17}{13}$ of 2; $\frac{4}{7}$ of $4\frac{2}{3}$ of $3\frac{1}{2}$.
3. $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{2}{3}$ of 6; $\frac{3}{5}$ of $\frac{3}{7}$ of $1\frac{3}{7}$ of $5\frac{5}{6}$ of $2\frac{1}{4}$; $\frac{17}{18}$ of $1\frac{2}{17}$ of $4\frac{3}{4}$ of $1\frac{6}{19}$.
4. $\frac{12}{105}$ of $\frac{35}{36}$ of $\frac{160}{209}$ of $\frac{4}{7}$; $\frac{16}{17}$ of $\frac{85}{123}$ of $2\frac{1}{8}$ of $\frac{1}{8}$ of 19; $\frac{18}{19}$ of $\frac{102}{112}$ of $5\frac{1}{7}$ of $81\frac{1}{4}$.

95. To reduce a complex fraction to a simple one.

EXAMPLES.

Ex. 1. $\frac{3\frac{1}{8}}{4} = \frac{\frac{25}{8}}{\frac{4}{1}} = \frac{25}{8 \times 4} = \frac{25}{32}$.

Ex. 2. $\frac{4\frac{1}{4}}{6\frac{1}{3}} = \frac{\frac{17}{4}}{\frac{17}{3}}$

Here the numerator and denominator of the complex fraction being expressed as mixed numbers, they are reduced to improper fractions, and thus we have numerator $4\frac{1}{4} = \frac{17}{4}$, and denominator $6\frac{1}{3} = \frac{19}{3}$; and if we divide $\frac{17}{4}$ by 19 we shall evidently divide by a number which is 3 times too great, and the answer will consequently be 3 times too small, and will therefore require to be multiplied by 3 to bring it to its required value.

Now $\frac{17}{4}$ can be divided by 19 by simply multiplying its denominator by 19, thus—

$$\frac{17}{4} \div 19 = \frac{17}{4 \times 19}$$

and $\frac{17}{4 \times 19}$ can be multiplied by 3 by simply multiplying its numerator by 3, thus—

$$\frac{17}{4 \times 19} \times 3 = \frac{17 \times 3}{4 \times 19} = \frac{51}{76}.$$

A consideration of these two examples enables us to lay down the following rule for reducing a complex fraction to a simple one.

RULE XXXII.

Express its numerator and denominator as fractions, keeping the denominator below the numerator; then multiply together the extremes (i.e., the top and bottom numbers) for the numerator of the answer, and the means (i.e., the middle numbers) for its denominator.

EXAMPLES.

Ex. 1. $\frac{5\frac{7}{9}}{9\frac{5}{11}} = \frac{\frac{52}{9}}{\frac{104}{11}} = \frac{52 \times 11}{9 \times 104} = \frac{572}{936}.$

The process of cancelling is thus applied in such cases,—

Since the answer $= \frac{\text{the product of the extremes,}}{\text{the product of the means,}}$ it follows that if either of the extremes is divisible by the same number as will divide one of the means, we may cancel by this common measure, thus,—

$$\frac{5\frac{7}{9}}{9\frac{5}{11}} = \frac{\frac{52}{9}}{\frac{104}{11}} = \frac{\cancel{52} \times 11}{\underset{2}{\cancel{104}} \times 9} = \frac{11}{18}$$

Ex. 2. Simplify the complex fractions $\frac{45}{4\frac{1}{2}}$

$$\frac{45}{4\frac{1}{2}} = \frac{\frac{45}{1}}{\frac{9}{2}} = \frac{\overset{5}{\cancel{45}} \times 2}{1 \times \cancel{9}} = 10$$

EXAMPLES FOR PRACTICE.

What are the simple fractions equal to the complex fractions—

1. $\frac{4\frac{3}{4}}{5\frac{3}{8}}$; $\frac{8\frac{4}{11}}{14\frac{6}{7}}$; $\frac{9\frac{7}{8}}{12\frac{4}{5}}$; $\frac{25\frac{7}{8}}{34\frac{11}{16}}$; $\frac{8}{3\frac{1}{4}}$; $\frac{5\frac{1}{2}}{7}$; $\frac{7\frac{5}{8}}{9\frac{7}{8}}$ of $\frac{14\frac{4}{11}}{13\frac{3}{7}}$; $\frac{6\frac{3}{8}}{10\frac{4}{7}}$ of $\frac{15}{12\frac{4}{7}}$ of $\frac{8\frac{1}{2}}{42}$; $\frac{\frac{3}{8}\text{ of }5}{\frac{5}{8}\text{ of }1\frac{3}{8}}$

96. To compare the value of different fractions, *i.e.*, to find out which is the greatest and which is the least.

RULE XXXIII.

Bring the given fractions to others of the same value, having a common denominator; then the respective values of the fractions will depend upon their numerators, that fraction being* greatest *which has the* greatest *numerator.

EXAMPLES.

Ex. 1. Then to find which is the greater, $\frac{6}{7}$ or $\frac{7}{8}$.

$$\frac{6}{7} = \frac{6 \times 8}{7 \times 8} = \frac{48}{56}$$

$$\frac{7}{8} = \frac{7 \times 7}{8 \times 7} = \frac{49}{56}$$

And sine $\frac{49}{56}$ is greater than $\frac{48}{56}$ it follows that $\frac{7}{8}$ is greater than $\frac{6}{7}$.

Ex. 2. Compare the fractions $\frac{11}{12}$, $\frac{15}{16}$, $\frac{23}{24}$, $\frac{31}{32}$.

$$\frac{11}{12} = \frac{11 \times 8}{12 \times 8} = \frac{88}{96}$$

$$\frac{15}{16} = \frac{15 \times 6}{16 \times 6} = \frac{90}{96}$$

$$\frac{23}{24} = \frac{23 \times 4}{24 \times 4} = \frac{92}{96}$$

$$\frac{31}{32} = \frac{31 \times 3}{32 \times 3} = \frac{93}{96}$$

Hence, the values of the given fractions are in the following order:—$\frac{88}{96}$, $\frac{90}{96}$, $\frac{92}{96}$, and $\frac{93}{96}$.

When fractions have a *common numerator* the *greater has the less* denominator. Thus—

Ex. $\frac{10}{7}$ is greater than $\frac{10}{8}$; $\frac{21}{15}$ greater than $\frac{21}{16}$.

97. It is a general rule that if we add the numerators of a number of fractions, and also the denominators the fraction given by $\frac{\text{sum of numerators}}{\text{sum of denominators}}$ lies between the greatest and least of the original fractions.

EXAMPLES.

Ex. 1. Show that the fraction $\frac{5+6}{6+7}$ is greater than $\frac{5}{6}$ and less than $\frac{6}{7}$.

$$\frac{5+6}{6+7} = \frac{11}{13}.$$

Hence, by comparing $\frac{5}{6}$ and $\frac{11}{13}$, we get—

$$\frac{5}{6} = \frac{5 \times 13}{6 \times 13} = \frac{65}{78}.$$

$$\frac{11}{13} = \frac{11 \times 6}{13 \times 6} = \frac{66}{78}.$$

therefore $\frac{11}{13}$ is greater than $\frac{5}{6}$.

But, comparing $\frac{11}{13}$ and $\frac{6}{7}$, we have—

$$\frac{11}{13} = \frac{11 \times 7}{13 \times 7} = \frac{77}{91}.$$

$$\frac{6}{7} = \frac{6 \times 13}{7 \times 13} = \frac{78}{91}.$$

therefore $\frac{11}{13}$ is less than $\frac{6}{7}$.

Ex. 2. $\frac{3+5}{8+7}$ lies between $\frac{3}{8}$ and $\frac{5}{7}$, and also between $\frac{3}{7}$ and $\frac{5}{8}$.

98. *Proper fractions* are *increased* and *improper fractions* are *decreased* by adding the same quantity to both numerator and denominator.

We may observe, in general, that the *higher* a number is the *less* relatively to another number is its increase made by the addition of 1.

Thus, 2 is double of 1, but 3 *not* double of 2; still less is 4 double of 3, or 100 double of 99.

So that by adding an unit, or any number of units, to each of two numbers, the increase to the smaller will be more in proportion than the increase to the larger. Hence, if a fraction be *proper*, *i.e.*, if its numerator be less than its denominator, by adding the same quantity to both, the increase to the numerator will be more in proportion than the increase to the denominator, and the value of the fraction will be *increased*; while, conversely, if the fraction be *improper*, the increase to the denominator will be less than the increase to the numerator, and the value of the fraction will be diminished.

Now, let us take the proper fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, where each successive fraction is made by adding 1 to the numerator and denominator of the fraction preceding it.

Reducing these to equivalent fractions having the same common denominator, they are respectively equal to $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$; and of these fractions the first is the least and the last the greatest; we see that by adding the same quantity to both numerator and denominator of proper fractions, their value is continually increased.

But, if the improper fractions $\frac{2}{1}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$, be taken, these are respectively equal to $\frac{120}{60}$, $\frac{90}{60}$, $\frac{80}{60}$, $\frac{75}{60}$, $\frac{72}{60}$; and as of these fractions the first is the greatest and the last the least, we see that by adding the same quantity to both numerator and denominator of improper fractions, their value is continually diminished.

Whence we conclude that we cannot *add* the same quantity to the numerater and denominator of *any* fraction without thereby altering its value.

Conversely, proper fractions are diminished and improper fractions increased by *subtracting* the same quantity from both numerator and denominator; whence we conclude that we cannot *subtract* the same quantity from the numerator and denominator of *any* fraction, without thereby altering its value.

Obs.—It is of great importance to remember from this, in reducing fractions, that, although we may *divide* both numerator and denominator by the same quantity, we may *not* take away the same quantity from both numerator and denominator by *subtraction*.

EXAMPLES FOR PRACTICE.

1. Find which is the greater fraction in each of the following pairs:—

   1. $\frac{5}{7}$ and $\frac{8}{11}$. 2. $\frac{6}{11}$ and $\frac{13}{25}$. 3. $\frac{36}{77}$ and $\frac{39}{82}$ 4. $\frac{34}{63}$ and $\frac{49}{85}$.

2. Find the greatest and least of the fractions—

   $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{4 \times 5}{5 \times 6}$; also of $\frac{4}{3}$, $\frac{5}{4}$, $\frac{9}{8}$, and $\frac{16}{14}$.

3. Of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{9}{10}$, find the greatest and the least.

4. Which is the greatest and which the least, $\frac{2}{13}$, $\frac{7}{45}$, $\frac{3}{20}$.

5. Arrange the following fractions in order of magnitude—

   $\frac{19}{25}$; $\frac{3}{4}$; $\frac{16}{21}$; $\frac{13}{17}$; $\frac{10}{13}$; $\frac{7}{9}$; $\frac{4}{5}$; $\frac{22}{29}$; $\frac{25}{33}$; $\frac{28}{37}$.

6. In the fractions, $\frac{8}{25}$, $\frac{9}{28}$, $\frac{17}{53}$, $\frac{26}{81}$, $\frac{25}{78}$, and $\frac{28}{87}$, show that the $\frac{\text{sum of numerators}}{\text{sum of denominators}}$ lies between the greatest and least of the original fractions.

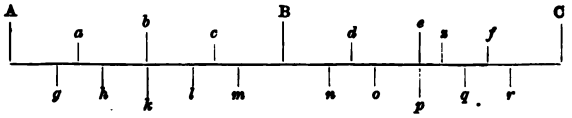
CHAPTER V.

THE ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF VULGAR FRACTIONS.

99. The addition of two or more fractions is effected by finding some single fraction which shall express the sum of all the given fractions. It is however impossible to find such a fraction unless all the given fractions be first expressed with a *common denominator*; for, since the denominator of a fraction expresses the number of equal parts into which the unit is divided, it follows that in two fractions which have not a common denominator, the unit is not divided into the same number of equal parts. Therefore, in endeavouring to add together two such fractions (for example, $\frac{3}{4}$ and $\frac{5}{6}$), so as to express their sum by a single fraction, if we did not first bring them to equivalent fractions with a common denominator we should have to seek for a new denominator which would express that the unit was to be divided into four equal parts and six equal parts, while the new numerator must express that three of the four equal parts, and five of the six equal parts, were to be taken; but no single numbers could express this, and the process could only be expressed symbolically, thus $\frac{3}{4} + \frac{5}{6}$; but if we reduce the fractions $\frac{3}{4}$ and $\frac{5}{6}$ to *others of the same value* having a common denominator (§ 92) they become $\frac{9}{12}$ and $\frac{10}{12}$, respectively, and the first fraction is made up of nine of the twelve parts into which the unit is now divided while the second fraction is made up of ten of these parts; the sum of the two fractions must therefore contain nine and ten, or nineteen, of these twelve parts; therefore, $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}$.

We may show how to add fractions together by the following illustration. Taking the foregoing example, viz.,—

$$\frac{3}{4} + \frac{5}{6}.$$



Let AB, BC, each represent unity. Divide AB, BC, each into 4 equal parts in *a*, *b*, *c*, *d*, *e*, *f*. Then A*c* represents $\frac{3}{4}$.

Again, divide AB, BC, each into 6 equal parts in *g*, *h*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*. Then A*m* represents $\frac{5}{6}$.

The divisions at *b*, *k*, coincide, because A*b* being 2 parts out of 4 is half AB, and so also is A*k*, being 3 parts out of 6.

Now the parts A*a*, *ab*, *bc*, &c., are greater than the parts A*g*, *gh*, *hk*, &c., so that the 3 parts of the former cannot be added immediately to the 5 parts of the latter. Now the L.C.M. of 4 and 6 is 12. Divide each of the parts A*a*, *ab*. *bc*, &c., into 3 equal parts, that is, the units AB, BC, each into 12 parts, and each of the parts A*g*, *gh*, *hk*, &c., into 2 parts, that is, the units AB, BC, each into 12 parts. Then these latter divisions will all coincide with the former divisions, and A*c* contains 9 such parts, and A*m* contains 10 such parts, and A*c*, A*m*, together are equal to A*z*, and contain 19 such parts, that is,—

$$\tfrac{3}{4} + \tfrac{5}{6} = \tfrac{9}{12} + \tfrac{10}{12} = \tfrac{19}{12}.$$

In the same manner we should proceed if we had several fractions. Hence we have the following rule:—

RULE XXXIV.

If the fractions have a *common* denominator:

Add the numerators together and place the sum over the common denominator.

But if the fractions have different denominators,

Find the L.C.M. *of all the denominators, and reduce each of the fractions to equivalent fractions having the* L.C.M. *as their denominators, and then* add *the numerators thus found together for the new numerator, the new denominator being the* L.C.M.

EXAMPLES.

Ex. 1. Add together $\tfrac{1}{8}$, $\tfrac{3}{8}$, $\tfrac{5}{8}$, and $\tfrac{7}{8}$ (*i.e., find the value of* $\tfrac{1}{8} + \tfrac{3}{8} + \tfrac{5}{8} + \tfrac{7}{8}$).

Here 1 *eighth* + 3 *eighths* + 5 *eighths* + 7 *eighths* = 16 *eighths* = $\tfrac{16}{8}$ = 2.

Ex. 2. Add together $\tfrac{3}{8}$, $\tfrac{7}{12}$, $\tfrac{5}{18}$, $\tfrac{11}{15}$, and $\tfrac{19}{20}$.

Since we can only add together quantities of the same kind, we must express these given fractions as equivalent ones having one common denominator, before we can find their sum. The L.C.M. of all the denominators = 360, and, reducing each of the fractions to equivalent ones having the L.C.M. as their denominators, we have—

$$\tfrac{3}{8} + \tfrac{7}{12} + \tfrac{5}{18} + \tfrac{11}{15} + \tfrac{19}{20} = \tfrac{135}{360} + \tfrac{210}{360} + \tfrac{100}{360} + \tfrac{264}{360} + \tfrac{342}{360} = \tfrac{1051}{360} = 2\tfrac{331}{360} \textit{ Ans.}$$

(1.) If any of the given quantities are whole or mixed numbers it is best to take separately the sum of the integral and fractional parts, then add the two results together.

EXAMPLE.

Ex. Find the sum of $24\tfrac{1}{2}$, $36\tfrac{3}{4}$, $28\tfrac{1}{4}$, and $112\tfrac{7}{8}$.

Adding together the *whole* numbers separately we have—

$$24 + 36 + 28 + 112 = 200,$$

and, adding together the *fractions* separately, we have—

$$\tfrac{1}{2} + \tfrac{3}{4} + \tfrac{1}{4} + \tfrac{7}{8} = \tfrac{4}{8} + \tfrac{6}{8} + \tfrac{2}{8} + \tfrac{7}{8} = \tfrac{19}{8} = 2\tfrac{3}{8}$$

$$\text{therefore, } 24\tfrac{1}{2} + 36\tfrac{3}{4} + 28\tfrac{1}{4} + 112\tfrac{7}{8} = 200 + 2\tfrac{3}{8} = 202\tfrac{3}{8} \textit{ Ans.}$$

(2.) If some of the given fractions are improper, they should be reduced to mixed numbers, and the integral parts of the mixed numbers should be added together separately, as follows:—

EXAMPLE.

$$\text{Ex. } \tfrac{21}{16} + 3\tfrac{5}{8} + 4\tfrac{7}{15} + \tfrac{9}{80} \text{ (L.C.M. 240)}$$

$$= 1\tfrac{5}{16} + 3\tfrac{5}{8} + 4\tfrac{7}{15} + \tfrac{9}{80} = 8\,\frac{75 + 150 + 112 + 27}{240}$$

$$= 8\tfrac{364}{240} = 8 + 1\tfrac{124}{240} = 9\tfrac{31}{60}.$$

NOTE.—In this example, to save labour, instead of writing the new denominator 240 under each of the new numerators, these last have all been written together at once, since they have to be afterwards added together.

(3.) When all, or any, of the quantities to be added together **consist of** compound or complex fractions, we first reduce them to simple **fractions, or** mixed numbers, and then add them together as such.

EXAMPLES.

Ex. 1. The sum of $\frac{2}{5}$ of $\frac{1}{8}$, $2\frac{1}{5}$ of 8, and $\frac{7}{8}$ of $2\frac{1}{2}$ of $1\frac{1}{7}$

$$=\left(\frac{\cancel{2}}{5}\text{ of }\frac{1}{\underset{4}{\cancel{8}}}\right)+\left(\frac{11}{5}\text{ of }\frac{8}{1}\right)+\left(\frac{\cancel{7}}{\cancel{8}}\text{ of }\frac{5}{2}\text{ of }\frac{\cancel{8}}{\cancel{7}}\right)$$

$$=\frac{1}{20}+\frac{88}{5}+\frac{5}{2}=\frac{1}{20}+\frac{352}{20}+\frac{50}{20}$$

$$=\frac{1}{20}+17\frac{12}{20}+2\frac{10}{20}=20\frac{3}{20}\ \textit{Ans.}$$

Ex. 2. The sum of $\frac{4\frac{1}{9}\text{ of }2\frac{1}{7}}{2\frac{2}{3}+5\frac{5}{9}}$, $\frac{\frac{1}{4}\text{ of }(\frac{1}{8}\text{ of }1\frac{6}{7})}{2\frac{1}{2}}$, $\frac{41}{63}$ and $\frac{7}{18}$ of 9.

$$\frac{\frac{37}{\underset{3}{\cancel{9}}}\text{ of }\frac{\overset{5}{\cancel{15}}}{7}}{(2+5)+(\frac{2}{3}+\frac{5}{9})}+\frac{\frac{1}{4}+(\frac{1}{8}\text{ of }\frac{13}{7})}{\frac{5}{2}}+\frac{41}{63}+\left(\frac{7}{\underset{2}{\cancel{18}}}\text{ of }\frac{\cancel{9}}{1}\right)=$$

$$\frac{\frac{185}{21}}{8\frac{2}{9}}+\frac{\frac{1}{4}+\frac{13}{56}}{\frac{5}{2}}+\frac{41}{63}+\frac{7}{2}=$$

$$\frac{\frac{\overset{5}{\cancel{185}}}{\underset{7}{\cancel{21}}}}{\frac{\underset{2}{\cancel{74}}}{\underset{3}{\cancel{9}}}}+\frac{\frac{27}{\underset{28}{\cancel{56}}}}{\frac{5}{\cancel{2}}}+\frac{41}{63}+\frac{7}{2}=$$

$$\frac{15}{14}+\frac{27}{140}+\frac{41}{63}+3\frac{1}{2}=$$

$$5\frac{523}{1260}.$$

EXAMPLES FOR PRACTICE.

Find the value of—

1. $\frac{1}{5}+\frac{2}{5}$; $\frac{2}{11}+\frac{5}{11}+\frac{4}{11}$; $\frac{1}{2}+\frac{1}{3}$; $\frac{2}{3}+\frac{3}{4}$; $\frac{4}{7}+\frac{2}{7}+\frac{6}{7}+\frac{5}{7}+\frac{3}{7}$.
2. $\frac{3}{7}+\frac{2}{5}+\frac{4}{35}$; $\frac{1}{2526}+\frac{1}{4344}$; $\frac{41}{18}+\frac{21}{13}+\frac{18}{15}$; $\frac{10}{14}+\frac{2}{15}+\frac{18}{70}$.
3. $\frac{8}{35}+\frac{19}{25}+\frac{20}{21}+\frac{83}{84}$; $\frac{121}{171}+\frac{15}{171}+\frac{98}{171}+\frac{117}{171}+\frac{162}{171}$; $\frac{1}{2526}+\frac{1}{4344}$.
4. $\frac{14}{99}$, $\frac{15}{28}$, $\frac{17}{77}$, $\frac{101}{126}$, and $\frac{1000}{1001}$; $\frac{6}{19}$, $\frac{18}{605}$, $\frac{271}{273}$, $\frac{1}{1008}$, $\frac{4132}{9275}$, $\frac{13}{209}$, and $\frac{1}{19152}$.
5. $\frac{37}{226}+3\frac{112}{565}+5\frac{701}{1130}+567031\frac{73}{113}$; $2\frac{1}{3000}+1\frac{6}{1000}+5\frac{1}{6000}+2\frac{7}{8000}$.
6. $\frac{11}{70}$, $\frac{5}{21}$, $\frac{1}{5}$, $\frac{17}{42}$; $\frac{2}{5}+\frac{35}{80}$, $\frac{14}{100}$, $\frac{3}{140}$; $\frac{1}{2}+\frac{2}{3}+\frac{3}{5}+\frac{7}{15}+\frac{23}{30}+\frac{11}{40}+\frac{23}{60}+\frac{41}{120}$.
7. $\frac{4}{11}$ of $\frac{3}{8}$ of $\frac{1}{2}+\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{10}{33}+\frac{2}{5}$ of $1\frac{2}{3}+2\frac{5}{7}$; $1+\frac{1}{6}+\frac{1}{24}+\frac{61}{5040}+\frac{277}{72576}$.
8. $\frac{301}{30}+\frac{919}{10}+\frac{71}{15}+\frac{86}{23}+\frac{101}{76}+\frac{460}{48}+\frac{1800}{10}+17\frac{3}{4}$.

SUBTRACTION OF VULGAR FRACTIONS.

100. We may show by reasoning, similar to that used in the addition of fractions, that it is impossible to express by a single fraction the *difference* between two fractions, unless these be first reduced to equivalent fractions having a common denominator. Hence the following rule for subtraction of vulgar fractions.

RULE XXXV.

If the fractions have not *a common denominator, reduce them to other equivalent fractions having a common denominator; then the parts into which the unit is supposed to be divided are the same in both fractions, and the* difference *of the* numerators placed over *the* common denominator *will give the* difference *of the fractions.*

EXAMPLES.

Ex. 1. From $\frac{14}{15}$ take $\frac{4}{15}$ (*i.e.*, find the value of $\frac{14}{15} - \frac{4}{15}$).

Since $14 - 4 = 10$

14 *fifteenths* $-$ 4 *fifteenths* $=$ 10 *fifteenths* $= \frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$.

Ex. 2. Take $\frac{10}{21}$ from $\frac{9}{14}$ (*i.e.*, find the value of $\frac{9}{14} - \frac{10}{21}$).

As in addition of fractions, we first reduce the given fractions to equivalent ones having one common denominator, and we thus find that

$$\frac{9}{14} - \frac{10}{21} = \frac{9 \times 3}{14 \times 3} - \frac{10 \times 2}{21 \times 2} = \frac{27}{42} - \frac{20}{42} = \frac{7}{42} = \frac{7 \div 7}{42 \div 7} = \frac{1}{6} \textit{ Ans.}$$

(*a*) If the given quantities are both mixed numbers, or consist of a whole and a mixed number, it is best to take separately the difference of the integral and the fractional parts, and then add the two results together.

Ex. 3. From $5\frac{5}{8}$ take $2\frac{1}{2}$ (*i.e.*, find the value of $5\frac{5}{8} - 2\frac{1}{2}$).

$$\left.\begin{array}{l}\text{Here } 5 - 2 = 3 \\ \frac{5}{8} - \frac{1}{2} = \frac{5}{8} - \frac{4}{8} = \frac{1}{8}\end{array}\right\} \therefore 5\frac{5}{8} - 2\frac{1}{2} = 3 + \frac{1}{8} = 3\frac{1}{8} \textit{ Ans.}$$

Ex. 4. From $17\frac{1}{8}$ take 8.

$17 - 8 = 9$.

$\therefore 17\frac{1}{8} - 8 = 9 + \frac{1}{8} = 9\frac{1}{8}$ *Ans.*

Ex. 5. $5\frac{4}{9} - 3\frac{5}{24} = 5 - 3 + \frac{4}{9} - \frac{5}{24} = 2 + \frac{32 - 15}{72} = 2\frac{17}{72}$ *Ans.*

In the cases above the whole numbers have been subtracted separately from the fractions. The mixed numbers might have been reduced to improper fractions first, but the plan above is more expeditious.

Ex. 6. From $5\frac{5}{24}$ take $3\frac{4}{9}$ (*i.e.*, find the value of $5\frac{5}{24} - 3\frac{4}{9}$).

$5\frac{5}{24} - 3\frac{4}{9} = 2 + \frac{15 - 32}{72} = 2 - \frac{17}{72} = 1\frac{55}{72}$ *Ans.*

In this case, on subtracting the fractional parts, we find that the subtrahend is the larger. We have therefore to subtract the difference, $\frac{17}{72}$, of the fractions from 2 of the whole numbers. The student will easily see that when unity is divided into 72 parts, of which 17 have been taken, there are 55 parts left, and, therefore, that $1 - \frac{17}{72} = \frac{55}{72}$.

NOTE.—From the last three examples we see that, after the fractions have been reduced to a common denominator, *if the fractional part of the subtrahend is less than that in the minuend*, the difference of the new numerators is taken as the numerator of a fraction which shall have the common denominator for its denominator; set the sign + (*plus*) before this fraction, and in *subtraction of mixed numbers* place it with its sign after the difference of the integers, and perform the operation indicated by this expression (see Exs. 3 and 4); *but if the fractional part in the subtrahend is* greater *than that in the minuend*, take the difference of the new numerators as the numerator of a fraction which shall have the common denominator of the given fraction for its denominator. Set the sign — (*minus*) before the fraction, and in *subtraction of mixed numbers* place it with its sign after the difference of the integers, and perform the operation indicated by this expression (see Ex. 6).

(*b*) When either the subtrahend or the minuend, or both, consists of an improper fraction.

RULE XXXVI.

***Reduce improper fractions to mixed numbers, and then proceed as in* subtraction of mixed numbers.**

EXAMPLES.

Ex. 7. $2\frac{17}{19} - \frac{16}{15} = 2\frac{17}{19} - 1\frac{1}{15} = 2 - 1 + \frac{255 - 19}{285} = 1 + \frac{236}{285} = 1\frac{236}{285}$.

Ex. 8. $\frac{24}{19} - 1\frac{1}{8} = 1\frac{5}{19} - 1\frac{1}{8} = 1 - 1 = \frac{40 - 19}{152} = \frac{21}{152}$.

(*c*) When either the subtrahend or the minuend, or both, consists of a compound or complex fraction.

RULE XXXVII.

Reduce compound or complex fractions to their simplest forms, and then proceed as in subtraction of simple fractions, or of mixed numbers, as the case may be.

EXAMPLES.

Ex. 9. $6\frac{1}{7} - (\frac{3}{8} \text{ of } 5\frac{1}{3}) = 6\frac{1}{7} - \left(\frac{\cancel{3}}{\cancel{8}} \text{ of } \frac{\overset{2}{\cancel{16}}}{\cancel{3}}\right) = 6\frac{1}{7} - 2 = 4\frac{1}{7}$.

Ex. 10. $4\frac{11}{12} - \frac{\frac{2}{3} \text{ of } 16\frac{1}{2}}{6\frac{2}{7}} = 4\frac{11}{12} - \frac{\frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\overset{11}{\cancel{33}}}{\cancel{2}}}{\frac{44}{7}} = 4\frac{11}{12} - \frac{7}{4} = 4\frac{11}{12} - 1\frac{3}{4} = 4\frac{55}{60} - 1\frac{45}{60} =$ $3\frac{10}{60}$ or $3\frac{1}{6}$.

Ex. 11. $(2\frac{4}{9} \text{ of } 20\frac{4}{7}) - 4\frac{1}{2} = \left(\frac{22}{\cancel{9}} \text{ of } \frac{\cancel{144}}{7}\right) - 4\frac{1}{2} = \frac{352}{7} - 4\frac{1}{2} = 50\frac{2}{7} - 4\frac{1}{2} = 45\frac{11}{14}$.

101. Sometimes an expression involving several fractions, some to be added, and some to be subtracted, occurs.

EXAMPLE.

$$\begin{aligned} \text{Ex. } & 2\tfrac{1}{2} + 3\tfrac{1}{3} - 5\tfrac{1}{6} + 4\tfrac{3}{5} - 2\tfrac{7}{10} \\ &= 2\tfrac{1}{2} + 3\tfrac{1}{3} + 4\tfrac{3}{5} - 5\tfrac{1}{6} - 2\tfrac{7}{10} \\ &= 9 - 7 + \frac{15 + 10 + 18}{30} - \frac{5 + 21}{30} \\ &= 2 + \frac{43 - 26}{30} = 2\tfrac{17}{30} \textit{ Ans.} \end{aligned}$$

EXAMPLES FOR PRACTICE.

1. $\frac{3}{4} - \frac{1}{4}$; $\frac{23}{25} - \frac{21}{25}$; $3 - \frac{1}{2}$; $4 - \frac{2}{3}$; $\frac{1}{2} - \frac{1}{3}$; $\frac{2}{3} - \frac{3}{5}$.
2. $\frac{5}{6} - \frac{23}{42}$; $\frac{101}{200} - \frac{49}{80}$; $\frac{21}{29} - \frac{31}{290}$; $\frac{6}{11} - \frac{10}{121}$; $\frac{101}{108} - \frac{14}{27}$; $\frac{860}{919} -$ [illegible].
3. [illegible] $-$ [illegible] $(\frac{14}{15} + \frac{2}{15} + \frac{3}{35}) - \frac{17}{15}$; [illegible] $-$ [illegible]; [illegible] $-$ [illegible].
4. $\frac{83417}{80000} - \frac{83417}{190000}$; $\frac{43}{93} -$ [illegible]; $2\frac{1}{3} - 1\frac{1}{4}$; $4\frac{1}{6} - 3\frac{2}{7}$; $2\frac{12}{13} - 1\frac{16}{28}$.
5. $4\frac{11}{20} - 3\frac{16}{25}$; $2 - 1\frac{19}{25}$; $20\frac{21}{22} - 9\frac{3}{4}$; $1\frac{4}{5} - \frac{2}{7}$; $3\frac{1}{3} - 1\frac{31}{1040}$; $16 - 14$ [illegible].
6. $\frac{22}{7} - \frac{333}{106}$; $\frac{15}{16} - \frac{14}{15} + \frac{13}{14} - \frac{11}{12}$; $\frac{1}{3} + \frac{2}{7} - \frac{5}{9} + \frac{71}{63}$; $\frac{1}{3} - \frac{4}{21} + \frac{1}{2} - \frac{7}{11}$.
7. $3 - \frac{1}{5} + \frac{1}{45} -$ [illegible]; $3\frac{3}{4} - 2\frac{5}{7} - 8\frac{13}{21} + 75\frac{9}{28} - 14\frac{11}{12} - 39\frac{59}{84} + 11\frac{53}{60} - 23\frac{44}{45}$.
8. From $12\frac{13}{30}$ take in succession $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{8}{9}$; [illegible] $-$ [illegible] $+$ [illegible].
9. $1 - \frac{1}{6} + \frac{1}{24} -$ [illegible] $+$ [illegible]; [illegible] $-$ [illegible] $-$ [illegible] $+$ [illegible].
10. $1 + \frac{1}{34} -$ [illegible] $-$ [illegible] $-$ [illegible] $+$ [illegible] $-$ [illegible].

MULTIPLICATION OF VULGAR FRACTIONS.

102. We have defined multiplication to be an abbreviated method of performing addition when one or two given quantities is to be taken as many times as there are units in the other.

Now applying this to fractions, to multiply one fraction by another, *e.g.* to multiply $\frac{3}{8}$ by $\frac{4}{5}$, will be to take $\frac{3}{8}$ as many times, or parts of a time, as there are units, or parts of a unit, in $\frac{4}{5}$; but as the proper fraction $\frac{4}{5}$ is less than one, this will be to take $\frac{3}{8}$ not as much as once, but four-fifths of once, *i.e.*, to find the value of $\frac{4}{5}$ parts of $\frac{3}{8}$. But to take $\frac{4}{5}$ parts of $\frac{3}{8}$ is to divide $\frac{3}{8}$ into 5 equal parts, and to take 4 of these parts. Now the division of $\frac{3}{8}$ into 5 equal parts is effected by multiplying the denominator by 5 (§ 86) also taking 4 of these parts is effected by multiplying the numerator by 4 (§ 84).

Thus $\frac{3}{8} \div 5 = \frac{3}{40}$
and $\frac{3}{40} \times 4 = \frac{12}{40}$
$\therefore \frac{3}{8} \times \frac{4}{5} = \frac{12}{40}$.

Hence the rule for the multiplication of fractions.

RULE XXXVIII.

Multiply the numerators together for a new numerator and the denominators together for a new denominator.

Note.—It will be seen that the rule for the multiplication of fractions is the same as that laid down for the reduction of a compound fraction to a simple one.

Example.

Ex. Multiply $\frac{3}{7}$ by $\frac{5}{8}$.

$$\frac{3}{7} \times \frac{5}{8} = \frac{3 \times 5}{7 \times 8} = \frac{15}{56} \textit{ Ans.}$$

103. Since the value of a fraction is not altered by dividing both its numerator and denominator by the same quantity (§ 89), we may "cancel" in multiplying fractions together, *i.e.*, may strike out of both numerator and denominator any common factor before we multiply the numerators and denominators together; thus,—

$$\frac{9}{10} \times \frac{15}{27} = \frac{135}{270},$$

and this reduced to its lowest terms (§ 90) is $\frac{1}{2}$; but we might have obtained this result without multiplying 9 and 15, 10 and 27 respectively together, by observing that 9 and 27 are both measured by 9, 10 and 15 both measured by 5, and we may write—

$$\frac{9}{10} \times \frac{15}{27} = \frac{\cancel{9}}{\underset{2}{\cancel{10}}} \times \frac{\overset{\cancel{3}}{\cancel{15}}}{\underset{\cancel{3}}{\cancel{27}}} = \frac{1}{2}.$$

104. Again, to find the *continued product* of any number of fractions, as $\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} \times \frac{8}{9}$*, instead of multiplying together all the numerators and all the denominators, we may write—

$$\frac{1}{\cancel{3}} \times \frac{1}{\cancel{2}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{\cancel{2}}{\cancel{8}}}{9} = \frac{1}{9}$$

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* This expression means that of $\frac{8}{9}$ we are to take $\frac{3}{4}$; of that result we are to take $\frac{1}{2}$, and of that result again we take $\frac{1}{3}$.

Multiply $\frac{8}{9}$, $\frac{16}{24}$, $\frac{27}{30}$, $\frac{45}{60}$ together.

$$\text{Product} = \frac{8\times16\times27\times45}{9\times24\times30\times60} = \frac{\cancel{8}\times\overset{\overset{2}{\cancel{4}}}{\cancel{16}}\times\overset{\cancel{3}}{\cancel{27}}\times\overset{\cancel{3}}{\cancel{45}}}{\cancel{9}\times\underset{\cancel{3}}{\cancel{24}}\times\underset{\underset{5}{\cancel{10}}}{\cancel{30}}\times\underset{\cancel{4}}{\cancel{60}}} = \frac{2}{5}\ \text{the product in its lowest terms.}$$

(a) Whenever the quantities whose product is to be found do not all consist of simple fractions, they must first be reduced to simple fractions, and then multiplied together, by the rule given above, thus—

EXAMPLES.

Ex. 1. Find the value of $4\frac{1}{8} \times 5\frac{1}{6}$.

$$\left.\begin{matrix}4\frac{1}{8} = \frac{33}{8}\\ 5\frac{1}{6} = \frac{31}{6}\end{matrix}\right\} \quad \therefore 4\frac{1}{8}\times5\frac{1}{6} = \frac{\overset{11}{\cancel{33}}}{8}\times\frac{31}{\underset{2}{\cancel{6}}} = \frac{341}{16} = 21\frac{5}{16}.$$

Ex. 2. Find the value of $\frac{4}{7}$ of $5\frac{3}{5} \times 3\frac{1}{2}$ of $9 \times 1\frac{8}{9}$.

$$\frac{4}{7}\text{ of }5\frac{3}{5}\times3\frac{1}{2}\text{ of }9\times1\frac{8}{9} = \frac{4}{\cancel{7}}\times\frac{\overset{14}{\cancel{28}}}{5}\times\frac{\cancel{7}}{\cancel{2}}\times\frac{\cancel{9}}{1}\times\frac{17}{\cancel{9}} = \frac{952}{5} = 190\frac{2}{5}.$$

Ex. 3. Find the value of $2\frac{1}{2} \times 4\frac{2}{3} \times 7\frac{1}{7} \times \frac{9}{25}$.

$$2\frac{1}{2}\times4\frac{2}{3}\times7\frac{1}{7}\times\frac{9}{25} = \frac{5}{\cancel{2}}\times\frac{\overset{\cancel{2}}{\cancel{14}}}{\cancel{3}}\times\frac{\overset{2}{\cancel{50}}}{\cancel{7}}\times\frac{\overset{3}{\cancel{9}}}{\cancel{25}} = 30.\ \textit{Ans.}$$

105. When any number or fraction is multiplied by a proper fraction, it is taken so many parts of a time as there are parts of a unit in the proper fraction; this result is still called the product of the two quantities; but whereas in whole numbers the product is made by taking a number a certain number of times, and a whole number is therefore *increased* by being multiplied by any number larger than unity, so on the other hand by multiplying a whole number or a fraction by a proper fraction, the product is *less* than the original multiplicand; and the number or fraction, being only taken some part or parts of once, is *diminished* by being multiplied by a proper fraction: *e.g.*, to multiply 2 by $\frac{1}{2}$ is to take 2 one-half of a time; or the product is $\frac{1}{2}$ of 2, *i.e.*, is 1: to multiply $\frac{1}{2}$ by $\frac{1}{2}$ is to take $\frac{1}{2}$ only one-half of a time; or the product is one-half of $\frac{1}{2}$, *i.e.*, is $\frac{1}{4}$.

EXAMPLES FOR PRACTICE.

1. Multiply $\frac{3}{4}$ by $\frac{2}{5}$; $\frac{7}{9}$ by $\frac{2}{3}$; $1\frac{1}{2}$ by $1\frac{1}{3}$; $3\frac{1}{2}$ by $2\frac{1}{3}$; 21 by $\frac{5}{7}$; 42 by $\frac{5}{6}$.
2. $100001 \times \frac{9090}{9091}$; $1000001 \times \frac{9900}{9901}$; $\frac{5}{7} \times \frac{21}{45}$; $\frac{8}{9} \times \frac{11}{24}$; $\frac{4}{9} \times \frac{27}{16}$; $\frac{14}{15} \times \frac{15}{14}$.
3. $\frac{112}{365} \times \frac{221}{115}$; $\frac{412}{167} \times \frac{397}{777}$; $\frac{1}{169} \times \frac{225}{963}$; $\frac{93}{4600} \times \frac{125}{662}$; $\frac{169}{289} \times \frac{4913}{6591}$.
4. $\frac{423}{616} \times \frac{52568}{18189} \times \frac{5}{4}$; $\frac{13}{27} \times 1\frac{4}{5} \times \frac{15}{82} \times 3\frac{2}{9} \times \frac{49}{69}$; $\frac{11}{21} \times \frac{70}{93}$ of $3\frac{1}{10} \times 2\frac{1}{4} \times \frac{8}{55}$.
5. $\frac{324}{361} \times \frac{1444}{1296} \times \frac{441}{529} \times \frac{2116}{1764}$; $\frac{60}{61} \times \frac{60}{61} \times \frac{60}{61} \times \frac{60}{61} \times \frac{60}{61} \times \frac{60}{61}$.
6. $1\frac{2}{9} \times 1\frac{9}{11} \times 1\frac{4}{17} \times 1\frac{9}{16}$; $1\frac{8}{29} \times 1\frac{6}{37} \times 1\frac{4}{43} \times 1\frac{6}{47}$; $2\frac{1}{19} \times 2\frac{6}{19} \times 2\frac{12}{37} \times 1\frac{23}{43}$.
7. $13\frac{5}{9}$ of $7\frac{1}{3}$ by $\frac{3}{4}$ of $\frac{4}{9}$ of $12\frac{1}{2}$; $2\frac{3}{5} \times \frac{6\frac{1}{4}}{1\frac{3}{4}} \times \frac{3\frac{2}{5}}{5\frac{4}{7}}$; $\frac{2\frac{3}{4}}{5\frac{7}{9}}$ of $\frac{1}{3} \times \frac{3}{8}$ of $\frac{4\frac{3}{8}}{7\frac{1}{2}}$ of $\frac{7\frac{3}{7}}{5\frac{1}{5}}$
8. $\frac{11\frac{2}{3}}{29}$ of $\frac{4\frac{1}{6}}{13}$ of $\frac{3\frac{1}{4}}{10\frac{5}{8}} \times 5\frac{1}{5}$ of 6 of $20\frac{3}{4}$; $\frac{2}{3\frac{3}{16}}$ of $\frac{6\frac{1}{14}}{8} \times \frac{2}{3}$ of $8\frac{1}{4}$ of $\frac{35}{36}$.
9. Find the value of $\frac{3}{22} \times \frac{11}{61} \times 2\frac{1}{3}$ of $3\frac{7}{16} \times \left\{4\frac{1}{8} + 2\left(\frac{3}{7} + 4\frac{2}{7}\right) + 1\frac{168}{188}\right\} \times \frac{33}{56}$.

DIVISION OF FRACTIONS.

106. The rule for the Division of Fractions will be seen from an examination of the following example:—

EXAMPLE.

Ex. Suppose it is required to divide $\frac{2}{3}$ by $\frac{5}{7}$, then it is evident (§ 86) that $\frac{2}{3} \div 5 = \frac{2}{15}$, which is 7 times too *small*, because the divisor has been taken 7 times too *great*, viz., 5 instead of the *seventh part* of 5; hence the quotient $\frac{2}{15}$ must be increased 7 times, and $\frac{2}{15} \times 7 = \frac{14}{15}$ (§ 84) is the quotient required. Hence—

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15}.$$

Hence the following rule for dividing one fraction by another—

RULE XXXIX.

Divide the numerator and denominator of the dividend *by the numerator and denominator of the* divisor *respectively,* if they will exactly divide; but if not, *invert the terms of the divisor, cancelling all the factors common both to the numerators and denominators, and the* product *of those remaining among the numerators placed over the product of those remaining among the denominators will be the answer required.*

(*a*). Before applying this rule, mixed numbers must be reduced to improper fractions, and compound fractions to simple ones. If a fraction is to be divided by a compound fraction all the fractions of the compound fraction may be inverted as multipliers, and cancelling takes place as in multiplication.

When two fractions have a common denominator, it is plain *one numerator* can be divided by the *other*, as well as *one whole* number by another; for the parts of the two fractions are of the *same denomination*. When the fractions have not a common denominator, the *reason* that inverting the divisor and proceeding as in multiplication will produce the true answer, is because this process in effect reduces the two fractions to a *common denominator*, and then the numerator of the dividend is divided by the numerator of the divisor. For, reducing the two fractions to a common denominator, we multiply the numerator of the dividend by the denominator of the divisor, and the numerator of the divisor by the denominator of the dividend; and then dividing the *former* product by the *latter*, we have the *same combination* of the *same numbers* as in the rule above, which will consequently produce the *same result*. We do not multiply the two denominators together for a common denominator; for, in dividing, no use is made of a common denominator when found, therefore it is unnecessary to obtain it. The object of *inverting the divisor* is simply for convenience in multiplying.

EXAMPLES.

Ex. 1. Divide $\frac{20}{9}$ by $\frac{4}{3}$.

$\frac{20}{9} \div \frac{4}{3} = \frac{5}{3} = 1\frac{2}{3}$ *Ans.*

Ex. 2. Divide $\frac{4}{5}$ by $\frac{1}{5}$.

$\frac{4}{5} \div \frac{1}{5} = \frac{4}{1} = 4$ *Ans.*

Ex. 3. Divide $\frac{2}{11}$ by $\frac{3}{5}$.

$$\frac{2}{11} \div \frac{3}{5} = \frac{2}{11} \times \frac{5}{3} = \frac{10}{33} \text{ Ans.}$$

Ex. 4. Divide $4\frac{1}{3}$ by $2\frac{3}{4}$.

$$4\frac{1}{3} \div 2\frac{3}{4} = \frac{13}{3} \div \frac{11}{4} = \frac{13}{3} \times \frac{4}{11} = \frac{52}{33} = 1\frac{19}{33}.$$

Ex. 5. Divide $4\frac{1}{6}$ by $7\frac{1}{4}$.

$$4\frac{1}{6} \div 7\frac{1}{4} = \frac{25}{6} \div \frac{29}{4} = \frac{25}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{4}}}{29} = \frac{50}{87} \text{ Ans.}$$

Ex. 6. Divide $\frac{2}{3}$ of $3\frac{3}{4}$ by $\frac{7}{8}$ of $\frac{4}{15}$ of $1\frac{3}{5}$.

$$\tfrac{2}{3} \text{ of } 3\tfrac{3}{4} \div \tfrac{7}{8} \text{ of } \tfrac{4}{15} \text{ of } 1\tfrac{3}{5} = \tfrac{2}{3} \text{ of } \tfrac{15}{4} \div \tfrac{7}{8} \text{ of } \tfrac{4}{15} \text{ of } \tfrac{8}{5} = \tfrac{2}{3} \times \tfrac{15}{4}$$

$$\div \frac{7 \times 4 \times 8}{8 \times 15 \times 5} = \tfrac{2}{3} \times \tfrac{15}{4} \times \frac{8 \times 15 \times 5}{7 \times 4 \times 8}$$

$$= \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{4}}} \times \frac{\cancel{8}}{7} \times \frac{15}{4} \times \frac{5}{\cancel{8}} = \tfrac{375}{56} = 6\tfrac{39}{56} \textit{ Ans.}$$

Ex. 7. $(\frac{7}{16}$ of $\frac{8}{9}$ of $3\frac{1}{2}) \div (\frac{14}{15}$ of $2\frac{7}{8}$ of $1\frac{4}{7})$

$$= \left(\frac{7}{\underset{2}{\cancel{16}}} \times \frac{\cancel{8}}{9} \times \frac{7}{2}\right) \div \left(\frac{\overset{2}{\cancel{14}}}{15} \times \frac{23}{\underset{4}{\cancel{8}}} \times \frac{11}{\cancel{7}}\right)$$

$$= \frac{7 \times 7}{2 \times 9 \times 2} \div \frac{23 \times 11}{15 \times 4} = \frac{7 \times 7}{\cancel{2} \times \underset{3}{\cancel{9}} \times \cancel{2}} \times \frac{\overset{5}{\cancel{15}} \times \overset{2}{\cancel{4}}}{23 \times 11} = \frac{7 \times 7 \times 5}{3 \times 23 \times 11} = \frac{245}{759} \textit{ Ans.}$$

EXAMPLES FOR PRACTICE.

Divide—

1. $\frac{2}{3}$ by $\frac{4}{5}$; $\frac{3}{4}$ by $\frac{1}{2}$; $\frac{6}{7}$ by $\frac{9}{11}$; $\frac{2}{7}$ by $\frac{3}{8}$; $\frac{3}{5}$ by $\frac{4}{5}$; $\frac{4}{11}$ by $\frac{4}{11}$ by $\frac{9}{16}$; $\frac{12}{5}$ by $\frac{27}{10}$.
2. $\frac{26}{51}$ by $\frac{91}{34}$; $\frac{17}{2}$ by $\frac{6}{5}$; $\frac{7}{8}$ by $\frac{1}{10}$; 1001 by $\frac{7}{13}$; 1001 by $\frac{137}{365}$; $\frac{26}{15}$ by $\frac{13}{25}$; 10001 $\div$ $\frac{286}{137}$.
3. $\frac{11}{7} \div 1\frac{1}{10}$; $\frac{14}{5} \div 2\frac{1}{10}$; $1\frac{1}{2} \div 1\frac{1}{3}$; $1\frac{1}{3} \div 1\frac{1}{4}$; $1\frac{1}{6} \div 1\frac{1}{4}$; $1\frac{1}{5}$ of $2\frac{1}{4} \div 3$.
4. $4\frac{2}{3} \div 9\frac{1}{3}$; $2\frac{1}{5} \div 3\frac{2}{5}$; $10\frac{4}{5} \div 13\frac{3}{7}$; $17\frac{2}{3} \div 7\frac{13}{14}$; $\frac{81}{28} \times \frac{44}{39} \div \frac{3}{91}$.
5. $2\frac{1}{19} \times 2\frac{6}{19} \times 2\frac{13}{37} \times 1\frac{23}{43} \div 1\frac{2}{7} \times 1\frac{2}{11} \times 1\frac{4}{17} \times 1\frac{2}{19}$; $\frac{19}{187} \times \frac{17}{341} \div 1\frac{8}{11}$.
6. $3\frac{4}{5}$ by $\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{5}{6}$; $3\frac{1}{125}$ of $3\frac{4}{7} \div \frac{47}{315}$ of 9; $15\frac{7}{11}$ of $8\frac{4}{5} \div \frac{4}{5}$ of $\frac{6}{11}$ of $15\frac{5}{8}$.
7. $2\frac{10}{11}$ of $5\frac{1}{16}$ of $1331 \div 3\frac{11}{12}$ of $\dfrac{44}{13\frac{3}{7}}$ of $202\frac{1}{2}$; $\dfrac{7\frac{2}{7}}{40\frac{5}{9}} \div \dfrac{17\frac{1}{3}}{73}$; $11\frac{2}{5} \div 1\frac{91}{111} \div 2\frac{136}{185}$.

107. When any whole number or fraction is divided by a proper fraction, the number of times, or the fractional part of a time, that the divisor can be subtracted from the dividend is called the quotient. But whereas in the division of whole numbers the quotient is always *less* than the dividend; on the other hand, in the division of fractions, whenever the divisor is a *proper* fraction, the quotient will be *greater* than the dividend, *e.g.*, $2 \div \frac{1}{4} = 2 \times \frac{4}{1} = 8$, that is, $\frac{1}{4}$ may be subtracted exactly 8 times from 2. Again, $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$, a fraction which is greater than the dividend $\frac{2}{3}$.

108. **A complex fraction may be reduced to a simple fraction by division.**

For if $\dfrac{7\frac{1}{3}}{5\frac{1}{2}}$ be the complex fraction, then its value will be the same as $7\frac{1}{3} \div 5\frac{1}{2}$. But $7\frac{1}{3} = \frac{22}{3}$, and $5\frac{1}{2} = \frac{11}{2}$, therefore,

$$\tfrac{22}{3} \div \tfrac{11}{2} = \frac{\overset{2}{\cancel{22}}}{3} \times \frac{2}{\cancel{11}} = \tfrac{4}{3}.$$

109. It may be here explained that *all examples in Division of Fractions may be worked out by the rule given for reducing Complex Fractions to Simple Fractions*, thus:—

EXAMPLES.

Ex. 1. $\frac{2}{3} \div \frac{5}{9} = \dfrac{\frac{2}{\cancel{3}_{1}}}{\frac{5}{\cancel{9}_{3}}} = \frac{6}{5}$. *Ans.*

Ex. 2. $7\frac{1}{3} \div 5\frac{1}{2} = \dfrac{\frac{\overset{2}{\cancel{22}}}{3}}{\frac{\cancel{11}_{1}}{2}} = \frac{4}{3}$. *Ans.*

MISCELLANEOUS EXAMPLES IN PRACTICE.

110. We have now shown how the fundamental operations of arithmetic may be severally performed in the case of fractions, but in practice these operations are commingled in various ways. No special rules can be given in such cases, but the following examples should be carefully examined, and each step verified by the student.

EXAMPLES.

Ex. 1. $2\frac{1}{2}$ of $\frac{3}{4} + 5\frac{7}{12} - \frac{3}{7}$ of $5\frac{3}{5} - 2\frac{23}{24}$

$$= \frac{5}{2} \text{ of } \frac{3}{4} + 5\frac{7}{12} - \frac{3}{\cancel{7}} \text{ of } \frac{\overset{4}{\cancel{28}}}{5} - 2\frac{23}{24}$$

$$= \frac{15}{8} + 5\frac{7}{12} - \frac{12}{5} - 2\frac{23}{24} = 1\frac{7}{8} + 5\frac{7}{12} - 2\frac{2}{5} - 2\frac{23}{24} \text{ (L.C.M. 120)}$$

$$= 2\frac{205 + 70 - 48 - 115}{120} = 2\frac{175 - 163}{120} = 2\frac{12}{120} = 2\frac{1}{10}. \text{ Ans.}$$

Here the compound fractions are reduced to simple ones before the operations of addition and subtraction are performed.

Ex. 2. $(2\frac{1}{2}$ of $\frac{4}{5} - \frac{3}{4}$ of $1\frac{3}{5}) \times (1\frac{1}{2} + 2\frac{1}{3})$

$$= \left(\frac{\cancel{5}}{\cancel{2}} \text{ of } \frac{\overset{2}{\cancel{4}}}{\cancel{5}} - \frac{3}{\cancel{4}} \text{ of } \frac{\overset{2}{\cancel{8}}}{5}\right) = \left(3\,\frac{3+2}{6}\right) = \left(2 - \frac{6}{5}\right) \times 3\frac{5}{6}$$

$$= \frac{\overset{2}{\cancel{4}}}{5} \times \frac{23}{\cancel{6}_{3}} = \frac{46}{15} = 3\frac{1}{15}. \text{ Ans.}$$

Here the parts enclosed in the brackets (....) are reduced to simple fractions before the brackets are removed, the multiplication of the bracketed parts is then performed.

Ex. 3. $$\frac{3\frac{1}{5} + 4\frac{1}{3} - 5\frac{1}{4} \text{ of } \frac{6}{7}}{3\frac{1}{2} - 4\frac{1}{3} \text{ of } \frac{7}{26} + 4\frac{1}{5} \text{ of } \frac{3}{7}} = \dfrac{3\frac{1}{5} + 4\frac{1}{3} - \frac{\overset{3}{\cancel{21}}}{\cancel{4}_{2}} \text{ of } \frac{\overset{3}{\cancel{6}}}{\cancel{7}}}{3\frac{1}{2} - \frac{\cancel{13}}{3} \text{ of } \frac{7}{\cancel{26}_{2}} + \frac{\overset{3}{\cancel{21}}}{5} \text{ of } \frac{3}{\cancel{7}}}$$

$$= \frac{3\frac{1}{5} + 4\frac{1}{3} - \frac{9}{2}}{3\frac{1}{2} - \frac{7}{6} + \frac{9}{5}} = \text{(A)}\ \frac{3\frac{1}{5} + 4\frac{1}{3} - 4\frac{1}{2}}{3\frac{1}{2} - 1\frac{1}{6} + 1\frac{4}{5}} = \frac{3\frac{6 + 10 - 15}{30}}{3\frac{15 - 5 + 24}{30}} = \frac{3\frac{1}{30}}{3\frac{34}{30}} = \frac{91}{30} \times \frac{30}{124} = \frac{91}{124} \text{ Ans.}$$

In a sum of this kind, when the step marked (A) has been reached, the following method is very useful:—The L.C.M. of all the denominators, 5, 3, 2, 6, is 30; multiplying numerator and denominator by 30, and observing that the integers in each amount to 3, the fraction becomes—

$$\frac{3\times 30+\frac{1}{5}\times 30+\frac{1}{3}\times 30-\frac{1}{2}\times 30}{3\times 30+\frac{1}{2}\times 30-\frac{1}{6}\times 30+\frac{4}{5}\times 30}$$

(This step may be omitted in practice).

$$=\frac{90+6+10-15}{90+15-5+24}=\tfrac{91}{124}\ \textit{Ans.}$$

Ex. 4. Reduce the expression $\dfrac{\frac{1}{2}+\frac{1}{3}\text{ of }\frac{1}{4}+\frac{5}{6}}{\frac{1}{13}\text{ of }(1+5\frac{1}{2})+\frac{5}{6}\text{ of }\frac{1}{23}\text{ of }(7-2\frac{2}{5})-\frac{1}{3}}$

$$\frac{\frac{1}{2}+\frac{1}{3}\text{ of }\frac{1}{4}+\frac{5}{6}}{\frac{1}{13}\text{ of }(1+5\frac{1}{2})+\frac{5}{6}\text{ of }\frac{1}{23}\text{ of }(7-2\frac{2}{5})-\frac{1}{3}}=\frac{\frac{1}{2}+\frac{1}{12}+\frac{5}{6}}{\frac{1}{13}\times 6\frac{1}{2}+\frac{5}{6}\times\frac{1}{23}\times 4\frac{3}{5}-\frac{1}{3}}$$

$$=\frac{\frac{6+1+10}{12}}{\frac{1}{\cancel{13}}\times\frac{\cancel{13}}{2}+\frac{\cancel{5}}{6}\times\frac{1}{\cancel{23}}\times\frac{\cancel{23}}{5}-\frac{1}{3}}=\frac{\frac{17}{12}}{\frac{1}{2}+\frac{1}{6}-\frac{1}{3}}=\frac{\frac{17}{12}}{\frac{2}{6}}=\tfrac{17}{12}\times\tfrac{3}{1}=4\tfrac{1}{4}\ \textit{Ans.}$$

Observe that when two quantities, connected by the sign of multiplication, are combined with others by the sign +, or —, these quantities must be first multiplied together, and the result then added to, or subtracted from, the other quantities; for instance, the numerator of the given fraction means that to ½ is to be added the product of ⅓ and ¼, and to this again ⅚ is to be added; *not* that the sum of ½ + ⅓ is to be multiplied by the sum of ¼ + ⅚; for, had this been meant, the expressions would have been written (½ + ⅓) of (¼ + ⅚), brackets being used, as they are in the denominator of the given fractions. It is therefore important to remember that the signs ×, ÷, and of, *connect* the terms between which they are found into one quantity, while the signs +, —, ~, *separate* the quantities between which they occur into distinct quantity.

Ex. 5. Reduce the expression $\dfrac{5\frac{6}{7}-\frac{1}{5}\text{ of }\frac{2}{3\frac{1}{2}}}{2\frac{13}{14}-\frac{\frac{1}{2}+\frac{1}{4}-\frac{1}{5}-\frac{1}{7}}{7\frac{1}{8}}}$

$$\frac{5\frac{6}{7}-\frac{1}{5}\text{ of }\frac{2}{3\frac{1}{2}}}{2\frac{13}{14}-\frac{\frac{1}{2}+\frac{1}{4}-\frac{1}{5}-\frac{1}{7}}{7\frac{1}{8}}}=\frac{5\frac{6}{7}-\frac{1}{5}\text{ of }\frac{4}{7}}{2\frac{13}{14}-(\frac{1}{2}+\frac{1}{4}-\frac{1}{5}-\frac{1}{7})\div 7\frac{1}{8}}$$

$$=\frac{5\frac{6}{7}-\frac{4}{35}}{2\frac{13}{14}-\frac{70+35-28-20\div\frac{57}{8}}{140}}=\frac{5\frac{30}{35}-\frac{4}{35}}{\frac{41}{14}-\frac{57}{140}\times\frac{8}{57}}=5\tfrac{26}{35}\div(\tfrac{41}{14}-\tfrac{8}{140})$$

$$=\tfrac{201}{35}\div\frac{410-8}{140}=\frac{\cancel{201}}{\cancel{35}}\times\frac{\overset{\overset{2}{\cancel{4}}}{\cancel{140}}}{\underset{\cancel{2}}{\cancel{402}}}=2\ \textit{Ans.}$$

Ex. 6. Simplify the expression, $\frac{1}{13}$ of $\cfrac{1}{1+\cfrac{\frac{1}{3}}{3+\frac{1}{4}}}$

$$\text{Now }\cfrac{1}{1+\cfrac{\frac{1}{3}}{3+\frac{1}{4}}}=\cfrac{1}{1+\cfrac{\frac{1}{3}}{\frac{13}{4}}}=\frac{1}{1+\frac{4}{39}}=\frac{39}{39+4}=\tfrac{39}{43},$$

$$\text{therefore, }\tfrac{1}{13}\text{ of }\cfrac{1}{1+\cfrac{\frac{1}{3}}{3+\frac{1}{4}}}=\tfrac{1}{13}\text{ of }\tfrac{39}{43}=\tfrac{3}{43}\ \textit{Ans.}$$

A fraction in this form is called a continued fraction.

Ex. 7. Find the value of $6\frac{5}{9}$ of $\cfrac{7}{2+\cfrac{\frac{6}{7}}{3+4\frac{1}{2}}}$

$$\text{Since } 3+4\tfrac{1}{2}=7\tfrac{1}{2}=\tfrac{15}{2}\ \therefore\ \frac{7}{3+4\frac{1}{2}}=\frac{7}{7\frac{1}{2}}=\frac{\frac{7}{1}}{\frac{15}{2}}=\tfrac{14}{15},$$

$$\text{therefore, } \frac{\frac{6}{7}}{3+4\frac{1}{2}}=\frac{\frac{\cancel{6}\,3}{1}}{\frac{\cancel{14}\,7}{15}}=\frac{3\times15}{7}=\tfrac{45}{7}=6\tfrac{3}{7},$$

$$\text{therefore, } \frac{7}{2+\frac{\frac{6}{7}}{3+4\frac{1}{2}}}=\frac{7}{2+6\frac{3}{7}}=\frac{7}{8\frac{3}{7}}=\frac{\frac{7}{1}}{\frac{59}{7}}=\frac{7\times7}{59}=\tfrac{49}{59}$$

$$\text{therefore, } 6\tfrac{5}{9}\text{ of }\tfrac{49}{59}=\frac{\cancel{59}}{9}\times\frac{49}{\cancel{59}}=\frac{49}{9}=5\tfrac{4}{9}\ \textit{Ans.}$$

111. In the following examples of addition, &c., of concrete quantities, fractions occur.

Addition.	£	s.	d.
	27	13	$6\frac{3}{8}$
	47	19	$8\frac{4}{5}$
	23	14	$6\frac{7}{10}$
	26	0	$1\frac{7}{20}$
	£125	7	$11\frac{9}{40}$

Here, adding the fractions of a penny we have—

$$\tfrac{3}{8}+\tfrac{4}{5}+\tfrac{7}{10}+\tfrac{7}{20}=\frac{15+32+28+14}{40}=\tfrac{89}{40}=2\tfrac{9}{40};$$

we put down the $\frac{9}{40}$ and carry the 2 to the column of pennies, and proceed as in common addition.

Subtraction.	mls.	fur.	yds.	ft.	in.
	5	2	114	2	$4\frac{3}{10}$
	2	3	118	2	$5\frac{7}{12}$
	2	6	215	2	$10\frac{43}{60}$

Here, on subtracting the fractions we find the lower one is the greatest, we have, therefore, to borrow 1 from the 4 inches; then—

$$1\tfrac{3}{10}-\tfrac{7}{12}=\tfrac{13}{10}-\tfrac{7}{12}=\frac{78-35}{60}=\tfrac{43}{60}.$$

Multiplication.	cwt.	qrs.	lbs.	ozs.	
	3	2	7	$13\frac{3}{8}$	$\times\ 33$
					$3\times11=33$
	10	2	23	$8\frac{1}{8}$	
				11	
	117	3	6	$10\frac{1}{8}$	*Ans.*

Here, $\frac{3}{8}\times3=\frac{9}{8}=1\frac{1}{8}$; we put down the $\frac{1}{8}$ and carry the 1 to the column of ounces. Similarly $\frac{1}{8}\times11=2\frac{1}{8}$.

Division.		yrs.	dys.	hrs.	mins.	secs.
	7)	2	312	15	24	$23\frac{1}{3}$
			148	22	46	$20\frac{10}{21}$

Here we divide as in common division till we come to the seconds, when we have 3 remaining. $3\frac{1}{3}\div7=\frac{10}{3}\div7=\frac{10}{21}$.

REDUCTION OF FRACTIONS OF CONCRETE NUMBERS.

112. Our attention has hitherto been confined to fractions considered *generally*, without regard to the particular species of their *units*, and it remains to apply what has been said to such *concrete* quantities as constitute the principal subjects of practical computation.

113. To reduce a given quantity to the fraction of another given quantity. In order to render possible the division of one concrete quantity by another concrete quantity, it is necessary that both should be in the *same denomination*. Hence the following rule:—

RULE XL.

Bring the proposed quantities into the same *(not necessarily the* lowest*) denomination, and take the* result *of the quantity that is to be* reduced *for the numerator, and the other* result *for the* denominator of the required fraction.

Examples.

Ex. 1. Reduce 16s. 5d. to the fraction of £1.

Here 16s. 5d. = 197 pence, and £1 = 240 pence. Therefore, the 197 pence in 16s. 5d. are to be divided by the 240 pence; or, $\frac{197}{240}$ is the required fraction of £1. The reason for this is as follows:—Since £1 contains 240 pence, and 16s. 5d. contains 197 pence, if the pound be divided into 240 equal parts and 197 of them be taken, these 197 parts will be represented by 16s. 5d.; but the fraction £$\frac{197}{240}$ represents that the pound has been divided into 240 equal parts and 197 of them taken;

therefore, 16s. 5d. = £$\frac{197}{240}$.

Ex. 2. Reduce 3s. 4d. to the fraction of £1.

$$\frac{3\text{s. }4\text{d.}}{£1} = \frac{40\text{d.}}{240\text{d.}} \therefore 3\text{s. }4\text{d.} = \tfrac{40}{240} \text{ of } £1 = \tfrac{1}{6} \text{ of } £1.$$

Or thus:—

3s. 4d. = $3\frac{1}{3}$s. = $\frac{10}{3}$s.,
and £1 contains 20s.,

therefore, $\frac{10}{3} \div 20 = \frac{10}{3} \times \frac{1}{20} = \frac{1}{6}$, the required fraction of £1.

Ex. 3. Reduce half-a-crown to the fraction of half-a-guinea.

Reducing them to pence we have the required fraction $= \frac{30}{126}$; but reducing to sixpences we have the same fraction in lower terms $= \frac{5}{21}$.

$$\frac{2\text{s. }6\text{d.}}{10\text{s. }6\text{d.}} = \frac{30\text{d.}}{126\text{d.}} \therefore 2\text{s. }6\text{d.} = \tfrac{30}{126} \text{ of half-a-guinea.}$$

$$\text{or} = \frac{2\text{s. }6\text{d.}}{10\text{s. }6\text{d.}} = \frac{5 \text{ sixpences}}{11 \text{ sixpences}} \therefore 2\text{s. }6\text{d.} = \tfrac{5}{11} \text{ of half-a-guinea.}$$

Note.—$\frac{5}{21}$ expresses what is called the *ratio* of 2s. 6d. to 10s. 6d.

Ex. 4. Reduce 3 qrs. 14 lbs. to the fraction of a ton.

$$\frac{3\text{ qrs. }14\text{ lbs.}}{1\text{ ton}} = \frac{98\text{ lbs.}}{2240\text{ lbs.}} \therefore 3\text{ qrs. }14\text{ lbs.} = \tfrac{98}{2240} \text{ of } 1 \text{ ton.}$$

Again, 3 qrs. 14 lbs. = $3\frac{1}{2}$ qrs. = $\frac{7}{2}$ qrs.,
and a ton contains 20 × 4 qrs.;
therefore, $\frac{7}{2} \times \frac{1}{20} \times \frac{1}{4} = \frac{7}{160}$ ton.

Ex. 5. Reduce 18s. 8d. to the fraction of half-a-crown.

$$18\text{s. }8\text{d.} = 18\tfrac{2}{3}\text{s.} = \tfrac{56}{3}\text{s.}$$
$$\text{and half-a-crown} = \tfrac{5}{2}\text{s.}$$

therefore, $\frac{56}{3} \div \frac{5}{2} = \frac{56}{3} \times \frac{2}{5} = \frac{112}{15} = 7\frac{7}{15}$ half-a-crown.

Ex. 6. What fraction of £3 10s. 6d. is £2 15s. 3d.?

Reducing both the quantities to threepenny pieces, we find that £3 10s. 6d. contains 282, and £2 15s. 3d. contains 221. Hence, if £3 10s. 6d. be divided into 282 parts, and 221 of these parts are taken, the result will be £2 15s. 3d. Hence, £2 15s. 3d. is the value of $\frac{221}{282}$ of £3 10s. 6d. The answer then will be $\frac{221}{282}$.

Ex. 7. Express 3 weeks 4 days 6 hours as the fraction of a year of 365¼ days.

$$3\text{ weeks }4\text{ days }6\text{ hours} = 25\tfrac{1}{4}\text{ days} = \tfrac{101}{4}\text{ days,}$$
$$\text{and the year consists of }365\tfrac{1}{4}\text{ days} = \tfrac{1461}{4}\text{ days;}$$

therefore, $\frac{101}{\cancel{4}} \times \frac{\cancel{4}}{1461} = \frac{101}{1461}$ of a year.

EXAMPLES FOR PRACTICE.

What fraction is—

1. 6s. 8d.; 12s. 6d.; 18s. 4d.; 17s. 11½d.; 19s. 10¾d., and £1 13s. 11½d. ½f. of £1.

2. 3s. 4d. of 16s. 8d.; 7½d. of 2s.; £4 0s. 6d. of £7 14s.; ¾d. of 6s. 8d.; £1 3s. 4d. of £9 6s. 8d.; 6s. 8½d. of 1¾d.; 6s. 7⅝d. of 7s. 9d.; 3s. 4d. of half-a-guinea; 16s. of £200.

3. 2 cwt. 1 qrs. 16 lbs. of a ton; 14^h 15^m of 3½ days; 1 ton of 3 cwt. 3 qrs. 21 lbs.; 20 ft. 7½ in. of 1 mile; 17 lbs. of 1 qr. 14½ lbs.; 23 tons 11 cwt. 1 qr. 1 lb. 1 oz. of 25 tons 6 cwt. 3 qrs. 10 lbs.; 439 mls. 7 fur. 211 yds. of 445 mls. 21 yds.; 2 wks. 5 dys. 7 hrs. 27 mins. of a day.

4. 126 yds. 2 ft. 6 in. of a mile; 6 cubic ft. 100 cubic in. of 1 cubic yd.; 6 ft. 3⅝ in. of 13 ft. 8$\frac{9}{10}$ in.; 1½ yd. of 1½ in.; 1 ft. ⅞ in. of sq. yd.

Reduce to lowest terms the following fractions—

5. $\frac{\text{£19 16s. 7¾d.}}{\text{£20 16s. 8¾d.}}$; $\frac{\text{217 mls. 5 fur. 18 pls. 2 yds. 2 ft. 1 in.}}{\text{506 mls. 2 fur. 23 pls. 1 yd. 2 ft. 7 in.}}$; $\frac{\text{1011 tons 18 cwt. 3 qrs. 10 lbs. 13 oz.}}{\text{70 tons 7 cwt. 0 qrs. 27 lbs. 1 oz.}}$

6. $\frac{\text{208 sq. miles 181 acres 0 ch. 93 yds. 4 ft.}}{\text{767 acres 9 ch. 279 yds. 4 ft.}}$; $\frac{\text{258 dys. 9 hrs. 19 mins. 57 secs.}}{\text{446 dys. 7 hrs. 23 mins. 33 secs.}}$

7. What fraction of 5 mls. 5 fur. 7 poles 11$\frac{1}{7}$ in. is $\frac{8}{17}$ of a league.

114. The value of a fraction of a concrete number is easily determined in terms of the same or lower denominations. For since ⅔ of a pound may be obtained either by dividing one pound into 3 equal parts, and taking 2 of these parts, or by taking the concrete unit twice, viz., 2 pounds, and dividing it into 3 equal parts; therefore we have—

⅔ of £1 = ⅓ of £2 = ⅓ of 40s. = 13s. 4d., and ⅔ of a mile = ⅓ of 3 miles, &c.

Hence—

To find the value of the fraction of a concrete quantity in terms of the same or lower denominator.

RULE XLI.

Multiply the given concrete quantity of the numerator of the fraction, and divide the product by the denominator, reducing the remainder (if any) *always to the lower denominations, as in compound division.*

EXAMPLES.

Ex. 1. Find the value of $\frac{4}{15}$ of a pound; that is, how many shillings and pence are in $\frac{4}{15}$ of a pound.

Here, $\frac{4}{15}$ of a £ $= \frac{4}{\underset{3}{\cancel{15}}} \times \frac{\overset{4}{\cancel{20}}}{1} = \frac{16}{3}$s., or $5\frac{1}{3}$s.

But $\frac{1}{3}$ of a shilling $\times \frac{1}{\cancel{3}} = \frac{\overset{4}{\cancel{12}}}{1}$ or 4d.

Therefore, $\frac{4}{15}$ of a £ = 5s. and 4d.

$$
\begin{array}{r|rrr}
 & £ & s. & d. \\
15 \left\{ \begin{array}{r} 3) \\ 5) \end{array} \right. & \begin{array}{r} 4 \\ \hline 1 \end{array} & \begin{array}{r} 0 \\ \hline 6 \end{array} & \begin{array}{r} 0 \\ \hline 8 \end{array} \\
\hline
 & 0 & 5 & 4
\end{array}
$$

Ex. 2. Find the value of $\frac{3}{7}$ of £24 12s. 4d.

$$
\begin{array}{rrr}
£ & s. & d. \\
24 & 12 & 4 \\
 & & 3 \\
\hline
7)73 & 17 & 0 \\
\hline
10 & 11 & 0
\end{array}
$$

Hence $\frac{3}{7}$ of £24 12s. 4d.
= £10 11s. 0d.

Ex. 3. Find the value of $\frac{3}{5}$ of 17 cwt. 3 qrs. 7 lbs.

$$
\begin{array}{rrr}
\text{cwt.} & \text{qrs.} & \text{lbs.} \\
17 & 3 & 7 \\
 & & 3 \\
\hline
5)53 & 1 & 21 \\
\hline
10 & 2 & 21
\end{array}
$$

Hence $\frac{3}{5}$ of 17 cwt. 3 qrs. 7 lbs.
= 10 „ 2 „ 21 „

Compound fractions must be reduced to simple ones before the application of this rule; but in the case of mixed numbers it is best to multiply separately and add the result to that obtained by the rule for the fractional parts.

Ex. 4. Multiply £2 10s. 4d. by $3\frac{5}{12}$.

$$
\begin{array}{rrrl}
£ & s. & d. & \\
2 & 10 & 4 & \times 3 \\
 & & 5 & \\
\hline
12)12 & 11 & 8 & \\
\hline
1 & 0 & 11\frac{2}{3} & = \frac{5}{12} \text{ of } £2\ 10s.\ 4d. \\
7 & 11 & 0 & = 3 \text{ times } \ldots\ \ldots\ \ldots \\
\hline
8 & 11 & 11\frac{2}{3} & = 3\frac{5}{12} \text{ times } 2\ \ 10\ \ 4
\end{array}
$$

Ex. 5. Multiply 17 cwt. 3 qrs. 12 lbs. by $5\frac{3}{5}$.

$$
\begin{array}{rrrl}
\text{cwt.} & \text{qrs.} & \text{lbs.} & \\
17 & 3 & 12 & \times 5 \\
 & & 3 & \\
\hline
5)53 & 2 & 8 & \\
\hline
10 & 2 & 24 & = \frac{3}{5} \text{ of } 17 \text{ cwt. } 3 \text{ qrs. } 11 \text{ lbs.} \\
89 & 1 & 4 & = 5 \text{ times } \ldots\ \ldots\ \ldots \\
\hline
100 & 0 & 0 & = 5\frac{3}{5} \text{ times } 17\ „\ 3\ „\ 12\ „
\end{array}
$$

Hence 17 cwt. 3 qrs. 12 lbs. × $5\frac{3}{5}$ = 100 cwt. = 5 tons. *Ans.*

Ex. 6. Divide 1 ton 13 cwt. 15 lbs. by $1\frac{1}{3}$.

Since $1\frac{1}{3} = \frac{4}{3}$, we have (§ 106) to multiply by $\frac{3}{4}$. We may do this as in Ex. 1, or (which is more convenient) by first dividing by 2, which gives $\frac{1}{2}$ of the quantity, and then dividing this half by 2, which gives $\frac{1}{4}$ of it, and adding the two results together, we shall have $\frac{3}{4}$ of it.

$$
\begin{array}{rrrrl}
\text{tons} & \text{cwt.} & \text{qrs.} & \text{lbs.} & \\
1 & 13 & 0 & 15 & \\
 & & & 3 & \\
\hline
4)4 & 19 & 1 & 17 & \\
\hline
1 & 4 & 3 & 11\frac{1}{4} & \textit{Ans.}
\end{array}
\qquad
\begin{array}{r|rrrrl}
 & \text{tons} & \text{cwt.} & \text{qrs.} & \text{lbs.} & \\
 & 1 & 13 & 0 & 15 & \\
\hline
\text{for } \frac{1}{2} & 0 & 16 & 2 & 7\frac{1}{2} & \\
\text{for } \frac{1}{4} & 0 & 8 & 1 & 3\frac{3}{4} & \\
\hline
 & 1 & 4 & 3 & 11\frac{1}{4} & \textit{Ans.}
\end{array}
$$

Ex. 7. Divide 20 hours 33 min. 20 sec. by $4\frac{1}{9}$.

$$20 \text{ hrs. } 33 \text{ min. } 20 \text{ sec.} \div 4\tfrac{1}{9} = 20 \text{ hrs. } 33 \text{ min. } 20 \text{ sec.} \div \tfrac{37}{9}$$

$$= 20 \text{ hrs. } 33 \text{ min. } 20 \text{ sec.} \times \tfrac{9}{37} = \frac{20 \text{ hrs. } 33 \text{ min. } 20 \text{ sec.} \times 9}{37}$$

$$= \frac{185 \text{ hrs.}}{37} = 5 \text{ hrs. } \textit{Ans.}$$

EXAMPLES FOR PRACTICE.

Find the value of—

1. £$\frac{3}{8}$; £$\frac{4}{15}$; £$\frac{13}{240}$; $\frac{3}{4}$ of £27 14s. 0d.; $\frac{4}{7}$ of £31 10s. 0d.; $19\frac{3}{4}$ of £5 1s. $1\frac{1}{4}$d.; $\frac{15}{16}$ of $2\frac{2}{3}$ of £8 14s. $2\frac{7}{12}$d. ÷ $8\frac{4}{5}$.

2. $\frac{4}{11}$ of £32 17s. $5\frac{3}{4}$d.; $\frac{2}{3}$ of $\frac{4}{5}$ of £17 16s. $10\frac{1}{2}$d.; $\frac{22}{35}$ of £128 9s. $10\frac{1}{2}$d.

3. $\frac{25}{36}$ of £140 3s. 6d.; $11\frac{7}{8}$ of 6s. $11\frac{1}{4}$d.; $\frac{4}{11}$ of a mile; $\frac{3}{16}$ of a day; $\frac{3}{8}$ of a yard.

4. $\frac{4}{7}$ of 1 ton 15 cwts.; $\frac{7}{15}$ of 3 tons; $\frac{5}{84}$ of 4 tons 16 cwt.; $\frac{7}{125}$ of 3 tons 15 cwt.

5. 1 cwt. 2 qrs. 13 lbs. × $3\frac{1}{24}$; $\frac{487}{576}$ of 1 ton 14 cwt. 3 qrs.; $\frac{1000000000}{1003003001}$ of 8288181 square miles.

6. 20 cwts. 3 qrs. 7 lbs. × $5\frac{1}{3}$; £38 2s. 6d. ÷ $3\frac{13}{16}$; $2\frac{1}{2}$ × $3\frac{1}{3}$ of $9\frac{1}{7}$ of £721 17s. 6d.

7. $\dfrac{1\frac{2}{3} \times 7\frac{1}{5}}{2\frac{1}{7} \text{ of } 2\frac{2}{3}} - \dfrac{4\frac{1}{3} \text{ of } 2\frac{2}{7}}{2\frac{4}{11} \div 7\frac{5}{8}}$ of £270; $2\frac{1}{2}$ × $3\frac{1}{3}$ of $9\frac{1}{7}$ of £721 17s. 6d; $\frac{4}{7}$ of $\frac{1}{3}$ of $\dfrac{13\frac{4}{5}}{3\frac{4}{7}}$ of a ton.

8. $5\frac{1}{3}$ times $\frac{2}{3}$ ÷ $3\frac{1}{5}$ × $\frac{121}{144}$ × 84 times $6\frac{3}{185}$ of £972; ($9\frac{4}{7}$ of $1\frac{1}{7}$ ÷ $3\frac{2}{15}$) × 41 times 140 tons 12 cwt. 1 qr.

9. $\dfrac{3\frac{1\frac{2}{15}}{4}}{20}$ of ton in cwts., &c.; $\dfrac{7\frac{5\frac{1}{2}}{12}}{2\frac{20}{88}}$ of £50; $\dfrac{39\frac{1}{2}}{\frac{1\frac{1}{4}}{8}}$ of 2 cwt.; $6\dfrac{\frac{11\frac{20}{80}}{12}}{14}$ of a week.

115. A fraction of a concrete number may be reduced to the fraction of another concrete number of a higher or lower denomination, by means of the principle employed in the reduction of integers from one denomination to another (see § 54, Rules XI and XII, pages 39 and 41).

EXAMPLE.

Ex. Let the fraction be £$\frac{2}{3}$, where the unit is one *pound;* then if it be required to find the fraction when the unit is one *farthing;* then since any integer number of pounds is reduced to farthings by multiplying the number of pounds by 20, and the product by 12, and this last by 4, so any fraction of a pound is reduced to the fraction of a farthing by multiplying the numerator by 20 × 12 × 4 or 960, that is, in order to retain the same *absolute* value, we must have 20 × 12 × 4 times as great a *fraction* as the original one: hence—

$$£\tfrac{2}{3} = \tfrac{2}{3} \times \tfrac{20}{1} \times \tfrac{12}{1} \times \tfrac{4}{1} = \tfrac{1920}{3} \text{ farthings},$$

and the value of the unit in the latter fraction being $\frac{1}{960}$th part of that in the former, the same *absolute* value is retained by taking 960 times as many parts in the latter as in the former.

Again, reversing the operation we have—

$$\tfrac{1920}{3} \text{ far.} = \tfrac{1920}{3} \times \tfrac{1}{4} \times \tfrac{1}{12} \times \tfrac{1}{20} = \tfrac{1920}{2880} = £\tfrac{2}{3},$$

the divisors 4, 12, 20 being inverted, according to the rule laid down in Division of Fractions (Rule XXXIX, page 83).

The preceding examples enable us to lay down the following rules for reducing a fraction of any given quantity to a fraction of another given quantity.

RULE XLII.

1°. If the given fraction has to be reduced from a *higher* to a *lower* denomination.

> *Multiply the* given fraction *by those numbers which in* whole numbers *would reduce the denomination in which the fraction stands to the denomination required.*

2°. But if it is required to reduce a given fraction of a *lower* to a *higher* denomination.

> *Divide the* given fraction *by those numbers which in* whole numbers *would reduce the denomination in which the fraction stands to the denomination required.*

Or we may reduce a number, or fraction, of one name to a fraction of another denominator by the following

RULE XLIII.

Express by (Rule XL) *the first quantity as a* fraction *of the* second, *and the fraction required will then be found by reducing the resulting compound fraction to a simple one.*

EXAMPLES.

Ex. 1. Reduce $\frac{2}{3}$ of a pound to the fraction of a penny.

Here we have a given fraction of a *higher* denomination, viz., the denomination of pounds, and it has to be expressed as an equivalent fraction of a *lower* denomination, viz., pence. Hence we *multiply* the given fraction by 20 and 12.

Thus, £$\frac{2}{3} = \frac{2}{3} \times \frac{20}{1} \times \frac{12}{1}$ of a penny $= \frac{480}{3}$, or $\frac{160}{3}$ of a penny.

Or (by Rule XL), £$1 = \frac{240}{1}$ of a penny, ∴ £$\frac{2}{3} = \frac{2}{\underset{3}{\cancel{9}}}$ of $\frac{\overset{80}{\cancel{240}}}{1} = \frac{160}{3}$ of a penny.

Ex. 2. Reduce $\frac{2}{3}$s. to the fraction of £1.

$\frac{2}{3}$s. $= \frac{2}{3}$s. $\div \frac{20}{1} = \frac{\cancel{2}}{3} \times \frac{1}{\underset{10}{\cancel{20}}} = \frac{1}{30}$ of £1.

The divisor $\frac{20}{1}$ is inverted, and the process is the same as in Multiplication of Fractions.

1s. $= \frac{1}{20}$ of £1, ∴ $\frac{2}{3}$s. $= \frac{\cancel{2}}{3}$ of $\frac{1}{\underset{10}{\cancel{20}}} = \frac{1}{30}$ of £1.

Ex. 3. Reduce $\frac{7}{8}$ of an hour to the fraction of a day.

$\frac{7}{8}$ hour $= \frac{7}{8} \div \frac{24}{1} = \frac{7}{8} \times \frac{1}{24} = \frac{7}{192}$ of a day.

The divisor $\frac{24}{1}$ is inverted, and we proceed as in Multiplication of Fractions.

Or (by Rule XL), $1^h = \frac{1}{24}$ of 1^d ∴ $\frac{7}{8}^h = \frac{7}{8}$ of $\frac{1}{24} = \frac{7}{192}$ of 1^d.

Ex. 4. Reduce $1\frac{2}{15}^h$ to the fraction of 10^m.

$$1\tfrac{2}{15}^h = \tfrac{17}{15}^h = \frac{17}{\underset{5}{\cancel{15}}} \times \frac{\overset{2}{\cancel{6}}}{1} = \tfrac{34}{5} = 6\tfrac{4}{5} \text{ of } 10^m.$$

$1^h = 6$ times 10^m, therefore multiply the fraction $\frac{17}{15}^h$ by $\frac{6}{1}$, the product is an equivalent [illegible]tion of 10^m.

Or (by Rule XL), $1^h = \frac{60}{10}$ of 10^m $\therefore$ $1\frac{2}{15}^h = \frac{17}{\underset{5}{\cancel{15}}} =$ of $\frac{\overset{2}{\cancel{6}}}{1} = \frac{34}{5} = 6\frac{4}{5}$ of 10^m.

Ex. 5. What fraction of £12 7s. 6d. is $\frac{11}{17}$ of £3 3s. 9d.

$$\frac{£3\ 3s.\ 9d.}{£12\ 7s.\ 6d.} = \frac{765}{2970}d. \therefore \frac{\cancel{11}}{\cancel{17}} \text{ of } \frac{\overset{\cancel{45}}{\cancel{765}}}{\underset{\underset{6}{\cancel{270}}}{\cancel{2970}}} = \frac{1}{6}. \textit{ Ans.}$$

Or, $\frac{11}{17}$ of £3 3s. 9d. $= \frac{11}{17} \times 3\frac{3}{16} = \frac{11}{17} \times \frac{51}{16}$.

And £12 7s. 6d. $=$ £$12\frac{3}{8} =$ £$\frac{99}{8}$,

$$\text{Therefore, } £\frac{\cancel{11}}{\cancel{17}} \times \frac{\overset{\cancel{3}}{\cancel{51}}}{\underset{2}{\cancel{16}}} \times \frac{8}{\underset{\underset{3}{\cancel{9}}}{\cancel{99}}} = \frac{1}{6}. \textit{ Ans.}$$

Ex. 6. Reduce $\frac{2}{7}$ of a pound to the fraction of a guinea.

$\frac{2}{7}$ of £1 $= \frac{2}{7} \times \frac{20}{1} = \frac{40}{7}$ of a shilling.

$\frac{40}{7}$ of 1 shilling $= \frac{40}{7} \times \frac{1}{21} = \frac{40}{147}$ of a guinea.

EXAMPLES FOR PRACTICE.

Reduce

1. £$\frac{3}{8}$ to the fraction of a guinea; $1\frac{3}{4}$s. to the fraction of £1; $\frac{2}{3}$ to the fraction of 15s.
2. $\frac{7}{8}$ hr. to fraction of 1 day; $\frac{4}{135}$ sq. yd. to fraction of sq. ft.; $\frac{24}{37}$ day to fraction of 1 hr.
3. $\frac{8}{2784}$ of 1 mile to fraction of a rood; $\frac{4}{9}$ sq. ft. to fraction of pole; $2\frac{3}{80}$ ton to fraction of 1 qr.
4. $\frac{5}{8}$ lb. troy to fraction of lb. av.; $\frac{4}{7}$ day to fraction of 7 weeks; $\frac{5}{1408}$ miles to fraction of a yard.
5. $\frac{2}{7}$ oz. troy to fraction of lb. troy, also to fraction of lb. av.; $\frac{4}{9}$ lb. to fraction of a cwt.
6. $\frac{7}{110}$ of a pole to fraction of a sq. yard; $\frac{88}{131}$ of a mile to fraction of a million inches.

116. By means of these reductions we may find the sum and difference of fractions of concrete numbers consisting of one or more denominations. Thus, to find the sum of $\frac{3}{4}$ of a pound, $\frac{5}{6}$ of a shilling, and $\frac{3}{4}$ of a crown, we may either reduce any two of them to the fraction of the denominator of the other, and then find the sum of these fractions, or we may find the value of each and then take their sum.

EXAMPLES.

Ex. 1. What is the difference between $\frac{5}{12}$ of £1 and $\frac{5}{14}$ of a guinea?

$$\frac{5}{12} \text{ of } £1 = \frac{5}{\underset{3}{\cancel{12}}} \times \frac{\overset{5}{\cancel{20}}}{1} = \frac{25}{3}s. = 8s.\ 4d.$$

$$\frac{5}{14} \text{ of a guinea} = \frac{5}{\underset{2}{\cancel{14}}} \times \frac{\overset{3}{\cancel{21}}}{1} = \frac{15}{2}s. = 7s.\ 6d.$$

Therefore the difference is 10d.

Ex. 2. Find the amount of $\frac{1}{3}$ of £1 + $\frac{2}{7}$ of a guinea — $\frac{4}{9}$ of 15s.

$\frac{1}{3}$ of 20s. = 6s. 8d.
$\frac{2}{7}$ of 21s. = 6s. 0d.

Sum of these is 12s. 8d.

$$\text{Subtract } \frac{4}{\cancel{9}_{3}} \text{ of } \frac{\overset{5}{\cancel{15}}}{1} \text{ or } \frac{20}{3}\text{s., or 6s. 8d.}$$

The required amount is 6s. 0d.

Ex. 3. How many shillings should be given in exchange for $\frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}}$ of a pound?

$$\frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}} = \frac{\frac{7}{12}}{\frac{5}{6}} = \frac{7}{\underset{2}{\cancel{12}}} \times \frac{\cancel{6}}{5} = \frac{7}{10}£$$

and $\frac{7}{10}$ of 20s. = 14s. *Ans.*

Ex. 4. Out of £4$\frac{3}{8}$, one-third is paid to A, and one-seventh to B; after this $\frac{4}{11}$ths of the remainder is paid to A, and the rest to B; find the sums respectively received by A and B.

£	s.	d.		£	s.	d.
3)4	7	6		7)4	7	6
1	9	2 the first sum paid to A.		0	12	6 the first sum paid to B.

Therefore, £2 1s. 8d. being paid away, there was (£4 7s. 6d.) — (£2 1s. 8d.) = £2 5s. 10d. left as remainder.

And $\frac{4}{11}$ × (£2 5s. 10d.) = 4 × (4s. 2d.) = 16s. 8d. the second sum paid to A.

Therefore, (£2 5s. 10d.) — (16s. 8d.) is all the rest, which is £1 9s. 2d., the second sum paid to B.

Therefore, £1 9s. 2d. + 16s. 8d. = £2 5s. 10d., sum received by A, and 12s. 6d. + £1 9s. 2d. = £2 1s. 8d., sum received by B.

Ex. 5. From $\frac{1}{2}$ of $\frac{1}{3}$ of a penny subtract $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of a shilling.

$$(\tfrac{1}{2} \times \tfrac{1}{3}\text{d.}) - (\tfrac{1}{4} \times \tfrac{1}{5} \times \frac{1}{\cancel{6}} \times \overset{2}{\cancel{12}}\text{d.})$$
$$= \tfrac{1}{6} - \tfrac{1}{10} = \frac{5-3}{30} = \tfrac{2}{30} = \tfrac{1}{15}\text{d.}$$

Ex. 6. Compare the values of $\frac{1}{96}$ of a pound, $\frac{7}{36}$ of a shilling, and $\frac{1}{112}$ of a guinea.

$$\tfrac{1}{96} \text{ of } £1 = \frac{1}{\underset{24}{\cancel{96}}} \text{ of } \overset{5}{\cancel{20}}\text{s.} = \tfrac{5}{24}\text{s.}$$

$$\tfrac{1}{112} \text{ of 1 guinea} = \frac{1}{\underset{16}{\cancel{112}}} \text{ of } \overset{3}{\cancel{21}}\text{s.} = \tfrac{3}{16}\text{s.}$$

Therefore, reducing to a common denominator the fractions $\frac{5}{24}$s., $\frac{7}{36}$s., and $\frac{3}{16}$s., they become respectively $\frac{30}{144}$s., $\frac{28}{144}$s., and $\frac{27}{144}$s., and the comparative value of the fractions are as 30, 28, and 27.

EXAMPLES FOR PRACTICE.

Find the value of—

1. $\frac{3}{5}$ of a ton + $\frac{5}{8}$ cwt. + $\frac{3}{8}$ lb.; $\frac{4}{5}$ lb. troy — $\frac{3}{8}$ oz. troy; $\frac{7}{10}$ mile — $\frac{5}{8}$ fur. + $\frac{4}{11}$ pol.
2. $\frac{8}{121}$ cub. yd. + 2$\frac{4}{7}$ cub. ft.; 7$\frac{3}{8}$ of 365$\frac{1}{4}$ dys. + 3$\frac{9}{10}$ of $\frac{5}{8}$ wks. + $\frac{3}{4}$ of 5$\frac{5}{6}$ hrs.
3. 1$\frac{2}{35}$ tons + $\frac{27}{37}$ cwt. + [illegible] lbs. in cwt.; $\frac{3}{40}$ ton + $\frac{4}{14}$ cwt. + 11[illegible] lbs. in tons.
4. Find the difference between $\frac{3}{5}$ of 1$\frac{7}{75}$ of 16 times £8 17s. 6d. and ($\frac{4}{5}$ — $\frac{1}{7}$ + 2$\frac{1}{5}$) of ($\frac{11}{12}$ × 1$\frac{3}{5}$ ÷ $\frac{41}{225}$) of £100, and find what fraction this difference is of £91 12s. 8d.
5. Compare the values of £$\frac{31}{40}$, $\frac{11}{12}$ of a guinea, and 3 × 4$\frac{1}{2}$ shillings; also of $\frac{1}{35}$ of a day, and $\frac{4}{7}$ of an hour.
6. Bring $\left(\frac{5\frac{1}{4} - \frac{1}{3} \text{ of } 2\frac{5}{6}}{\frac{3}{5} \times 4\frac{1}{3} + \frac{1}{12}} + \frac{2\frac{4}{7}}{4\frac{2}{7}}\right) \div 21\frac{23}{25}$ of 3$\frac{19}{208}$ cwt. to fraction of 4$\frac{1}{7}$ ton.
7. $\frac{\frac{1}{3} \text{ of } \frac{2}{5} + \frac{1}{4} \text{ of } \frac{5}{8}}{\frac{1}{5} + \frac{4}{7} - \frac{3}{8} + \frac{71}{83}}$ of $\frac{6\frac{1}{5}}{4\frac{2}{5}}$ of $\frac{1}{3}$ of a sq. foot; $\frac{\frac{1}{6} \text{ of } 1\frac{1}{3} - \frac{1}{3} \text{ of } \frac{2}{5}}{\frac{1}{3} - \frac{2}{5} + \frac{4}{5} - \frac{1}{12}}$ ÷ $\frac{3}{4}$ of $\frac{4}{7}$ of $\frac{3}{8}$ of a rood to the fraction of a furlong.

CHAPTER V.

DECIMAL FRACTIONS.

117. In the Notation of Integers, or common numbers, the actual value of each figure depends upon its position with respect to the place of units, its value in any one position being one-tenth of what it would be if it stood one place further to the left; thus the number 1111 denotes one thousand, one hundred, one ten, and one unit, or 1000 + 100 + 10 + 1, where the second unit beginning with the right-hand one is ten times the first, the third is ten times the second, the fourth ten times the third, and so on; or, beginning with the first on the left, the second is the tenth part of the first, the third the tenth part of the second, and so on, till we come down to the last unit, which is merely one; or, in other words, the figures decrease in a tenfold ratio from left to right.

118. Now we may evidently extend this principle still further, and on the same plan may represent one-tenth of *one*, one-tenth of this, or one-hundredth of *one*, one-thousandth of *one*, and so on, by simply putting some mark of separation between the *integers* and these *fractions*. The mark actually used is a ***dot*** or ***full stop*** and is called the *decimal point*,* thus 1111·1111. The unit (or 1) next the dot, on the left, is 1, the unit, one place from this on the left, is 10; the next is 100; the next 1000, and so on. In like manner, the unit next the decimal point, on the right, is $\frac{1}{10}$, the next $\frac{1}{100}$, the next $\frac{1}{1000}$, and so on. In other words, any figure one place to the right of the unit's place will be one-tenth of what it would be if it were in the unit's place; any figure two places to the right of unit's place will be one-hundredth of what its value would be if it were in the unit's place, and so on for any number of figures. Such fractions are called Decimals.

We shall in this manner have a decimal scale of notation extended below unity.

The whole system of Decimal Notation may, therefore, be denoted thus,—

7 Millions.	6 Hundreds of Thousands.	5 Ten Thousands.	4 Thousands.	3 Hundreds.	2 Tens.	1 Units.	2 Tenths.	3 Hundredths.	4 Thousandths.	5 Tens of Thousandths.	6 Hundred of Thousandths.	7 Millionths.

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* The decimal point should be put at the top of the line of figures thus, 5·7, because 5.7, with a stop at the bottom, is used in most works to mean 5 × 7 = 35.

119. Returning now to the values of the figures on the right of the decimal point, it will be observed that each is a *fraction* having ten, or *some power** of ten, *for its denominator*, although in the decimal system of notation these denominators are never expressed.

The following definition is now clear—

Def. *A decimal fraction is one whose denominator—never expressed as such—is either ten or some power of ten.*

NOTE 1.—It is not necessary that there should be any whole numbers in order that a fraction may be expressed in the decimal system, for

one one hundredth may be expressed thus, ·01
five ten thousandths, ·0005

NOTE 2.—Decimal Fractions are also called *Decimals*.

120. A decimal fraction may either be considered as the sum of as many fractions as it contains digits, or as a single fraction, thus,—

EXAMPLES.

Ex. 1. $\cdot 567 = \frac{5}{10} + \frac{6}{100} + \frac{7}{1000}$.

(The L.C.M. of their denominators is 1000).

$$\therefore = \frac{500 + 60 + 7}{1000} = \frac{567}{1000}.$$

Ex. 2. $\cdot 03057 = \frac{0}{10} + \frac{3}{100} + \frac{0}{1000} + \frac{5}{10000} + \frac{7}{100000}$.

(The L.C.M. of their denominators is 100000).

$$\therefore = \frac{3000 + 50 + 7}{100000} = \frac{3057}{100000}.$$

Ex. 3. $18\cdot 204 = 18 + \frac{2}{10} + \frac{0}{100} + \frac{4}{1000}$.

(The L.C.M. of their denominators is 1000).

$$\therefore = 18 + \frac{200 + 4}{1000} = \frac{18204}{1000}.$$

Hence we see,—

1st. *That a decimal is always equivalent to the vulgar fraction which has for its numerator the significant*† *figures of the decimal fraction, and for its denominator the denominator of the last figure on the right of the decimal.*

2nd. *That this denominator may also be found by writing a cypher for every figure in the decimal fraction and placing a* 1 *before it,*

$$\text{Thus, } \cdot 648 = \frac{648}{1000};\ \cdot 0091 = \frac{91}{10000};\ 7\cdot 01 = 7\tfrac{1}{100}.$$

3rd. *That any number composed of integers and decimals may be expressed as a vulgar fraction having all the figures of the number for its numerator and the denominator of the last decimal on the right for its denominator,*

$$\text{Thus, } 814\cdot 0315 = \frac{8140315}{10000}.$$

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* A power of a number is found by multiplying that number by itself a certain number of times. When a number is multiplied by itself *once*, the product is called the *second* power of that number; if the number be multiplied by itself *twice*, the product is said to be the *third* power of that number, &c.

† The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, in a decimal are sometimes called *significant* digits; thus, in such a decimal as ·0002345, we should say that 2345 are the significant digits, being those which, taken by themselves alone, have some real value, because they indicate a number, the cyphers only serving to fix the place in which the 2 occurs. Again, in ·0407 the figures 407 are the significant digits.

121. Conversely, *any vulgar fraction having* 10, *or any power of* 10, *for its denominator, may be expressed as a decimal.*

Thus,

Ex. 1. $\frac{826}{10} = 82\frac{6}{10} = 82{\cdot}6$ (§ 117-118).

Ex. 2. $\frac{82349}{100} = 823\frac{49}{100} = 823{\cdot}49$ (§ 117-118).

We generally speak of any figure in a decimal as being in *such a place of decimals;* thus, for instance, in 3·14159, we should say that the 5 is in the fourth place of decimals, the 9 in the fifth place, and so on, reckoning from left to right.

122. To read off, or express in words, decimal fractions, *read the decimal figures as if* whole *numbers, and to the last figure* add *the* name *of the order determined by the place it occupies;* thus, ·734 is read *seven hundred and thirty-four* thousandths; 58·64327 is read fifty-eight, *together with sixty-four thousand three hundred and twenty-seven* hundred-thousandths.

In reading decimals as well as whole numbers, the *unit's* place should always be made the *starting* point. It is advisable for the learner to apply to every figure the name of its order, or the place which it occupies, before attempting to read them. Beginning at the unit's place he should proceed towards the right, thus, *units, tenths, hundredths, thousandths,* &c., pointing to each figure as he pronounces the name of its order. In this way he will be able to read decimals with as much ease as he can whole numbers.

123. The value of the decimal figures depending entirely on the place they occupy with respect to the point which separates the units from the tenths, any number of cyphers on their right may be annexed or effaced, without altering the value of the *significant* figures. For instance, 0·7 is the same as 0·70, because the number that expresses the decimal fraction becomes ten times greater while its parts become hundredths, and are therefore diminished ten times,

$$\text{thus, } \frac{7}{10} = \frac{70}{100} = \frac{700}{1000}, \text{ \&c.}$$

and hence it is evident that annexing cyphers to the right-hand of decimals does not change their value, for we only multiply both numerator and denominator by 10, 100, &c., and consequently do not alter the value at all. Again, take a decimal such as ·56, which as already explained, means 5 tenths 6 hundredths, it will follow that ·560 means 5 tenths, 6 hundredths, *no* thousandths; whence the addition of the cypher to the right-hand has made no alteration in the value of the decimal. In fact

$${\cdot}56 = \frac{56}{100} \text{ and } {\cdot}560 = \frac{560}{1000} = \frac{56}{100}.$$

Similarly, ·23, ·230, and ·2300, are all of equal value, for expressed as fractions they are respectively $\frac{23}{100}$, $\frac{230}{1000}$, and $\frac{2300}{10000}$.

The effect of prefixing and affixing cyphers to decimals is thus exactly the opposite of the effect produced by prefixing and affixing cyphers to integers.

124. It is evident that noughts on the extreme left, or on the extreme right, of any set of figures, are only of use where the unit point cannot be properly

marked without them. Thus, in 0089·3200 the noughts are useless; but in the case of 8900· they are needed to distinguish eight thousand nine hundred from 89, and conversely in ·0043 they are needed to distinguish $\frac{4}{1000} + \frac{3}{10000}$ from $\frac{4}{10} + \frac{3}{100}$.

125. But placing cyphers between the decimal point and the other decimal figures does alter the value of the decimal, because this alters the place of the significant digits, the value being diminished ten times for each cypher that is prefixed;

thus, $·7 = \frac{7}{10}$, $·07 = \frac{7}{100}$, $·007 = \frac{7}{1000}$, and so on.

126. We infer from this, that as the value of a decimal is *decreased* ten-fold for every cypher added to the left-hand, we do in fact *divide* a decimal by 10, by 100, by 1000, &c., as we shift the decimal point one, two, three, &c., places to the *left;* and that conversely by shifting the decimal point one, two, three, &c., places to the *right*, we multiply the decimal by 10, by 100, by 1000, &c. For instance, the expression 56·789 is divided by 10 if written 5·6789, is divided by 100 if written ·56789, and is divided by 1000 if written ·056789; whereas the expression ·00723 is multiplied by 10 if written ·0723, is multiplied by 100 if written ·723, and is multiplied by 1000 if written 7·23.

127. The advantage arising from the use of decimals consists in this, viz., that the addition, subtraction, multiplication, and division of *decimal* fractions are much more easily performed than those of *vulgar* fractions; and although all vulgar fractions cannot be reduced to finite decimals, yet we can find decimals so near their true value, that the error arising from using the *decimal* instead of the vulgar fraction is not perceptible.

EXAMPLES FOR PRACTICE.

Express as decimals—

1. $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{33}{100}$, $\frac{7}{100}$, $\frac{3}{1000}$, $\frac{11}{10000}$, $\frac{2178}{10000}$, $\frac{2178}{100000}$, and $\frac{2178}{100}$.
2. $\frac{23}{1000000}$, $\frac{1041}{1000000}$, $\frac{6}{100000000}$, $\frac{9}{1000000}$, $\frac{4203}{10}$, $\frac{29}{100}$, and $\frac{20143}{10000}$.
3. $\frac{672819}{100000}$, $\frac{672819}{10000000000}$, $\frac{67281900}{10000}$, $\frac{3}{10000000}$, $\frac{40001}{100}$, $\frac{4200416}{100000}$, $\frac{40000101}{\text{millionths}}$, $\frac{401}{\text{billionths}}$.

In the following mixed numbers write the fractional parts in decimals:—

4. $7\frac{89}{1000}$, $43\frac{2143}{100000}$, $9\frac{7823457}{1000000000}$, $1\frac{1}{10000000}$, and $35\frac{721341}{1000000}$.
5. $53\frac{9}{10}$, $6\frac{39}{1000}$, $8\frac{4}{10000000}$, $3\frac{7000}{100000}$, $9\frac{409537}{1000000000000}$, $902\frac{80401}{10000000}$.

Express as vulgar fractions, and reduce to lowest terms:—

6. ·7, ·07, ·007, ·000007, ·327, 3·27, ·45697, 456·97, ·893, ·0000893, 47·256, ·0050005.
7. ·0000001024, ·000000000576, ·9241, 67·09, ·2064919, ·000000000000000065536.

Express as decimal fractions the following:—

8. Seventy-three *thousandths;* one hundred and ninety-seven *ten thousandths;* one *millionth;* two hundred and sixty-one *hundred-thousandths;* one thousand and one *ten millionths.*
9. One, and fifty-four *hundredths;* twenty-four, and seventy-nine *thousandths;* three hundred and fifteen, eight *thousandths;* and fifty *millionths;* eleven hundred *millionths;* nine *thousandths.*

10. One *tenth*; three *hundredths*; five *thousandths*; one hundred and five *thousandths*; two *millionths*; sixty *millionths*; forty-one, and eight *hundredths*; one thousand, and one *thousandths*; thirty, and six *millionths*; one hundred *thousandths*; two thousand three hundred and seventy-five hundred *millionths*.

11. Four thousand and four, and three hundred and forty-five *ten-millionths*; three, and four hundred *thousandths*; five hundred and twenty-four millions six hundred and twenty-four, and eight thousand and thirty-four *ten-millionths*; one hundred thousand, and one *ten-millionth*.

Express in words the following:—

12. ·6, ·92, ·5498, 7·07, 26·405, ·000201, ·00037, 11·101101, ·0440308, ·82344, ·13236.

13. 9·0457, 4004·0000345, 3·4001, 524000534·0008034, ·000003705, ·000024056, 7005·000000674, 100000·0000001, 10·001, 9·000028, 1·0006003.

14. 1·000001, ·1000001, ·00000001, 1·13004, 9·203167, 4·3008004, 27·4627350.

15. Multiply each of the quantities ·0007453, 48·95621, and 8·76430071 successively by one thousand, by ten thousand, and one hundred thousand; and divide each of 531·674, ·000317, and 902030401 successively by one million, by ten million, and by one hundred million.

16. Multiply 47·83 by 10, 100, 1000, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, ·5 × 1000, 75.

ADDITION.

128. Decimals, or integers and decimals mixed, may be added together precisely as in whole numbers, care being taken so to arrange the figures that all the decimal points fall exactly under one another. This will ensure that *tenths* fall under *tenths*, *hundredths* under *hundredths*, *&c.* The reason of this arrangement will appear from the following consideration; if this rule were *not* observed, tenths would fall under hundredths, or hundredths under thousandths, as the case might be; and we should be attempting to add together fractions which had not common denominators. But if we arrange the decimal points all exactly beneath one another, tenths fall under tenths, hundredths under hundredths, &c., in other words, by so arranging them we at once bring the several fractions to a common denominator, and can proceed to add them together. The decimal point in the answer will fall exactly beneath the decimal points in the quantities to be added. When the sum of any figures exceeds 10, 20, &c., *carrying* to the next denominator will be performed exactly as in whole numbers, whether the given quantities are all decimals or are mixed integers or decimals. For as the value of each figure decreases tenfold as we proceed from left to right, the rules of ordinary addition are immediately applicable. For instance, let it be required to add together ·8, ·78, and ·678.

$$\begin{array}{r} \cdot8 \\ \cdot78 \\ \cdot678 \\ \hline 2\cdot258 \\ \hline \end{array}$$

Where we see that after writing in the answer 8 in the place of *thousandths*, that 7 *hundredths* and 8 *hundredths* added together make 15 *hundredths*; but 15 *hundreds* are 1 *tenth* and 5 *hundredths*, writing 5 in the place of *thousandths*, and carrying one to the place of *tenths*, we obtain 22 *tenths*; but 22 *tenths* are properly written as 2 *integers* and 2 *tenths*.

Again, where integers and decimals are mixed.

$$\begin{array}{r} 3\cdot007 \\ 42\cdot6 \\ \cdot3975 \\ \hline 46\cdot0045 \\ \hline \end{array}$$

Where writing 5 in the place of *ten thousandths*, the sum of 7 *thousandths* and 7 *thousandths* is 14 *thousandths*; writing 4 in the place of *thousandths*, and carrying one to the place of *hundredths* we obtain 10 as the sum in the *hundredths* place; but 10 *hundredths* are 1 *tenth*, carrying 1 to the place of *tenths*, we have 10 *tenths*; but as 10 *tenths* are 1 *unit*, we carry 1 to the place of integers, and write 6 in the place of *units*, and 4 in the place of *tens*.

RULE XLIV.

1°. ***Place the quantities so that their decimal points shall be in the same vertical line;*** **for then the quantities of the same denomination will stand together.**

2°. ***Then proceed as in addition of whole numbers.***

EXAMPLES.

Ex. 1. Add together 27·5037, ·042, 342, and 2·1.

$$\begin{array}{r@{\cdot}l} 27&5037\\ &042\\ 342&\\ 2&1\\ \hline 371&6457\end{array}$$

Ex. 2. Add together 23·628, 4·1056, ·0137, and ·0042.

$$\begin{array}{r@{\cdot}l} 23&628\\ 4&1056\\ &0137\\ &0042\\ \hline 27&7515\end{array}$$

Ex. 3.

$$\begin{array}{r@{\cdot}l} 1234&6789\\ 13&\\ 170&\\ &0054\\ &5\\ 87&142\\ \hline 1505&3263\end{array}$$

Ex. 4.

$$\begin{array}{r@{\cdot}l} 66199&3226\\ &301\\ 54&5\\ &00632\\ 1000&\\ &07\\ 32745&80008\\ \hline 100000&\end{array}$$

EXAMPLES FOR PRACTICE.

Add together

1. **·295, 3·086, 12·87, ·0051, and 729·54; ·08, 165, 1·327, ·0003, 2760·1, and 9.**
2. **8·1, 40·652, 98·51, 695·7, and 43·9706; 69·75, 0·97, 0·059, 672·5, 4·8, and 932·6.**
3. **897·4, 63·18, 400·03, 7·9, 63·9, and 4·0079.**
4. **3608·26, 360·826, 36·0826, 3·60826 and ·22314.**
5. **36·053, ·0079 ·000952, 417, 85·5803, and ·0000501.**
6. **87·1 + 0·376 + ·0056 + 49 + 3·009 + ·709; 21·74, ·075, 103·00375, ·0005495, and 4957·5**
7. **·8 + ·046 + 9·1 + 3·09, + 8·6409 + 32.**
8. **·0003025, 29·99987, 143·2, 5·000025, 9000, and 3·4073.**
9. **.1 + ·1 + ·01 + ·001 + ·0001; 1 + ·2 + ·03 + ·004 + ·0005.**
10. **19000·00019 + 208·20808 + 64·39007 + 210860·8000341 + 9·637.**
11. **Eighteen hundredths; seven hundred and forty-five hundred thousandths; nine thousandths; forty-three millionths; five hundred and eight thousandths; one hundred and thirty-two thousandths; one thousand and forty-four ten millionths; twenty-five hundredths; five tenths; and six hundred and five thousandths.**

SUBTRACTION.

129. In subtraction of decimals, or of integers and decimals mixed, for reasons precisely similar the decimal points must be arranged to fall exactly beneath one another; and then the smaller quantity can be subtracted from the larger in the same manner as in whole numbers, ***thousandths*** **being taken from** ***thousandths, hundredths*** **from** ***hundredths, tenths*** **from** ***tenths.*** **The decimal point in the answer will fall exactly beneath the decimal points in the subtrahend and minuend, cyphers may be added (or supposed to be added) to the** ***right*** **of the decimal figures in the minuend, as this will not alter the value (§ 123), and the subtraction may proceed as in whole numbers.**

RULE XLV.

1°. *Place the quantities so that their decimal points shall be in the same vertical line.*

2°. *Next proceed as in subtraction of whole numbers.*

Examples.

Ex. 1. Subtract ·756 from ·897.

·897
·756

·141

Here the difference between 6 *thousandths* and 7 *thousandths* is 1 *thousandth*, between 9 *hundredths* and 5 *hundredths* is 4 *hundredths*, between 8 *tenths* and 7 *tenths* is 1 *tenth*.

Ex. 2. From 37·6 take ·907.

In this instance 37·6 may be written 37·600.

37·600
·907

36·693

Ex. 3. From 98765·4321 take 99·99.

98765·4321
99·99

98665·4421

In this instance 2 cyphers may be supposed annexed to the right of 99·99 as 99·99 = 99·9900.

Ex. 4. Subtract ·97658 from 5·1394.

5·1394
·97658

4·16282

In subtracting 8, 0 is supposed to occupy the place above it, as 5·13940 = 5·1394.

Ex. 5.

·01
·0000999

·0099001

Ex. 6.

1·
·47712

·52288

In examples of this kind (4, 5, 6, and 8), when the number of decimal figures in the lower lines exceeds the number of figures in the upper, it is advisable to mentally supply cyphers to make up the deficiency in the upper line. This may be done without altering the value of the upper line.

Ex. 7.

356·285457
256·285458

99·999999

Ex. 8.

1
·000001

·999999

Examples for Practice.

Subtract

1. 364·3123 from 456·0546; 32·756218 from 1460·39; ·009623478 from ·076345; ·87623 from 24681.

2. 3·07 from 6·501; ·99 from 1; 2·9989 from 3; ·000001 from 10; and 10·415 from 200.

3. ·0090806 from 39·857; ·000001 from ·0001; ·876534 from 8·21314; ·000004 from ·0004; ·9682347 from 65·00001; ·79999 from 9; 7 hundredths from 700; and 65·84003 from 603.

4. ·999999 from 9; ·00000099 from 99; 9·163 from 81·6823401; ·999 from 3·29.

5. From 42 hundredths take 42 thousandths; from 96 thousandths take 909 ten thousandths; from 7 take 7 tenths; from 7 tenths take 7 millionths.

Find the results of the following mixed additions and subtractions:—

6. 18·79 + 1·003 — 19 + 271·94 — ·000125; 243·12 + ·0101 — 79·89 + 8·451 — 129·99.

7. ·30103 + ·47712 — ·84510 + ·22185 — ·15490.

8. 814·21903 — 1007·612 — 203·05566 + 6002·1918 — 50·13826605.

MULTIPLICATION.

130. We have stated that for every place we shift the decimal point to the *right* we increase the value of the decimal *ten-fold*, for every place we shift it to the *left* we decrease it *ten-fold*. Now, in multiplying two decimals together, since the law of *local value* holds good with regard to the digits comprising the decimals, the process of multiplication will be performed exactly as in ordinary whole numbers; the only matter requiring consideration will be the proper position of the decimal point. Suppose we have to multiply 4·935 by 6·28, and let us suppose the decimal point in each case removed to the extreme right. Then (§ 123) we have multiplied the number 4·935 by 1000, and the number 6·28 by 100, and we have obtained the numbers 4935· and 628· respectively. Now, 4935 × 628 = 3099180, but as we have increased our original numbers one thousand and one hundred fold respectively, it is evident our product is increased 1000 × 100, or one hundred thousand fold. Dividing, therefore, the above result, 3099180 by 100000, or which is the same thing (§ 123), writing it 3099180· and removing the decimal point 5 places to the left, we get for the product of the numbers 4·935 and 6·28 the result 30·99180. It will be seen that the number of decimal places in the product, namely 5, is the sum of the number of decimal figures in the two given numbers.

We have, therefore, the following rule for multiplication:—

RULE XLVI.

Multiply the numbers together, as whole numbers, and point off as many decimal places in the product (beginning at the right) as there are decimal places in the multiplier and multiplicand together.

When the decimal places to be pointed off are more in number than the figures of the product, make up the proper number by prefixing cyphers to the product.

EXAMPLES.

Ex. 1. Multiply ·784 by ·8.

·784
·8
·6272

There are *three* decimal places in the multiplicand and *one* in the multiplier, *i.e.*, *four* in all; and, since there are also *four* fiures in the product, the whole product is a decimal.

Ex. 2. Multiply ·0048 by ·000012.

·0048
·000012
·0000000576

The product of 48 and 12 is 576. The number of decimal places in the multiplicand and multiplier together being *ten*, while the product contains but *three*, it is necessary to prefix *seven* cyphers to the product to give the ten places of decimals required.

Ex. 3. Multiply 236000 by ·48.

236000
·48
1888
944
113280·00

The product of 236 by 48 is 11328; in 236000 are *no* decimals; in ·48 are *two* decimals; therefore *two* places are pointed off in the product.

Ex. 4. Multiply 5·63 by ·00005.

5·63
0·00005
0·0002815

In 5·63 are *two* decimals; in ·00005 are *five*; therefore three cyphers must be prefixed to the product 2815, and *seven* places of decimals marked off.

Ex. 5. Find the value of 19·01 × 4·308.

$$\begin{array}{r} 4·308 \\ 19·01 \\ \hline 4308 \\ 38772 \\ 4308 \\ \hline 81·89508 \\ \hline \end{array}$$

In 19·01 are *two* decimals; in 4·308 are *three* decimals; therefore *five* decimal places must be marked off from the product.

Ex. 6. Find the value of 1·005 × ·005 × ·0064.

$$\begin{array}{r} 1·005 \\ ·005 \\ \hline ·005025 \\ ·0064 \\ \hline 20100 \\ 30150 \\ \hline ·0000321600 \end{array}$$

EXAMPLES FOR PRACTICE.

Find the value of

1. 2·5 × 4; ·25 × 40; 2·5 × 476; 2·5 × 4·76; ·0025 × 4·76; ·025 × ·0476.
2. ·0002 × ·00101; 90·01 × 0·034; ·0008 × ·00014; ·6005 × ·0035.
3. ·0783 × ·461; ·2764 × 96; ·06948 × ·0087; ·00043 × 4700.
4. 230·015 × 5·002; 521801 × ·801; 507 × ·507; 2020 × ·202; ·05 × ·0086.
5. 1·075 × ·0101; 8·004 × ·004; ·0006 × ·00012; 923521 × ·28629151.
6. 100·0008 × ·000306; 7535060 × 62·3906; 31·50371 × 17·0352.
7. ·000713 × 2·30561; 42·10062 × 3·821013; 1·0142034 × ·0620034.
8. 25067823 × ·0030001; 394·2003 × ·00000003; ·834567834 × ·00000008.
9. 22·5 × ·0241 × ·024; ·0003 × ·01 × 500000; ·12 × 1·2 × ·012 × 120.
10. 2·7 × ·27 × ·027 × 270; ·2 × ·04 × ·008 × 64000.

DIVISION.

131. Let it be required to divide 37·015 by 6·73.

By shifting the decimal point to the right of the dividend and divisor, so as to turn both into whole numbers, we increase the dividend 1000-fold and the divisor 100-fold. The former of these alterations will have the same effect as multiplying the quotient by 1000, the latter the same as dividing by 100, so that the quotient will be 10 times too great, and must be further divided by 10, *i.e.*, one decimal place must be pointed off to give a correct result.

Had it been required to divide 370·15 by 6·73, where there is the same number of decimal places in both dividend and divisor, by shifting the decimal points so as to make both whole numbers, we should increase the dividend 100-fold and the divisor 100-fold; this would not affect the value of the result, and the quotient would be a whole number requiring no decimal point at all.

If the given qualities had been 370·15 and ·673, so that there had been fewer decimal places in the dividend than in the divisor by converting both into whole numbers, we should have increased the dividend 100-fold and the divisor 1000-fold, and to obtain the correct result we should had to *multiply* the quotient by 10.

We can hence determine the following practical rule for the division of decimals.

RULE XLVII.

Divide as in whole numbers. The rule for placing the decimal point is, that the quotient must have as *many decimal figures* as the decimal places in the dividend *exceed* those in the divisor; that is, the *quotient* and divisor together **must contain as many decimals as the dividend.**

Examples.

Ex. 1. Divide 17·68 by 3·4.

3·4)17·68(5·2
170
68
68

Here 17·68 contains two decimals; 3·4 contains one; therefore 52 must contain the remaining one required, and be written 5·2.

Ex. 2. Divide 547·8 by 66.

66)547·8(8·3
528
198
198

Here 547·8 contains one decimal; 66 none; hence 83 must contain one, and be written 8·3.

Ex. 3. Divide 4·784 by 9·2.

9·2)4·784(·52
460
184
184

Here 4·784 contains three decimals, and 9·2 one, the remaining two required must therefore be obtained by pointing off both figures thus, ·52.

Ex. 4. Divide 1353·6 by 37·6.

37·6)1353·6(36
1128
2256
2256

Here the dividend has one decimal, and the divisor also one, or as many, and the quotient is therefore an integer.

(a) *When the dividend has no decimals, cyphers must be annexed, preceded by the decimal point.*

Examples.

Ex. 1. Divide 38 by ·08.

·08)38·00
475

Annexing two cyphers to 38 gives the complete quotient 475.

Ex. 2. Divide 132 by 0·7.

0·7)132·00000
1885714

Annexing five cyphers (decimals) gives quotient 1885714. Then the number which added to one decimal in 0·7 to make up five is four. *Ans.:* 188·5714.

(b) *When the number of figures in the quotient is not sufficient to make up the required number of decimals, cyphers must be prefixed.*

Examples.

Ex. 1. Divide ·30285 by 67·3.

67·3)·30285(45
2692
3365
3365

Here ·30285 contains five decimals, and 67·3 contains only one; the quotient 45 contains only two, and four are required; hence two cyphers must be prefixed, and the quotient written as ·0045.

Ex. 2. Divide 4·784 by 92.

92)4·784(52
460
184
184

Here 4·784 contains three decimals, and 92 none; the quotient therefore must contain three, and becomes ·052.

(c) When the number of decimal places in the *dividend* is *less* than the number in the *divisor*, *annex as many cyphers to the dividend as will make the number equal to the number in the divisor, and then proceed as in whole numbers.*

EXAMPLES.

Ex. 1. Divide 14·4 by ·0012.

$$\cdot 0012)14\cdot 4000$$

$$12000$$

The divisor contains *four* decimal places and the dividend only *one*, annex *three* cyphers to the dividend, so as to make the number of decimal places in both divisor and dividend equal, and then divide by 12.

Ex. 2. Divide 3028·5 by ·673.

$$\begin{array}{l} \cdot 673)3028\cdot 500(4500 \\ \quad\ \ 2692 \\ \quad\ \ \overline{\ \ 3365} \\ \quad\ \ \ \ 3365 \\ \quad\ \ \overline{\ \ \ \ \ \ 00} \end{array}$$

In this instance the divisor contains *three* decimal places and the dividend only *one*; therefore *two cyphers* are annexed to make the number of decimal places in each equal.

The divison may always be carried to any degree of accuracy by annexing cyphers to the dividend, as is seen in example 2 (a).

The decimal point may be removed altogether from both the divisor and dividend, by continually multiplying each by 10; for the quotient will thus remain unaltered. The first decimal in the quotient will then appear only with the first cypher annexed to carry on the division.

Ex. Divide 55·80 by 0·04. Multiplied by 10 they become 558 and 0·4, multiplied again they become 5580 and 4, the quotient of which is 1395.

This easy process furnishes a complete security against wrongly placing the decimal point in the quotient.

EXAMPLES FOR PRACTICE.

1. Divide 99·12105 by 5, 7, 9, 36, and 88; 6·66816 by 4, 8, 25, and 630.
2. „ 6541·234567 by 21; 7461·30765 by 112; 2188·054 by 983; 8727588 by 1620.
3. „ ·0029202 by 157 and by 1·57; 302·85 by 6·73; 3028·5 by ·673; 30·285 by 6·73.
4. „ ·00033 by ·011; 140·02564 by 1·871; 4·32067 by ·001; ·59 by 8000; 167342 by ·002; ·001024 by 30517578125; 1001 by 16384; 412 by 515; 17 by 4·25.
5. „ 325·67543 by 20·02; 684234·6 by 2682; 73·8243 by ·061; ·00006 by ·003.
6. „ 992 by ·37; ·1599 by 4100; ·019 by 190; 1 by 152; ·2 by 23·2.
7. „ 1·95 by ·00013; 9·614 by ·0000019; ·25 by 31·25; 8·92 by 237·6567.
8. „ 61000 by ·825; 570 by ·005; 51·5 by 412; ·0468 by ·1755; ·1755 by ·0468.
9. „ ·012550 by 1004000; 12·55 by ·01004; 1255 by 10·04; ·02916 by ·0012.
10. „ ·001255 by 1004; ·032016 by ·004; ·000000202 by ·00101; ·002 by 34·2.
11. „ 7821 by ·079; ·10304 by 9200; ·00015 by ·016; 500 by ·25.
12. „ ·6 by 6; 6 by ·6; ·06 by 60; ·006 by 6; 600 by ·6; 600 by ·06; ·006 by 600.
13. „ (1) $\frac{1}{17}$ to 10 decimal places; (2) $\frac{10\cdot8}{113\cdot6}$ to 12 decimals; (3) $\frac{\cdot001}{16\cdot31}$ to 25 decimals; (4) $\frac{\cdot001}{\cdot1111}$ to 25 decimals; (5) $\frac{\cdot08661}{531441}$ to 15 decimals; (6) $\frac{1}{333\cdot33333}$ to 6 decimals; (7) $\frac{\cdot0021461}{\cdot0078536}$ to 6 decimals; (8) $\frac{\cdot0070856329}{\cdot0321461}$ to 6 decimals.

Contracted Multiplication of Decimals.

If, in the ordinary way, we multiply a number containing four figures of decimals by another number containing four figures of decimals, the product will contain eight decimal figures. Now decimals as they occur in practice are not exact but approximate quantities, and the use of four figures of decimals implies, speaking roughly, an uncertainty of five units either way in the fifth decimal place; that is to say, ·7854 may mean anything between ·78535 and ·78545. Similarly, ·4142 may mean anything between ·41415 and ·41425. If we have to multiply ·7854 and ·4142 together, we get, in the ordinary way, ·32531268. But since these numbers are uncertain, instead of exact, their product is uncertain also, and may fall anywhere between

·78535 × ·41415 and ·78545 × ·414125,

or between

·3252527025 and ·32537266625.

We can therefore only be certain of the product as far as these agree, viz., that it is ·3253 nearly. Hence, in order not to throw away labour, it is desirable to use some method which will give us this product without superfluous results. We proceed by

RULE XLVIII.

Invert[*] *the order of the figures of the* multiplier, *and write them* under *those of the* multiplicand *in such a way that what was the* unit's *figure of the* multiplier *may come under the* last *place of* decimals, *which is to be retained. Multiply as usual, with this exception, that* each *place of the* multiplier begins *with* the *figure of the* multiplicand, *which comes* immediately over it, *the figure* next *to that being only used to* carry *from* (as in the subsequent example). *Put the several lines directly under one another, instead of removing each one place to the* left.

EXAMPLES.

Ex. 1. To multiply 147·3861 by ·6457, retaining only *three* places of decimals. The second factor, written so as to show a unit's place, is 0·6457; and in reversing, the 0 must fall under the *third* decimal place of the other factor, thus:—

1473861 7546·0	Multiplier reversed; unit's place 0 falling under third decimal 6 of the upper line.
88432	Multiplier 6; figure to begin with, 8, figure to carry from, 6. Six times 6 is 36, nearest ten, *four* tens, carry *four*. Six times 8 is 48, and 4 is 52, put down 2 and carry 5. The rest as usual.
5895	Multiplier 4; figure to begin with, 3, figure to carry from, 8. Four times 8 is 32; nearest ten, *three* tens, carry *three*. Four times 3 is 12, and 3 is 15, &c. The rest as usual.
737	Multiplier 5; figure to begin with, 7, figure to carry from, 3. Five times 3 is 15; nearest[†] ten, *two* tens, carry *two*. Five times 7 is 35, and 2 is 37, &c. The rest as usual.
103	Multiplier 7; figure to begin with, 4, figure to carry from, 7. Seven times 7 is 49; nearest ten, *five* tens, carry *five*. Seven times 4 is 28, and 5 is 33, &c. The rest as usual.
95·167	Add as usual, and mark off three places (the number proposed) for decimals.

The full product of 147·3861 and ·6457 is 95·16720477, which in thousandths only is nearest to 95·167, our result.

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[*] The object of reversing the figures of the multiplier is simply to get the product figures into their proper places with ease and certainty.

[†] In this case 15 is equally near to *one* ten and *two* tens. It is usual, and generally more correct, to take the higher of the two.

Ex. 2. Multiply ·456798 by ·23456, retaining *five* decimal places in the answer. The multiplier written so as to show a unit's place is 0·23456.

```
 ·456798
 65432·0
 -------
  9136
  1370
   183
    23
     2
 -------
 ·10714
```

Multiplier reversed; unit's place 0 falling under fifth decimal 9 of the upper line.

Begin multiplying the 2 into the 7, which is the fourth figure of the multiplicand. We must, however, *carry* from the product of the rejected figures (always carrying to the first figure set down in each row of multiplication as many *units* as are equal to the *nearest number of tens* derived from the multiplication of the last two rejected figures of the multiplicand). Hence the process will be,—twice 8 are 16; twice 9 are 18, and 1 are 19; but 19 is nearer to 20 than to 10, therefore carry 2; then say, twice 7 are 14, and 2, 16, set down 6, and finish the line of multiplication in the ordinary way. For the next row multiply by 3, rejecting this time 7 also from the multiplicand, but carrying 2, as the number of units equal to the nearest number of tens derived from the multiplication of the rejected 9 and 7. 3 times 6, 18, and 2, 20; place the 0 under the last figure in the upper row of multiplication, and finish the line in the ordinary way. Rejecting every time one figure from the multiplicand, in the next row multiply the 4 into the 5, carrying 3 (for 4 times 7 are 28; 4 times 6 are 24, and 2, 26; and 26 is nearer 30 than to 20, so carry 3). In the next row, for a similar reason, 3 has likewise to be carried, and multiplying 5 into 4 and carrying 3, we obtain 23. In the last row, 6 times 4 is 24, of which the 2 only is set down. These rows added together as they stand will give the required product correct to 5 decimal places.

Ex. 3. Multiply ·007853 by ·00476, retaining *seven* places of decimals.

```
 ·0078530
  67400·0
  -------
      314
       55
        4
  -------
 ·0000373
```

Ex. 4. Multiply ·00579 by 3796·8 retaining *four* decimal places.

```
 ·0057900
   8·6973
  -------
   173700
    40530
     5211
      347
       46
  -------
  21·9834
```

The following multiplications have the proper arrangement and result given. *No* decimal places means that the whole number of the result is required, without fractions.

Multiplication required.	No. of Decimals retained.	Arrangement of Multiplier and Multiplicand.	Result.
25·54545 × 13·2436	three	25·545454 6342·31	338·314
36·3771 × 9·99339	three	36·3771 933999	363·529
19·081137 × 523·36	two	19·081137 63·325	9986·30
·0699268 × ·9975641	seven	·0699268 1·4657990	·0697565
13763819 × 0·5877853	one	13763819·0 35877850·0	809017·0
753554·1 × 7·986355	none	753554·1 553689·7	6018150
1·2799416 × ·6156615	seven	1·2799416 5166516·0	·7880108
26·57032 × 47·6314	four	26·57032 4136·74	1265·5815

Where the figures of the multiplier extend to the left of the multiplicand, continue as long as there is either multiplication or carriage. Thus, in the second example, the first 9 of the arranged multiplier has no figure above it; but the carriage from the 3 (9 × 3 = 27) is *three* tens, and three must be written under the right-hand column of the preceding lines.

Contracted Division of Decimals.

RULE XLIX.

Proceed, as usual, until the number of quotient *figures remaining to be found does not exceed the number of figures in the* divisor. *Then instead of annexing a* cypher, *or bringing a figure down from the* dividend, *cut off the* last *figure of the* divisor; *that is, do not employ it except to* carry *from,* as in the last rule. *See how often this abridged* divisor *is contained in the* remainder; multiply, carrying *from the figure cut off; find a* new remainder; *cut off another figure from the* divisor, *and repeat the process until all figures of the* divisor *are cut off. When the abridged* divisor *is not contained in the* remainder, *cut off a second figure from the* divisor, *put a* cypher *in the* quotient, *and proceed. We subjoin a detailed example.*

EXAMPLES.

Ex. 1. To divide ·1299494 by 9915206 as far as nine decimal places. The first quotient figure being a decimal, and there being seven places in the divisor, two quotient figures must be found by the usual method; after which, the process as explained.

Divisor, afterwards abridged.	Dividend.	Quotient.	
9915206)	12994940 9915206 30797340 29745618	(·131060717	
991520 6	1051722 991521		Cut off the 6, reserving it to carry from 991520 is contained in 1051722 once; one 6 is 6, nearest ten, one ten, carry one. The rest as usual.
99152 06 9915 206	60201 59491		Figure to carry from, 0; 99152 not contained in 60201, cut of another figure from divisor, and put 0 in quotient. Figure to carry from, 2; 9915 contained in 60201, six times. Six times 2 is 12; nearest ten, one ten, carry 1. Six times 5 is 30, and 1 is 31. The rest as usual.
991 5206 99 15206	710 694		991 not contained in 710; cut off one more figure, and put 0 in quotient, carrying figure 1; 99 contained in 710, seven times. Seven times 1 is 7, nearest ten, one ten, carry one. Seven times 9 is 63, and 1 is 64. The rest as usual.
9 915206	16 10		Carrying figure 9. Divisor 9, contained once in 16. Once 9 is 9; nearest ten, one ten, carry one. Once 9 is 9 and 1 is 10
9915206	6 6 0		No divisor, carrying figure 9. What number of times 9 will carry 6, or be most nearly 60? Seven times 9 is 63, put 7 in the quotient, and carry 6; which finishes the process.

Ex. 2. Let it be required to divide 3 by ·643258 to *find* places of decimals.

```
·643528)3 00000(·46618
         257411
         ------
          42589
          38611
          -----
           3978
           3861
           ----
            117
             64
            ---
             53
             51
             --
              2
```

In this case, as there are *more* figures in the divisor than are required in the quotient, we may at once cut off the 8, the extreme right-hand figure of the divisor, *carrying*, however, from the multiplication of 4 into 8, and saying 4 times 8 is 32, carry 3; 4 times 2 is 8, and 3 is 11, &c.

Ex. 3. Let it be required to divide ·197241937 by ·254 correctly to five places of decimals.

In this case we shall not be able to use the contracted form at first, as there are *fewer* figures in the divisor than are required in the quotient. We must, therefore, perform the first three rows of division in the ordinary way, or we should not obtain five figures in the quotient; after that, instead of bringing down more figures, we can proceed in the last two rows by the contracted method.

```
·254)·197241937(·77654
      1778
      ----
      1944
      1778
      ----
      1661
      1524
      ----
       137
       127
       ---
        10
        10
        --
```

EXAMPLES FOR PRACTICE.

No.	Dividend.	Divisor.	No. of decimals to be retained.	Quotient.
1.	1	3·14159265	7	·3183099
2.	1	2·7182818	7	·3678794
3.	2992·9	51·77771	5	57·80347
4.	171·8	414·487636	9	·414487636
5.	·273	74·529	9	·003663004
6.	·0008202	·67272804	8	·00121922
7.	1·6866591	·4471618	5	3·77192
8.	85·643825	6·321	5	13·54909
9.	6001·58373	1732·508	5	3·46410
10.	7·2117562	2·257432	5	3·19467

132. When the divisor itself contains more places than are required in the quotient, as many places may be cut from the right as will make the two the same; and the dividend may be cut down in the same way until no more places are left than will give one figure in the quotient, to the abridged divisor, remembering the rule for increasing the last figure. Thus ·1648267 ÷ ·7263, to two places only, may be found from ·16483 ÷ 73, by the rule exemplified above.

133. Both in the multiplication and division, it is the best to retain one more place than is absolutely required to be correct.

REDUCTION.

144. The great convenience of decimals makes it often desirable to reduce vulgar fractions to the decimal form.

To reduce a vulgar fraction to a decimal.

We have to change the fraction to another equivalent fraction whose denominator is of the form 10, 100, 1000, &c. To do this we multiply the numerator and denominator of the fraction by 10, 100, 1000, &c., as may be necessary (*i.e.* we add a certain number of cyphers, the same to each); we then divide the numerator and denominator by the original denominator. These operations will not alter the value of the fraction. If the numerator, by the addition of the cyphers, becomes divisible by the denominator, without remainder, the required decimal is found; if not, a circulating or recurring decimal is produced, as is shown in the following examples:—

EXAMPLES.

Ex. 1. To reduce $\frac{5}{8}$ to a decimal.

$$\frac{5}{8} = \frac{5000}{8000} \div 8 = \frac{625}{1000} = \cdot625.$$

Here we multiply numerator and denominator by 1000 and divide them by 8. The resulting fraction $\frac{625}{1000}$ represented as a decimal is ·625.

Ex. 2. To reduce $\frac{123}{625}$ to a decimal.

$$\frac{123}{625} = \frac{1230000}{6250000} \div \frac{1968}{10000} = \cdot1968.$$

Here we multiply numerator and denominator by 10000, and then divide them by 625. The resulting fraction $\frac{1968}{10000}$ represented as a decimal is ·1968.

```
625)1230000(·1968
    625
   -----
    6050
    5625
    -----
     4250
     3750
     -----
      5000
      5000
      -----
```

The work is shortened thus:—We put down the numerator 123 as dividend, and denominator 625 as a divisor, and adding cyphers as often as required, we obtain as a quotient the significant digits of the decimals; and the number of cyphers added to the dividend will be the number of places to be marked off in the question.

Hence, to convert vulgar fractions into decimals, we proceed by the following rule.

RULE I.

Annex a cypher to the numerator, and then divide by the denominator; if there be a remainder, annex another cypher, and continue the division, still annexing a cypher, either till the division terminates without a remainder, or till as many decimals as are considered necessary are obtained; the quotient, with a decimal point before it, will be the value of the fraction in decimals.

EXAMPLES.

Ex. 1. Reduce $\frac{1}{5}$ to a decimal fraction.

```
5)1·0
  ---
  0·2
  ---
```

Dividing 10 by ·5 (the cypher being added) we find that $\frac{1}{5} = 0\cdot2$. That $\frac{1}{5} = 0\cdot2$ is easily proved, for $\frac{1}{5} = \frac{10}{50}$; consequently, by dividing both the numerator and denominator by 5, we have $\frac{1}{5} = \frac{2}{10} = \cdot2$.

Ex. 2. Convert $\frac{3}{8}$ into a decimal fraction.

```
8)3·000
  -----
   ·375
  -----
```

That $\frac{3}{8} = \cdot375$, is proved thus, $\frac{3}{8} = \frac{3000}{8000}$; consequently, dividing the numerator and denominator by 8 (the denominator of the fraction), we have $\frac{3}{8} = \frac{375}{1000} = \cdot375$.

Ex. 3. Express $\frac{7}{16}$, $\frac{5}{64}$, and $\frac{18}{128}$ as decimals.

$$16\left\{\begin{array}{l} 4)\underline{7{\cdot}0000} \\ 4)\underline{1{\cdot}7500} \\ \quad {\cdot}4375 \text{ Ans.} \end{array}\right.
\qquad
64\left\{\begin{array}{l} 8)\underline{5{\cdot}0000} \\ 8)\ \underline{{\cdot}6250} \\ \quad {\cdot}078125 \text{ Ans.} \end{array}\right.$$

$$\frac{18}{128} = \frac{18 \div 2}{128 \div 2} = \frac{9}{64}
\qquad
64\left\{\begin{array}{l} 8)\underline{9{\cdot}0000} \\ 8)\underline{1{\cdot}125000} \\ \quad {\cdot}140625 \text{ Ans.} \end{array}\right.$$

It is sometimes necessary to refer to the rule for Division of Decimals in order to determine the position of the decimal point, thus:—

Ex. 4. Convert $\frac{313}{512}$ to a decimal.

$$\begin{array}{l}
512)313{\cdot}000000000({\cdot}611328125 \text{ Ans.} \\
\qquad \underline{3072} \\
\qquad\ \ 580 \\
\qquad\ \ \underline{512} \\
\qquad\ \ \ 680 \\
\qquad\ \ \ \underline{512} \\
\qquad\ \ \ 1680 \\
\qquad\ \ \ \underline{1536} \\
\qquad\ \ \ \ 1440 \\
\qquad\ \ \ \ \underline{1024} \\
\qquad\ \ \ \ \ 4160 \\
\qquad\ \ \ \ \ \underline{4096} \\
\qquad\ \ \ \ \ \ 640 \\
\qquad\ \ \ \ \ \ \underline{512} \\
\qquad\ \ \ \ \ \ 1280 \\
\qquad\ \ \ \ \ \ \underline{1024} \\
\qquad\ \ \ \ \ \ \ 2560 \\
\qquad\ \ \ \ \ \ \ \underline{2560}
\end{array}$$

The same example might have been worked as follows:—

$$512\left\{\begin{array}{l} 8)\underline{313{\cdot}} \\ 8)\ \underline{39{\cdot}125} \\ 8)\ \ \underline{4{\cdot}890625} \\ \quad {\cdot}611328125 \text{ Ans.} \end{array}\right.$$

145. Sometimes the division will not terminate, but the same figures will be repeated over again continually.

Ex. 5. Reduce $\frac{1}{3}$ to a decimal fraction.

$$\begin{array}{l} 3)\underline{1{\cdot}0000} \\ \ \ {\cdot}3333, \text{ \&c.}, = \frac{1}{3} \end{array}$$

Ex. 6. Reduce $\frac{6}{11}$ to a decimal fraction.

$$\begin{array}{l} 11)\underline{6{\cdot}00000} \\ \ \ \ {\cdot}545454, \text{ \&c.} \end{array}$$

In these examples the fractions cannot be *exactly* reduced to decimals, for there will continue to be a remainder after each division as long as we continue the operation of division. In the first the remainder is always 1, and in the other, after obtaining the first two figures in the quotient, the remainder is the same as the given numerator, and the next two figures are the same as the first two, when the same remainder recurs again. The same remainders, and consequently the same figures in the quotient, will thus continually recur.

Ex. 7. **Reduce $\frac{25}{36}$ to a decimal.**

$$36\left\{\begin{array}{l}4)\underline{25{\cdot}00}\\ 9)\underline{\ \ 6{\cdot}25}\\ \quad\ {\cdot}69444,\ \&c.\end{array}\right.$$

Hence $\frac{25}{36} = {\cdot}69\dot{4}$ which is called a *mixed recurring* or *circulating* decimal, consisting of the non-recurring part 69 and the recurring part 444, and usually written with a point or dot above the figure which is repeated.

Ex. 8. **Reduce $\frac{3}{14}$ to a decimal.**

$$14\left\{\begin{array}{l}2)\underline{3{\cdot}000}\\ 7)\underline{1{\cdot}500}\\ \quad{\cdot}2142857\end{array}\right.$$

Hence $\frac{3}{14} = {\cdot}2\dot{1}4285\dot{7}$; the recurring part $\dot{1}4285\dot{7}$ having a point above its first and last figures being called its period. If the whole decimal recurs, it is called a pure *circulator*.

146. Decimals of this kind, in which the same figures are continually repeated without end, are called *Circulating*, *Repeating*, or *Recurring Decimals*; and the part of the decimal which is repeated is called the *Period*, or *Repetend*.

EXAMPLES FOR PRACTICE.

Reduce the following vulgar fractions to decimals:—

1. $\frac{3}{16}$, $\frac{9}{40}$, $\frac{139}{625}$, $\frac{17}{125}$, $\frac{106}{125}$, $\frac{1\frac{1}{10}}{62\frac{1}{2}}$, $\frac{7}{16}$, and $\frac{11}{32}$.

Find the circulating periods arising out of the following:—

2. $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{11}$, $\frac{1}{17}$, $\frac{10}{11}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{13}{99}$, $\frac{122}{66}$, $\frac{8}{900}$, and $\frac{17}{99}$.
3. $\frac{1}{39}$, $\frac{11}{13}$, $\frac{6}{117}$, $\frac{41}{492}$, $\frac{1}{9091}$, $\frac{61}{4649}$, $\frac{17}{9901}$, $\frac{8}{78}$, and $2\frac{1}{700}$.

147. When a vulgar fraction can be exactly expressed by a decimal it is said to be *terminate*, or *finite*; but when it cannot be exactly expressed by a decimal it is said to be *interminate*, or *infinite*.

The term infinite does not refer to the *value* of the fraction, but only to the number of decimal figures required to *express* its value.

148. It becomes important to observe what fractions will produce terminating decimals. Suppose a fraction in its lowest terms; then in reducing it to a decimal we multiply the numerator by 10, 100, 1000, &c. Now the numerator contains no factor common to the denominator, and by this multiplication we introduce the factors 2 and 5 as often as we please, and no others. Unless, then, the denominator contains no other factor except twos and fives, this multiplication cannot render the numerator divisible by it. Hence the only fractions which will produce terminating decimals are those whose denominators contain only 2 and 5 as prime factors. All other fractions will produce *Circulating Decimals where the number of figures in the recurring period will be less than the denominator.*

149. *To explain the reason of the occurrence of the figures of the quotient in the same order, when reducing a vulgar to an interminate decimal fraction.*

In attempting to reduce a vulgar fraction to a decimal, the *remainder* of each step of the division must always be *less* than the divisor, *i.e.*, than the denominator of the vulgar fraction; and the number of remainders different from each other which can arise can only be a number less than the units in the divisor. If, therefore, the remainder never become 0, by carrying on the division far enough one remainder must occur again, and as a cypher is added to every remainder, when the dividend becomes the same as has occurred before, the quotient will necessarily be again the same, and the process from that point will be repeated.

EXAMPLE.

Ex. Reduce $\frac{1}{7}$ to a decimal fraction.

$$\begin{array}{r} 7\overline{)1\cdot0} \\ \hline \cdot142857 \\ \hline \end{array}$$

Remainders 326451

Now we observe that we have obtained every possible remainder except 0, consequently, the remainder after the next step must be either 0 or one of the remainders that have occurred already. The next remainder being 3, the whole process will recur again from the beginning, and we have $\frac{1}{7}$ = 142857142857, &c.

150. Circulating Decimals are of two kinds, viz. :—

1. *Pure Circulating Decimals, i.e.*, those which recur from the beginning;

Thus, ·444, &c.; ·232323, &c.; ·04090409 0409, &c., which are respectively written thus, $\cdot\dot{4}$, $\dot{3}\dot{2}$, $\dot{0}40\dot{9}$.

2. *Mixed Circulating Decimals, i.e.*, those which do not begin to recur till after a certain number of figures;

Thus, ·25343434, &c.; ·342555, &c.; ·314200920092009, &c., which are respectively written $\cdot25\dot{3}\dot{4}$, $\cdot342\dot{5}$, $314\dot{2}00\dot{9}$.

151. To reduce a circulating decimal to an equivalent vulgar fraction.

Let it be required to convert $\cdot\dot{5}\dot{7}$ into a vulgar fraction. Remembering that we effect the multiplication of a decimal by 10, by 100, &c., by shifting the decimal place 1, 2, &c., places towards the *right* hand, we may say—

Let F denote the true value of the equivalent fraction,

Then F = ·575757, the period 57 being supposed continued *ad infinitum.*

If we multiply F by 100, and also the decimal by 100, the results will still be equal.

Hence, 100 F = 57·5757, &c.

The decimal point being moved two places to the right, and the period 57 being still continued *ad infinitum*, on the right of the decimal point as before.

But F = ·575757, &c.

Now the difference of 100 F and F, *i.e.*, 99 F must be equal to the difference of the decimals to which they are respectively equal. Now this difference is 57·5757, &c., — ·575757, &c., or 57, because the infinite recurrence of the decimal part is the same in each decimal, and therefore vanishes when the subtraction is performed.

Hence, 100 F = 57·575757, &c.

and F the fraction required = $\frac{57}{99} = \frac{19}{33}$.

This is a case of a pure circulating decimal, in which there is no non-recurring part.

We shall next take a case of a mixed circulating decimal. To find the vulgar fraction equivalent to ·34567.

Denoting the true value of the equivalent fraction by F, we have F = ·34568568568, &c., the period 568 being supposed to be continued *ad infinitum.*

If we multiply F by 100000, and also the decimal by 100000, the results will still be equal.

Hence, 100000 F = 34568·568568568.

The decimal point being removed five places to the right, and the period being still continued *ad infinitum* on the right hand of the decimal point as before.

Similarly, 100 F = 34·568568568.

Now the difference of 100000 F and 100 F, *i.e.*, 99900 F must be equal to the difference of the decimals to which they are respectively equal. And this difference is 34568 — 34, because the infinite recurrence of the period $\dot{5}6\dot{8}$ on the right hand of the decimal point is the same in each decimal, and therefore vanishes when the subtraction is performed.

Hence, 99900 F = 34568 — 34, and F, the fraction required, $= \frac{34568-34}{99900} = \frac{34534}{99900}$. This, being reduced to its lowest terms, is the fraction required.

Now, let it be carefully observed how each part of this fraction has arisen. The numerator is obtained by writing the figures of the decimal, as far as the end of the first period, without the decimal point, and then subtracting from the number so obtained the figures that occur before the period, or it is usually called the non-recurring part. The denominator 99900 results from subtracting 100 (*i.e.*, 10 raised to the same power as the number of figures in the non-recurring part) from 100000 (*i.e.*, 10 raised to the same power as there are figures in the non-recurring part and period together).

This subtraction will necessarily produce a number 99900, containing, that is to say, as many nines as there are figures in the period, and as many cyphers as there are figures in the non-recurring part and period together.

152. **From the foregoing we obtain the following practical rules for converting a circulating decimal into its equivalent vulgar fraction.**

1. In the case of a pure circulator.

RULE LI.

Put the recurring *period for the* numerator, *and a* number *expressed by as* many nines *as there are* digits in the recurring period *for the* denominator, *and reduce the resulting fraction to its lowest terms.*

Note.—The fraction is only reduced to its lowest terms for the sake of exhibiting it in its simplest form. It is not of course actually necessary so to reduce it.

Example.

Ex. Reduce the following pure circulating decimals, $\dot{3}$, $\dot{2}\dot{7}$, $\cdot\dot{8}5714\dot{2}$, to their respective equivalent vulgar fractions.

$$\dot{3} = \tfrac{3}{9} = \tfrac{1}{3} \textit{ Ans.}$$

$$\cdot\dot{2}\dot{7} = \tfrac{27}{99} = \tfrac{3}{11} \textit{ Ans.}$$

$$\cdot\dot{8}5714\dot{2} = \tfrac{857142}{999999} = \tfrac{95238}{111111} = \frac{6 \times 15873}{7 \times 15873} = \tfrac{6}{7} \textit{ Ans.}$$

2. In the case of a mixed circulator.

RULE LII.

From the circulating decimal written as a whole number subtract *the figures which do* not recur, *and place the* remainder *for the* numerator *of the vulgar fraction; and for the* denominator *as many* nines *as there are* recurring figures, *followed by as* many cyphers *as there are* non-recurring *figures; and reduce the fraction to its lowest terms.*

Example.

Ex. Reduce the following mixed circulating decimals, $\cdot 1\dot{4}$, $\cdot 013\dot{8}$, $\cdot 2\dot{4}1\dot{8}$, and $2\cdot 578\dot{9}\dot{3}$, to their respective equivalent vulgar fractions.

$$\cdot 1\dot{4} = \frac{14-1}{90} = \tfrac{13}{90} \textit{ Ans.}$$

$$\cdot 013\dot{8} = \frac{138-13}{9000} = \tfrac{125}{9000} \textit{ Ans.}$$

$$\cdot 2\dot{4}1\dot{8} = \frac{2418-2}{9990} = \tfrac{2416}{9990} = \tfrac{1208}{4995} \textit{ Ans.}$$

$$2\cdot 578\dot{9}\dot{3} = 2\frac{57893-578}{99000} = 2\tfrac{57315}{99000} = 2\tfrac{11463}{19800} = 2\tfrac{3821}{6600} \textit{ Ans.}$$

EXAMPLES FOR PRACTICE.

Reduce to their equivalent vulgar fractions:—

1. $\dot{3}$, $\dot{7}$, ${\cdot}\dot{7}\dot{2}$, $\dot{6}\dot{3}$.
2. ${\cdot}\dot{7}\dot{2}$, ${\cdot}\dot{0}\dot{9}$, $\dot{9}\dot{0}$, ${\cdot}5\dot{6}$.
3. ${\cdot}008$, ${\cdot}08\dot{3}$, ${\cdot}1\dot{3}\dot{6}$.
4. $7{\cdot}\dot{4}\dot{8}$, $106{\cdot}0\dot{9}$, $10{\cdot}\dot{6}$.
5. ${\cdot}42\dot{6}2\dot{3}$, $3{\cdot}1\dot{4}1\dot{6}$.
6. $357{\cdot}00\dot{3}12\dot{9}$, $27{\cdot}52\dot{3}\dot{8}$.
7. $21{\cdot}00030\dot{8}$, $52{\cdot}\dot{3}1415\dot{9}$.
8. $3{\cdot}0101\dot{0}\dot{3}$, ${\cdot}052\dot{1}00$.
9. $105\dot{7}6923\dot{0}$, ${\cdot}9\dot{8}4615\dot{3}$.
10. ${\cdot}60\dot{7}1428\dot{5}$, $23\dot{4}2857\dot{1}$.
11. $2000{\cdot}0908\dot{6}$, $7030{\cdot}\dot{8}\dot{2}$.
12. $106{\cdot}90\dot{8}0\dot{9}$, ${\cdot}51\dot{4}1\dot{3}$.
13. ${\cdot}\dot{2}335766\dot{4}$.
14. ${\cdot}\dot{0}0010099989\dot{9}$.
15. $2{\cdot}00\dot{1}4285\dot{7}$.
16. $10{\cdot}000\dot{1}4285\dot{7}$.
17. $1{\cdot}0021052631578947368\dot{4}$.
18. ${\cdot}\dot{0}58823529411764\dot{7}$.
19. ${\cdot}\dot{1}50943396226\dot{4}$.
20. ${\cdot}\dot{0}34482758620689655172413793\dot{1}$.

153. **As a general rule, all sums involving circulating decimals may be worked out by reducing the decimals to their equivalent fractions, and after performing the required operations on the fractions, reducing, if necessary, the result back to a decimal. This is the only method in some cases, but the labour of it can generally be saved, as we will now show.**

ADDITION.

154. **In most practical cases it is sufficient to take the decimals to 6 or 7 places of figures. To ensure accuracy to 6 places it is better to take 8 figures in the decimals, as there may be something carried from the 8th and 7th places so as to affect the 6th place.**

If the recurring period of the sum is required, we must put down the figures of each decimal to be added till the vertical columns of figures begin to recur.

EXAMPLE.

Ex. Add together $3{\cdot}4\dot{7}5\dot{6}$, $5{\cdot}\dot{9}83\dot{4}$, $72{\cdot}36\dot{8}4392\dot{5}$.

$$\begin{array}{r|l|l} 3{\cdot}47 & 567567567567 & 56 \\ 5{\cdot}98 & 349834983498 & 34 \\ 72{\cdot}36 & 843925843925 & 84 \\ \hline 81{\cdot}82 & \dot{7}6132839499\dot{1} & 74 \end{array}$$

Here the first two decimal places are not recurring in all the numbers; the column 5, 3, 8 recurs after 12 figures of the recurring periods; two figures more are used to ensure accuracy in the last figures of the recurring period of the answer. If circulating decimals, whose periods contain 3, 4, and 6 figures, respectively, are to be added, the period of the sum will generally contain 12 figures (12 being the L.C.M. of 3, 4, and 6).

SUBTRACTION.

155. **Exactly the same observations will apply as in addition.**

EXAMPLE.

Ex. From $32{\cdot}5\dot{6}8\dot{3}$ **take** $14{\cdot}378\dot{4}\dot{5}$.

$$\begin{array}{r|l|l} 32{\cdot}568 & 368368 & 36 \\ 14{\cdot}378 & 454545 & 45 \\ \hline 18{\cdot}189 & \dot{9}1382\dot{2} & 91 \end{array}$$

MULTIPLICATION.

156. When *only one* of the decimals is a recurring decimal, we proceed as follows:—

Example.

Ex. Multiply $\cdot 3\dot{5}4\dot{5}$ by $3\cdot 5$.

$$\begin{array}{r|r|l} \multicolumn{3}{r}{\cdot 35455455} \\ \multicolumn{3}{r}{3\cdot 5} \\ \hline 1 & 772 & 77275 \\ 10 & 636 & 6365 \\ \hline 1\cdot 2 & \dot{4}0\dot{9} & 40 \end{array}$$

Ans. $1\cdot 2\dot{4}0\dot{9}$.

Here we take a sufficient number of figures to be quite sure of obtaining the recurring periods in each line of the multiplication, and add as before. The product will generally have the same number of figures in its recurring period as the multiplicand.

DIVISION.

157. When only the dividend is a recurring decimal we may proceed as in reducing a fraction to a decimal. We must, however, continue the division till not only the remainder recurs, but the next figure brought down to that.

Example.

Ex. Divide $\cdot 37\dot{4}\dot{5}$ by $2\cdot 5$.

$$\begin{array}{l} 2\cdot 5)\cdot 37\dot{4}\dot{5}(\cdot 149\dot{8}\dot{1} \\ \quad\;\; \underline{25} \\ \quad\;\; 124 \\ \quad\;\; \underline{100} \\ \quad\;\;\; 245 \\ \quad\;\;\; \underline{225} \\ \quad\;\;\;\; 204 \\ \quad\;\;\;\; \underline{200} \\ \quad\;\;\;\;\;\; 45 \\ \quad\;\;\;\;\;\; \underline{25} \\ \quad\;\;\;\;\;\; 20 \end{array}$$

158. **Multiplication and Division of Circulating Decimals** may be performed, as above, by carrying out the repetend; but these operations are more usually performed by reducing the decimals to vulgar fractions, then multiplying or dividing, as in vulgar fractions, and reducing the results once more to decimals.

EXAMPLES.

Ex. 1. Multiply $2\cdot\dot{3}\dot{4}$ by $5\cdot4$.

$$\text{The value of } 2\cdot\dot{3}\dot{4} \times 5\cdot4 = \frac{2\cdot34-2}{99} \times 5\tfrac{4}{10} = \frac{232}{\underset{11}{\cancel{99}}} \times \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{54}}}{\underset{5}{\cancel{10}}} = \frac{696}{55} = 12\tfrac{36}{55}.$$

Ex. 2. Multiply $107\cdot2\dot{2}\dot{5}$ by $\cdot01\dot{4}3\dot{1}$.

$$\text{The value of } 107\cdot2\dot{2}\dot{5} \times \cdot01\dot{4}3\dot{1} = \frac{107225-1072}{999} \times \frac{1431-1}{99900} \text{ (§ 152)}$$

$$= \frac{\overset{2869}{\cancel{106153}}}{\underset{9}{\cancel{999}}} \times \frac{\overset{13}{\cancel{1430}}}{\underset{2700}{\cancel{99900}}} = \frac{37297}{24300} = 1\tfrac{12997}{24300} = 1\cdot53\dot{4}8559670781893004115226337\dot{4} \textit{ Ans.}$$

N.B.—The latter part of the above example furnishes a good illustration of (§ 145), thus—

To express $\frac{12997}{24300}$ as a decimal.

$$\text{Here, } \frac{12997}{24300} = \frac{12997}{243 \times 100} = \frac{1}{100} \text{ of } \frac{12997}{243},$$

$$\text{Now, } \frac{12997}{243} = \frac{12997}{9 \times 9 \times 3}$$

$$\therefore\ 243 \left\{ \begin{array}{llll} 9)\underline{12997} & & & \\ 9)\ \underline{1444\cdot\dot{1}} & & & \\ 3)\ \ 160\cdot45679012\dot{3} & & 456790123 & 456790123 \\ \hline \quad 53\cdot485596707 & & 818930041 & 152266374\dot{} \\ \quad {\scriptstyle 1}\qquad\qquad {\scriptstyle 2} & & \qquad\quad {\scriptstyle 0} & \end{array} \right.$$

We thus find that $\frac{12997}{243} = 53\cdot\dot{4}8559670781893004115226374\dot{}$.

$$\therefore\ \frac{1\cdot2997}{243 \times 100} = \cdot53\dot{4}8559670781893004115226374\dot{}.$$

Ex. 3. Divide $84\cdot332\dot{4}7\dot{6}$ by $12\cdot734$.

$$\text{The value of } 84\cdot332\dot{4}7\dot{6} \div 12\cdot734 = \frac{84\cdot332476-84332}{999000} \div \frac{12734}{1000}$$

$$= \frac{\overset{6616}{\cancel{84248144}}}{999\cancel{000}} \times \frac{\cancel{1000}}{\cancel{12734}} = \frac{6616}{999} = 6\tfrac{622}{999} = 6\cdot\dot{6}2\dot{2} \textit{ Ans.}$$

Ex. 4. Divide $3\cdot41\dot{4}$ by $93\cdot\dot{1}\dot{2}$.

$$\text{The value of } 3\cdot41\dot{4} \div 93\cdot\dot{1}\dot{2} = \frac{3414-341}{900} \div \frac{9312-93}{99} = \frac{3073}{900} \div \frac{9219}{99}$$

$$= \frac{\cancel{3073}}{\underset{100}{\cancel{900}}} \times \frac{99}{\underset{3}{\cancel{9219}}} = \frac{11}{300} = \cdot03\dot{6} \textit{ Ans.}$$

159. To reduce any quantity or fraction of one denominator to the decimal of another denominator.

RULE LIII.

Reduce the number of the lowest *denomination to a decimal of the next* higher *denomination,* prefix *to this decimal the number of its denomination given in the* *question, if any, and reduce this also to a decimal of the next* higher *order, and so on till all the numbers of the given denominations are exhausted, and the decimal of the required denomination has been obtained;* the last result will be the answer.

EXAMPLES.

Ex. 1. Let it be required to express 17s. 5¼d. as the decimal of £1.

The process will be first to express the *fractional* part of a penny as a *decimal* of a penny; placing the 5 as a whole number before this decimal, to divide that result by 12, in order to reduce it to the decimal of a shilling; placing the 17 as a whole number before this decimal, to divide that result by 20 in order to reduce it to the decimal of a pound. This will be written as follows:—

4)1·

12)5·25 pence.

20)17·4375 shillings.

·871875 of a pound.

It will be seen from this, that whatever we should divide by in whole numbers in order to bring pence into shillings, or shillings into pounds, that we must likewise divide by in this case, only marking off correctly the decimal results.

Ex. 2. Let it be required to express 2 cwt. 3 qrs. 13·832 lbs. as the decimal of a ton.

Dividing severally by 28, 4, and 20, in order to bring pounds into qrs., qrs. into cwts., and cwts. into tons, the process will be—

28 { 4)13·832
 { 7) 3·458

4) 3·494 qrs.

20) 2·8735 cwts.

·143675 tons.

Ex. 3. Find what decimal of 1 mile (nautical) is 700 feet.

There are 6080 feet nearly in the nautical mile; hence 1 foot is $\frac{1}{6080}$ of 1 mile, and 700 feet are $\frac{700}{6080}$ of 1 mile, which gives 0·115 of 1 mile nearly.

6080)700·0(0·115
6080

9200
6080

31200
30400

Ex. 4. Find what decimal of 1 foot is 8¾ inches.

First, ¾ is 0·75 of 1 inch, hence 8¾ inches are 8·75 inches. Then 1 inch is $\frac{1}{12}$ of 1 foot; hence 8·75 inches are $\frac{8·75}{12}$, or 0·729 of 1 foot.

12)8·75

0·729

Ex. 5. Reduce £23 17s. 6¾d. to the decimal of £25.

4)3·00 farthings.

12)6·75 pence.

20)17·5625 shillings.

25 { 5)23·878125 pounds.
 { 5) 4775625

·955125 of £25.

Here, dividing the 3 farthings by 4, and expressing the result as a decimal, viz., ·75, we replace the 6¾d. by 6·75 pence. Dividing this by 12 we reduce the pence to shillings, and 17s. 6¾d. is 17·5625 shillings, and so for the pounds, and £25.

Answer ·955125.

Ex. 6. What decimal of a mile is 3 fur. 100 yds. 2 ft. 3 in.?

$$\begin{array}{l}
12\overline{)3{\cdot}0}\text{ inches.}\\
3\overline{)2{\cdot}25}\text{ feet.}\\
220\overline{)100{\cdot}75}({\cdot}4579\dot{5}\dot{4}\text{ furlongs.}\\
\quad 880\\
\quad \overline{1275}\\
\quad 1100\\
\quad\ \overline{1750}\\
\quad\ 1540\\
\quad\ \ \overline{2100}\\
\quad\ \ 1980\\
\quad\ \ \ \overline{1200}\\
\quad\ \ \ 1100\\
\quad\ \ \ \ \overline{1000}\\
\quad\ \ \ \ \ 880\\
\quad\ \ \ \ \ \overline{120}\\
8\overline{)3{\cdot}4579\dot{5}\dot{4}}\text{ furlongs.}\\
\quad {\cdot}43224431\dot{1}\dot{8}\text{ mile.}
\end{array}$$

Answer ·43224431$\dot{1}\dot{8}$ mile.

160. When the above rule is impracticable because of the occurrence of several terms in the second given quantity, the following must be used.

RULE LIV.

Express the first *quantity as a vulgar fraction of the* second (§ 113), *and then* reduce *this vulgar fraction to a decimal.*

EXAMPLE.

Ex. Reduce 2s. 9¾d. to the decimal of 7s. 9¾d.

$$\frac{2\text{s. }9\tfrac{3}{4}\text{d.}}{7\text{s. }9\tfrac{3}{4}\text{d.}} = \frac{135\text{ farthings}}{375\text{ farthings}} = \tfrac{9}{25},\text{ and }\begin{array}{l}25\overline{)9{\cdot}00}({\cdot}36\ \textit{Ans.}\\ \quad\ 75\\ \quad\ \overline{150}\\ \quad\ 150\\ \quad\ \overline{}\end{array}$$

EXAMPLES FOR PRACTICE.

What decimal is

1. 6s. 4d.; 8s. 8½d.; 4s. 7½d.; 15s. 11¼d.; 19s. 10¾d.; £1 13s. 11½d. $\frac{19}{21}$f. of £1.
2. 3s. 4d. of 16s. 8d.; 7½d. of 2s.; £4 0s. 6d. of £7 14s.; ¾d. of 6s. 8d.
3. 3 qrs. 21 lbs.; 12 cwt. 1 qr. 16 lbs; 14 cwt. 1 qr.; 10 cwt. 1 qr. 14 lbs.; 14 cwt. 3 qrs. 17 lbs. 5 ozs.; 13 tons 6 cwt. 7 lbs.; 1 ton 13 cwt.; 17 tons 18 cwt. 3½ lbs. of a ton; and 3 tons 12 cwt. 3 qrs. of 18 tons.
4. 6 fur. 127 yds. of 1 mile; 118·37 feet (1) of a mile, (2) of a furlong, (3) of a yard.
5. 1 oz. avoirdupoise of 1 oz. troy; 13 ozs. 5 drams 7 scruples 19 grains of a pound and of an ounce.

6. Express 12 lbs. 7 ozs. 6 dwt. 11 grs. troy in decimals of 1 pound troy; 17 lbs. 15 ozs. 400 grs. avoirdupoise in decimals of 1 lb. troy.
7. $5^d\ 12^h\ 25^m\ 39{\cdot}92^s$ in decimals of a week; $29^d\ 12^h\ 44^m\ 2{\cdot}82^s$ to decimals of a day.
8. Reduce $\frac{3}{7}$ of $\frac{1}{10}$ of 40 yards to the decimal of $\frac{1}{8}$ of 2 miles; $\frac{1}{4}$ of $3\frac{1}{2}$ sq. yards to the decimal of 2 acres 1 rood; $\frac{3}{8}$ of $4\frac{4}{5}$ hours to the decimal of $365\frac{1}{4}$ days; $\frac{21}{10000}$ of a year to the decimal of a day; $\frac{23}{18}$ oz. to the decimal of 1 lb. troy; $3\frac{1}{4}$ inch to the decimal of $\frac{1}{2}$ mile.
9. Express as decimals (correct to 7 places of figures) the fractions in Exs. 5, 6, and 7, page 89.

161. **To find the value of a decimal in terms of the inferior denominations.**

RULE LV.

1°. *Note the number of parts which the unit or integer of the given quantity contains of the next inferior denomination, and multiply the given decimal by this number;* the product is the given quantity expressed in that denomination.

2°. *If this product has a decimal part, multiply this decimal by the number of parts which the unit of the present denomination contains of the next inferior denomination to that just before employed;* this product is the quantity which the given decimal contains of the *next denomination.*

3°. *Proceed (if there still be decimals), in like manner, to the lowest denomination in which the decimal is required to be expressed.*

EXAMPLES.

Ex. 1. Find the value of £·36875.

£·36875
20
$7{\cdot}375\cancel{00}$ shillings.
12
$4{\cdot}5\cancel{00}$ pence.
4
2·0 farthings.
7s. $4\frac{1}{2}$d. *Ans.*

Here we multiply the denomination pounds by 20, to bring it into shillings, we obtain 7 shillings and three hundred and seventy-five *thousandths* of a shilling. Multiplying this ·375 of a shilling by 12, to bring it into pence, we obtain 4 pence and five-*tenths* of a penny. Multiplying ·5 of a penny to bring it into farthings, we obtain 2 farthings. In the ·37500 of the line of shillings, and ·500 of the line of pence, the cyphers are crossed out as they have no value.

Ex. 2. Find the value of ·3945 of a day.

·3945 day.
24
15780
7890
$9{\cdot}468\cancel{0}$ hours.
60
$28{\cdot}08\cancel{0}$ minutes.
60
4·80 seconds.
$9^h\ 28^m\ 4{\cdot}8^s$ *Ans.*

We here multiply the denomination days by 24, to find the number of hours, and then multiply the resulting decimal part of an hour by 60, to find the number of minutes, and again by 60 to find the number of seconds.

Ex. 3. What is the value of ·625 cwt.

	·625 cwt.
The next inferior denominator to that of a cwt. is qrs. of which the number in a cwt. is	4
	2·500 qrs.
The next inferior denominator to qrs. is lbs. of which the number in a quarter is	28
2 qrs. 14 lbs. *Ans.*	14·0 lbs.

Ex. 4. Find the number of feet in 0·115 of a mile.

	0·115
The next inferior denomination to that of miles is here feet, of which the number in one mile is	× 6082
	230
	920
	6900
Ans. (in the lowest denomination required)	699·430 feet.

Ex. 5. Find the number of inches and eighths in 0·48 of a foot.

	0·48
The next inferior denomination to that of feet is inches, of which the number in a foot is	× 12
	5·76 inches.
The next proposed inferior denomination to inches is eighths, of which the number in an inch is	× 8
	6·08 eighths.

Ex. 6. Find the value of ·3̇4̇ of £7.

·343434 . . .
7

2·404088 pounds.
20

8·0800 shillings.
12

·96 pence.

8s. 0·96d. *Ans.*

Here the student must be careful to take a sufficient number of figures to obtain the recurring period in each line of the multiplication.

Ex. 7. What is the value of 2·046̇ of £1?

£ 2·0466666 &c.
20

s. ·09333320
12

d. 11·199984

£2 0s. 11·19d. *Ans.*

Since ·6̇ = $\frac{6}{9}$ = $\frac{2}{3}$
∴ £2·046 = £204$\frac{2}{3}$
Or thus: £ 2·04$\frac{2}{3}$
20

s. 0·93$\frac{1}{3}$
12

d. 11·20

N.B.—When the figure "9" circulates by itself it is usual to omit it, and add 1 to the figure on the left.

Thus, ·7$\dot{9}$ is called ·8; 2·$\dot{9}$ is called 3, &c., because

$$·7\dot{9} = \frac{79-7}{90} = \frac{72}{90} = \frac{8}{10} = 8$$

$$2·9 = 2\tfrac{9}{9} = 2 + 1 = 3.$$

In reducing such quantities as that in Ex. 7 above. it is often preferable to reduce the decimal to a vulgar fraction and then find its value by § 144.

$$\text{Thus, } 2·04\dot{6} \text{ of } £1 = \frac{2·046 - 204}{900} \text{ of } 20\text{s.} = \frac{\overset{614}{\cancel{1842}}}{\underset{\underset{15}{\cancel{45}}}{\cancel{900}}} \times \frac{\cancel{20}}{1}$$

$$= \tfrac{614}{15}\text{s.} = 40\tfrac{14}{15}\text{s.} = £2 \text{ 0s. } 11\tfrac{1}{5}\text{d.}$$

In reducing any given quantities to the decimal of a higher denomination, a method similar to that used is the rule of Practice may sometimes be conveniently adopted; the method will be more clearly understood after "Practice" has been studied; meanwhile, in order to explain the process, one or two examples are subjoined.

Ex. 8. Reduce 19 cwt. 3 qrs. 10 lbs. to the decimal of a ton.

1 cwt.	$\frac{1}{20}$	= ·05	of a ton.
		19	
19 cwt.		= ·95	„
2 qrs.	$\frac{1}{2}$ of 1 cwt.	= ·025	„
1 qr.	$\frac{1}{2}$ of 2 qrs.	= ·0125	„
{4 lbs.	$\frac{1}{7}$ of 1 qr.	= ·0017857, &c.	„
{4 lbs.	$\frac{1}{7}$ of 1 qr.	= ·0017857, &c.	„
2 lbs.	$\frac{1}{2}$ of 4 lbs.	= ·0008928, &c.	„
		·9919642, &c.	„

Ex. 9. Reduce 3 weeks 3 days 13 hours 36 minutes to the decimal of a month.

3 weeks	$\frac{3}{4}$	= ·75	of a month.
3 days	$\frac{1}{7}$ of 3 weeks	= ·10714285, &c.	„
12 hours	$\frac{1}{6}$ of 3 days	= ·01785714, &c.	„
1 hour	$\frac{1}{12}$ of 12 hours	= ·00148809, &c.	„
30 minutes	$\frac{1}{2}$ of 1 hour	= ·00074404, &c.	„
6 minutes	$\frac{1}{5}$ of 30 minutes	= ·00014880, &c.	„
		·87738092, &c.	„

162. **If the given quantity be expressed in more than one denomination, it should be reduced to *one* before applying the rule.**

Ex. 10. Find the value of ·07 of £2 10s.; and of ·7365 of 6s. 8d.

Here £2 10s. = 50s., and 6s. 8d. = 80d.

·07	·7365
50	80
3·50 shillings.	58·9200 pence.
12	*Ans.* 4s. 10·92d.
6·0 pence.	
Ans. 3s. 6d.	

EXAMPLES FOR PRACTICE.

Find the value of the following expressions:—

1. £9; £2·4; £17·375; £10·625; £2·8$\dot{3}$.
2. ·25 cwt.; ·375 qr.; ·625 lb.; ·975 ton; ·7768 ton; ·24956 ton; ·0675 cwt.
3. ·3375 acre; ·874 lb. troy; 1·425 dwt.; ·575 lb. apoth.
4. ·472 oz. apoth.; ·315 dram apoth.; ·492 day; ·343 week; ·534 year.
5. 3·$\dot{3}$ days; ·8$\dot{3}$ mile; ·316 fur.; ·$\dot{5}9\dot{2}$ sq. yd.; ·43428793 year (of 365 days).
6. 9·01 × 50·01 of £600; 2·106 × ·2106 × ·375 of a ton.
7. Reduce 3·14159265 tons to cwt. qrs. lbs. ozs., and to ozs. only; also to grains, troy.
8. ·318309886 of the semi-circumference of a circle to degrees, minutes, and seconds, and also to seconds.
9. ·57721314987 sq. mile in acres and decimals, and in feet and decimals.
10. ·487956324 tons of sea water in cubic feet and inches (35 ft. = 1 ton).
11. Express the same in gallons of 277·274 cubic inches.
12. ·03125 of £20 + ·$\dot{7}2\dot{9}$ of 6s. 2d. + ·$\dot{7}2\dot{9}$ of £2 1s. 3d.
13. ·45 of £1; ·16875 of £1; 2·36875 of £1; ·0474609375 of £10 13s. 4d.; 46·$\dot{9}\dot{0}$ of 1 mile 6 fur. 20 poles; 13·27$\dot{5}$ of 5½ acres; $\dot{2}8571\dot{4}$ of a cwt.

163. Decimals are most frequently used to make calculations on numbers that have been obtained by observations of some kind, by measuring, for instance, or weighing; and it is very seldom indeed that the accuracy of the set observations can be relied on to within one five-thousandth part of the unit employed. Now, if we cannot rely on the measurement beyond three decimal places, it is needless to carry the result derived from it any farther. In all operations with decimals, then, whether terminating or repeating, we may usually stop at the third or fourth place, and need very rarely go beyond the fifth or sixth. We may, however, attain any degree of exactness that may be required, by carrying the decimal far enough, for although any number of figures of a recurring decimal is not exactly equal to the fraction from which it arises, yet they are practically useful; for, taking only 6 figures in the decimal, the error will not exceed one-millionth, *i.e.*, in a calculation involving a mile, the error will be less than one-tenth of an inch.

With respect to repeating decimals, if perfect accuracy be necessary, they must in most cases be reduced to vulgar fractions before they are added, subtracted, multiplied, or divided. In almost all the applications of decimals, however, an approach to accuracy is sufficient, and this is attained by carrying the decimal only to a moderate number of digits, and omitting the rest. If, in converting a vulgar fraction into a decimal, we stop after the third digit, for instance, adding unity to that digit, if the next be 5 or upwards, it does not differ from its exact value by more than one five-thousandth part of the unit employed. Thus, ·172 differs from ·17$\dot{2}43\dot{7}$ by ·000$\dot{4}37\dot{2}$, which is less than ·0005. Similarly, ·938 differs from ·9$\dot{8}27\dot{6}$ by ·000$\dot{2}31\dot{7}$, which is also less than ·0005.

MISCELLANEOUS EXAMPLES FOR PRACTICE.

1. Divide the sum of $(11\frac{13}{100} - \cdot35) \div (\cdot05 - \frac{1}{75})$ and $\frac{7}{125} + 6\cdot007$ by the difference of $\frac{2}{3}(1\cdot35 - \cdot72)$ and 5·0004, and express the result as a decimal.

2. What decimal multiplied by ·125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375, and 2·46?

3. Show that ·718283 is more nearly equal to ·7183 than to ·7182, and find how much.

4. Which is the nearest to 3·1415, $\frac{22}{7}$, or $\frac{333}{106}$, and by how much?

5. From a rod 2·078 inches long, portions are cut off, each equal to ·0037 of an inch long, find how many such portions can be cut off, and what will be the length of the remainder?

6. If a gallon contains 277·27 cubic inches, and water weighs 62·42 lbs. a cubic foot; find the weight of a pint of water.

7. Reduce £$\cdot0\dot{2}\dot{1}$ to the fraction of ($1\cdot\dot{7}\dot{6}$s. — 3·2d.)

8. Find the value of ·03125 of £20 + $\cdot\dot{7}2\dot{9}$ of 6s. 2d. + $7\dot{2}\dot{9}$ of £2 1s. 3d.

9. Reduce $\frac{482}{20}$ and $\frac{1}{3125}$ to decimal fractions, then multiply them together, and divide the product by ·0000075. Reduce the quotient to a vulgar fraction.

10. Arrange in order of magnitude ·3456, $\cdot\dot{}456$, $\cdot34\dot{5}\dot{6}$, $\cdot3\dot{4}5\dot{6}$, and $\cdot345\dot{6}$.

11. What is the quotient of the sum by the difference of ·6 and $\cdot\dot{6}$.

12. The sidereal year being $365^d\ 6^h\ 9^m\ 9^s\cdot6$, and the tropical year $365^d\ 5^h\ 48^m\ 49^s\cdot7$; reduce their difference to the decimal of a tropical year.

13. The driving wheel of a locomotive engine 5 feet in diameter turned 2500 times in going 6 miles; supposing the circumference of a circle to be 3·1416 times the diameter; find what distance was lost owing to the slipping of the wheel.

14. The height of the highest mountain is 28000 feet; what decimal is this of the earth's diameter, which is 8,000 miles?

15. A penny is ·08975 inches thick; find the height of a pile of 1000 pennies.

16. The circumference of a circle is 3·14159 times its diameter; find the circumference of circles whose diameters measure 13·7 feet, 1·96 yards, and 28·342 miles respectively.

17. A cubic inch of water weighs 252·458 grains, and the weight of an imperial gallon of water is 10 lbs. avoirdupoise; find (to three places of decimals) the number of cubic inches in an imperial gallon, there being 7000 grains in the lb. avoirdupoise.

18. A cubic foot weighs 445 lbs; what does a pound measure in inches?

19. A gallon of water weighs 10 lbs. and measures 277·274 cubic inches; what is the weight of a cubic foot and the measure of a ton?

20. What multiplier will convert grains troy, to the cubic inch into avoirdupoise pounds per foot?

21. Reduce [illegible] of £233 9s. 8d. to the decimal of £99 3s. 4d.

22. It having been calculated that the air-pump of a marine engine can lift 8616·96 tons of salt water in 6 hours; by considering that a cubic foot of salt water weighs 64 lbs., what is the error and actual weight of the salt water when the true weight of a cubic foot is 64·16875 lbs.?

23. If a sovereign weighs 123·274 grains, how many sovereigns will weigh 10 lbs. 8 ozs. 8 dwt. 5 grs.?

24. Simplify—

(1) $$\frac{\cdot17 \text{ of } (\cdot00617 - \cdot00532) + \cdot119 \times \cdot007}{\cdot17 \times \cdot17}$$

(2) $$(1568\cdot774396\dot{4}1\dot{8} + 28\cdot0093 + 23\cdot31000\dot{8}\dot{1}) \div 22\cdot817.$$

(3) $$\left[(28 \times 6\cdot09 - 31\cdot206 \div 7 + \frac{\cdot019 \times \cdot013}{\cdot019 \div \cdot013} - 5 \text{ times } 33\cdot1474338 + \tfrac{1}{8}) \text{ of } 2\tfrac{1}{2}\right]$$
$$\div \left\{3\,(8\cdot1 \times 6\cdot3 - 50\cdot72\dot{3} - \cdot16) + (\cdot1\dot{5} + \cdot\dot{2}\dot{5} + \cdot035) \times \cdot071 + 5\cdot168501\dot{8}\right\}$$

25. Supposing the velocity of electricity to be 288,000 miles per second, and the earth's circumference to be 25000 miles; calculate to seven places of decimals the time of transmission of an electric telegraph to the antipodes.

26. The parallax of the star *a* Centauri is given as 0"·9187, or $\frac{11}{12}$ths of a second; show by how much the vulgar fraction differs from the decimal fraction.

27. Water expands when freezing so that a cubic foot of water becomes 1·089 feet of ice; find how many cubic feet of water there are in an iceberg which is estimated to be 900 feet long, 88 feet broad, and 220 feet high.

28. A nautical mile is 6082·66 feet, and an imperial mile is 5280 feet; express each of these miles as decimals of the other.

29. In Fahrenheit's thermometer there are are 180 degrees between the boiling and freezing points, in that of Reaumer's only 80; what fraction of a degree in the latter expresses a degree in the former?

RECIPROCALS.

164. The quotient arising from the division of unity by any number is called the reciprocal of that number. Thus, the reciprocal of 14 is ·076923, or $\frac{1}{14}$. The reciprocal of ·5 is 2. The reciprocal of 80 is ·0125.

The use of reciprocals answers the same purpose in mixed multiplications and divisions, as the arithmetical complement does in a mixture of additions and subtractions. For it is evident, that to multiply by the reciprocal of a number is equivalent to division by the number itself; and, therefore, if we have a mixture of multipliers and divisors, we may take the reciprocals of the divisors, and thus convert them into multipliers.

This is nothing more than, for example, multiplying by $\frac{4}{5}$ or ·8 instead of dividing by $\frac{5}{4}$ or 1·25. Large tables of reciprocals have been calculated. Those most used in England are Barlow's. The tables of reciprocals are very useful in such cases as the following:—

To find the decimal value of $\frac{365}{7988} + \frac{273}{5079}$.

The reciprocal of 7988 is ·0001251878.
The reciprocal of 5079 is ·0001968892.

Multiplying these by the numerators of the fractions, and adding them, we have ·05375075 + ·04569355 = ·099443.

CHAPTER VI.

PRACTICE.

165. **Practice** is a compendious method of finding the value of a number of articles, when the value of a unit of any denomination is given.

Thus we may find by this rule the cost of 182 tons of sugar at £3 15s. per ton; of 21 sq. yards of carpet at 2s. 9d. per sq. foot; or 30 tons of coal at 1s. 2d. per cwt., &c.

Note.—It is evident that all the results obtained by Practice may also be found by Compound Multiplication, but the former rule has the advantage of arriving at these results by shorter methods. The rule of Practice does not involve any principle beyond the rules of Compound Multiplication and Division, Fractions and Decimals. It depends for its use on the readiness with which the above rules can be applied, and on the dexterity and quickness which the calculator acquires by practice.

166. The common difficulty of working sums in Practice consists in finding out what are the proper fractional parts into which to sub-divide either the money which is the price, or the quantities of the goods purchased; the tables below will supply this information. The separation of the *price*, or the *quantities*, of the goods into the different parts may be made in a great variety of ways, but in all of them each of the parts must be an *aliquot* part of some one which precedes it, and the first is usually an aliquot part of the unit of the highest denomination.

167. **Def. An Aliquot Part** of a number, or fraction, is a part which is contained in that number, or fraction, an exact number of times,

Thus, 4 is an aliquot part of 12, 6 of 18; 10s. is an aliquot part of £1, for it is ½ of £1; 2s. 6d. is an aliquot part of 10s., since it is ¼ of 10s; 14 lbs. is an aliquot part of 1 cwt., for it is ⅛ of a cwt., and so on.

The first table contains the aliquot parts of £1, so arranged that the simple aliquot parts, or the aliquot part of an aliquot part, may be found at once by inspection.

	Half.	Third.	Fourth.	Fifth.	Sixth.	Eighth.	Tenth.	Twelfth.	Twentieth	Fortieth.
Of £1	10s. 0d	6s 8d	5s 0d	4s 0d	3s 4d	2s 6d	2s 0d	1s 8d	1s 0d	6d
Half . . 10s 0d	5	3 4	2 6	2 0	1 8	1 3	1 0	0 10	0 6	3
Third . . 6 8	3 4		1 8	1 4		0 10	0 8		0 4	2
Fourth . . 5 0	2 6	1 8	1 3	1 0	0 10	0 7½	0 6	0 5	0 3	1½
Fifth . . 4 0	2 0	1 4	1 0		0 8	0 6		0 4		
Sixth . . 3 4	1 8		0 10	0 8		0 5	0 4		0 2	1
Eighth . . 2 6	1 3	0 10	0 7½	0 6	0 5	0 3¾	0 3	0 2½	0 1½	0¼
Tenth . . 2 0	1 0	0 8	0 6		0 4	0 3		0 2		
Twelfth . 1 8	0 10		0 5	0 4		0 2½	0 2		0 1	0½
Twentieth. 1 0	0 6	0 4	0 3		0 2	0 1½		0 1		
Fortieth . 0 6	0 3	0 2	0 1½		0 1	0 0¾		0 0½		

Here the simple aliquot parts of a pound, the half, third, &c., are found in the *left*-hand column, and in the *upper* column; the aliquot part of an aliquot part is found in the square *opposite to* the one and *under* the other part; thus opposite to *eighth* and under *fourth* is found 7½d., which shows that 7½d. is the *eight* of a *fourth* of a pound, *i.e.*, of 5s.

The following tables of aliquot parts will be found useful for reference:—

Of a shilling.

1d.	is	$\frac{1}{12}$
1½d.	are	$\frac{1}{8}$
2d.	,,	$\frac{1}{6}$
3d.	,,	$\frac{1}{4}$
4d.	,,	$\frac{1}{3}$
6d.	,,	$\frac{1}{2}$

Of a pound troy.

6 oz.	are	$\frac{1}{2}$
4 oz.	,,	$\frac{1}{3}$
3 oz.	,,	$\frac{1}{4}$
2 oz.	,,	$\frac{1}{6}$
1 oz. 10 dwts.	,,	$\frac{1}{8}$
1 oz.	is	$\frac{1}{12}$

Of an ounce troy.

10 dwts.	are	$\frac{1}{2}$
6 dwts. 16 grs.	,,	$\frac{1}{3}$
5 dwts.	,,	$\frac{1}{4}$
4 dwts.	,,	$\frac{1}{5}$
3 dwts. 8 grs.	,,	$\frac{1}{6}$
2 dwts. 12 grs.	,,	$\frac{1}{8}$
2 dwts.	,,	$\frac{1}{10}$
1 dwt. 16 grs.	,,	$\frac{1}{12}$

Of an acre.

2 roods	are	$\frac{1}{2}$
1 rood	is	$\frac{1}{4}$
20 poles	are	$\frac{1}{8}$
16 poles	,,	$\frac{1}{10}$

Avoirdupoise.

Of a ton.

10 cwt.	are	$\frac{1}{2}$
5 cwt.	,,	$\frac{1}{4}$
4 cwt.	,,	$\frac{1}{5}$
2 cwt.	,,	$\frac{1}{10}$
1 cwt.	is	$\frac{1}{20}$

Of a cwt.

2 qrs.	are	$\frac{1}{2}$
1 qr.	is	$\frac{1}{4}$
16 lbs.	are	$\frac{1}{7}$
14 lbs.	,,	$\frac{1}{8}$
8 lbs.	,,	$\frac{1}{2}$ of $\frac{1}{7}$
7 lbs.	,,	$\frac{1}{2}$ of $\frac{1}{8}$
3½ lbs.	,,	$\frac{1}{4}$ of $\frac{1}{8}$
2 lbs.	,,	$\frac{1}{7}$ of $\frac{1}{8}$
1¾ lbs.	,,	$\frac{1}{8}$ of $\frac{1}{8}$

Of a quarter.

14 lbs.	are	$\frac{1}{2}$
7 lbs.	,,	$\frac{1}{4}$
4 lbs.	,,	$\frac{1}{7}$
3½ lbs.	,,	$\frac{1}{8}$

Of a pound.

8 oz.	are	$\frac{1}{2}$
4 oz.	,,	$\frac{1}{4}$
2 oz.	,,	$\frac{1}{8}$
1 oz.	is	$\frac{1}{16}$

168. Practice may be separated into two cases, **Simple** and **Compound.**

I. SIMPLE PRACTICE.

In this case the given quantity is expressed in the same denomination as the unit whose value is given.

Under this head will occur such examples as the following:—

86 cwts. at £3 10s. per cwt.; 54 lbs. at £2 13s. 4d. per lb.; 73 ozs. at £5 17s. 6d. per oz., &c., where the unit, whose value is *given*, is of the same denomination as the quantity whose value is *required.*

The rule for Simple Practice will be easily shown by the following examples.

Examples.

Ex. 1. Find the cost of 324 articles at 16s. 8½d. each.

£324 = the cost of 324 articles at £1 each.

Therefore—

				£	s.	d.
The cost of 324 articles at	10s. each = ½ their cost at	£1 each =		£162	0	0
Their cost at	5s. each = ½ their cost at	10s. each =		81	0	0
Their cost at	1s. 3d. each = ¼ their cost at	5s. each =		20	5	0
Their cost at	5d. each = ¼ their cost at	1s. 3d. each =		6	15	0
Their cost at	½d. each = $\frac{1}{10}$ their cost at	5d each =		0	13	6
Therefore, by adding up the vertical columns, we have their cost at 16s. 8½d. each			=	£270	13	6

The usual form of writing the process is as follows:—

	£324			= the cost at £1 each.
10s. = ½ of £1	162	0	0	= the cost at 10s. 0d. each.
5s. = ½ of 10s.	81	0	0	= the cost at 5s. 0d. each.
1s. 3d. = ¼ of 5s.	20	5	0	= the cost at 1s. 3d. each.
5d. = ⅓ of 1s. 3d.	6	15	0	= the cost at 0s. 5d. each.
½d. = $\frac{1}{10}$ of 5d.	0	13	6	= the cost at 0s. 0½d. each.
Therefore,	£270	13	6	= the cost at 16s. 8½d. each.

Answer £270 13s. 6d.

N.B.—In taking the *aliquot parts* a little judgment should be exercised to prevent an awkward quantity from remaining at last—*e.g.*, we might have taken the aliquot parts in the above example thus:—

10s. 0d. = ½ of £1.
6s. 8d. = ⅓ of £1.

We should now have had ½d. to calculate for. But ½d. is $\frac{1}{160}$ of 6s. 8d., so that we should have been compelled to divide "the cost at 6s. 8d. by 160 to find the cost at ½d." This would have been a comparatively awkward operation, and it will be found better to avoid such remainders when it is possible to do so. When a remainder of this kind does occur, however, it is sometimes easier to make a separate calculation for it, thus:—

324 articles at ½d. each = 162 pence = 13s. 6d.

Ex. 2. Find the cost of 8794 at 2s. 0¼d. each.

In this instance, instead of taking 2s. 0d. = $\frac{1}{10}$ of a £, and a farthing = $\frac{1}{96}$ of 2s., the last being an inconvenient divisor, we take 1s. 8d. = $\frac{1}{12}$ of £, 4d. = ⅕ of that, and a ¼ = $\frac{1}{16}$ of that.

The cost of 8794 articles at £1 each is £8794.

	£8794			= the cost at £1 each.
1s. 8d. = $\frac{1}{12}$ of a £	732	16	8	= the cost at 1s. 8d. each.
4d. = ⅕ of 1s. 8d.	146	11	4	= the cost at 4d. each.
¼d. = $\frac{1}{16}$ of 4d.	9	3	2¼	= the cost at ¼d. each.
Answer	£888	11	2¼	= the cost at 2s. 0¼d. each.

169. There is another method of finding the value of a number of articles when the price of one is less than £1, thus,

Ex. 3. Find the cost of 186 articles at 7s. 3½d. each.

The cost of 186 articles at 1s. each = 186s. = £9 6s.

Therefore, we say

	£9	6	0	= the cost at 1s. 0d. each.
			7	
	65	2	0	= the cost at 7s. 0d. each.
3d. = ¼ of 1s.	2	6	6	= the cost at 0s. 3d. each.
½d. = ⅙ of 3d.	0	7	9	= the cost at 0s. 0½d. each.
Answer	£67	16	3	= the cost at 7s. 3½d. each.

Ex. 4. Find the cost of 750 articles at £12 18s. 10¾d. each.

The cost of 750 articles at £1 each = £750.

Therefore £750

12

	9000	0s.	0d.	= the cost at	£12	0s.	0d.	each.
10s. = ½ of £1	375	0	0	= the cost at	0	10	0	each.
6s. 8d. = ⅓ of £1	250	0	0	= the cost at	0	6	8	each.
2s. = ⅕ of 10s.								
(∴ take ⅕ of £375)	75	0	0	= the cost at	0	2	0	each.
2d. = $\frac{1}{12}$ of 2s.	6	5	0	= the cost at	0	0	2	each.
½d. = ¼ of 2d.	1	11	3	= the cost at	0	0	0½	each.
¼d. = ½ of ½d.	0	15	7½	= the cost at	0	0	0¼	each.
Answer	£9708	11	10½	= the cost at	£12	18	10¾	each.

Since the 750 articles at £1 each would obviously cost £750, if we multiply that sum by 12 we obtain the cost at £12. As 10s. is ½ of £1, by dividing 750 by 2, we find the cost at 10s. Next, 6s. 8d. is ⅓ of £1, by dividing 750 by 3 we get the cost at 6s. 8d. Next, we see by the table that 2s. is ⅕ of 10s.; hence, by dividing the cost at 10s. by 5 we find the cost at 2s.

170. If the price be less than a shilling, the best way is to take parts of a shilling so as to find the price in shillings, and afterwards reduce to pounds, thus—

Ex. 5. Find the cost of 10374 articles at 9¾d.

The cost of 10374 articles at 1s. = 10374s.

	10374s.		= the cost at 1s. 0d. each.
6d. = ½ of 1s.	5187	0	= the cost at 0s. 6d. each.
3d. = ½ of 6d.	2593	6	= the cost at 0s. 3d. each.
¾d. = ¼ of 3d.	648	4½	= the cost at 0s. 0¾d. each.
2,0	842,8	10½	= the cost at 0s. 9¾d. each.
	£421 8	10½	

171. The following artifice may sometimes be employed to simplify the solution of questions in practice, viz.—

Taking the price as if somewhat higher than that which is given, and subtracting a convenient aliquot part; thus,—

Ex. 6. Find the price of 218 cwt. at £5 18s. 4d. per cwt.

Here the given price, viz., £5 18s. 4d. = £6, less 1s. 8d.

Therefore, find the cost at £6, and subtract the cost at 1s. 8d.

	£218			= the cost of 218 cwt. at	£1	0	0 per cwt.
	6						
Therefore, 1s. 8d. = $\frac{1}{12}$ of £1	1308	0	0	= the cost at	£6	0	0 per cwt.
Subtracting	18	3	4	= the cost at	£0	1	8 per cwt.
Answer	£1289	16	8	= the cost at	£5	18	4 per cwt.

172. When the price is an *even* number of *shillings*, multiply the number of articles by *half* the number of shillings, cut off the *unit's* figure of the result, and *double* it: reckon this doubled figure as the shillings, and the rest of the result as the *pounds* of the answer.

Ex. 7. Required the cost of 313 articles at 18s. each.

	313	
	9	
Answer	£281	14

Here the $\frac{1}{2}$ of 18s. is 9s.; then 9 times 3 are 27, and doubling this we have 54; 54 shillings are 2 pounds 14 shillings; write 14s. down and carry the 2 pounds: 9 times 1 are 9, and 2 carried are 11; set down 1 and carry 1: 9 times 3 are 27, and 1 carried are 28, set down 28; hence the answer is £281 14s.

173. When it is required to find the value of a certain number of things, and a fraction, we proceed as follows:—

Ex. 8. 270$\frac{1}{8}$ cwt. at £2 13s. 8d. each.

Here the cost of 270$\frac{1}{8}$ cwt. at £1 each = £270$\frac{1}{8}$ = £270 2s. 6d.

	£270	2	6	= the cost at	£1	0	0	each.
			2					
	540	5	0	= the cost at	£2	0	0	each.
10s. = $\frac{1}{2}$ of £1	135	1	3	= the cost at	0	10	0	each.
3s. 4d. = $\frac{1}{3}$ of 10s.	45	0	5	= the cost at	0	3	4	each.
4d. = $\frac{1}{10}$ of 3s. 4d.	4	10	0$\frac{1}{2}$	= the cost at	0	0	4	each.
Answer	£724	16	8$\frac{1}{2}$	= the cost at	£2	13	8	each.

The above is the most expeditious method of finding the answer, which, however, might have been obtained as follows,—

Since 270 cwt. will cost 270 times £2 13s. 8d.,

and $\frac{1}{8}$ cwt. will cost $\frac{1}{8}$ times of £2 13s. 8d.,

it follows that 270$\frac{1}{8}$ cwt. will cost 270 times £2 13s. 8d. + $\frac{1}{8}$ of £2 13s. 8d.

Now 270 times £1 = £270.

therefore	£270							
	2							
	540			= the cost of 270 cwt. at	£2	0	0	per cwt.
10s. = $\frac{1}{2}$ of £1	135	0	0	= the cost of 270 cwt. at	0	10	0	per cwt.
3s. 4d. = $\frac{1}{3}$ of 10s.	45	0	0	= the cost of 270 cwt. at	0	3	4	per cwt.
4d. = $\frac{1}{10}$ of 3s. 4d.	4	10	0	= the cost of 270 cwt. at	0	0	4	per cwt.
	724	10	0	= the cost of 270 cwt. at	2	13	8	per cwt.
$\frac{1}{8}$ of £2 13s. 8d. =		6	8$\frac{1}{2}$	= the cost of $\frac{1}{8}$ cwt. at	2	13	8	per cwt.
Answer	£724	16	8$\frac{1}{2}$	= the cost of 270$\frac{1}{8}$ cwt. at	2	13	8	per cwt.

Ex. 9. 256$\frac{7}{8}$ ounces at £3 16s. 11$\frac{1}{2}$d.

Here cost of 256$\frac{7}{8}$ ozs. at £1 each = £256 17s. 6d.

10s. = $\frac{1}{2}$ of £1	£256	17	6	= the cost at	£1	0	0	each.
			3					
	770	12	6	= the cost at	3	0	0	each.
	128	8	9	= the cost at	0	10	0	each.
5s. = $\frac{1}{2}$ of 10s.	64	4	4$\frac{1}{2}$	= the cost at	0	5	0	each.
1s. 3d. = $\frac{1}{4}$ of 5s.	16	1	1$\frac{1}{8}$	= the cost at	0	1	3	each.
7$\frac{1}{2}$d. = $\frac{1}{2}$ of 1s. 3d.	8	0	6$\frac{9}{16}$	= the cost at	0	0	7$\frac{1}{2}$	each.
1d. = $\frac{1}{15}$ of 1s. 3d.	1	1	4$\frac{7}{8}$	= the cost at	0	0	1	each.
	£988	8	8$\frac{1}{16}$	= the cost at	£3	16	11$\frac{1}{2}$	each.

In the last line of division the student must observe that the 1d. is the $\frac{1}{15}$ of 1s. 3d., and that £16 1s. 1$\frac{1}{8}$d. is divided by 15, instead of the line immediately above.

174. We may sometimes take aliquot parts of the ***multiplied*** quantity instead of the original quantity, thus—

Ex. 10. Required the cost of 324 articles at £7 17s. 6d.

	£324	0	0	= the cost at	£1	0	0	each.
	7							
	2268	0	0	= the cost at	7	0	0	each.
17s. 6d. = $\frac{1}{8}$ of £7	283	10	0	= the cost at	0	17	6	each.
Answer	£2551	10	0	= the cost at	7	17	6	each.

Here 17s. 6d., which is $\frac{7}{8}$ of £1, is $\frac{1}{8}$ of £7; therefore divide 2268 by 8, the quotient is the cost at 17s. 6d. each.

II. COMPOUND PRACTICE.

175. In this case of Practice, the given number is not wholly (sometimes not at all) expressed in the same denomination as the unit whose value is given; as, for instance, 1 cwt. 2 qrs. 14 lbs. at £2 2s. per cwt.; 19 ton 17 cwt. 2 qrs. at 1s. 2d. per cwt.

The rule for Compound Practice will be easily seen by the following examples.

Examples.

Ex. 1. Find the value of 84 cwt. 3 qrs. 14 lbs. of sugar at £12 11s. 8d. per cwt.

The method of working such an example is the following:—

The value of 1 cwt. of sugar being £12 11s. 8d,

the value of 84 cwt. of sugar =	£1057	0	0
2 qrs. = $\frac{1}{2}$ (value of 1 cwt.) =	6	5	10
1 qr. = $\frac{1}{2}$ (value of 2 qrs.) =	3	2	11
14 lbs. = $\frac{1}{2}$ (value of 1 qr.) =	1	11	5½
therefore, by adding up the vertical columns,			
the value of 84 cwt. 3 qrs. 14 lbs. =	£1068	0	2½

The operation is usually written thus—

	£12	11	8	= value of 1 cwt.			
			12				
	151	0	0	= value of 12 cwt.			
			7				
	1057	0	0	= value of (12 × 7) or	84 cwt.	0 qrs.	0 lbs.
2 qrs. = $\frac{1}{2}$ of 1 cwt.	6	5	10	= value of	0	2	0
1 qr. = $\frac{1}{2}$ of 2 qrs.	3	2	11	= value of	0	1	0
14 lbs. = $\frac{1}{2}$ of 1 qr.	1	11	5½	= value of	0	0	14
	£1068	0	2½	= value of	84	3	14

Here, as £12 11s. 8d. is the cost of 1 cwt., we multiply that sum successively by 12 and 7, the factors of 84, to obtain the cost of 84 cwt.; then as 2 qrs. are $\frac{1}{2}$ of 1 cwt., $\frac{1}{2}$ of the top line will be the cost of 2 qrs., and as 1 qr. is the $\frac{1}{2}$ of 2 qrs., we divide the cost of 2 qrs. by 2. Next, 14 lbs. being $\frac{1}{2}$ of 1 qr., we divide the value of 1 qr. by $\frac{1}{2}$. The results being added together give the cost of 84 cwt. 3 qrs. 14 lbs.

Ex. 2. Find the value of 7 cwt. 3 qrs. 11 lbs. at £2 13s. 1d. per quarter.

The given unit being a quarter, we reduce 7 cwt. 3 qrs. to 31 qrs., and find the value of them by multiplying by 31; then to find the value of 11 lbs., we consider that 7 lbs. are $\frac{1}{4}$ qr., and 4 lbs. are $\frac{1}{7}$ qr.; so that, dividing £2 13s. 1d., the value of 1 qr., by 4 and 7, and adding up, we have the value of 7 cwt. 3 qrs. 11 lbs.

	£	s.	d.		
10 × 3 + 1 = 31	£2	13	1		
			10		
	26	10	10	=	value of 10 qrs.
			3		
	79	12	6	=	value of 30 qrs. 0 lbs.
	2	13	1	=	value of 1 0
7 lbs. = $\frac{1}{4}$ of 1 qr.	0	13	$3\frac{1}{4}$	=	value of 0 7
4 lbs. = $\frac{1}{7}$ of 1 qr.	0	7	7	=	value of 0 4
Answer	£83	6	$5\frac{1}{4}$	=	value of 31 qrs. 11 lbs.

Ex. 3. Find the cost of 6 cwt. 3 qrs. $7\frac{1}{2}$ lbs. at £7 13s. 6d. per cwt.

	£	s.	d.			cwt.	qrs.	lbs.
2 qrs. = $\frac{1}{2}$ of 1 cwt.	£7	13	6	=	the cost of 1 cwt.			
			6					
	46	1	0	=	the cost of	6 cwt.	0 qrs.	0 lbs.
	3	16	9	=	the cost of	0	2	0
1 qr. = $\frac{1}{2}$ of 2 qrs.	1	18	$4\frac{1}{2}$	=	the cost of	0	1	0
{4 lbs. = $\frac{1}{7}$ of 1 qr.	0	5	$5\frac{11}{14}$	=	the cost of	0	0	4
{$3\frac{1}{2}$ lbs. = $\frac{1}{8}$ of 1 qr.	0	4	$9\frac{9}{16}$	=	the cost of	0	0	$3\frac{1}{2}$
	£52	6	$4\frac{95}{112}$	=	the cost of	6	3	$7\frac{1}{2}$

Here, as £7 13s. 6d. is the cost of 1 cwt., we multiply that sum by 6 to obtain the cost of 6 cwt.; then, as 2 qrs. are $\frac{1}{2}$ of 1 cwt., $\frac{1}{2}$ of the top line will be the cost of 2 qrs.; and as 1 qr. would cost $\frac{1}{2}$ of the cost of 2 qrs., we divide the cost of 2 qrs. by 2. Next, 4 lbs. being $\frac{1}{7}$ and $3\frac{1}{2}$ lbs. being $\frac{1}{8}$ of 1 qr., we divide the cost of 1 qr. by 7 and 8 successively. These results being added give the cost of 6 cwt. 3 qrs. $7\frac{1}{2}$ lbs.

$$\frac{1}{2} + \frac{11}{14} + \frac{9}{16} = \frac{56 + 88 + 63}{112} = \frac{207}{112} = 1\frac{95}{112}.$$

In cases where there are fractional parts of a penny remaining after division, it is easier *not* to reduce them to farthings, but to leave them as fractions of a penny. If greater accuracy be thought necessary, the fraction in the answer may be reduced to farthings. Thus the answer of this example may be written £52 6s. 4d. $3\frac{11}{28}$f.

[N.B.—If any fractional parts of a farthing remain, it is better *not* to write the farthings as a fraction of a penny, to be followed again by another fraction, which is meant to represent the fraction of a farthing. But write the farthings as whole numbers of a separate denomination, followed by their own fraction].

Ex. 4. Find the cost of 294 yards at 8s. $9\frac{1}{2}$d. per foot.

Here the given unit is a foot; then since 3 feet make 1 yard, 294 yards = 294 × 3 = 882 feet; and the cost of 882 feet at 1s. per foot = 882s.; therefore,

	s.	d.			s.	d.	
	882s.		=	the cost at	1s.	0d.	per foot.
	8						
	7056		=	the cost at	8s.	0d.	per foot.
6d. = $\frac{1}{2}$ of 1s.	441		=	the cost at	0	6	per foot.
3d. = $\frac{1}{2}$ of 6d.	220	6	=	the cost at	0	3	per foot.
$\frac{1}{2}$d. = $\frac{1}{6}$ of 3d.	36	9	=	the cost at	0	$0\frac{1}{2}$	per foot.
2,0	775,4	3	=	the cost at	8	$9\frac{1}{2}$	per foot.
	£387 14	3					

The above example might also have been worked thus:—

Since 3 feet = 1 yard, we have

8s. $9\frac{1}{2}$d. per foot = 3 (8s. $9\frac{1}{2}$d.) per yard = £1 6s. $4\frac{1}{2}$d. per yard.

Therefore,	£	s.	d.			£	s.	d.	
	£294	0	0	=	the cost at	£1	0s.	0d.	per yard.
5s. = $\frac{1}{4}$ of £1	73	10	0	=	the cost at	0	5	0	per yard.
1s. 3d. = $\frac{1}{4}$ of 5s.	18	7	6	=	the cost at	0	1	3	per yard.
$1\frac{1}{2}$d. = $\frac{1}{10}$ of 1s. 3d.	1	16	9	=	the cost at	0	0	$1\frac{1}{2}$	per yard.
	£387	14	3	=	the cost at	£1	6	$4\frac{1}{2}$	per yard.

Ex. 5. Find the value of 76 sq. yds. 5 sq. ft. 108 sq. in. at 3s. 9½d. per sq. ft.

Here 3s. 9½d. per sq. ft. = 3s. 9½d. × 9, *i.e.*, £1 14s. 1½d. per sq. yd.

Therefore,	£1	14	1½	= the cost of 1 sq. yd.			
			12				
	20	9	6	= the cost of 12 sq. yds.	0 sq. ft.	0 sq. in.	
			6				
	122	17	0	= the cost of 72 sq. yds.	0 sq. ft.	0 sq. in.	
	6	16	6	= the cost of 4	0	0	
3 sq. ft. = $\frac{1}{3}$ of 1 sq. yd.	0	11	4½	= the cost of 0	3	0	
1 sq. ft. = $\frac{1}{3}$ of 3 sq. ft.	0	3	9½	= the cost of 0	1	0	
1 sq. ft. = $\frac{1}{3}$ of 3 sq. ft.	0	3	9½	= the cost of 0	1	0	
72 sq. in. = $\frac{1}{2}$ of 1 sq. ft.	0	1	10¾	= the cost of 0	0	72	
36 sq. in. = $\frac{1}{2}$ of 72 sq. in.	0	0	11⅜	= the cost of 0	0	36	
Answer	£130	15	3⅝	= the cost of 76 sq. yds.	5 sq. ft.	108 sq. in.	

176. The principles of Practice may be applied to any Compound Multiplication sum, as in the following examples.

Ex. 6. Find the gross weight of 179 cast iron pipes, each weighing 2 cwt. 3 qrs. 7 lbs.

The weight of 179 pipes at 1 ton each = 179 tons.

	tons.	cwt.	qrs.	lbs.				
	179	0	0					
2 cwt. 2 qrs. = $\frac{1}{8}$ of 1 ton.	22	7	2	0	= weight at 2 cwt.	2 qrs.	0 lbs.	each.
1 qr. 7 lbs. = $\frac{1}{8}$ of 2 cwt. 2 qrs.	2	15	3	21	= weight at 0	1	7	each.
Answer	25	3	1	21	= weight at 2	3	7	each.

Ex. 7. Multiply 3 tons 7 cwt. 3 qrs. by 4793.

	tons.	cwt.	qrs.
	4793	0	0
			3
	14379	0	0
5 cwt. = $\frac{1}{4}$ of 1 ton.	1198	5	0
2 cwt. 2 qrs. = $\frac{1}{2}$ of 5 cwt.	599	2	2
1 qr. = $\frac{1}{10}$ of 2 cwt. 2 qrs.	59	18	1
Answer	16236	5	3

Ex. 8. A bankrupt owes £3600 and pays 12s. 6½d. in the £, what is the value of his assets?

This means that for every £ he owes he can pay only 12s. 6½d.; here then we have to find the value of 12s. 6½d. × 3600, which must have been the value of his whole estate.

If he paid 20s. in the £ his assets would be worth £3600.

Therefore,	£3600	0	0	= assets when 20s.		are paid in £.
10s. 0d. = $\frac{1}{2}$ of £1	1800	0	0	= assets when 10s.	0d.	are paid in £.
2 6 = $\frac{1}{4}$ of 10s. 0d	450	0	0	= assets when 2	6	are paid in £.
0 0½ = $\frac{1}{60}$ of 2 6	7	10	0	= assets when 0	0½	is paid in £.
Answer	£2257	10	0	= assets when 12	6½	are paid in £.

Ex. 9. Find the weight of 252 sacks of wheat, each sack weighing 1 cwt. 2 qrs. 23 lbs.

Here 252 sacks of wheat, each weighing 1 cwt., weight 252 cwt.

Therefore,	252 cwt.	0 qrs.	= weight at 1 cwt.	0 qrs.	0 lbs.	to the sack.
2 qrs. = $\frac{1}{2}$ of 1 cwt.	126	0	= weight at 0	2	0	to the sack.
14 lbs. = $\frac{1}{4}$ of 2 qrs.	31	2	= weight at 0	0	14	to the sack.
7 lbs. = $\frac{1}{2}$ of 14 lbs.	15	3	= weight at 0	0	7	to the sack.
2 lbs. = $\frac{1}{7}$ of 14 lbs.	4	2	= weight at 0	0	2	to the sack.
Answer	429	3	= weight at 1	2	23	to the sack.

177. We may sometimes introduce a subsidiary aliquot part to facilitate calculation, and erase it from the result when the other parts have been found from it, *e.g.*,—

Ex. 10. Find the cost of a silver-gilt goblet weighing 3 lbs. 4 oz. 15 grs. at £2 7s. 8d. per ounce.

3 lbs. 4 ozs. = 40 ozs.

1 dwt. = $\frac{1}{20}$ of 1 oz.	£2	7	8 40	= the cost of 1 oz.		
	95	6	8	= the cost of	40 ozs.	0 grs.
	0	$\cancel{2}$	$\cancel{4\frac{3}{5}}$	= the cost (to be erased) of 1 dwt.		
12 grs. = $\frac{1}{2}$ of 1 dwt.	0	1	$2\frac{3}{10}$	= the cost of	0	12
3 grs. = $\frac{1}{4}$ of 12 grs.	0	0	$3\frac{23}{40}$	= the cost of	0	3
Answer	£95	8	$1\frac{7}{8}$	= the cost of	3 lbs. 4 ozs.	15 grs.

In this example, as the denomination of *dwts.* does not occur in the question, and as a *grain* is $\frac{1}{480}$ part of an *ounce*, it is convenient to introduce the price of 1 *dwt.*; not, however, to affect the answer, but only to derive from it the price of 15 *grs.* When this has been done, the cost of the 1 *dwt.* must be erased before the products, which form the answer, are added together.

$$\tfrac{3}{10} + \tfrac{23}{40} = \frac{12 + 23}{40} = \tfrac{35}{40} = \tfrac{7}{8}.$$

178. When both the factors in the question contain fractional parts which are complicated, it is a good plan to turn the money factor into pounds and the *decimal* of a pound, and then to take the aliquot parts of the other factor; *e.g.*, let it be required to find the price of 11 tons 17 cwt. 1 qr. 19 lbs. at £7 7s. 4½d. per ton.

Here, reducing 7s. 4½d. to the decimal of a pound, we find it is £·36875.

Here the cost of a ton is £7·36875.

10 cwt.	$\frac{1}{2}$	7·36875 11
		81·05625
5 cwt.	$\frac{1}{2}$	3·68437, &c.
2 cwt.	$\frac{1}{5}$	1·84218, &c.
		·73687, &c.
1 qr.	$\frac{1}{8}$	·09210, &c.
14 lbs.	$\frac{1}{2}$	·04605, &c.
4 lbs.	$\frac{2}{7}$	·01317, &c.
1 lb.	$\frac{1}{4}$	·00329, &c.
	£	87·47428, &c. 20
	s.	9·48560 12
	d.	5·8272 4
	f.	3·3088

∴ £87 9s. 5¾d. is the cost required.

EXAMPLES FOR PRACTICE.

Find the value of—

1. 785 at 10s.; 785 at 5s.; 785 at 4s.; 785 at 6s. 8d.; 785 at 1s. 8d.
2. 785 at 1s. 3d.; 234 at £1 3s. 4d.; 895 at 6d.; 895 at 4d.; 895 at ¾d.
3. 1275 at £1 10s.; 1275 at £1 5s.; 1275 at £1 7s. 6d.; 2101 at £2 12s.
4. 8943 at £3 17s. 9d.; 4782 at £7 13s. 9d; 890 at £2 0s. 1½d.; 164 at 8s. 5½d.
5. 640 at £7 4s. 7½d.; 320 at £3 0s. 3¾d.; 582 at £12 10s. 2½d.; 1603 at 16s. 10½d.
6. 150 at 9½d. per doz.; 265 at £63 3s. 1½d. per score; 85 at £8 17s. 6d. for every 40.
7. 111 at £11 11s. 11d. for every 11; 425⅝ at £3 17s. 10½d.; 1243⅞ at £3 14s. 2d.
8. 3 tons 12 cwt. 2 qrs. × 379; 2 lbs. 7 oz. 5 dwt. × 856; 14 yds. 1 ft. 6 in. × 248; 178⅔ at 6s. 7½d.
9. 750 tons iron plates at £8 15s. 6d.; 215 tons angle iron at £11 14s.; 30 tons rivets at £22 1s.
10. 72 cwt. 3 qrs. 17 lbs. at 6s. 1¼d. per qr.; 250 acres 3 roods 28 poles at £2 15s. 6d. per acre; 30848 at 6¾d.
11. 10000 at 8½d.; 7895 at 10¼d.; a million at 11¾d.; 31426 at 2s. 11d; 22828 at 14s. 9d.
12. A mile at 3s. 11d. per yard; a ton at 8s. 7½d. per lb.; a lb. troy at 1¾d. per grain.
13. The weight of a cubic yard at 1 lb. 13 oz. to the inch; the weight in tons of 257354 cubic ft. of sea water at 64 lbs.; the income tax at 5d. in the pound for £27,000,000.
14. What is the dividend on £2045 15s. 9d. at 5s. 11½d. in the £.
15. The weight of a sheet of lead 35 × 10½ feet at 6 lb. 11 oz. to the foot, and its cost at 3½d. per foot.
16. The weight of 3 miles of iron pipes 189 lbs. per foot run, and the cost at 8s. 10d. per cwt.
17. 12 tons 13 cwt. 3 qrs. 4 lbs. copper at 95s. 6d. per cwt.; 7 tons 17 cwt. 2 qrs. spelter at 37s. 9d. per cwt.
18. What is the difference in weight, in tons, &c., between 100 fathoms of chain cable weighing 76 lbs. to the foot, and 300 fathoms wire rope at 18 lbs. What will be the difference of cost, the chain cable being 15s. 6d. per cwt., and the wire rope £1 3s. 6d.
19. Required the cost of breaking metal for 176 miles 3 furlongs 19 chains of road at 3½d. per yard run.
20. How much property tax must be paid on £7964 at 2s. 9¾d. in the £.
21. Find the value of 157 tons at 7 guineas per ton, at £2⅝ per ton, and at £4 11s. 8d. per ton, using one aliquot part only. In the same way calculate the cost of 158 tons at 8 guineas per ton, at £1$\frac{11}{12}$ per ton, and at £4 7s. 6d. per ton.
22. Find (using decimals) the price of 10 lbs. 11 oz. 16 dwts. 16 grs. of gold at £3 17s. 10½d. per oz.
23. Find (introducing a subsidiary aliquot part) the cost of making a road whose length is 3 miles 30 poles 5 yards at £72 17s. 6d.
24. Find the cost of 3 qrs. 3 bushels 3 pecks at £6 16s. 8d. per quarter (artifice).
25. Find (using only one aliquot part) the cost of 18 lbs. 7 oz. 4 dwt. at £5 5s. 8d. per lb.
26. A contractor having commenced a railway finds that when he has yet 36 miles 3 fur. 22 yards to finish, his working expenses are increased by a strike of his men by £14 13s. 4d. per mile, and by a rise in the price of materials, by £9 13s. 4d. per mile. If he had reckoned on clearing £5000 by the contract, what will he be able to clear after these misfortunes?

CHAPTER VII.

RATIO, PROPORTION, AND RULE OF THREE.

179. We may compare one number or quantity with another, or ascertain the relation which one bears to the other in respect of magnitude, in two different ways; either by considering how much one is greater or less than the other; or by considering what multiple, part, or parts, one is of the other, that is, how many times or parts of a time, or both, one number is contained in the other. Thus, if we compare the number 12 with the number 3, we observe, adopting the first mode of comparison, that 12 is greater than 3 by the number 9; or, adopting the second mode of comparison, that 12 contains 3 four times, and is thus $\frac{12}{3}$, or four times as great as 3. Again, if we compare the number 5 with the number 11, we observe, according to the first mode of comparison, that 5 is less than 11 by the number 6, and, according to the second, that as 1 is one-eleventh part of 11, so 5 is five-thirteenth parts of 11, or $\frac{5}{11}$ths of 11.

180. The relation of one number to another in respect of magnitude is called **Ratio**; and when the relation is considered in the first of the above methods, that is, when it is estimated by the difference between the two numbers, it is called **Arithmetical Ratio**; but when it is considered according to the second method, that is, when it is estimated by considering what multiple, part, or parts, one number is of the other, or, which is seen from the above to be the same thing, by the fraction which the first number is of the second, it is called **Geometrical Ratio***. Thus, for instance, the arithmetical ratio of the number 12 and 3 is 9, while their geometrical ratio is $\frac{12}{3}$ or 4. In like manner, the arithmetical ratio of 5 and 11 is 6, while their geometrical ratio is $\frac{5}{11}$.

181. It is more common, however, in comparing one number with another to estimate their relation to one another in respect of magnitude according to the second method, and to call that relation so estimated by the name of **Ratio**. According to this mode of treatment, which we shall adopt in what follows, "**Ratio** is the relation which one number has to another, the comparison being made by considering what multiple, part or parts, the first number is of the second, or how many times, or parts of a time, or both, the second is contained in the first."

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* The term *arithmetical* ratio is now generally abandoned, so that when the word *ratio* is used, it signifies that which is divided by the *quotient* of one divided by the other.

182. The ratio of one number to another is usually expressed by placing two points, or a colon (:), between the numbers compared. Then the ratio of 8 to 4 is written 8 : 4; the ratio of 12 to 3 is 12 : 3. The two numbers are called the *terms* of the ratio, the former being called the *antecedent*, and the latter the *consequent*.

But to determine the value of a ratio the antecedent is written as the numerator, and the consequent as the denominator, of a fraction; for this will determine how many times the first contains, or is contained in the second, since a fraction (Def. § 81, page 63), is a simple manner of expressing the division of the numerator by the denominator; and the magnitude of the fraction thus determines the value of the ratio. The ratio of 6 to 2 is expressed by the fraction $\frac{6}{2}$ or $\frac{3}{1}$, which denotes that 2 is *one-third* of 6, *i.e.*, 3 times less than 6. By this method ratios can be treated arithmetically, their values determined, and so compared with one another.

183. A *direct* ratio is that which arises from dividing the antecedent by the consequent, as 6 ÷ 2. An *inverse*, or *reciprocal* ratio, is the ratio of the *reciprocals* of two numbers. Thus, the direct ratio of 9 to 3 is 9 : 3, or $\frac{9}{3}$; the reciprocal ratio is $\frac{1}{9} : \frac{1}{3}$, or $\frac{1}{9} \div \frac{1}{3} = \frac{3}{9}$, that is, the consequent 3 is divided by the antecedent 9. An *inverse, or reciprocal ratio, is expressed by inverting the fraction which expresses the direct ratio; or, when the notation is by points, by inverting the order of the terms.* Thus, 8 is to 4 directly as $\frac{1}{4}$ to $\frac{1}{8}$, or inversely, as 4 to 8.

184. *If the terms of a Ratio be multiplied or divided by the same quantity, the magnitude of the ratio will not be altered.*

Let the ratio be 3 : 8, then its magnitude is $\frac{3}{8}$, which is equivalent to

$$\frac{6}{16}, \text{ or } \frac{9}{24}, \text{ or } \frac{12}{32}, \text{ \&c.,}$$

that is, the ratio 3 : 8 is equal to each of the ratios 6 : 16, 9 : 24, 12 : 32, &c., which arise from the equal *multiplication* of its terms; and, conversely, each of the latter ratios is reducible to the original one by the equal *division* of its terms.

185. If the antecedents of two or more ratios be multiplied together for a new antecedent, and their consequents be multiplied together for a new consequent, the resulting ratio is said to be *compounded* of the others, and it is called their *Compound Ratio*.

Thus, if the ratios be 2 : 3, 4 : 7, and 8 : 13, the ratio which arises from their composition will be

$$2 \times 4 \times 8 \ : \ 3 \times 7 \times 13, \text{ or } 64 \ : \ 273.$$

186. If the two numbers compared are *equal*, the *ratio* is a *unit*, or 1, and is called a ratio of *equality*.

If the antecedent of a ratio is *greater* than the consequent, the ratio is *greater* than a unit, and is called a ratio of *greater inequality*. Thus, the ratio of 12 : 4 is 3; for the value of $\frac{12}{4} = 3$.

If the antecedent is *less* than the consequent, the ratio is *less* than a unit, and is called a ratio of *less inequality*. Thus, the ratio of 3 : 6 is $\frac{3}{6}$, or $\frac{1}{2}$, for $\frac{3}{6} = \frac{1}{2}$.

187. We have hitherto given instances only of the ratios of *abstract* quantities or *numbers* to one another; but we may similarly obtain the ratios of *concrete* quantities.

Thus, the ratio of £54 : £72, of 9 cwt. : 12 cwt., of 15 gals. : 20 gals., of 33 ft. : 44 ft., are by (§ 113, Rule XL) represents $\frac{54}{72}$, $\frac{9}{12}$, $\frac{15}{20}$, $\frac{33}{44}$, each of which reduces to $\frac{3}{4}$, and we say, therefore, that the ratio of £54 : £72 is the same as that of 3 : 4, or $\frac{3}{4}$, meaning that £54 is $\frac{3}{4}$ of £72, and so on with the other ratios; but *Ratio can exist only between quantities of the kind*, for otherwise, one of them could not be a fraction of the other.

The idea of relative or comparative magnitude which is essential to the correct notion of proportion, forbids our considering the term *ratio* in the same unrestricted sense as the term *quotient*; the two terms are to be regarded as meaning the same thing only when dividend and divisor are quantities of the same kind; *ratio is always an abstract number*; but the name quotient, as we have seen, is applied not only to abstract numbers, but to the concrete quantities that arise from taking the third, fourth, fifth, &c., part of concrete quantities. Be careful to observe this distinction, and to remember that the ratio of an antecedent to its consequent *always* has reference to the *number of times*, and parts of a time, which the former contains the latter; so that it would be absurd to speak of the ratio of one thing to another of a different kind, as for instance, of the ratio of £6 to the number 3, of 8 cwt. to £4, or 9 cwt. to 12 gals., &c.

188. *The ratio between two quantities of the same kind cannot be expressed by an abstract number unless both are expressed in one denomination.*

If the concrete numbers, though of the same kind, be not in the same denomination of that kind, they must be reduced to one and the same denomination, in order to find their ratio. Thus, if one number be 7 days and the other 13 hours, the ratio of the former to the latter will not be that of 7 to 13 but that of 7 days to 13 hours, that is, 168 hours to 13 hours, which will clearly be that of the abstract number 168 to the abstract number 13, and so will be expressed, not by $\frac{7}{13}$, but by $\frac{168}{13}$. We see that 7 dys. : 13 hrs. is the same as 168 : 13, and that each is $\frac{168}{13}$. Thus it is plain that when the numbers are concrete, we may reduce them to one and the same denomination, and then, in considering their ratio, treat them as abstract numbers.

EXAMPLES FOR PRACTICE.

1. What is the simplest expression of the magnitude of the ratio 3 : 5, 4 : 12, 9 : 21, $\cdot 3\dot{9} : \cdot 9\dot{3}$.
2. Which of the ratios is greatest, 5 : 9 or 7 : 11, 10 : 17 or 17 : 23, and 34 : 27 or 37 : 31.
3. Which of the three ratios is least, 7 : 15, $1\frac{3}{4} : 2\frac{5}{8}$, and ·75 : ·96.
4. What is the ratio of £2 6s. 8d. to £3 10s., of 3 tons 5 cwt. 2 qrs. to 5 tons 7 cwt., of an oz. troy to an oz. avoirdupoise.
5. Find which is the greatest ratio, $\frac{3}{5}$ or $\frac{37}{60}$, $\frac{11}{9}$ or $\frac{16}{17}$, $\frac{12}{31}$ or $\frac{42}{124}$.

189. **Proportion in an equality of ratios.**

If two ratios are equal (*i.e*, "If the fraction expressed by the antecedent and consequent of the first ratio be equal to the fraction expressed by the antecedent and consequent of the second ratio") they are said to form a *Proportion*, and the four terms of which it is composed are called *Proportionals*, or said to be *proportional to one another.*[*]

Thus, if we have two numbers, as 6 and 2, of which we know the first is 3 times greater than the second, and two other numbers, 15 and 5, of which the first is again 3 times greater than the second, the ratio of 6 to 2 is equal to the ratio of 15 to 5, and the four numbers 6, 2, 15, 5, constitute a proportion which is usually written—

$$6 : 2 :: 15 : 5, \text{ or } 6 : 2 = 15 : 5$$

and is read—

as 6 is to 2, so is 15 to 5.

And since a ratio may be written as a fraction the proportion may be written $\frac{6}{2} = \frac{15}{5}$, and is read—6 divided by 2 is equal to 15 divided by 5.

190. The first and last terms of a proportion are called the *Extremes*, the two middle terms the *Means*.

While the two terms of a ratio, if they be not abstract numbers, must be quantities of the *same kind*, it is not necessary that all the four terms of a proportion should be of the same kind; it will be sufficient that the quantities in the first ratio be of one kind, and the quantities in the second ratio of one kind; thus 3 cwt. : 12 cwt. = 7 in. : 28 in.; and generally we may say that any four quantities are proportionals *when the first is the same number of times greater or less than the second that the third is greater or less than the fourth.*

191. Since a proportion expresses the equality of ratios, and the value of these ratios may be denoted by fractions, the properties of ratios are made to depend directly upon the properties of fractions.

Hence, if we take any two ratios which are equal to one another, for instance, 4 : 12 and 7 : 21, where it is true that 4 : 12 :: 7 : 21, we may say,

$$\frac{4}{12} = \frac{7}{21}$$

next, reducing these fractions to equivalent fractions having a common denominator, we have—

$$\frac{4 \times 21}{12 \times 21} = \frac{7 \times 12}{21 \times 12}$$

$$\text{or } 4 \times 21 = 7 \times 12, \textit{ i.e.,}$$

$$84 = 84,$$

and, as the numbers taken were not particular but general, we deduce the following general rule, viz., that in every proportion *the product of the extremes is equal to the product of the means.*

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[*] The following definition should be remembered—"Four quantities are said to be proportionals when the first is the same number of times greater or less than the second that the third is greater or less than the fourth."

This property affords the test by which we learn the various alterations that may be made in a proportion, the original proportionality being still preserved.

192. The following variations in the order of the four terms of a proportion occur the most frequently:—

		In like manner.
Given form	18 : 6 :: 12 : 4	4 : 6 :: 12 : 18
Alternately	18 : 12 :: 6 : 4	6 : 4 :: 18 : 12
Reversing	6 : 18 :: 4 : 12	12 : 18 :: 4 : 6
Or	4 : 12 :: 6 : 18	12 : 4 :: 18 : 6

193. From this it follows, as a necessary consequence, that *if the product of the* means *be divided by* one *extreme, the quotient will be the* other *extreme.*

Thus, 18 : 6 :: 12 : 4

$$6 = \frac{18 \times 4}{12} \text{ and } 12 = \frac{18 \times 4}{6}$$

Also, if the product of the extremes *be divided by* one *mean, the quotient will be the* other *mean.*

Thus, 18 : 6 :: 12 : 4

$$18 = \frac{6 \times 12}{4} \qquad 4 = \frac{6 \times 12}{18}$$

Hence, if any of three terms of a proportion be given, the fourth may be found by *dividing the product of two of them by the other term.*

It is important to notice this, because upon this property of proportion depends the solution of questions in the Rule of Three; where three known terms of a proportion are given, to find a fourth unknown one.

EXAMPLES FOR PRACTICE.

Find the value of x in the following proportions:—

1. 4 : 9 :: 12 : x
2. 7 : 28 :: 16 : x
3. 35 : 5 :: x : 2
4. 63 : x :: 14 : 2
5. 136 : 17 :: 64 : x
6. x : 361 :: 171 : 19
7. x : 324 :: 360 : 6480
8. $13\frac{3}{4}$: $137\frac{1}{2}$ :: 2·5 : x
9. 7 : 8 :: x : 9
10. x : 7 :: 8 : 9.
11. $\frac{3}{7}$: $\frac{3}{8}$ :: $\frac{4}{21}$: x.
12. $\frac{13}{16}$: $\frac{1}{2}$ :: 7·5 : x.
13. 2·56 : 7·5 :: x : 29·5.
14. $\frac{3}{4}$ of $2\frac{1}{2}$: x :: 1·25 : 1.

£ s. d. d. yds.

15. 1 12 6 : $2\frac{1}{2}$ :: 156 : x.
16. 3s. : £2 :: 2 yd. : x.
17. £3 10s. : £5 10s. :: 3 tons : x.
18. 3 cwt. 3 qrs. : 5 cwt. 2 qrs. :: £3 17s. 6d. : x.

19. Express in the form of a proportion so that each term shall in turn be the fourth term, the question, "If $6\frac{3}{4}$ yds. cost 5s. 3d., then $73\frac{1}{2}$ yds. cost £2 17s. 2d."

20. Express as a proportion so that each term is in turn the fourth term, the question, "If 12 men do a piece of work in 20 days, then 15 men do it in 16 days."

194. It is often required to *increase* or diminish a quantity in a *certain ratio.* For example, to increase the number 12 in the ratio of 3 to 1 is to multiply by 3. For the increased quantity (which being yet unknown we will call x) is to be to the given quantity as 3 to 1, or x : 12 :: 13 : 1. Whence $1 \times x = 12 \times 3$.

Again, to reduce a number, as 13 in the ratio of 5 to 7, is to multiply by 5 and then divide by 7, for the required number (x) is to be to the given number (13) as 5 is to 7, whence $x = \frac{13 \times 5}{7}$.

For example, if certain provisions last 122 men a given time, it is evident that, in order to last 146 men the same time, they must be *increased in the ratio of* 146 : 122; that is, multiplied by 146 and then divided by 122. Again, if certain provisions suffice 106, and they are required to serve 74 men, must be *diminished in the ratio* of 74 to 106, that is, × 74 ÷ 106.

195. One quantity is said to vary as another, when, if the second quantity is changed, the first is changed in the same proportion, *i.e.*, if the second is halved, the first is also halved; if the second is trebled, the first is also trebled, and so on.

Thus the price of any article varies as the quantity of the article; the wages of a labourer vary as the time he has been working; the income-tax varies as the amount of income. So also the quantity of an article varies as the price, the time as the wages, the income as the tax, that is, if one quantity varies as another, the second also varies as the first.

When one quantity varies as another a proportion can always be found by them, thus, the price of one quantity of coal : the price of another quantity, as the first quantity : the second quantity.

Ex. Price of 14 tons : price of 18 tons = 14 tons : 18 tons.

Ex. If 8 men earn 56s., how much will 48 men earn?

It is here supposed that the money earned *varies directly* with the number of men engaged in earning it; *i.e.*, that when *more men* are at work *more money* will be earned, and when *fewer* men are engaged *less money* will be earned.

With this assumption we are enabled to establish this ratio, that—

As 8 men : 48 men :: the money earned by the 8 men : the money earned by the 48 men, in other words, that—

8 men : 48 men :: 56s. : *the Answer*;

therefore,

the Answer $= \frac{56 \times 48}{8} = 336$s. = £16 16s.

In this example we have the three terms 8 men, 48 men, and 56s. given, and we are required to find the fourth term, which will also be expressed in shillings; for the third term being the antecedent to the fourth, must necessarily be of the same kind and of the same denomination as the fourth.

196. One quantity is said to vary *inversely* as another, when, as one quantity is increased the other is diminished in the same proportion, *i.e.*, if the second quantity be halved, the first is doubled; if the second is trebled, the first is divided by 3, and so on. Thus, the time of doing a piece of work varies inversely as the number of men employed to do it, the length of carpet required to cover a given room varies inversely as its breadth.

In such a case a proportion will be formed thus:—Time occupied by 11 men : time occupied by 15 men = 15 men : 11 men. And if carpets A B are each capable of covering a given room : length of carpet A : length of B = breadth of B : breadth of A. The second ratio in these proportions have their terms corresponding to the terms in the first ratio in an *inverted* order, whence the expression, *vary inversely*.

In all cases of Rule of Three four quantities are involved, the three given quantities and the answer; two of these quantities are of one kind, and two of another; the quantities of the first kind must, in the nature of the case, vary directly, or inversely, as these of the second, or a proportion cannot be formed amongst them, and the process of the Rule of Three is not applicable.

From the preceding illustrations and principles we deduce the following general rule for **Simple Proportion.**

To find the fourth term of a proportion when three terms are given.

RULE LVI.

Place the number for the third term which is of the same kind as the answer or number required. Then, if by the nature of the question it appears that the answer must be greater than the third term, place the greater *of the other* two numbers for the second term, *and the* less *for the* first; *but if the fourth term is to be* less *than the third term, place the* less *of the other two numbers for the* second *term, and the* greater *for the* first. *Then multiply the* second *and* third *terms together,* divide *the* product *by the* first, *and the* quotient *will be the answer in the same denomination as the third term.*

The arrangement of the given terms in the manner mentioned at the beginning of the rule, is commonly called *stating the question*. Sometimes a word or two, or a letter, or other symbol, will be added to represent the fourth or required term.

NOTE I.—The process denoted by the above rule may often be much abbreviated by dividing the first and second, or the first and third terms, (*but never the second and third*), by any number which will divide each of them without a remainder, and using the quotients instead of the numbers themselves.

For 9 : 12 :: 21 : 28 is the same as $\frac{9}{12} = \frac{21}{28}$, which is the same as $\frac{3}{4} = \frac{21}{28}$, which is the same as 3 : 4 :: 21 : 28, which represents tho first proportion after its first and second terms have each been divided by the same number 3.

Again, 9 : 12 :: 21 : 28 is the same as $\frac{9}{12} = \frac{21}{28}$, which is the same as $\frac{3}{12} = \frac{7}{28}$, which is the samo as 3 : 12 :: 7 : 28, which represents the first proportion after the first and third terms have each been divided by 3.

Again, 9 : 12 :: 21 : 28 is the same as $\frac{9}{12} = \frac{21}{28}$, but this is *not* the same as $\frac{9}{4} = \frac{7}{28}$, which is the same as 9 : 4 :: 7 : 28, which represents the first proportion after the second and third terms have each been divided by 3. Moreover, $\frac{9}{4}$ is *not* equal to $\frac{7}{28}$, and of course 9 : 4 :: 7 : 28 is not a true proportion.

NOTE II.—Although we have said in the rule multiply the second and third terms together, and then divide their product by the first, it will be found in most cases advisible not to perform the actual multiplication until we have discovered by putting the expression in the form of a fraction, whether there be any factor or factors common to the numerator and denominator, and if so, have rejected such factor or factors.

EXAMPLES.

Ex. 1. If 27 sheep cost £45, what will be the cost of 36 sheep?

Arranging the three terms so that the last term is the odd one, and the first that term by which the last has to be divided; thus:—

$$\begin{array}{ccccc} \text{sheep.} & & \text{sheep.} & & \pounds \\ 27 & : & 36 & :: & 45 \end{array}$$

Now it is indifferent whether we first divide 45 by 27 and then multiply by 36, or first multiply by 36 and divide by 27, the answer will be $\frac{45 \times 36}{27}$.

But any factor common to either 45 or 36 and 27 may be struck out without affecting the value of the fraction; this is called cancelling, and the sum will be worked thus—

$$\begin{array}{cccccc} \cancel{27} & : & \cancel{36} & :: & & \cancel{45} \\ \cancel{3} & & 4 & & & 15 \\ & & & & & \underline{4} \\ & & & & \textit{Ans.} & \pounds 60 \end{array}$$

9 divides 27 and 36, giving quotients 3 and 4 respectively; 3 divides the 3 and 45, giving quotients 1 (which is not put down) and 15. The answer will now be 15 × 4 divided by 1, that is, £60.

Or, by Note II—

$$\text{Cost of 36 sheep} = \frac{\overset{15}{\cancel{45}} \times \overset{4}{\cancel{36}}}{\underset{\cancel{3}}{\cancel{27}}} = 60.$$

Ex. 2. If 2 qrs. 7 lbs. of sugar cost £1.11s. 6d., what will be the cost of 5 cwt. 3 qrs.?

$$\begin{array}{cccccccccc} \text{qrs.} & \text{lbs.} & & \text{cwt.} & \text{qrs.} & & \pounds & \text{s.} & \text{d.} \\ 2 & 7 & : & 5 & 3 & :: & 1 & 11 & 6 \\ \underline{4} & & & \underline{4} & & & \underline{20} & & \\ \cancel{9} & & & 23 & & & 31 & & \\ & & & \underline{4} & & & \underline{2} & & \\ & & & 92 & & & \cancel{63} & & \\ & & & \underline{7} & & & & & \\ & & & 2)644 & & & 7 & & \\ & & & 2{,}0)32{,}2 & & & & & \\ & & & \pounds 16\ 2 & & & & & \end{array}$$

Here the first two terms have been reduced each to 7 lbs. weight, and the last term to sixpences. The student must observe that labour is always saved by reducing to as large a denomination as possible; but the first two terms *must* be reduced to the *same* denomination. The question, after reduction, really becomes—If 9 seven-pound weights cost 63 sixpences, how much will 92 seven-pound weights cost? *Answer*, 644 sixpences. The second term, 92, has been multiplied by the 7 for convenience. The student must remember that the answer is in the denomination to which the third has been reduced.

Or, by Note II—

$$\text{Cost of 5 cwt. 3 qrs.} = \frac{92 \times \overset{7}{\cancel{63}}}{\cancel{9}} = 644 \text{ sixpences} = \pounds 16\ 2\text{s.}$$

Ex. 3. If 135 yards of cloth cost £27, how many can be purchased for £33?

$$\begin{array}{ccccc} \pounds & & \pounds & & \text{yards.} \\ \cancel{27} & : & 33 & :: & \cancel{135} \\ & & \underline{5} & & 5 \\ & & 165 & & \textit{Answer}, \text{ 165 yards.} \end{array}$$

The first and third terms are divisible by 27. The 33 is, for convenience, multiplied by 5.

Or, by Note II—

$$\text{yards that can be purchased} = \frac{33 \times \overset{5}{\cancel{135}}}{\cancel{27}} = 165 \text{ yards.}$$

Ex. 4. If a piece of cloth is 20 yards in length and $\frac{3}{4}$ yard in breadth, how broad is another piece which is 12 yards long, and which contains as much cloth as the other?

As the length of the second piece is less than that of the first, its breadth must necessarily be greater, in order that the content may be the same. Therefore, in this case a less length requires a greater breadth, and so the example belongs to Rule of Three Inverse.

We have the *breadth* of the second piece to find. That of the first piece is $\frac{3}{4}$ yard; place this, therefore, as the third term. Now the required *breadth* is to be greater than this; therefore place the 20 yards as the second term, and the 12 yards as the first.

$$\begin{array}{ll}
\text{12 yds. : } & \text{20 yds. :: 3 qrs. of a yd. : required breadth in qrs. of a yd.}\\
 & 3\\
\hline
 & 12)60\\
\hline
 & \text{5 qrs. of a yard} = 1\tfrac{1}{4}\text{ yard.}
\end{array}$$

Or thus:—

$$\text{12 yds. : 20 yds. :: } \tfrac{3}{4}\text{ yd. : required breadth in yds.}$$

$$\therefore \text{ required breadth} = \frac{20 \times \frac{3}{4}}{12}\text{ yds.} = \frac{5 \times 3}{12} = \frac{5}{4}\text{ yds.} = 1\tfrac{1}{4}\text{ yd.}$$

The required breadth is therefore a yard and a quarter.

Ex. 5. If a man whose income is £638 6s. 8d. pays an income-tax of £15 19s. 2d., how much ought to be paid on an income of £227 3s. 4d.?

$$\begin{array}{rrrcrrrcrrr}
£ & s. & d. & & £ & s. & d. & & £ & s. & d.\\
638 & 6 & 8 & : & 227 & 3 & 4 & :: & 15 & 19 & 2\\
6 & & & & 6 & & & & 20 & & \\
\cancel{3830} & & & & 2)1363 & & & & 319 & & \\
2 & & & & & & & & 6 & & \\
 & & & & 6)681 & \text{1d.} & & & \times\cancel{2}\times\cancel{5} & & \\
 & & & & 2{,}0)11{,}3 & \text{7d.} & & & & & \\
 & & & & £5 & \text{13s.} & \text{7d.} & & \textit{Ans.}\ £5\ \text{13s. 7d.} & &
\end{array}$$

Here, the first two terms have been reduced to sums of 3s. 4d. each, and the last term to twopences. It would in this case have saved trouble.

Ex. 6. If 27 men do a piece of work in 20 days, in how many days will 45 men do it?

$$\begin{array}{ccccc}
\text{men.} & & \text{men.} & & \text{days.}\\
\cancel{45} & : & \cancel{27} & :: & \cancel{20}\\
\cancel{5} & & 3 & & 4\\
 & & & & 3\\
\hline
 & & & & \text{12 days.}
\end{array}$$

Ans. 12 days.

Here, the *first* and *second* terms are each divided by 9, and then the *first* and *third* are divided by 5.

Ex. 7. If 5·25 yards of calico cost 3s. 8·625d., what will be the cost of 367·5 yards at the same rate?

5·25 yds. : 367·5 yds. :: 3s. 8·625d. : cost of 367·5 yds.
or 44·625d.

The cost of 367·5 yds. $= \frac{44 \cdot 625 \times 367 \cdot 5}{5 \cdot 25}$ = 3123¾d. = £13 0s. 3¾d. *Ans.*

Ex. 8. If I can travel 198 miles by railway for £2 9s. 6d., how far at the same rate of charge ought I to be carried for £8 0s. 10½d.

$$
\begin{array}{cccccccccccc}
£ & \text{s.} & \text{d.} & & £ & \text{s.} & \text{d.} & & \text{miles.} & & \\
2 & 9 & 6 & : & 8 & 0 & 10\frac{1}{2} & :: & 198 & : & \text{required distance.}\\
20 & & & & 20 & & & & & & \\
\hline
49 & & & & 160 & & & & & & \\
12 & & & & 12 & & & & & & \\
\hline
594 & & & & 1930 & & & & & & \\
4 & & & & 4 & & & & & & \\
\hline
2376 & & & & 7722 & & & & & &
\end{array}
$$

$$\therefore \text{required distance} = \frac{\overset{3861}{\cancel{7722}} \times \cancel{198}}{\underset{\underset{6}{\cancel{1188}}}{\cancel{2376}}} = \frac{3861}{6} \text{ miles.}$$

$$= 643\tfrac{3}{6} \text{ miles} = 643\tfrac{1}{2} \text{ miles.}$$

Ex. 9. If 12½ tons cost £3 15s., what will 5 cwt. cost?

Instead of reducing each of the given quantities to their lowest terms, fractions may be commonly used with advantage; care being taken to express the quantities in each ratio in the *same* denomination. Thus, in the example given, express 5 cwt. as a fraction of a ton, and 15s. as the fraction of £1, we have—

$$
\begin{array}{llll}
12\frac{1}{2} \text{ tons} : \frac{1}{4} \text{ ton} & :: & £3\frac{3}{4} : \text{cost of } \frac{1}{4} \text{ ton.}\\
2 & & 4 \\
\hline
25 & & 15 \\
\hline
2 & & 4
\end{array}
$$

$$\tfrac{25}{2} \times \textit{Ans.} = \tfrac{1}{4} \times \tfrac{15}{4}.$$

$$\textit{Ans.} = \frac{1}{4} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{4}}} \times \frac{2}{\underset{5}{\cancel{25}}} = \frac{3}{4 \times 2 \times 5} = £\tfrac{3}{40} = 1\text{s. } 6\text{d.}$$

Ex. 10. If ⅞ of a share of a speculation be worth £21 17s. 6d., what would ⅔ of a share be worth?

$$
\begin{array}{lllll}
\frac{7}{8} : \frac{2}{3} & :: & £21\frac{7}{8} & : & \textit{Ans.}\\
 & & 8 & & \\
\hline
 & & 175 & & \\
\hline
 & & 8 & &
\end{array}
$$

$$\tfrac{7}{8} \times \textit{Ans.} = \tfrac{2}{3} \times \tfrac{175}{8} = \tfrac{2}{3} \times \frac{\overset{25}{\cancel{175}}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{7}} = \tfrac{50}{3}$$

$$\tfrac{50}{3} = 16\tfrac{2}{3} = £16 \text{ } 13\text{s. } 4\text{d.}$$

Ex. 11. If 24 men can do a piece of work in 8 days, in what time can the same work be performed by 64 men.

It is clear that 64 men can do the work in a less time than 24 men, and so the *time* required will be less than 8 days, the third term in our proportion. We must therefore place the 24 as the second term, and the 64 as the first.

64 : 12 :: 8 days : required time in days.

$$\begin{array}{r} 12 \\ 8 \\ \hline 64)96(1 \\ 64 \\ \hline 32 \end{array} \qquad \frac{32}{64} = \frac{1}{2}$$

∴ the required time is $1\frac{32}{64}$ days $= 1\frac{1}{2}$ day.

Or thus:—

$$\text{Required time} = \frac{\overset{3}{\cancel{12}} \times \cancel{8}}{\underset{\underset{2}{\cancel{8}}}{\cancel{64}}} = \frac{3}{2} = 1\tfrac{1}{2} \text{ day.}$$

Ex. 12. If 42 cwt. 2 qrs. cost £73 16s., what is the cost of 41 cwt. 20 lbs.?

Here, the answer is to be *cost*. Put the given *cost* in the third place. The ratio of 3rd : 4th will clearly be a ratio of *greater* inequality, or the 4th will be ∠ the 3rd. Therefore the 2nd must be ∠ the 1st. Hence—

cwt.	qrs.		cwt.	lbs.		£	s.		
42	2	:	41	20	::	73	16	:	required cost.
4			4			20			
170			164			1476			
28			28						
1360			1332						
340			328						
4760			4612						

$$\text{required cost} = \frac{\overset{369}{\cancel{1476}} \times \overset{2306}{\cancel{4612}}}{\underset{\underset{595}{\cancel{1190}}}{\cancel{4760}}} = \frac{850914}{595}\text{s.} = 1420\text{s. } 1\tfrac{173}{595}\text{d.}$$

$$= £71 \ 10\text{s. } 1\tfrac{173}{595}\text{d.}$$

But a shorter and preferable method is as follows:—

$$\text{Required cost} = \frac{£73\frac{4}{5} \times 41\frac{5}{28}}{42\frac{1}{2}} = \frac{369 \times 1153 \times \cancel{2}}{5 \times \underset{14}{\cancel{28}} \times 85} = £\frac{425457}{5950} = £71 \ 10\text{s. } 1\tfrac{173}{595}\text{d.}$$

There are certain examples in which at first sight more than three terms appear to be given, but they, nevertheless, in certain cases, come under this rule, as in the following instance.

Ex. 13. If the carriage of 5 cwt. 7 lbs. for 84 miles cost me £3 18s. 4d., what will it cost me to have 21 cwt. 1 qr. 14 lbs. carried the same distance?

The 84 miles may evidently be left out of consideration, since the distance in both cases is the same.

Proceeding then according to our rule—

5 cwt. 7 lbs. : 21 cwt. 1 qr. 14 lbs. :: £3 18s. 4d. : required cost,

whence it will be found that—

Required cost = £16 10s. 8¼d. ⅝q.

197. Questions in proportion can always be worked independent of the artificial rule of *stating*, and although sometimes not so conveniently, yet always in a satisfactory way as regards simplicity of demonstration. It will appear from the following examples that a knowledge of first principles, or the fundamental rules of arithmetic, is sufficient for the solution of all problems in the Rule of Three.

EXAMPLES.

Taking Ex. 1, page 143, we have—

Ex. 1. If 27 sheep cost £45, what will 36 sheep cost?

The cost of 27 sheep = £45,
Therefore the cost of 1 sheep = £45 ÷ 27 = $\frac{45}{27}$,
Therefore the cost of 36 sheep = $\frac{45}{27} \times 36$

$$= \frac{\overset{15}{\cancel{45}} \times \overset{4}{\cancel{36}}}{\underset{\cancel{3}}{\cancel{27}}} = 60.$$

Ans. £60.

The question may also be worked by the method indicated in § 194, page 140, thus—

By Ratio: Here £45 has to be increased in the ratio of 36 to 27, or of 4 to 3.

45 × 4 ÷ 3 = £60. *Ans.* £60.

Ex. 2. If 42s. be the wages of 6 men, what will be the wages of 36 men?

The wages of 6 men = 42s.
Therefore the wages of 1 man = 42s. ÷ 6 = $\frac{42}{6}$,
Therefore the wages of 36 men = $\frac{42}{6} \times 36 = \frac{42 \times \overset{6}{\cancel{36}}}{\cancel{6}} = 252$s. *Ans.*

Ex. 3. If 4 tons 3 cwt. of coal cost £6 4s. 6d., how much coal may be had for £4 16s.?

For (£6 4s. 6d., *i.e.*, for) 249 sixpences we can purchase (4 tons 3 cwt., *i.e.*,) 83 cwt. of coal,

Therefore for 1 sixpence we can purchase $\frac{83}{249}$ cwt. of coal,

And for (£4 16s., *i.e.*, for) 192 sixpences we can purchase $\frac{83 \text{ cwt.} \times 192}{249}$ of coal,

$$\frac{\cancel{83} \times \overset{64}{\cancel{192}}}{\underset{\cancel{3}}{\cancel{249}}} = \frac{64}{1} = 64 \text{ cwt. } \textit{Ans.}$$

Ex. 4. If the railway fare for travelling 85 miles is £1 9s. 4d., what will be the fare for 130 miles?

The fare for 85 miles is $29\frac{1}{3}$s,
" 1 mile is $\frac{1}{85}$ of $29\frac{1}{3}$s., or
" 130 miles is $\frac{130}{85}$ of $29\frac{1}{3}$s. = $\frac{130}{85}$ of $\frac{88}{3}$s.,

$$\frac{\overset{26}{\cancel{130}} \times 88}{\underset{17}{\cancel{85}} \quad 3} = \frac{2288}{51}\text{s.} = 44\text{s. } 10\tfrac{6}{17}\text{d. } \textit{Ans.}$$

By Ratio: Here £1 9s. 4d. has to be increased in ratio of 130 to 85, or 26 to 17.

£1 9s. 4d. × 26 ÷ 17 = 44s. $10\frac{6}{17}$d. *Ans.*

Ex. 5. If $10\frac{5}{7}$ lbs. of sugar cost $4\frac{11}{16}$s., what will be the cost of $3\frac{2}{3}$ cwt.?

$$\text{The cost of } 10\tfrac{5}{7}\text{ lbs. } (= \tfrac{75}{7}\text{ lbs.}) = 4\tfrac{11}{16}\text{s. } (= \tfrac{75}{16}\text{s.}),$$

$$\text{,,} \quad 1\text{ lb.} = \frac{7}{\cancel{75}} \text{ of } \frac{\cancel{75}}{16} = \tfrac{1}{16} \text{ of } 7\text{s.},$$

$$\text{,,} \quad 112 \times 3\tfrac{2}{3}\text{ lbs.} = \frac{112 \times 3\frac{2}{3}}{16} \text{ of } 7\text{s.},$$

$$\frac{7 \times \overset{7}{\cancel{112}} \times 11}{\cancel{16} \times 3} = \frac{7 \times 7 \times 11}{3} = £8\ 19\text{s. } 8\text{d.}$$

Ex. 6. A steam vessel consumes 13 tons of coal in $1\frac{3}{4}$ days: how long will 98 tons last?

Method of Unity: 13 tons in $1\frac{3}{4}^{d}$, or $\frac{7}{4}^{d}$, is 1 ton in $\frac{7}{4 \times 13}$ or $\frac{7}{52}^{d}$, and 98 tons is

$$98 \times \tfrac{7}{52} = 13\tfrac{5}{26}^{d}, \text{ or } 13^{d}\ 5^{h} \text{ nearly.}$$

$$13 \ : \ 98 \ :: \ 1\tfrac{3}{4} \ : \ \text{days required.}$$

$$= \frac{98 \times 1{\cdot}75}{13} = 13{\cdot}2.$$

By Ratio: Here $1\frac{3}{4}$ days is to be increased in the ratio of 98 to 13.

$$1{\cdot}75 \times 98 \div 13 = 13{\cdot}2 \text{ days.}$$

198. Examples, such as the following, are easily worked out by the Rule of Three.

Examples.

Ex. 1. A clock gains $3\frac{1}{4}$ minutes in 15 seconds under the 24 hours; at noon it is 2 minutes too slow; when will it indicate true time?

Take 15 seconds from 24 hours and there remains $23^{h}\ 59^{m}\ 45^{s}$. The question, therefore, is, "if a clock gains $3\frac{1}{4}$ minutes in $23^{h}\ 59^{m}\ 45^{s}$, in what time will it gain 2 minutes?"

$$3\tfrac{1}{4} \text{ minutes} : 2 \text{ minutes} :: 23\tfrac{717}{720} \text{ hours} : \textit{Ans.} \text{ hours.}$$

$$:: \tfrac{17277}{720} \qquad : \textit{Ans.} \text{ hours.}$$

$$\textit{Ans.} = \cancel{2} \times \frac{17277}{\underset{\substack{\cancel{180}\\ 90}}{\cancel{720}}} \times \frac{\cancel{4}}{13} = \tfrac{17277}{1170} = 14 \text{ hours } 16 \text{ minutes.}$$

Ex. 2. Given that the diameter of a circle is to the circumference as $1 : 3\frac{1}{7}$; find the number of revolutions made by a wheel whose radius is 1 foot 9 inches in a journey of 8 miles.

When the radius is 1 foot 9 inches, the diameter is 3 feet 6 inches, and we are told that—

$$\text{diameter} : \text{circumference} :: 1 : 3\tfrac{1}{7}.$$

$$\text{therefore, } 3\tfrac{1}{2} : \text{circumference} :: 1 : 3\tfrac{1}{7}.$$

$$\text{circumference} = \frac{\cancel{7}}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} = 11 \text{ feet.}$$

The question is now only how many times a length of 11 feet is contained in 8 miles; hence, bringing miles to feet, we have—

$$\textit{Ans.} = \frac{8 \times \overset{160}{\cancel{1760}} \times 3}{\cancel{11}} = 3840 \text{ times.}$$

Ex. 3. If $(\frac{2}{3} + \frac{1}{21})$ of a steamer be worth £14053 19s. 2d., what is the value of $(\frac{1}{3} + \frac{1}{15})$ of it?

$$\text{Here } \tfrac{2}{3} + \tfrac{1}{21} = \frac{14+1}{21} = \tfrac{15}{21} = \tfrac{5}{7},$$

$$\text{And } \tfrac{1}{3} + \tfrac{1}{15} = \frac{5+1}{15} = \tfrac{6}{15} = \tfrac{2}{5},$$

Therefore the question stands thus:—

"If $\frac{5}{7}$ of a steamer is worth £14053 19s. 2d., what is the value of $\frac{2}{5}$ of it?"

By the rule:—

$$\tfrac{5}{7} : \tfrac{2}{5} :: £14053\ 19\text{s. } 2\text{d.} : \text{the } \textit{Answer}.$$

$$\begin{aligned} \therefore \text{the } \textit{Answer} &= £14053\ 19\text{s. } 2\text{d.} \times \tfrac{2}{5} \div \tfrac{5}{7} \\ &= £14053\ 19\text{s. } 2\text{d.} \times \tfrac{2}{5} \times \tfrac{7}{5} \\ &= £14053\ 19\text{s. } 2\text{d.} \times \tfrac{14}{25} \\ &= \frac{£14053\ 19\text{s. } 2\text{d.} \times 14}{25} \\ &= \frac{£196755\ 8\text{s. } 4\text{d.}}{25} \\ &= £7870\ 4\text{s. } 4\text{d.} \end{aligned}$$

Ex. 4. If after paying an income tax of 7d. in the pound a person has £776 13s. 4d. remaining, what is his actual gross income?

Every pound of 240 pence is reduced by paying a tax of 7d. to 233 pence, and the whole income is reduced in the same ratio; hence—

$$\begin{aligned} 240 : 233 &:: \text{original income} : \text{reduced income} \\ &:: \textit{Answer} \quad : £776\tfrac{2}{3} \end{aligned}$$

$$\begin{array}{r} 776\tfrac{2}{3} \\ 3 \\ \hline 2330 \\ \hline 3 \end{array}$$

$$\textit{Answer} = 240 \times \tfrac{2330}{3} \div 233$$

$$= \overset{80}{\cancel{240}} \times \frac{\overset{10}{\cancel{2330}}}{\cancel{3}} \times \frac{1}{\cancel{233}} = £800.$$

Examples for Practice.

1. If the cost of 54 tons of coal is £41 17s., what will 72 tons cost?
2. If a score of sheep cost £43, what would be the price of a flock of 3580 sheep?
3. What would be the price of 7632 articles at 3¾d. for every 144 of them?
4. If 2 cwt. 3 qrs. 14 lbs. of sugar cost £6 14s. 2d., what quantity of the same quality of sugar can be bought for £29 15s.?
5. If 1 cwt. 1 qr. cost £3 3s. 4d., what is the cost of 15 cwt. 2 qrs. 3½ lbs.?
6. What will be the cost of 2¼ tons of merchandise, if 3 cwt. 27 lbs. cost £35 8s. 10d.?
7. Fifteen ounces of copper ore are found to contain 1¼ oz. of pure copper; how much pure copper is there in a heap of 27 tons of copper ore?
8. If a ship can steam a distance of 245 miles in 2 days 7 hours, how long should she take to go 2600 miles?
9. If 8 men can do a piece of work in 18 working days, how long should it take 12 men to do it?

10. If one fibre of silk will sustain 50 grains, how many would be required to support 97 lbs.?
11. If 4·4 articles cost £2·86, what is the value of 7·375 such?
12. If 5 yds. 7 in. cost £3 15s., what will 23 yds. 1 ft. cost?
13. If 27 sovereigns weigh 3341·25 grs., how many lbs., ozs., &c., will 1000 sovereigns weigh?
14. There are provisions in a town sufficient to support 4000 soldiers for 3 months, how many must be sent away to make them last 8 months?
15. How many pairs of gloves, at 1s. $10\frac{1}{2}$d. a pair, will cost as much as 141 yards of silk at 4s. 8d. per yard?
16. Find a quantity which bears the same ratio to 5 yds. 3 qrs. 3 nls. that 30 days 3 hrs. 29 min. 29 sec. bears to 13 days 16 hrs. 51 min. 35 sec.
17. Find the value of 7 tons 8 cwt. when 5 cwt. are worth £18 13s. 4d.
18. If a bankrupt pays 13s. 7d. in the pound, what will he pay on a debt of £452? Work the sum also by Practice.
19. Shrubs are planted a yard apart upon two acres of ground; how much will they cost at 13s. 6d. per 1000?
20. How many yards of carpet, three-quarters of a yard wide, will cover a room 27 feet square?
21. Water consists of eight parts by weight of oxygen and one of hydrogen; what weight of oxygen is there in 19 gallons of water?
22. Given that the diameter of a circle is to the circumference as 1 : 3·1425; find correctly to the thousandth part of an inch the circumference of a circle whose radius is 2·1 feet?
23. The sloth does not advance more than 100 yards in a day; how long would it take to crawl from Dublin to Cork, allowing the distance to be 160 English miles?
24. An acre of coals 2 feet thick yields 3000 tons, and one 5 feet thick 8000; how many acres of 5 feet thick would give the same quantity as 48 of 2 feet thick?
25. One piece of silk a mile long weighs but 12 grains; how many miles would 4 millions of pounds, annually consumed in England, reach?
26. A tank contains 406 gallons of water when $\frac{3}{7}$ full, how many gallons does it contain when quite full?
27. A bar of iron, an inch and a quarter square and 2 feet long, is rolled down to $\frac{3}{4}$ inch square; how long will it be?
28. If $\frac{11}{12}$ of a truck of coal weighs 26 cwt. 1 qr. 19 lbs., what is the weight of all the coal in a truck?
29. If 72 men dig a trench in 63 days, in how many days will 42 men do the same?
30. If 12 men can reap a field in 4 days, in what time can the same work be done by 32 men?
31. What weight ought to be carried $25\frac{3}{4}$ miles for the same sum for which 3 cwt. are carried 40 miles?
32. If a pound weight of standard gold were worth £46 14s. 6d., how much should one sovereign weigh?
33. If a treasure of silver was worth £1800000 in the time of Henry VII, when silver was worth £1 17s. 6d. a pound, what would it be worth now when silver is 5s. 6d. an ounce?
34. A steamer, whose average rate of sailing is 8 knots an hour, makes a voyage in 56 days; how long will another steamer be in making the same voyage, whose average rate is 7 knots?

35. If $4\frac{5}{8}$ ozs. of tea cost $1\frac{1}{20}$s., what will $29\frac{2}{7}$ lbs. cost?

36. If $\frac{3}{7}$ cwt. cost £$7\frac{5}{8}$, what will $6\frac{3}{4}$ lbs. cost?

37. If $\frac{1}{4}$ of a ton cost £$\frac{3}{8}$, what will $\frac{5}{8}$ of a ton cost?

38. A watch set accurately at noon indicates 10 min. to 5 at 5 o'clock the same afternoon, what is the time when the watch indicates 5 o'clock?

39. Two clocks point to 2 o'clock at the same instant, one loses 7 sec. and the other gains 8 sec. in 24 hours; when will one be half an hour before the other?

40. The supply of a certain number of persons with bread at $7\frac{1}{2}$d. a loaf costs £27 18s., what will be the cost of supplying $\frac{2}{3}$ of that number at $6\frac{1}{4}$d. a loaf?

41. A garrison of 1000 men is victualled for 30 days, after 10 days it is reinforced and then the provisions were exhausted in 5 days; what was the number of the reinforcement?

42. A garrison was victualled for 30 days, and after 10 days it was reinforced by 3000 men, and then the provisions were exhausted in 10 days; what was the original number of the garrison?

43. If a pole 10 feet high cast a shadow 12 feet 8 inches long, how high is a tower whose shadow at the same time is 57 feet long?

44. If one tower, known to be 99 feet high, cast a shadow 73 feet 3 inches long, what length of shadow will another tower 100 feet high cast at the same time?

45. If $4\frac{5}{8}$ oz. avoir. cost $8\frac{31}{32}$s., what will $8\frac{13}{24}$ lbs. cost?

46. If $\frac{1}{192}$ of $\frac{2}{3}$ of $2\frac{1}{2}$ of 40 lbs. of beef cost $1\frac{3}{80}$d., how many lbs. may be bought at the same rate for 6s. $7\frac{1}{2}$d.?

47. Lead weighs $11\frac{4}{10}$ times, and platinum 21 times, as much as water; what weight of platinum will be equal in bulk to 112 lbs. of lead?

48. A watch which gains uniformly tells the true time once in every 200 days; how much does it gain or lose per hour?

49. A leaden shot of $4\frac{1}{2}$ inches in diameter weighs 17 lbs., but the size of a shot 4 inches in diameter is to that of one $4\frac{1}{2}$ inches in diameter as 64000 : 91125; what is the weight of a leaden ball 4 inches in diameter?

50. The ratio of the *vibrations* of a pendulum to the *number of seconds in which the vibrations are made* is $\frac{1\frac{2}{3}}{\frac{2}{3}}$; how many times will it vibrate in 24 hours?

COMPOUND PROPORTION.

199. Questions frequently arise in which five quantities are given to find a sixth, or seven quantities given to find an eighth, and so on. In such cases it becomes necessary to repeat the process already adverted to in the Rule of Three, or to combine two or more proportions so as to reduce them all to a simple proportion, and then, if any three terms of the reduced proportion be known, the fourth can be found as before; and the method by which the fourth term of a proportion is found when the antecedent and the consequent consists each of more than one quantity, is called *Compound Proportion.*

The following example is one of Compound Proportion.

Example.

Ex. If the expenses of 7 persons for 3 months be 70 guineas, what will be the expenses of 10 persons for 12 months at the same rate?

Here we must find what the expenses of 10 persons for 3 months will be.

This is done by the Rule of Three in the following manner:—

7 persons : 10 persons :: 70 guineas : 100 guineas (A),

$$\frac{\overset{10}{\cancel{70}} \times 10}{\cancel{7}} = 100 \text{ guineas.}$$

Now, suppose 10 persons expend 100 guineas in 3 months, we must next inquire how much they will expend in 12 months, and by the same rule we get—

3 months : 12 months :: 100 guineas : 400 guineas (B),

$$\frac{\overset{4}{\cancel{12}} \times 100}{\cancel{3}} = 400 \text{ guineas.}$$

Instead of repeating the operation of the Rule of Three, the terms of the proportion (A) may be multiplied by the corresponding terms of the proportion (B), and the product will still be proportional (§ 184).

Hence—

$$7 \times 3 \ : \ 10 \times 12 \ :: \ 70 \times 100 \ : \ 100 \times 400,$$

But as the ratio of the third term to the fourth will not be altered by dividing each of the terms by 100, it is obvious that the operation for finding the fourth term (100) in A is superfluous and may be dispensed with entirely.

The statement of the terms of the operation will then be as follows:—

7 persons : 10 persons :: 70 guineas
3 months : 12 months,

then $\frac{\overset{10}{\cancel{70}} \times 10 \times \overset{4}{\cancel{12}}}{\cancel{7} \times \cancel{3}} = 10 \times 10 \times 4 = 400$ guineas. *Answer.*

200. For the sake of convenience we may divide each question into two parts, the *supposition* and the *demand;* the former being the part which expresses the condition of the question, and the latter the part which mentions the thing demanded or sought. In the question, for example, "If the carriage of 15 cwt. for 17 miles cost me £4 5s., what would the carriage of 21 cwt. for 16 miles cost me?" form the demand. Adopting this distinction we may give the following rule for working out examples in Compound Proportion.

RULE LVII.

1°. *Take from the supposition that quantity which corresponds to the quantity sought in the demand, and write it down as a third term. Then take one of the other quantities in the supposition and the corresponding quantity in the demand and consider them with reference to the third term* only *(leaving all the other given quantities out of consideration) when the two quantities are so considered, if from the nature of the case the fourth term would be greater than the third, then, as in the Rule of Three, put the larger of the two quantities in the second term, and the smaller in the first term; but if less, put the smaller in the second term, and the larger in the first term. From the remaining quantities select two others, and retaining the same third term, proceed in the same way to make one of these quantities a first term, and the other a second term.*

2°. *If there be other quantities in the supposition and demand, proceed in like manner with them.*

3°. *In each of these statings reduce the first and second terms to the same denomination. Let the common third term be also reduced to a single denomination if it be not already in that state. The terms may then be treated as abstract numbers.*

4°. *Multiply all the first terms together for a final first term, and all the second terms for a final second term, and retain the former third term. Multiply the third term by the product of the quantities in the second term, aud divide the result by the product of all the quantities in the first term.*

NOTE.—In dealing with the final statement obtained by the Rule, the two notes to Rule LVI, page 142, will often be found useful.

EXAMPLES.

Ex. 1. If 6 men in 24 days of 8 hours each set up 576 yards of telegraph wire, how many yards will be set up by 5 men in 20 days of 10 hours each?

In such questions the first thing is to get a distinct idea of the *supposition* as distinguished from the *demand.*

Here the supposition is that 6 men set up 576 yards of wire in 24 days of 8 hours each. If, now, we arrange the demand in the same way as the supposition, it will run—5 men set up ? yards in 20 days of 10 hours each, and the ? is the number of yards we have to fill up.

Since the third term *must always be of the same kind as the answer*, in this example the third term will therefore be "576 yards."

We next consider thus:—

"*If 6 men can set up 576 yards of wire, how many yards can be set up by 5 men?*"

This proportion would be stated thus:—

As 6 men : 5 men :: 576 yards of wire : the yards set up by 5 men.

Again, "*If 576 yards of wire are set up in 24 days, how many yards will be set up in 20 days?*"

The proportion would be stated thus:—

As 24 days : 20 days :: 576 yards : the yards set up in 20 days.

And, therefore, resuming the statement obtained above, we have—

24 days : 20 days :: 576 yards : the *Answer*.

Again, in the above example consider thus:—

"*If 576 yards of wire are set up when the men work 8 hours a day, how many yards will be set up when the men work 10 hours a day?*"

This proportion would be stated thus:—

As 8 hours : 10 hours :: 576 yards : the yards set up in days of 10 hours each.

And, therefore, resuming the statement obtained above, we have—

$$\left.\begin{array}{rcl} 6 \text{ men} & : & 5 \text{ men} \\ 24 \text{ days} & : & 20 \text{ days} \\ 8 \text{ hours} & : & 10 \text{ hours} \end{array}\right\} :: 576 \text{ yards} : \text{the } \textit{Answer}.$$

Let us now consider the separate effect of altering each term of the supposition to suit the question.

(1) Employing 5 men instead of 6 men will reduce the number of yards set up in the ratio of 6 to 5; therefore, as far that change alone is concerned, the 576 yards will have to be multiplied by the fraction $\frac{5}{6}$.

(2) The number of days is decreased from 24 to 20. This will decrease the number of yards in the ratio of 24 to 20. The effect of that change upon the answer will therefore be to multiply it by $\frac{20}{24}$.

(3) The alteration of the working day from 8 to 10 hours will increase the number of yards in the ratio of 8 to 10. Its effect, therefore, will be equivalent to multiply by the fraction $\frac{10}{8}$.

We have thus found the separate effects of those changes, each of which acts independently of the other, and simply introduces its own multiplier. The conjoint effect upon the number of yards will evidently be to multiply that number by

$$\frac{5}{6} \times \frac{20}{24} \times \frac{10}{8};$$

that is to say, the answer will be

$$\frac{5}{6} \times \frac{20}{24} \times \frac{10}{8} \times 576 \text{ yards} =$$

$$\frac{5 \times 20 \times \overset{5}{\cancel{10}} \times \overset{\overset{\cancel{3}}{\cancel{72}}}{\cancel{576}}}{\underset{\cancel{2}}{\cancel{6}} \times \cancel{24} \times \cancel{8}} = 500 \text{ yards. } \textit{Ans.}$$

Ex. 2. If 15 men working 8 hours a day can plough 240 acres of land in 10 days, how many days will it take 20 men working 10 hours a day to plough 600 acres?

Here the supposition is that 15 men plough 240 acres in 10 days of 8 hours. If, now, we arrange the demand in the same way as the supposition it runs—20 men plough 600 acres in ? days of 10 hours, and the ? in the number of days is what we have to fill up.

Let us go on to consider the separate effect of altering each term of the supposition to suit the question.

We may take the first part of the question thus:—

"*If it take 15 men 10 days to plough a piece of land, how many days will it take 20 men?*"

By the rule of Simple Proportion we get the following proportion :—

20 men : 15 men :: 10 days : the number of days 20 men will take.

Hence, employing 20 men instead of 15 will evidently reduce the number of days in the proportion of 15 to 20; therefore, as far as that change alone is concerned the 10 days will have to be multiplied by $\frac{15}{20}$.

We next consider the second part of the question thus :—

"*If* 240 *acres are ploughed in* 10 *days, how many days will it take to plough* 600 *acres?*"

By the rule of Simple Proportion we get the following proportion :—

As 240 acs. : 600 acs. :: 10 dys. : the number of days required to plough 600 acs.

Or again, consider that the number of acres is increased from 240 to 600. This will increase the number of days in the ratio of 600 to 240. The effect of that change upon the answer will therefore be to multiply it by $\frac{600}{240}$.

We next consider the third part of the question thus :—

"*If it take* 10 *days to plough a certain number of acres when the working day is* 8 *hours, how many days will it take to do the same working* 10 *hours a day?*"

Here we have the following proportion :—

10 hours : 8 hours :: 10 days : the days of 8 hours each.

The alteration of the working day from 8 to 10 hours will reduce the number of days in the proportion of 10 to 8. Its effect, therefore, will be equivalent to multiply by the fraction $\frac{8}{10}$.

Therefore, resuming the statement obtained, we have—

$$\left.\begin{array}{rcl} \text{20 men} & : & \text{15 men} \\ \text{240 acres} & : & \text{600 acres} \\ \text{10 hours} & : & \text{8 hours} \end{array}\right\} :: \text{10 days} : \textit{Ans.}\text{, days.}$$

Having found the separate effects of three changes, each of which acts independently of the other, and simply introduces its own multiplier. The conjoint effect upon the number of days will evidently be to multiply that number by

$$\frac{15}{20} \times \frac{600}{240} \times \frac{8}{10},$$

$$\frac{15 \times \overset{\cancel{10}}{\cancel{600}} \times \overset{\cancel{2}}{\cancel{8}}}{\underset{\cancel{2}}{\cancel{20}} \times \underset{\cancel{4}}{\cancel{240}} \times \cancel{10}} \times \cancel{10} \text{ days} = 15 \text{ days.}$$

Answer 15 days.

and the answer is as shown above, 15 days.

Ex. 3. Some repairs to an engine have occupied 16 men 7 weeks, and have cost £243 16s. 9¼d. for wages. Some larger repairs are necessary, which is expected will occupy 20 men 11 weeks, how much will be required to pay them at the same rate of wages?

$$\begin{array}{rcl} \text{16 men} & : & \text{20 men} \\ \text{7 weeks} & : & \text{11 weeks} \\ \hline 112 & : & 220 \\ \text{or, } 28 & : & 55 \end{array} \quad :: \quad \begin{array}{rrr} \pounds & \text{s.} & \text{d.} \\ 243 & 16 & 9\frac{1}{4} \\ & & 5 \\ \hline 1219 & 3 & 10\frac{1}{4} \\ & & 11 \\ \hline 4)13411 & 2 & 4\frac{3}{4} \\ \hline 7)\ 3352 & 15 & 7\text{—}3 \\ \hline 478 & 19 & 4\frac{1}{4}\frac{23}{28} \end{array} \quad : \quad \textit{Answer.}$$

(The divisors 4 and 7 are bracketed together as 28.)

Ex. 4. If 1500 navvies working 10 hours a day can cut a canal 20 miles long, 20 feet broad, and 6 feet deep, how many soldiers will it take to make a trench 10 miles long, 30 feet wide, and 5 feet deep, supposing that they only work 6 hours a day, and that in the same space of time 7 navvies can do the work of 10 soldiers?

The arrangement of the work is as follows:—

$$\left.\begin{array}{lrcr} \text{hours per day} & 6 & : & 10 \\ \text{length, miles} & 20 & : & 10 \\ \text{breadth, feet} & 20 & : & 30 \\ \text{depth, feet} & 6 & : & 5 \\ \text{soldiers instead of navvies} & 7 & : & 10 \end{array}\right\} :: \; 1500 \text{ men} \; : \; \textit{Answer}, \text{ men.}$$

$$\textit{Answer} = \overset{125}{\overset{\cancel{250}}{\cancel{1500}}} \times \frac{\cancel{10}}{\cancel{6}} \times \frac{\cancel{10}}{\underset{\cancel{2}}{\cancel{20}}} \times \frac{\overset{5}{\overset{\cancel{15}}{\cancel{30}}}}{\underset{\cancel{2}}{\cancel{20}}} \times \frac{5}{\underset{\cancel{2}}{\cancel{6}}} \times \frac{\overset{5}{\cancel{10}}}{7} = \frac{15625}{7} = 2232\tfrac{1}{7} \text{ men. } \textit{Answer.}$$

Ex. 5. If 10 men dig a trench 80 yards long, 6 yards wide, and 2 feet 8 inches deep, in 6 days, how deep a trench can 96 men dig in 3 days if it be 240 yards long and 16 yards wide?

$$\left.\begin{array}{lrcr} \text{Men} & 10 & : & 96 \\ \text{Length} & 240 & : & 80 \\ \text{Width} & 16 & : & 6 \\ \text{Days} & 6 & : & 3 \end{array}\right\} :: \; \begin{array}{c} \text{depth.} \\ 2 \text{ ft. } 8 \text{ in.} \\ (= \tfrac{8}{9} \text{ yd.}) \end{array} \; : \; \textit{Answer.}$$

$$\frac{8 \times \overset{\cancel{3}}{\overset{\cancel{6}}{\cancel{96}}} \times \cancel{80} \times \cancel{6} \times \cancel{3}}{\underset{\cancel{3}}{\cancel{9}} \times \underset{5}{\cancel{10}} \times \underset{3}{\cancel{240}} \times \cancel{16} \times \cancel{6}} = \tfrac{8}{15} \text{ yd.} = 1 \text{ ft. } 7\tfrac{1}{5} \text{ in.}$$

Ex. 6. A can do a piece of work in 20 days, B in 12 days; A and B work at it together for 6 days, and then C joins it in 2 days; in how many days could C have done it?

(1) In 20 days A can do all the work.

∴ in 1 day A can do $\frac{1}{20}$ of the work.

∴ in 6 days A does $\frac{6}{20}$, *i.e.*, $\frac{3}{10}$ of the work.

(2) In 12 days B can do all the work.

∴ in 1 day B can do $\frac{1}{12}$ of the work.

∴ in 6 days B does $\frac{6}{12}$, *i.e.*, $\frac{1}{2}$ of the work.

Therefore,—

(3) In 6 days A and B, working together, do $(\frac{3}{10} + \frac{1}{2})$ of the work, *i.e.*, $\frac{4}{5}$ of the work.

And, therefore, when A and B cease working, $\frac{1}{5}$ of the work remains to be done.

The remaining fifth is done by C in 2 days.

Therefore C could do all the work in 2 days × 5, *i.e.*, in 10 days. *Answer*, 10 days.

201. The questions given in proportion are not always set in these perfectly simple terms; sometimes both care and ingenuity are required to prepare the question before it is *stated* as a proportion. We shall accordingly now proceed to give examples where the questions are not put exactly in the straightforward manner in which the previous examples have been given; yet we shall show how a little previous consideration will enable us to reduce such question to a plain proportion.

Ex. 7. If 12 oxen and 35 sheep eat 6 tons 7 cwt. of hay in 4 days, how much will it cost per week to feed 4 oxen and 6 sheep, the price of hay being £3 15s. per ton, and 2 oxen being supposed to eat as much as 5 sheep?

If the consumption of 2 oxen = that of 5 sheep,
then " 12 " = " 30 "
and " 4 " = " 10 "

Also the cost of $6\frac{7}{20}$ tons of hay at £$3\frac{3}{4}$ per ton will be

$$\frac{127}{\underset{4}{\cancel{20}}} \times \frac{\overset{3}{\cancel{15}}}{4} = £\tfrac{381}{16}.$$

Hence the question becomes—"If it cost £$\frac{381}{16}$ to feed 30 sheep + 35 sheep for 4 days, how much will it cost to feed 10 sheep + 6 sheep for 7 days?

$$65 \times 4 \ : \ 16 \times 7 \ :: \ \tfrac{381}{16} \ : \ \textit{Ans.}$$

$$65 \times 4 \times \textit{Ans.} = \cancel{16} \times 7 \times \frac{381}{\cancel{16}}$$

$$\textit{Ans.} = 7 \times 381 \times \tfrac{1}{65} \times \tfrac{1}{4} = \tfrac{2667}{260}.$$

$$= £10\ 5\text{s}.\ 1\tfrac{11}{13}\text{d}.$$

Ex. 8. If 6 horses are worth 14 cows, and 10 cows cost as much as 60 sheep, and 32 sheep cost £330, what is the value of 16 horses?

Such problems may more easily be solved by the following process:—

(1) State the several equations under each other, as below—

Let x = the number of £ in the *Answer.*
Then the value of 6 horses = the value of 14 cows,
" 10 cows = the value of 60 sheep,
" 32 sheep = £330,
and £x = the value of 16 horses.

(2) *Divide* the product of the numbers in the right hand column by the product of the numbers in the left-hand column.

$$\text{Thus, } \frac{\overset{7}{\cancel{14}} \times \overset{10}{\cancel{60}} \times \overset{33}{\cancel{330}} \times \cancel{16}}{\underset{1}{\cancel{6}} \times \cancel{10} \times \underset{2}{\cancel{32}}} = £2310 \ \textit{Answer}.$$

EXAMPLES FOR PRACTICE.

1. If 10 men can do a piece of work in 6 days, working 10 hours a day, how many hours a day must 8 men work to complete the same undertaking in 15 days?
2. If the carriage of 8 cwt. for 60 miles be 19s., what ought to be the charge for sending 36 cwt. a distance of 38 miles?
3. A railway company charges £1 8s. 6d. for the carriage of 19 cwt. for 30 miles, what weight ought to be carried 21 miles for 10s. 6d. at the same rate?
4. If the sixpenny loaf weighs 36 oz. when wheat is at 55s. per quarter, what would be the weight of an eightpenny loaf when wheat is 60s. per quarter?
5. If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of $12\frac{1}{2}$ sheets?
6. Some repairs to an engine have occupied 5 men during 3 weeks, and have cost £41 10s. wages; some larger repairs are necessary, which it is expected will occupy 22 men 27 days: how much will be required to pay them at the same rate of wages?

7. The repairs to an engine have taken 12 men 5 weeks and 3 days, and have cost £46 15s. wages; some larger repairs are necessary, which it is expected will occupy 16 men 7 weeks; how much will be required to pay them at the same rate?

8. If 6 iron bars 4 ft. long, 3 in. broad, and 2 in. thick, weigh 288 lbs., how much will 15 weigh, each 6½ ft. long, 4 in. broad, and 3 in. thick.

9. If a page of a newspaper, consisting of 6 columns of large print, contain 205,000 letters, how many letters will be contained on the page of another newspaper, which consists of 7 columns each as large as one of the former, there being on an average 48 letters in the second newspaper occupying the same space of 41 of the first.

10. A plate of copper 2 ft. long, 9 in. wide, and 2 in. thick, is rolled into a sheet 3 ft. 6 in. wide and 18 ft. long; how thick will it be?

11. Bar iron 1½ in. square and 22 ft. long is rolled down to ¾ inch square; how long will it be?

12. A horse-power will lift 33,000 pounds one foot high per minute; how many gallons of water per day will an engine of 120 horse-power lift to the height of 140 fathoms?

13. If 35 men working 8 hours a day can build a wall 1700 yards long, 13 ft. high, and 2 ft. thick, in 130 days; how many hours a day must 40 men work to construct a wall 1½ miles long, 17 ft. high, and 2½ ft. thick, in 210 days?

14. It has been calculated that a square degree (about 69 × 69 square miles) of water gives off by evaporation 33 millions of tons of water per day; how much may be supposed to rise from a square mile in a week?

15. When the mercury in the barometer stands at a height of 30 inches, the pressure of the air on every square inch of surface is 15 lbs.; what will be the pressure on the human body—supposing its whole surface to be 14 square feet, and that the barometer stands at 31 inches?

16. 5 compositors in 16 days of 14 hours long can compose 20 sheets of 24 pages, 50 lines in a page, and 40 letters in a line; in how many days of 7 hours long may 10 compositors compose a volume to be printed in the same letter, containing 40 sheets, 16 pages in a sheet, 60 lines in a page, and 50 letters in a line?

17. If 1100 men make 10 miles of railroad in 3 months, how long will it take 2750 men to make 75 miles?

18. The driving wheel of a locomotive engine 5 feet in diameter turned 2500 times in going 6 miles. Supposing the circumference of a circle to be 3·1416 times the diameter, find what distance was lost owing to the slipping of the wheel on the rail?

19. If a man travels 90 miles in 3 days by walking 8 hours a day, in what time will he travel 540 miles by walking 6 hours a day?

20. If 8 men can dig a trench 100 ft. long, 3 ft. broad, and 4 ft. 6 in. deep in 9 days, how many will be required to dig a trench 80 ft. long, 5 ft. broad, and 2 ft. deep in 5½ days?

21. If 7 fires burning 10 hours a day consume 4 tons 10 cwt. of coal in 30 days, how much coal will be consumed by 20 fires in 12 days burning for 14 hours a day?

22. If 5 pumps, each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of a mine, how many pumps with a length of stroke of 2½ feet, working 10 hours a day for 12 days will be required to empty the same mine, the strokes of the former set of pumps being performed four times as fast as those of the latter?

PROPORTIONAL PARTS.

202. As in some sort a sequel to the rule of proportion, we place next the method of finding, by proportional parts, into what portions any quantities should be divided, when the ratio which the several required parts bear to one another is given. Most questions under this head might be solved by making several statements in proportion; but the simpler process by which the result may be arrived at will now be explained.

EXAMPLES.

Ex. 1. Let it be required to divide the number into two parts which shall bear to one another the ratio of 6 : 7.

We have here to find two numbers which shall together make up 91, and shall be in the ratio of 6 : 7.

Now we may either say that the first of the two numbers in the given ratio is to the sum of those two numbers as the first of the required parts is to the whole number 91, which would give us the statement:—

$$6 : 13 :: Ans. : 91$$

$$Ans. = \tfrac{6}{13} \times 91 = 42.$$

Or we may directly apply the following rule:—

RULE LVIII.

"*Form fractions which shall have the numbers composing the given ratio as the numerators, and the sum of these numbers as the common denominator, take these fractional parts of the proposed quantity; they will be the parts required.*"

This would give us—

$$\tfrac{6}{13} \text{ of } 91 = 6 \times 7 = 42$$

$$\tfrac{7}{13} \text{ of } 91 = 7 \times 7 = 49$$

The reason of the above rule may be thus explained:—$\frac{6}{13}$ and $\frac{7}{13}$ are clearly in the ratio of 6 : 7, as likewise are $\frac{6}{13}$ of 91 and $\frac{7}{13}$ of 91; for we may multiply and divide both the terms of any ratio by the same quantities without thereby altering the value of the ratio. Again, $\frac{6}{13}$ added to $\frac{7}{13}$ makes $\frac{13}{13}$ or 1; therefore $\frac{6}{13}$ of 91 added to $\frac{7}{13}$ of 91 will make 91. But the conditions of the problem before us only required that we should find two numbers which were in the ratio of 6 : 7, and which, when added together, would make 91. Hence $\frac{6}{13}$ of 91 and $\frac{7}{13}$ of 91 are the numbers required.

Ex. 2. Divide the sum of £95 0s. 2d. among 3 persons in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

$$\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} = \tfrac{6}{12} + \tfrac{4}{12} + \tfrac{3}{12} = \tfrac{13}{12}.$$

Therefore the fractions are—

$$\tfrac{1}{2} \div \tfrac{13}{12}, \text{ or } \tfrac{6}{13};\quad \tfrac{1}{3} \div \tfrac{13}{12}, \text{ or } \tfrac{4}{13};\quad \tfrac{1}{4} \div \tfrac{13}{12}, \text{ or } \tfrac{3}{13}.$$

and $\frac{6}{13}$ of £95 0s. 2d. = 6 × (£7 6s. 2d.) = £43 17s. 0d.

$\frac{4}{13}$ of £95 0s. 2d. = 4 × (£7 6s. 2d.) = £29 4s. 8d.

$\frac{3}{13}$ of £95 0s. 2d. = 3 × (£7 6s. 2d.) = £21 18s. 6d.

Ex. 3. Divide £1278 among three persons in the ratio of 13, 17, and 41.

By the rule—$13 + 17 + 41 = 71$.

Therefore the fractions are $\frac{13}{71}$, $\frac{17}{71}$, and $\frac{41}{71}$.

$$\left.\begin{array}{l} \frac{13}{71} \text{ of } £1278 = £234 \\ \frac{17}{71} \text{ of } £1278 = £306 \\ \frac{41}{71} \text{ of } £1278 = £738 \end{array}\right\} \textit{Answers.}$$

Ex. 4. A bonus of £140 is to be divided among 3 engineers in proportion to their wages. The first has received £201, the second £341, and the third £185: what amount of the £140 does each receive?

1st £201
2nd £341
3rd £185

727

Therefore the fractions are $\frac{201}{727}$, $\frac{341}{727}$, and $\frac{185}{727}$.

$\frac{201}{727} \times 140 = £38$ 14s. $1\frac{497}{727}$d.
$\frac{341}{727} \times 140 = £65$ 13s. $4\frac{80}{727}$d.
$\frac{185}{727} \times 140 = £35$ 12s. $6\frac{150}{727}$d.

Total £140 0 0

If the terms of the given ratio are expressed as fractions, it is better to reduce them to integers, and then proceed as above.

Ex. 5. Divide £1580 into parts proportional to $1\frac{1}{2}$, $1\frac{1}{4}$, $1\frac{1}{5}$.

Multiplying by 20 : $1\frac{1}{2} : 1\frac{1}{4} : 1\frac{1}{5} = 30 : 25 : 24$ and the parts required are $\frac{30}{79}$ of £1580 = £600, $\frac{25}{79}$ of £1580 = £500, $\frac{24}{79}$ of £1580 = £480.

Ex. 6. Gunpowder is made up of nitre, sulphur, and charcoal, which are mixed in this proportion, 75 parts of nitre to 10 of sulphur and 15 of charcoal; in half a ton of powder how many pounds of each ingredient?

$75 + 10 + 15 = 100$, and $\frac{1}{2}$ ton $= 1120$ lbs.

Hence $\frac{75}{100}$ of $1120 = \frac{3}{4}$ of $11202 = 840$ lbs. nitre.
$\frac{10}{100}$ of $1120 = \frac{1}{10}$ of $1120 = 112$ lbs. sulphur.
$\frac{15}{100}$ of $1120 = \frac{3}{2}$ of $112 = 168$ lbs. charcoal.

Examples for Practice.

1. Divide the number 100 into two parts which shall have to one another the ratio of 2 : 3.
2. Divide the number 45 into three parts which shall be to one another in the ratio of 7, 5, and 3.
3. Divide the number 2679 into parts which shall be to one another in the ratio of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.
4. Of a certain dynasty $\frac{1}{3}$rd of the kings are of one name, $\frac{1}{4}$th of another name, $\frac{1}{5}$th of another, and $\frac{1}{12}$th of another, and there are 10 kings besides; what is the number of kings in the dynasty?
5. A bonus of £150 is to be divided among three engineers in proportion to their length of service in the employ. The 1st has served 2 years 2 months, the 2nd has served 1 year 9 months, and the 3rd only 3 months, how much should each receive?
6. A bonus of £120 is to be divided among three engineers in proportion to the pay received. The 1st has received £142, the 2nd has received £116, and the 3rd £42, how much will each get of the bonus?

CHAPTER VIII.

INTEREST, DISCOUNT, PER CENTAGE, &c.

INTEREST.

203. When one person lends another a sum of money, this sum is called the *Principal.*

The money which the borrower pays to the lender for the use of his money is called the *Interest.*

Interest is generally reckoned at so much for every £100 lent, for every year during which the loan continues, this is called the *Ratio per cent. per annum.*

Thus, if A lends B a sum of money at 3½ per cent. per annum, B will pay A £3 10s. for every £100 he has borrowed, every year as long as he keeps it.

When the Interest of the Principal alone is taken, it is called *Simple Interest;* but if the interest, as soon as it becomes due, be added to the principal, and interest be charged upon the whole, it is called *Compound Interest.*

204. The solution of questions in Simple Interest depends upon an easy practical rule, deduced from Simple Proportion, *e.g.*, if it be required to find the simple interest on £885 for 1 year at 4 per cent. we should say, "If £100 in one year gain £4 as interest, what will £885 gain in the same time?"

$$\pounds 100 : \pounds 885 :: \pounds 4 : \pounds, \textit{ Answer.}$$

$$\textit{Ans.} = \frac{885 \times 4}{100}$$

Hence we deduce the rule—

RULE LIX.

Multiply the Principal by the rate per cent., and divide by 100.

It is therefore unnecessary to state each sum as a proportion, because the division by 100 being effected in whole numbers by cutting off with a decimal point the last two figures, and in decimals by shifting the decimal point two places to the *left*, it is the easiest plan, generally, instead of reducing the quantities by cancelling, to multiply the principal at once by the rate per cent., and then effect the division by 100 (§ 58 (4), page 53).

EXAMPLES.

Ex. 1. Find the interest for a year of £375 at 4½ per cent.

£ s.
375 0
4½
1500 0
187 10
£ 16 | 87 10
20
s. 17 | 50
12
d. 6 | 00

Here, in dividing the £1687 10s. by 100, we do not put down the 100, but cut off the last two figures of the pounds, giving a quotient 16, and remainder 87. We then reduce to shillings, &c., as in ordinary Long Division (see page 53).

To find the simple interest for a term of years we may either multiply the interest found, as in the last example, by the number of years, or multiply the principal before we divide by the 100.

Ex. 2. Find the simple interest on £2275 for 3½ years, at 5 per cent.

£2275
5
113·75 = interest for 1 year.
3½
34·125
56·875
£398·125 = interest for 3½ years.
20
s. 2·500
12
d. 6·0 *Answer*, £398 2s. 6d.

DISCOUNT.

205. Discount is the abatement made upon a debt in consequence of money being paid before it was due, the value of this accommodation being reckoned in the same manner as simple interest.

EXAMPLE.

Ex. 1. What is the discount of £612 17s. 6d., due 9 months hence, at 5 per cent. per annum?

£612 17s. 6d.
5
£30 | 64 7 6
20
s. 12 | 87
12
d. 10 | 50
4
2 | 0

£30 12s. 4d.
3
4)91 17 0
22 19 3

9 months = $\frac{9}{12}$ = $\frac{3}{4}$ of a year.
Discount for 12 months = £30 1s. 10½d.
∴ " 9 months = $\frac{3}{4}$ of £30 1s. 10½d.
= £22 19s. 3d.

PER CENTAGE.

206. In the preceding examples in the chapter, wherever the term "Per Cent." occurs, it referred to £100 money; there are, however, cases in which the term Per Cent. occurs where the reference is not to £100 money, but to the number 100 where the unit is an abstract number of a different kind from the above-mentioned.

All such examples depend on the principles of proportion; some examples will now be worked by way of illustration, and others subjoined for practice.

EXAMPLES.

Ex. 1. Find how much per cent. 7 is of 16.

In other words, the question is, "Find what number bears the same ratio to 100 that 7 bears to 16."

$$16 : 7 :: 100 : \text{number required.}$$

$$\frac{7 \times 100}{16} = 43\cdot75.$$

Ex. 2. In standard gold 11 parts out of 12 are pure gold, how much per cent. is dross?

In every 12 parts 1 part is dross,

$$\therefore\ 12 : 1 :: 100 : \text{per cent. of dross,}$$

$$\therefore\ \text{per cent. of dross} = \frac{100 \times 1}{12} = 8\tfrac{1}{3}.$$

Ex. 3. If 8 per cent. of the persons employed in a factory in which 625 persons were engaged were women, what was the total number of the women employed?

As 8 : 100 :: the women in the factory : the total number of persons engaged therein,

i.e., As 8 : 100 :: the *Ans.* : 625 persons,

$$\textit{i.e.},\ \text{the } \textit{Ans.} = \frac{\overset{25}{\cancel{625}} \times \overset{2}{\cancel{8}}}{\underset{4}{\cancel{100}}} = 50 \text{ women.}$$

Ex. 4. Two years ago an engineer's wages were £16 per month, and were then increased 12 per cent., but now being slack times, they are to be decreased by 15 per cent., what will be the present wages?

$$\begin{array}{r} 100\\ 12\\ \hline 112 \end{array} \qquad 100 : \begin{array}[t]{r} 112\\ 16\\ \hline 672\\ 112\\ \hline \text{£}17\cdot92 \end{array} :: \text{£}16 : x \qquad \text{£}17\cdot92 \text{ increased wages.}$$

$$\begin{array}{r} 100\\ 15\\ \hline 85 \end{array} \qquad 100 : 85 :: \begin{array}[t]{rr} & \text{£}17\cdot92\\ & 85\\ \cline{2-2} & 8960\\ & 14336\\ \cline{2-2} & \text{£}15\cdot232\cancel{0}\\ & 20\\ \cline{2-2} \text{s.} & 4\cdot64\cancel{0}\\ & 12\\ \cline{2-2} \text{d.} & 7\cdot68 \end{array}$$

Answer, £15 4s. 7·68d.

Ex. 5. An engineer is to be allowed 3½ per cent. on all coal left at the end of the voyage; what will be the money value to him if he has 95 tons left, coals selling at £1 2s. 6d.?

95 tons of coal at £1 2s. 6d. per ton = £1 2s. 6d. × 95 = £106 17s. 6d.

£100 : £106 17s. 6d. :: £3½ : the *Answer.*

$$\begin{array}{rrrr}
 & 106 & 17 & 6 \\
 & & & 3\frac{1}{2} \\
\hline
 & 320 & 12 & 6 \\
 & 53 & 8 & 9 \\
\hline
£ & 3{\cdot}74 & 1 & 3 \\
 & 20 & & \\
\hline
s. & 14{\cdot}81 & & \\
 & 12 & & \\
\hline
d. & 9{\cdot}75 & & \\
 & 4 & & \\
\hline
q. & 3{\cdot}00 & &
\end{array}$$

Answer, £3 14s. 9¾d.

Ex. 6. The consumption of coal is 42 tons per day, after a few days it increases to 44 tons, what is the increase per cent.?

Here the increase is 2 tons on 42 tons.

42 : 100 :: 2 : *the Answer.*

$$\frac{100 \times \cancel{2}}{\underset{21}{\cancel{42}}} = \frac{100}{21} = 4{\cdot}76 \text{ per cent.}$$

Ex. 7. The distance sailed as found from the pitch and revolutions of the propeller is 46 miles, but the actual distance sailed as found by the chart is 44 miles: required the slip of the screw per cent.?

Distance from engines 46
" by chart 44
Slip 2

46 miles : 100 miles :: 2 : *Answer.*

$$\frac{100 \times 2}{46} = 4{\cdot}34 \text{ per cent slip.}$$

Examples for Practice.

Find the simple interest

1. On £475 for 3 years at 5 per cent.: on £936 7s. 3d. for 2 years at 4 per cent.
2. On £556 13s. 4d. for 6 years at 5 per cent. 3. On £945 10s. for 2 years at 4 per cent.

Find the discount of

4. £31 13s. 4d. due 4 months at 4 per cent: £132 3s. due 2½ years at 4½ per cent: £1649 due 6 months at 4 per cent.
5. How much per cent. is 15 of 96; 19 of 81; 23 of 256; 185½ of 7321·75.
6. An engineer's wages was £17 per month which was afterwards increased by 18 per cent., but times having become slack they are decreased by 16 per cent.: what is present rate of wages?

TO EXTRACT THE SQUARE ROOT.

207. If a number be multiplied by itself, the product is called the *second power*, or the *square*, of that number. If this also be multiplied by the same number, the product is called the *third power*, or the *cube*, of that number; and so on for the *fourth power*, *fifth power*, *&c.* This raising of powers, which is also called **Involution**, is therefore nothing more than multiplying together *equal factors*, and is easy enough.

Powers are denoted by a *small figure* placed above the given number at the right hand.

This figure is called the *index* or *exponent*. It shows how many times the given number is employed as a factor to produce the required power. Thus the index of the *first* power is 1; but this is commonly omitted, that is $2^1 = 2$. The index of the *second* power is 2, the index of the *third* power is 3, the index of the *fifth* is 5, &c. That is, taking 3 as the root:—

$3^1 = 3$, the first power of 3.
$3^2 = 3 \times 3$, the square, or second power of 3.
$3^3 = 3 \times 3 \times 3$, the cube, or third power of 3.
$3^4 = 3 \times 3 \times 3 \times 3$, the bi-quadrate, or fourth power of 3.
$3^5 = 3 \times 3 \times 3 \times 3 \times 3$, the fifth power of 3, &c.

But the reverse operation—that is, to find the factor which, being involved in this manner, shall produce a *given number*—is a problem not so readily disposed of. The factor referred to is called the *root* of the power; so that the reverse problem spoken of, *i.e.*, *the process of resolving numbers into equal factors*, is called **Evolution**, or the **Extraction of Roots.**[*]

Powers and Roots are therefore correlative terms. If one number is a *power* of another the latter is a *root* of the latter. Thus 27 is the cube of 3 and 3 is the cube root of 27. The learner will be careful to observe that in *subtraction* a number is resolved into *two parts*; in *division*, a number is resolved into *two factors*; in *evolution*, a number is resolved into *equal factors*.

Roots, as well as powers, are divided into *different orders*:—Thus, when a number is resolved into *two equal factors*, each of these factors is called the *second* or *square* root; when resolved into *three equal factors*, each of these factors is called the *third* or *cube* root, and so on. *The name of the root expresses the number of equal factors into which the given number is to be resolved.*

Roots are expressed in *two* ways; one by the *radical sign* ($\sqrt{}$) placed before a number, the other by a *fractional index* placed above the number on the right hand. Thus $\sqrt{4}$, or $4^{\frac{1}{2}}$, denotes the *square* or 2nd root of 4; $\sqrt[3]{27}$, or

---

[*] "The extraction of the square root answers the two following questions:—

1. Let there be three quantities, such that the first : second :: second : third, where the first and third are given, what is the second? Evidently the result of the proportion is that the product of the second into itself is equal to the product of the first and third, or, in other words, the second is the square root of the latter product. This is called finding a mean proportional. Thus, to find a mean proportional between 7 and 63, we take the square root of 7×63, or of 441, viz., 21. This evidently satisfies the proportion 7 : 21 :: 21 : 63.

2. Geometrically it solves the following question. Given the length and breadth of a rectangular figure, to find the side of a square which will have the same surface. This is merely the geometrical statement of the first question."

$27^{\frac{3}{3}}$, denotes the *cube* of 3rd root of 27; $\sqrt[4]{16}$, or $16^{\frac{1}{4}}$, denotes the 4th root of 16.

The figure placed over the radical sign denotes the *root*, or the number of *equal factors* into which the given number is to be resolved. The figure for the *square* root is usually omitted, and simply the radical sign $\sqrt{}$ is placed before the given number. Thus, the square root of 25 is written $\sqrt{25}$.

When a root is expressed by a *fractional* index, the *denominator*, like the figure over the radical sign, denotes the root of the given number. Thus, $(25)^{\frac{1}{2}}$ denotes the square root of 25: $(27)^{\frac{1}{3}}$ denotes the cube root of 27.

A fractional index whose *numerator* is greater than 1, is sometimes used. In such cases the *denominator* denotes the *root*, and the *numerator* the *power* of the given number. Thus, $8^{\frac{2}{3}}$ denotes the *square* of the *cube root* of 8, or the *cube root* of the *square* of 8, each of which is 4.

The radical sign $\sqrt{}$, is derived from the letter *r*, the initial of the Latin *radix*, a *root*.

208. To extract the *square* root of a given number is to find a number, which, when squared, or raised to the second power, or, which is the same thing, when multiplied by itself, shall reproduce the given number. The rule for this operation is as follows:—

RULE LX.

1°. *Divide the given number into periods of two figures each, by setting a point over the place of units, another over the place of hundreds, and so on, over every second figure, both to the* left *hand in* integers *and to the* right *hand in decimals. Always* begin to point at the place of units; *or if the number to be extracted be entirely decimal, put a* cypher *in the unit's place, and over it put the first point and mark successively over every second figure to the right.*

NOTE.—The reason for separating the figures of the dividend into periods or portions of two places each is, that the square of any single figure never consists of more than two places; the square of a number of two figures of not more than four places, and so on. So that there will be as many figures in the root as the given number contains periods, or divided, or parted off. If the number of figures be *even*, we shall therefore have two figures in every period; but if *odd*, the first period on the left will have but a single figure.

The number of points over the whole numbers and decimals will show respectively the number of figures in the integral and decimal parts of the root. Thus, if the given number were 8115·23, place the first over the 5, and mark from it to the right and left thus, $8\dot{1}1\dot{5}\cdot2\dot{3}$. If the given number were 85·342, first make the decimal places even in number thus, 85·3420, and then point $8\dot{5}\cdot3\dot{4}2\dot{0}$.

2°. *Find the greatest square in the first period on the left hand, and set its root on the right hand of a given number, after the manner of a quotient figure in division.*

3°. Subtract the square *thus found from the same period, and to the remainder annex the two figures of the next following period.*

4°. *Double the root above mentioned for a divisor, and find how often it is contained in the said dividend, exclusive of its right hand figure; and set that quotient figure both in the quotient and divisor.*

5°. *Multiply the whole augmented divisor by this last quotient figure, and subtract the product from the said dividend, bringing down to it the next period of the given number for a new dividend.*

6°. *Repeat the same process over again, viz., find another divisor by doubling*

all the figures now found in the root; from which, and the last dividend, find the next figure of the root as before; and so on through all the periods, to the last.

The best way of doubling the root figure, to form the new divisor, is by adding the last figure always to the last divisor, as appears in the following examples. Also, after the figures belonging to the given number are all exhausted, the operation may be continued into decimals at pleasure by adding any number of periods of cyphers, two in each period.

EXAMPLES.

Ex. 1. Extract the square root of 188356.

```
      ·  ·  ·
    188356(434
4 | 16
    ——
83 | 283
 3 | 249
—————————
864 | 3456
    | 3456
```

Explanation.—Divide the number into periods of two figures, thus: begin at the last figure 6, count two figures towards the left, and place a dot over the figure 3; count two more and place a dot over the 8; and so on, till there are no more figures. Take the figures to the right of the last dot 18 and find the number which being multiplied by itself will give 18; there is no number that will do so exactly, for 4 × 4 = 16, and 5 × 5 = 25, the former being too small, and the latter too large; we, however, take the smaller number, viz., 4, and place it in the quotient, and its square (16) under 18, take the difference, and bring down the next two figures 83. To get the next divisor double the root already obtained, viz., 4, which give 8, then 8 into 28 goes three times; then set that quotient figure 3 after quotient figure 4, and also after 8 in the divisor; multiply the whole augmented divisor 83 by the quotient 3, and place this product 249 under 283, and take the difference which is 34; next bring down the remaining period 56; add the last figure in the root, 3, to the last divisor, 83, which make 86; then see how often it is contained in 3456, four times, place the four in the quotient and also after the divisor 86, which make 864, then four times this is 3456; and since there is no remainder we have found the exact square root.

Ex. 2. *Extraction of the Square Root.*

Divisors.	Given number pointed.	Root found, figure by figure, as below.
	)$\dot{3}2\dot{1}9\dot{0}0\dot{0}0\dot{0}0$	(5·6736
	25	First *period* 32; nearest square, 25; root, 5. Put 5 in the root and subtract 25 from 32.
106)	719	Remainder 7; bring down next period, 19 double 5 (10), which place in divisor.
		Cut off one figure from 719,—71. This contains the divisor 10 seven times; *try* 7, as follows: annex it to divisor, 107; multiply by it, 107 × 7 is 749: this is greater than 719; 7 will not do. Try* 6.
	636	Then 106 × 6 is 636—less than 719. Put 6 in the root and in the divisor, and subtract 636 from 719: remainder, 83. Bring down
1127)	8300 7889	next period, 00; add 6 last found to 106, giving new divisor, 112. But one figure from 8300—830. This contains 112 seven times. Try 7, and 1127 × 7 is 7889. Put 7 in the root and in the divisor, and subtract 7889 from 8300.
11343)	41100	Remainder, 411. Bring down next period, 00; add 7 last found to 1127, giving new divisor 1134. Cut one figure from 41100 = 4110. This contains 1134 three times: trial* no longer necessary.
	34029	Put 3 in the root, annex 3 to divisor, giving 11343. Subtract 11343 × 3, or 34029.
113466)	707100	Remainder, 7071. Bring down last period, 00; add 3 last found to 11343, giving new divisor 11346. Cut one figure from 707100 =
	680796	70710. This contains 11346 six times. Put 6 in the root; annex 6 to divisor, giving 113466. Subtract 113466 × 6, or 680796.
	26304	Remainder, 26304; less than half of 113466, which shows that there is no occasion to change the last found 6 into 7, to have the nearest decimal of four places.

---

* This trial will rarely be necessary after the second step. So that having cut one figure from the increased remainder, the number of times which the divisor is therein contained may be written down on the right, and the whole division *thus altered*, multiplied by its last figure.

Ex. 3. Find the square root of 29506624.

```
       .  .  .  .
      29506624(5432
    5 | 25
     ----------
  104 | 450
    4 | 416
  ---------------
 1083 | 3466
    3 | 3249
  ---------------
10862 | 21724
        21724
```

Ex. 4. Find the square root of 31640625

```
       .  .  .  .
      31640625(5625
    5 | 25
     ----------
  106 | 664
    6 | 636
  ---------------
 1122 | 2806
    2 | 2244
  ---------------
11245 | 56225
        56225
```

Ex. 5. Extract the square root of 1004065523374249.

```
           .  .  .  .  .  .  .  .
       1 | 1004065523374249(1
       1 | 1
      -----------
       2 | 00
...............................................
      20 | 0040(10
...............................................
    2001 | 004065(1002
       2 |  4004
    ------------------
    2004 |    6152
...............................................
  200403 |  615237(100203
       3 |  601209
  --------------------
  200406 |  1402842
...............................................
20040607 | 1402842349(10020307
           1402842349
           ----------
```

Wherever a dotted line occurs, the augmented remainder, with the last figure cut off, is found not to contain the dividend, a new period is brought down below the line, a cypher is annexed to the divisor and to the root (also brought down), and the figure which, after this, answers the purpose, appears at the end of the divisor and of the root. There being no remainder the exact square root is 10020307.

The student should perform the preceding operation in this form:—

```
             .  .  .  .  .  .  .  .
1          | 1004065523374249(10020307
1          | 1
-----------------------
    2001   | 004065
       2   |   4004
-----------------------
  200403   |   615237
       3   |   601209
---------------------------
20040607   |   1402842349
               1402842349
               ----------
```

We must notice one more case in which a cypher may occur. We will first write the beginner's attempt, as it would be if he were not cautious.

Ex. 6. Extract the square root of 2034.

First Attempt.		*Corrected Process.*	
4	$2\dot{0}3\dot{4}{\cdot}0\dot{0}$, &c. (45·1	4	$2\dot{0}3\dot{4}{\cdot}0\dot{0}$, &c. (45·09, &c.
4	16	4	16
85	434	85	434
5	425	5	425
901	900	9009	90000
	901	9	81081
		2018	8919

In the first he has gone wrong, for though 900 stripped of its last figure, contains 90 once exactly, yet 901, the new figure being annexed, is not contained in 900. He therefore puts a cypher in the divisor and the root, and brings down another period. The decimal point of the root figure always precedes that root figure in forming which the first decimal period was used, annexed cyphers being always considered as decimals. If periods of cyphers be thrown away in the beginning of the operation, the root is all decimal, and has a cypher at the beginning for every period so thrown away; but this rule does not apply to the throwing away of a single cypher (not a whole period) at the beginning, or to a cypher in the unit's place.

EXAMPLES FOR PRACTICE.

In the following examples, the number whose root is to be extracted is in the first column; the pointing at full length in the second; the same with the decimal point and preliminary cyphers, if any, thrown away, in the third; and the answer in the last.

NO. GIVEN.	DO. POINTED.	DO. SIMPLIFIED.	SQUARE ROOT. (NEARLY).
·1	$\dot{0}{\cdot}1\dot{0}0\dot{0}$, &c.	$1\dot{0}0\dot{0}$, &c.	·31622776602
·85	$\dot{0}{\cdot}8\dot{5}0\dot{0}$, &c.	$8\dot{5}0\dot{0}$, &c.	·92195444573
·0683	$\dot{0}{\cdot}0\dot{6}8\dot{3}0\dot{0}$, &c.	$\dot{6}8\dot{3}0\dot{0}$, &c.	·261342686907
·0068	$\dot{0}{\cdot}0\dot{0}6\dot{8}0\dot{0}$, &c.	$6\dot{8}0\dot{0}$, &c.	·082462112512
9·79	$\dot{9}{\cdot}7\dot{9}0\dot{0}$, &c.	$9\dot{7}9\dot{0}0$, &c.	3·12889756943
97·9	$9\dot{7}{\cdot}9\dot{0}0\dot{0}$, &c.	$9\dot{7}9\dot{0}0\dot{0}$, &c.	9·89444288

209. When the root is to be extracted to many places of figures, the work may be considerably shortened, thus:—

RULE LXI.

Proceed in the extraction as in the common method, till there be found half the required number of figures in the root, or one figure more; then for the rest, divide the last remainder by its corresponding divisor, after the manner of contraction in decimals.

EXAMPLE.

Ex. 1. The following is the extraction of the square root of 12 to 12 decimal places by this method:—

```
               12·000000000000 (3·464101615138
                9
               ---
    64     | 300
     4     | 256
    ----------------
    686    | 4400
      6    | 4116
    ----------------
    6924   | 28400
       4   | 27696
    ----------------
    69281  | 70400
        1  | 69281
    -----------------
    6928201 | 11190000
            |  6928201
              -------
              4261799
              4156921
              -------
               104878
                69282
               ------
                35596
                34641
                -----
                  955
                  693
                  ---
                  262
                  208
                  ---
                   54
                   55  The nearest.  The 8 not so much too
                   --     great as 7 would be too small.
```

210. To extract the square root of a vulgar fraction, if the numerator and denominator of the fraction be perfect squares, we may find the square root of each separately, and the answer will thus be obtained as a vulgar fraction; if not, we can first reduce the fraction to a decimal, or to a whole number and decimal, and then find the root of the resulting number. The answer will thus be obtained either as a decimal or as a whole number and decimal, according to the case. Also a mixed number may be reduced to an improper fraction, and its root extracted in the same way.

EXAMPLES.

Ex. 1. Find the square root of $\frac{529}{2401}$.

```
     ·  · ·                         · ·
     2 | 529(23                 4 | 2401(49
     2 | 4                      4 | 16
    ----------                 -----------
    43 | 129                   89 |  801
       | 129                      |  801
         ---                         ---
```

therefore the square root required $= \frac{23}{49}$.

Ex. 2. $\sqrt{536\frac{117}{289}} = \frac{\sqrt{160801}}{\sqrt{289}} = \frac{401}{17} = 23\frac{10}{17}$.

```
  4 | 16̇08̇01̇(401                      1 | 2̇89̇(17
  4 | 16                               1 | 1
 ----------                           ---------
801 |  0801                           27 | 189
    |   801                              | 189
 ----------                           ---------
```

therefore the square root of $\frac{160801}{289} = \frac{401}{17} = 23\frac{10}{17}$.

211. In finding the square root of a fraction whose denominator is not a square number, either reduce the fraction to a decimal, or multiply the numerator and denominator by that factor which will make the latter an exact square.

Thus, $\sqrt{\frac{4\cdot7}{5}} = \sqrt{\frac{23\cdot5}{25}} = \frac{4\cdot847}{5} = 969$.

Ex. 3. Find the square root of 1049, 104·9, 10·49, 1·049, ·1049, and ·01049, to seven significant digits by the shortest process.

```
   3  | 1049(3238827
   3  |  9
  ---   ----
  62  | 149
   2  | 124
  ---   ----
 643  | 2500
   3  | 1929
 ----   -----
 6468 | 57100
    8 | 51744
 ----   ------
 6476 | 535600
      | 51808
        ------
          1752
          1295
          ----
           457
           453
           ---
             4
```

In the same way

$\sqrt{104\cdot9} = 10\cdot24206$.

From these two roots all the others may be found as follows:—

$\sqrt{10\cdot49} = \sqrt{\frac{1049}{100}} = \frac{1}{10}\sqrt{1049} = 3\cdot238827$.

$\sqrt{\cdot1049} = \sqrt{\frac{10\cdot49}{100}} = \frac{1}{10}\sqrt{10\cdot49} = \cdot3238827$.

$\sqrt{1\cdot049} = \sqrt{\frac{104\cdot9}{100}} = \frac{1}{10}\sqrt{104\cdot9} = 1\cdot024206$.

$\sqrt{\cdot01049} = \sqrt{\frac{1\cdot049}{100}} = \frac{1}{10}\sqrt{1\cdot049} = \cdot1024206$.

EXAMPLES FOR PRACTICE.

Extract the square roots of—

1. 10004569; 240168·6049; and 33232930569601.
2. 5; ·5; ·05; and ·005; each to 5 places of decimals.
3. 97199881; 9719988·1; 971998·81; 97199·881; to 7 decimal places.
4. 9719·9881; 971·9981; 97·199881; 9·7199881; to 7 decimal places.
5. ·97199881; ·097199881; ·0097199881; ·000997199881; to 7 decimals.
6. 904382075008804449001; and to 7 decimal places of 3·14159265.
7. Take the square root of 185302018885 1841, then take the square root of that square root until it can no longer be done without decimals.
8. $\frac{14161}{14641}$; $\frac{169}{256}$; $\frac{841}{1369}$; $\frac{2304}{8281}$; $4\frac{25}{36}$; $5\frac{11}{49}$; $345\frac{24}{25}$; $15061\frac{119}{121}$.
9. 9030025; ·0144; ·1; ·0064; ·005329; and of ·00053361.
10. 4020025; ·000009; ·9; and of the recurring decimals $\cdot\dot{1}$; $\cdot02\dot{7}$, and $\cdot\dot{0}4938271\dot{6}$.

CUBE AND CUBE ROOT.

212. The cube of a number is the product of a number and of its square, or the product of a number multiplied twice into itself. Thus, the cube of 3 is 3 × 3 × 3 = 27; the cube of 7 is 7 × 7 × 7 = 343 (see § 207). Taking the cube is the direct answer to such questions as the following:— A linear foot being 12 linear inches, how many cubic inches in a cubic foot? the answer is the cube of 12, or 12 × 12 × 12 or 1728. The process of taking the cube is merely one of multiplication, and calls for no special remark. The cube root answers the question: what is that number which multiplied twice by itself, makes up the number of which it is called the cube root. It answers the geometrical question: if a cube contains *so many* cubic units, how many linear units will its side contain? The cubes of the numbers up to 12 should be committed to memory; they are as follows:—

Numbers	1	2	3	4	5	6	7	8	9	10	11	12
Cubes	1	8	27	64	125	216	343	512	719	1000	1331	1728

Since 1000 is the cube of 10, the cube root of 1000 times any number will have the same figures as the cube root of the number itself. Thus the cube of 62·3 is 241804·367. However, the cube of 6·23 is 241·804367.

But if we multiply a number by 10, its cube root will have a different set of figures, and if we multiply the number by 100, we shall again have a different set of figures: thus

The cube root of 47 is 3·6088261
„ 470 „ 7·7749801
„ 4700 „ 16·7506869

Every number, therefore, considered with reference to its figures only, and not with reference to its decimal place, has 3 cube roots which differ from one another by the cube of 10 and the cube root of 100. The extraction of the cube root is a troublesome process, and much longer and more complicated than that of the square root.

TO EXTRACT THE CUBE ROOT.[*]

RULE LXII.

1°. *Divide the page into three columns*, and call them L M N, in *order from left to right, and so that* M *shall be double the breadth of* L, *and* N *double the breadth of* M. *At the head of* N *place the number* whose *root is to be extracted*,

---

[*] This method was invented by the late Mr. W. G. Horner, and forms only one single and simple application of a universal method of extracting all roots whatever, and even his method, as applied to roots, is only a particular application of his general method of solving numerical equations of all orders.

In the form here given it is rather more concise in the operation than is generally applied to equations of a higher order, and is put in this form for the use of those students who are not likely to proceed to the higher equations.

and mark off the place for the root as the quotient is marked in division, or the root in the extraction of the square root. Thus:—

L	M	N	
3 × root already found.	3 × square of root already found.	Number whose root is to be found.	Root.

2°. *Divide the number into periods of three figures* each, by setting a point* over *the place of* units, *and also* over every third figure *from thence to the left* hand *in whole numbers, and to the* right *in decimals. The number of periods will show the number of figures in the required root.*

NOTE.—If there be no whole number or integral part in the given number, we must in pointing begin with the *third* figure from that which would be the unit's place, if there were a whole number, and mark successively every *third* figure to the right. If there be a whole number as well as a decimal, it will be the safest method to begin at the unit's place, if there were a whole number, and mark successively every *third* figure to the right. If there be a whole number as well as a decimal, it will be the safest method to begin at the unit's place, and point over every third figure to the right and left of it; the number of points over the whole numbers and decimals will show respectively the number of figures in the integral and decimal part of the root. Thus, if the given number were 5623·453134, place the first point over the 3, and mark from it to the right and left thus $\dot{5}62\dot{3}\cdot45\dot{3}13\dot{4}$. If the given number were 5·23, make the number of decimal places equal to three by affixing a cypher thus $\dot{5}\cdot23\dot{0}$, place the first point over the 5 and the second over 0; if the root to more decimals than one be required, more cyphers must be affixed. Again, ·01, 24·1, and ·000007 would be pointed for the cube root of $\cdot01\dot{0}$, $2\dot{4}\cdot10\dot{0}$, $\cdot00\dot{0}00\dot{7}$.

3°. *Find the* greatest *number whose cube is contained in the first period at the* left, *this is the* first figure *in the required root, which set in the place appropriated for the root.* Subtract *its* cube *from the* first *period, and to the remainder bring down the second period.*

4°. *In the column* L *put three times the root already found, and in* M *put three times its square. With the last number as a trial divisor of the resolvend, omitting the two figures to the right hand, find the next figure of the root and annex it to the former one, and also to the number in the column. Multiply the number now in* L *by the new figure of the root, and place the product under that in* M, *but having its figure two places more to the right than the numbers already there.*

5°. Multiply *the corrected divisor by the root figure, and place it under the number in* N, *and subtract it therefrom.*†

---

* The reason for pointing the given number into periods of three figures each is, because the cube of one figure never amounts to more than three places; the cube of two figures to more than six, but always more than three; the cube of three figures never to more than nine, but always more than six; and so on to any extent.

† A little attention to the composition of the algebraic expression for the cube of a binominal will render the truth of this rule very obvious. For $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + [3a^2 + (3a+b)\,b]\,b$, the last form of which is exactly that whose composition is directed in Rule, 4° and 5°. Thus:—

L	M	N
$3a$	$3a^2$	$a^3 + 3a^2b + 3ab^2 + b^3 + B + C \ldots (a+b)$
$+b$	$(3a+b) \times b$	a^3
$3a+b$	$3a^2 + (3a+b)\,b$	$+3a^2b + 3ab^2 + b^3$
		$\{3a^2 + (3a+b)\,b\}\,b$
		$+B+C \ldots$

where B, C, &c., stand for other quantities to be afterwards brought down for continuing the process.

NOTE I.—If the number last found be greater than the number previously in N, the subtraction cannot be performed; the work dependent on the root figure which produced it must therefore be cancelled, and a smaller root figure tried instead of it. If necessary, the same process must be again repeated, till a number is found which will admit of being subtracted according to the rule.

NOTE II.—If at any part of the operation the number to be divided by the trial divisor be less than it, affix a cypher to the root, two cyphers to the trial divisor, bring down the next period and proceed according to the rule (See Example 3).

6°. *Write twice the last root figure under the number* L, *and the square of the last root figure under* M. *Add the two last written numbers in* L *together, and the three last written numbers in* M.* *These numbers will be respectively triple the new root and triple its* square, *and we may proceed with these to find a new root figure, and complete the operation as in Rule,* 4° *and* 5°.

NOTE.—After about one-third of the number of figures in the root have been obtained, (or one-third of the number intended to be taken into account if the root be interminable), we may contract the work very considerably by the following method:—

From column M cut off one figure, and from column L cut off two. Multiply by the new root figure, as before, keeping the columns of corrections in M and N as in Contracted Multiplication and Division of Decimals. This will generally ensure the number of figures specified correct in the root.

EXAMPLES.

Ex. 1. Extract the cube root of 4822854.

Dividing the figures of the given number N into groups of three we have

$$4\dot{8}22\dot{8}54\dot{4}$$

where the right hand figure of each group is indicated by a dot or point placed over it. It is evident, from reasons given above (page 173, note *), that the cube root of the number is a number of three figures.

L	M	N
9~~7~~		$4\dot{8}22\dot{8}54\dot{4}$(3~~7~~64
96	27	27
12	~~679~~	21228 = 1st resolvend.
1084	~~3379~~	~~23653~~
	576	19656
	3276	1572544 = 2nd resolvend.
	$6^2 =$ 36	
	3888	
	4336	1572544
	393136	

Ans. 364.

---

* That this method produces the triple of the new root in the column L, and the triple of its square in the column M may be thus immediately shown:—

L	M	N
⋮	⋮	⋮
$3a + b$	$(3a + b)b$	$(a + b)$
$2b$	$3a^2 + (3a + b)b$	
	$+ b^2$	
$3a + 3b$	$3a^2 + 6ab + 3b^2$	
$= 3(a + b)$	$= 3(a + b)^2$	

And we thus start from the last root $a + b$ as we did at first from a.

The first period is 48, and the greatest integral cube contained in it is 27. Its root 3 is put in the root place; then subtracting its cube we bring down the next period 228, making 21228 as the first resolvend. We now set the triple of the first figure in the root 3 under L (there is 97 in the sum, but the 7 will be accounted for by and by); then we multiply this triple by the first figure of the root and place the product under M. We now divide the resolvend—omitting the two right hand figures—by the number under M ($= \frac{212}{27} = 7$) giving as a trial quotient 7. Annex this to L, which gives 97; now we multiply this 97 by 7, the second figure of the root, set the product 679 in M under 27, but having its figures two places more to the right, add them up, multiply the sum 3379 by 7, which gives 23653. This cannot be subtracted from the resolvend, and hence the whole work dependent on 7 is to be obliterated and a lower number, 6, tried.

Performing the same operation with 6 as has just been described with 7, we get 96, 3276, and 19656 for L, M, and N, the last of which subtracted gives, after bringing down the next period, for the next resolvend 1572544. We next write twice the last root figure (2 × 6) under the number L, and the square of the last root figure (62) under M. Adding the two last written numbers in L, viz., 96 and 12, together, and the three last written numbers in M, viz., 376, 3276, and 36, we have the triple (= 108) and triple the square (= 3888) of 36 in L and M respectively. The trial divisor 3888 is found to be continued 4 times in 15725 (omitting the two right hand figures of the resolvend) and annexing this 4 to the former figures of the root makes 364, and the column L makes 1084. This multiplied by 4 and added to M gives 393136, and this last multiplied by 4 gives 1572544. No remainder occuring, the work is hence terminated at this step, and the root is 364.

Ex. 2. Extract the cube root of 312908547069.

Dividing the figures of the given number into groups of three, we have—

$31\dot{2}90\dot{8}54\dot{7}06\dot{9}$

where the right hand figure of each group is indicated by a dot placed over it.

L	M	N	
18~~8~~	108	$31\dot{2}90\dot{8}54\dot{7}06\dot{9}$(6~~8~~789	
		216	
	~~1504~~	96908	
	~~12304~~	~~98432~~	
187	1309	84763	
2×7 14	12109	12145547	
	$7^2 =$ 49		
2018	13467		
16	16144		
	1362844	10903752	
	$8^2 =$ 64	1242795069	
20349	137905200		
	183141		
	138088341	1242795069	*Ans.* 6789

Ex. 3. Extract the cube root of 223648543.

Dividing the figures of the given number into groups of three, we have—

$22\dot{3}64\dot{8}54\dot{3}$

where the right hand figure of each group is indicated by a dot placed over it.

L	M	N	
18	108	$22\dot{3}64\dot{8}54\dot{3}$(607	
		216	
		7648	Omitting two last figures of resolvend 76 is not divisible by 108
1807	10800		
	12649		
	1092649	7648543	bring down next period and affix 0 to root, then ~~$\frac{76485}{10800}$~~ goes 7 times.
		7648543	

Ex. 4. Find the cube root of ·000007 to three places of decimals.

```
L          M              ·000007000(·019
3          3                 1
           351               —
           ———               6000   Answer, ·019
           651               5859
                             ————
                              141
```

213. To extract the cube root of a vulgar fraction, if the numerator and denominator of the fraction be perfect cubes, we may find the cube root of each separately, and the answer will thus be obtained as a vulgar fraction; if not, first reduce the fraction to a decimal, or to a whole number and decimal, and then find the root of the resulting number. The answer will thus be obtained either as a decimal or as whole number and decimal; or we may multiply both numerator and denominator by such a number.

EXAMPLES.

Ex. 1. Find the cube root of $\frac{5}{9}$ to three places of decimals.

$$\tfrac{5}{9} = ·555555555·$$

```
L          M                     N
242        192                   ·555555555(·822
  4          484 ⎫               512
———        ————— ⎪               ————
246        19684 ⎬               43555
        2² =   4 ⎭               39368
           —————                 ———————
           20172                  4187555
             4924
           ———————
           2022124                4044248
                                  ———————
                                   143307
```

Ex. 2. Find the cube root of $\frac{3}{16}$.

$$\sqrt[3]{\tfrac{3}{16}} = \sqrt[3]{\tfrac{12}{64}} = \frac{\sqrt[3]{12}}{4}\ \frac{2·289428}{4} = 572357\ \textit{Ans.}$$

Ex. 3. Find the cube root of $\frac{5}{7}$.

$$\sqrt[3]{\tfrac{5}{7}} = \sqrt[3]{\frac{5 \times 49}{7 \times 49}} = \frac{\sqrt[3]{245}}{\sqrt[3]{343}} = \frac{6·25732}{7} = ·893904\ \textit{Ans.}$$

EXAMPLES FOR PRACTICE.

Extract the cube roots of

1. 3375; 10648; 74088; 3890177; 5735339; 324617597791.
2. 48228·544; 27054·036008; 12·977875; 27054036008; 21936532.
3. ·065939264; ·023639903; ·000030664297; 001030301; ·007645373.
4. 4 to three places of decimals; 7 to two places of decimals; $\dot{8}$ to two decimal places.
5. $\frac{64}{343}$; $\frac{729}{1404928}$; $49\frac{8}{27}$; $7558\frac{197}{512}$; $12\frac{19}{27}$; $31\frac{15}{343}$.
6. Extract the cube root of 3 to six places, and also the cube root of 6, and show how the cube root of 2, of 12, and of 18, may be obtained from these.
7. Extract the cube root of ·0090009009; and thence obtain the cube root of that cube root, and thence again the cube root of the last cube root to six and four places of decimals respectively.

ON FINDING THE VALUE OF FORMULA.

The following examples are intended to show the manner in which formulæ given in an algebraic appearance are to be treated.

1. Find the value of $\frac{D^2\,V}{6000}$* where D = 54, and V 180.

$$\frac{D^2V}{6000} = \frac{54 \times 54 \times 180}{6000} = \frac{524880}{6000} = 87{\cdot}48 \textit{ Answer.}$$

Here D² (D squared) has to be multiplied by V, and the product divided by 6000. Although no sign is written between D² and V, in all such cases the sign of multiplication is implied.

2. Given $\frac{C_{,}K - C\,K_{,}}{K_{,} - K}$ when $C_{,} = 13$, $K = 8$, $C = 10$, $K_{,} = 9$.

$$\frac{C_{,}K - C\,K_{,}}{K_{,} - K} = \frac{(13 \times 8) - (10 \times 9)}{(9 - 8)} = \frac{104 - 90}{1} = \frac{14}{1} = 14 \textit{ Answer.}$$

Here the product of C and $K_{,}$ has to be subtracted from the product of $C_{,}$ and K, and the remainder divided by the difference of $K_{,}$ and K.

3. Find $v = \frac{p\,A\,V}{f\,a}$ when $p = 6$, $A = 5$, $V = 4$, $f = 3$, and $a = 2$.

$$\text{Then } v = \frac{p\,A\,V}{f\,a} = \frac{6 \times 5 \times 4}{3 \times 2} = \frac{120}{6} = 20 \;\therefore\; v = 20.$$

Supposing the values attributed to v, p, A, V, f and a to be the same, and it is required to find a. By attentively reading Nos. 191 and 192, page 140, it will be seen that $v = \frac{p\,A\,V}{f\,a}$ means $v\,f\,a = p\,A\,V$, and by transposing, or in this case, dividing both sides of the equation by $v\,f$, it becomes $\frac{p\,A\,V}{v\,f}$ or $a = \frac{p\,A\,V}{v\,f}$ which in words means this:—The product of the quantities referred to as p A V, divided by the product or result produced by multiplication of the quantities referred to as $v\,f$, will give a as required. Thus,

$$a = \frac{p\,A\,V}{f\,v} = \frac{6 \times 5 \times 4}{3 \times 20} = \frac{120}{60} = 2 \;\therefore\; a = 2.$$

Similarly, the values of the other single letters are obtained in the same way.

4. Find the value of $B = \frac{2880\,d^3}{D^2\,S}$ $d = 14$, $D = 57$, $S = 36$.

$$B = \frac{2880\,d^3}{D^2\,S} = \frac{2880 \times 14 \times 14 \times 14}{57 \times 57 \times 36} = \frac{7902720}{116964} = 67{\cdot}5 \textit{ Answer.}$$

5. If $x = a - \left(\frac{s}{2} - p\right)$; where $a = 12$, $s = 5\frac{1}{2}$, and $p = 2\frac{1}{2}$: find the value of x.

$$x = 12 - \left(\tfrac{5{\cdot}5}{2} - 2{\cdot}5\right) = 12 - (2{\cdot}75 - 2{\cdot}5) = 12 - {\cdot}25 = 11{\cdot}75 \textit{ Answer.}$$

First find the quantity within the brackets—it is ·25—subtract this from 12.

6. $1 + \frac{T + t - 64}{1000}$ where $T = 90$, and $t = 46$.

$$1 + \frac{T - t - 64}{1000} = 1 + \frac{90 + 46 - 64}{1000} = 1 + \frac{72}{1000} = 1 + {\cdot}072 = 1{\cdot}072 \textit{ Ans.}$$

7. $12\,\frac{D^3\,d^3}{S}$; $D = 28$, $d = 14$, and $S = 25$.

$$12\,\frac{D^3 - d^3}{25} = \frac{28^3 - 14^3}{25} = 12\,\frac{21952 - 2744}{25} = 12\,\frac{19208}{25} = 12 \times 768{\cdot}32 = 9219{\cdot}84 \textit{ Ans.}$$

8. Find $B = \frac{f\,d^2 - 15\,S\,D^2}{S\,H^2}$; where $f = 4936$, $D = 41$, $H = 64$, and $S = 32$. *Ans.* 85·5.

---

* This was the Admiralty formula for N.H.P. for paddle engines.

USEFUL RULES IN MENSURATION.

MENSURATION OF SUPERFICIES.*

Def.—A **Parallelogram** is a four-sided figure, of which the opposite sides are parallel.

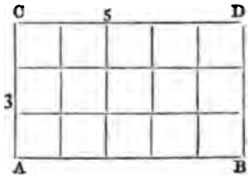
Def.—A **Square** is a four-sided figure which has all its sides equal, and all its angles right-angles.

Def.—A **Rectangle** (or oblong) is a four-sided figure which has its *opposite* sides equal, and all the angles right-angles.

Def.—The straight line joining two opposite angles of a quadrilateral is called a **Diagonal.**

Def.—The **Area** of any plane figure is the measure of the space contained within its extremes or bounds, without any regard to thickness. The area or the content of the plane figure is estimated by the number of little squares that may be contained in it, the side of these little measuring squares being an inch, a foot, a yard, or any other fixed quantity. And hence the area or content is said to be so many square inches, or square feet, or square yards, &c.

Thus, if the figure to be measured be the rectangle ABCD, and the little square E, whose side is one inch, is the measuring unit proposed; then, as often as the said little square is contained in the rectangle, so many square units the rectangle is said to contain, which in this case is 15.



214. To find the area of the square or any rectangular figure.

RULE LXIII.

Multiply the length by the breadth; the result is the required area.

Note.—The dimensions must be of the same denominations, such as both feet or both inches, &c.; and if the product be inches, divide by 144 for feet, and if feet by 9 for square yards, &c.

---

* *Measurement of Surfaces.*—The dimensions employed for areas of surfaces naturally follow from a length of the side of a square measured in linear feet and inches, and this brings us at once to square feet and square inches for surfaces or figures of two dimensions.

Circles are greatly employed in the forms of machinery, because turning is the cheapest and easiest class of machine work. Boring a cylinder and turning a piston, for example, give great facilities for fitting the two together in a cheap and ready manner. All calculations as to the horse-power of an engine or the strength of its parts involve a knowledge of the areas of circles.

Let l = length of figure, b = breadth, and A = area.

$$A = lb,\ l = \frac{A}{b}, \text{ and } b = \frac{A}{l} \text{ that is,}$$

Since the **AREA** = the **LENGTH** × the **BREADTH**, it follows that if any two of these be given, the third may be found as follows:—

1. The *Area* may be found by multiplying the *Length* by the *Breadth*.
2. The *Length* may be found by dividing the *Area* by the *Breadth*.
3. The *Breadth* may be found by dividing the *Area* by the *Length*.

EXAMPLES.

Ex. 1. Find the area of a square, the side of wich is 32 feet 9 inches.

9 inches = ·75 feet. $\frac{a}{12}$)9·0

·75

Area = 32·75 × 32·75 = 1072·5625 square feet.

$$\begin{array}{r} 32\cdot75 \\ \underline{32\cdot75} \\ 16375 \\ 22925 \\ 6550 \\ \underline{9825} \\ 1072\cdot5625 \end{array}$$

1072·5625 square feet.

Ex. 2. The sides of a rectangle contain 12 feet 6 inches and 37 feet 3 inches; find its area.

$\frac{a}{12}$)6·0 $\frac{a}{12}$)3·0

·5 ·25

Area = 12·5 × 37·25 = 465·625 square feet.

$$\begin{array}{r} 37\cdot25 \\ \underline{12\cdot5} \\ 18625 \\ \underline{44700} \\ 465\cdot625 \end{array}$$

465·625 square feet.

Ex. 3. How many yards of carpet 30 in. wide will be required for a room 28 feet long by 18½ feet wide.

The area of room = 28 feet × 18½ feet.
" = 336 inch. × 222 inch. = 74592 sq. in.

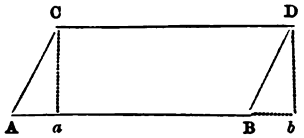
We want to know how long a piece of carpet must be to cover this area supposing the carpet to be 30 inches wide; in other words, we have the *area* given—viz., 74592 sq. in.—and the width given—viz., 30 inches—and from these we find the *length*, for

$$\text{The length} = \frac{\text{area}}{\text{width}} = \frac{74592 \text{ sq. in.}}{30 \text{ inches}} = 2486 \text{ inches} + 12 \text{ inches over.}$$

207 feet 2 inch = 69 yards 2 + inches. *Answer.*

The area of a parallelogram which is not right-angled is easily obtained from this consideration:—That we can convert it into a rectangle by cutting off a triangle at one end and putting it on at the other. Its area is thus seen to be the length multiplied into the breadth measured perpendicularly, or, as it is more commonly stated,

Area = base × altitude.



Def.—A Rhombus is a four-sided figure which has all its sides equal, but its angles are not right-angles.

Def.—A Rhomboid is a four-sided figure which has its *opposite* sides equal, but its angles are not right-angles.

Note.—It will be observed that the rhombus and rhomboid differ from the square and rectangle in one respect, viz., that their angles are not right-angles. They are, indeed, like squares and rectangles that have been forcibly wrenched (like a loose slate frame), from their true form. The rhombus being a distorted square, and the rhomboid a distorted rectangle.

These figures, because their opposite sides are parallel, are frequently called *parallelograms*, although that name really includes the square and rectangle.

Example.

Ex. A rhombus is 12·4 feet long, and its perpendicular breadth is 10·9 feet: find the area.

Area = length × perpendicular breadth = 12·4 × 10·9 = 135·16 sq. feet.

$$\begin{array}{r} 12{\cdot}4 \\ 10{\cdot}9 \\ \hline 1116 \\ 124 \\ \hline \end{array}$$

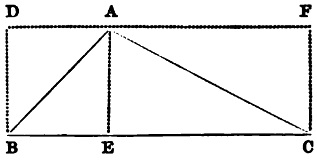
Area = 135·16 sq. feet.

Examples for Practice.

1. Find the area of a square whose side is 35 feet 9 inches.
2. The area of a square is 1278·0625 feet: required the side.
3. Required the area of a rectangle $27\frac{1}{2}$ inches long by 14 inches broad.
4. A board is $7\frac{3}{16}$ inches broad by 3 feet $5\frac{1}{4}$ inches long: required its surface.
5. The length of a parallelogram is $12\frac{1}{4}$ feet and its breadth $8\frac{1}{2}$ feet: what is its area.
6. A rectangle is to contain 133 square inches: what must be its length when the breadth is 7 inches.
7. What length must be cut off a rectangular board whose breadth is 9 inches to make a square yard.
8. What is the area of a rhombus whose side is 7 ft. 6. in., and perpendicular height 3 ft. 4 in.?
9. What is the area of a rhombus whose length is 3 yards, and perpendicular height 2 ft. 3 in.?

215. To find the area of a triangle.

The easiest way by which to find the area of a triangle is by measuring the base and perpendicular height. In the accompanying figure let us suppose that we know the length of B C and A E. Now, if we multiply these together their product gives the area of the enclosing rectangle D B C F, which we know to be exactly double of the triangle A B C (Euc. I, 41). The area of which can, therefore, readily be found by dividing by 2.



RULE LXIV.

Multiply its base and height together, and half the product will be the area.

To find the area of a triangle, the base of which is b and perpendicular p.

$$A = \tfrac{1}{2}\,b\,p, \text{ also } p\,\frac{2A}{b}, \text{ and } b = \frac{2A}{b}.$$

Example.

The base of a triangle is 76·5 feet, and perpendicular 92·2 feet, what is its area?

Area = 76·5 × 92·2 ÷ 2 = 3526·65 square feet.

EXAMPLES FOR PRACTICE.

1. The base of a triangle is 72·7, and the perpendicular height is 36·5, what is its area?
2. The base of a triangle measures 15 ft. 6 in., and the perpendicular 12 ft. 7 in., what is its area?

216. The area of a triangle may, however, be determined from its three sides by the following

RULE LXV.

From half the sum of the three sides, subtract each side separately; then multiply the half sum and the three remainders together, and the square root of the last product will be the area of the triangle.

For practical purposes this rule may be more conveniently stated in the form of the following formula, where s represents half the sum of the sides:—

$$\text{Area} = s\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

NOTE.—If the triangle be equilateral, all that is necessary is evidently to measure one side. The area may then be calculated from this by the following short and useful rule,—*multiply the square of the side by* 433, *and the product will be the area.*

EXAMPLE.

Ex. Find the area of a triangle whose sides are 18, 24, and 30.

Here we have according to Rule, half the sum of the three sides $= \frac{1}{2}(18 + 24 + 30) = 36$;

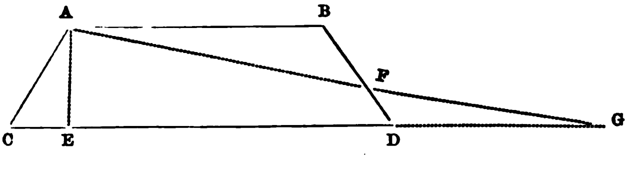
$$\text{also, } \left.\begin{aligned} 36 - 18 &= 18 \\ 36 - 24 &= 12 \\ 36 - 30 &= 6 \end{aligned}\right\} \text{are the three remainders.}$$

whence the area $= \sqrt{36 \times 18 \times 12 \times 6} = \sqrt{46656} = 216$ *Answer.*

EXAMPLES FOR PRACTICE.

1. What is the area of a triangle whose sides are 50, 40, and 30?
2. Required the area of a triangle whose sides are 13, 14, and 15 feet respectively.

217. To find the area of a trapezoid, which is a four-sided figure, having only two opposite sides parallel.



RULE LXVI.

Half the sum of the two parallel sides multiplied by the perpendicular distance between them is the area of the trapezoid.

The reasonableness of this rule may be demonstrated without any strictly mathematical investigation, by remembering that we calculate all areas by reference to the rectangle; for if we multiply the longer side CD by the perpendicular distance, the product will obviously be greater than the area of the trapezoid, while if we multiply the perpendicular distance by the shorter side AB the product will be too little; hence we find a side of mean or average length by dividing the sum of AB and CD by 2, and the product of this and the perpendicular distance is the exact area.

EXAMPLE.

Let A B C D be a trapezoid, the side A B = 25, C D = 40, A E = 18: required the area.

$$\begin{array}{r} AB = 25 \\ CD = 40 \\ \hline 2)65 \\ \hline 32{\cdot}5 \\ 18 \\ \hline 2600 \\ 325 \\ \hline 585{\cdot}0 \text{ the area.} \end{array}$$

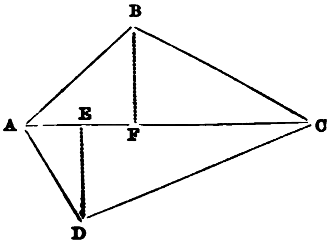
218. To find the area of a trapezium, which is a four-sided figure, having no sides parallel.

RULE LXVII.

1°. *Divide it into triangles* by a diagonal.

2°. *Next find the* sum *of the areas of the two triangles.*

This is an irregular figure, and its area is usually found by dividing it, as in the figure, into two triangles.



Now, the area of the triangle ABC is ½ AC BF, and area of the triangle ACD is ½ AC DE; and, therefore, the area of the trapezium (*i.e.*, of the two triangles together) is

$$\tfrac{1}{2} + AC\ BF + AC\ DE$$
$$= \tfrac{1}{2}\ AC\ (BF + DE).$$

EXAMPLE.

In the trapezium A B C D, the diagonal A C is 100, B F 35, and D E 30: what is its area?

$$\begin{array}{r} BF = 35 \\ DE = 30 \\ \hline 65 \\ AC = \quad 100 \\ \hline 2)6500 \\ \hline 3250 \text{ the area.} \end{array}$$

219. To find the area of a regular polygon, or figure of four or more sides, having all the sides equal.

RULE LXVIII.

Let fall a perpendicular from the centre upon one of its sides, multiply this by the side itself and by the number of sides in the polygon, and half the product is the area of the polygon.

220. To find the area of the sector of a circle or any part of a circle bounded by an arc and two radii.

RULE LXIX.

Half the radius multiplied by the length of the arc equals the area.

221. To find the length of an arc.

RULE LXX.

From eight times the chord of half the arc subtract the chord of the whole arc, and take one-third the remainder for the length of the arc, nearly.

222. To find the area of the segment or any part of a circle bounded by an arc and its chord.

RULE LXXI.

Find the area of the sector and the triangle: the difference *is the area of the segment.*

FINDING THE AREAS OF CIRCLES, &c.

We now pass on from rectangular figures to those which are called *curvilinear*, being bounded not by straight lines but by curved lines.

Def.—A circle is a plane figure contained by one line, which is called the *circumference*, and is such that all strait lines drawn from its centre to the circumference are equal to one another.

The straight line drawn from the centre of a circle to its circumference is called the *radius*.

The straight line drawn from the circumference through the centre till it meets the circumference again, is called the *diameter*, and is exactly equal to *twice the radius*.

The diameter divides the circle into two *semicircles*.

It is necessary to remember the numbers ·7854 and 3·1416; the first is one-fourth the length, and we may use it thus for areas:—

RULE LXXII.

As the area of a square whose side is 1	:	the area of its inscribed circle ·7854	::	the area of any other square	:	the area of its inscribed circle.

And again—

RULE LXXIII.

As a circle whose diameter is 1	:	its circumference 3·1416	::	the diameter of any other circle	:	its circumference.

N.B.—An inscribed square is half the area of the circumscribed square.

223. To find the circumference of a circle, the diameter being given.

RULE LXXIV.

Multiply the diameter by 3·1416 *and the* result will be the circumference.

Examples.

Ex. 1. The diameter of a piston is 94 inches: required the circumference.

$$\begin{array}{r} 3{\cdot}1416 \\ 94 \\ \hline 12{\cdot}5664 \\ 282744 \\ \hline \end{array}$$

Circum. = 295·3104 inches.

Ex. 2. The diameter of a circle is 2·5 inches: required the circumference.

$$\begin{array}{r} 3{\cdot}1416 \\ 2{\cdot}5 \\ \hline 157080 \\ 62832 \\ \hline \end{array}$$

Circum. = 7·85400 inches.

Ex. 3. Find the circumference of a valve whose diameter is 3½ inches.

Since circumference = 3·1416 × diameter = 3·1416 × 3·5 = 10·9956 inches.

Ex. 4. Find the length of the outer ring of a paddle wheel whose diameter is 26 feet.

3·1416 × 26 = 81·6816 feet.

Examples for Practice.

1. The diameter of a screw shaft is 9⅛ inches: required the circumference.
2. Required the circumference of a safety valve, the diameter being 4⅞ inches.
3. Given the diameter of a circle 7⅛ inches: find the circumference.
4. Find the circle described by the outer edge of a screw, the diameter being 18 feet.
5. Find the circumference of the outer ring of a paddle-wheel, the diameter of which is 30 feet 5 inches.
6. The diameter of a circle is 84 inches: find the circumference.

224. To find the diameter of a circle, the circumference being given.

RULE LXXV.

Divide the circumference by 3·1416 *and the* result is the diameter.

Examples.

Ex. 1. The circumference of a piston is 226·1952 inches: find the diameter.

$$\begin{array}{r} 3{\cdot}1416)226{\cdot}1952(72 \text{ inches.} \\ 219912 \\ \hline 62832 \\ 62832 \\ \hline \end{array}$$

Ex. 2. The circumference of a piston-rod is 28·2 inches: find the diameter.

$$\begin{array}{r} 3{\cdot}1416)28{\cdot}2000(8{\cdot}9 \text{ inches.} \\ 251328 \\ \hline 306720 \\ 282744 \\ \hline 23976 \\ \hline \end{array}$$

Examples for Practice.

1. The circumference of a circle being 50 feet: find the diameter.
2. The circumference of a paddle-wheel being 47·124 feet: required the diameter.
3. The circumference of a circle is 17·59296 inches: required the diameter.
4. The circumference or girth of a cylinder is 103·6728 inches: find the diameter.
5. The circumference of a piston is 329·8680 inches: find the diameter.
6. What is the diameter of a piece of shafting which measures 28¼ inches round its circumference.

225. To find the area of a circle, having given the diameter.

RULE LXXVI.

Multiply the square of the diameter by ·7854 *and the* result will be the area.

EXAMPLES.

Ex. 1. The diameter of a safety-valve is 5 inches: required the area.

Since area = (diameter^2) × ·7854 and $5^2 = 25$ ∴ ·7854 × 25 = 19·6350 square inches.

$5^2 = 25$
·7854
25

39270
15708

19·6350 square inches.

Ex. 2. Find the area of a circle whose diameter is 15 inches.

Since area × ·7854 × (diameter^2) and $15^2 = 225$ ∴ area = ·7854 × 225 = 176·715 square inches.

Ex. 3. The diameter of a circle is 74½ inches: required the area.

Since area = ·7854 × (diameter^2) and $74{\cdot}5^2 = 5550{\cdot}25$, ∴ area = ·7854 × 5550·25 = 4359·166350 square inches.

Ex. 4. The diameter of a piston-rod at the bottom of the thread is 5 inches, and the two bolts in the cross head journal 3 inches: what is the proportion of area of rod to that of the bolts?

Area of section of rod $= 5^2 \times {\cdot}7854 = 25 \times {\cdot}7854 = 19{\cdot}6350$ square inches.

" " bolts $= 3^2 \times {\cdot}7854 = 9 \times {\cdot}7854 = 14{\cdot}1372$ square inches.

Then 19·6350 : 14·1372 :: 1 : ·72. *Answer* ·72.

Ex. 5. The diameter of a piston-rod is 6 inches, and that of the piston 33 inches; compare the area of a section of the rod with the area of the piston. Since the areas of circles are as the square of their diameters, there areas will be as $46^2 : 323^2$;

or, as 16 : 1089;

or, as 1 : 68·06 nearly, *Answer*.

EXAMPLES FOR PRACTICE.

1. The diameter of a piston is 64 inches: required the area.
2. The diameter of a valve is 10 inches: find the number of square inches.
3. The diameter of a piston is 53⅜ inches: required the area.
4. The diameter of the barrel of a pump is 6⅞ inches: find the area of a horizontal section.
5. What is the sectional area of a shaft whose diameter is 9¾ inches?
6. How many square inches are there in both pistons of double-cylinder engines of H.M.S. "Terrible," the diameter being 72 inches.
7. The diameter of a cylinder is 104¼ inches: required the area.
8. What is the proportionate error of the following rough rule for finding the area of a circle, "Take 7/9 on the square of the diameter and add 1 per cent."

NOTE.—The area may also be found by multiplying the circumference by the diameter, and dividing by 4.

226. To find the diameter of a circle having given the area.

Since the area = the square of the diameter × ·7854 nearly ∴ diameter $= \sqrt{\dfrac{\text{area}}{{\cdot}7854}}$ hence the following Rule.

RULE LXXVII.

Divide the area by ·7854, *and take the square root of the quotient, which will give* the diameter.

EXAMPLES.

Ex. 1. The area of the piston of the steamship "Apostol" is 706·8600 square inches: find the diameter.

```
·7854)706·8600(900
      70686
      -----
         00
          —

      . .
      900(30 inches.  Ans.
      9
      --
      00
      --
```

Ex. 2. What is the diameter of a piston containing 4809 square inches?

Diameter $= \sqrt{\frac{\text{area}}{.7854}}$

$= \sqrt{\frac{4809}{.7854}} = \sqrt{6122{\cdot}99}$

$= 78{\cdot}2$ inches, nearly. *Ans.*

Ex. 4. The area of a piston is 4071·5136 square inches: find the diameter of the piston-rod, which is one-eighth of that of the piston.

Sq. inches.
$\sqrt{\frac{4071{\cdot}5136}{.7854}} = \sqrt{5184} = 72$ inches diameter.

```
·7854)4071·5136(5184               . .
      39270             √7    | 5184(72 in.
      -----              7    | 49
       14451            ---   | ---
        7854            142   |  284
       -----                  |  284
        65973
        62832
        -----
         31416
         31416
         -----
```

Then $\frac{1}{8}$th of 72 inches = 9 inches, the diameter of piston rod.

Ex. 3. The area of the cylinder of H.M.S. "Black Prince" is 8535·77628750 square inches: required the diameter.

```
·7854)8535·77628750(10868·0625
      7854
      -----
       68177
       62832
       -----
        53456
        47124
        -----
         63322
         62832
         ------
          49087
          47124
          -----
           19635
           15708
           -----
            39270
            39270
            -----

    1   | 10868·0625(104·25 in.  Ans.
    1   | 1
   -----------
   204  | 0868
     4  |  816
   -------------
   2082 |  5206
      2 |  4164
   ---------------
  20845 | 104225
          104225
```

EXAMPLES FOR PRACTICE.

1. The area of the cylinder of H.M.S. "Spiteful" is 3019·0776 inches: required the diameter.
2. The area of the cylinder of H.M.S. "Lion" is 2642·0856 square inches: required the diameter.
3. The area of a cylinder is 3605·03508750 square inches: find the diameter.
4. The area of a cylinder is 7013·81635 square inches: required the diameter.
5. The area of a piston is 4536 square inches: find the diameter of a piston-rod which is $\frac{1}{8}$ that of a piston.
6. The area of a piston is 1590·4350 square inches: find the diameter of the piston-rod which is $\frac{1}{10}$ that of the piston.

227. **Def.—A circular inch is a** *circle* whose diameter is an inch; consequently its area is less than a square inch, being the circle inscribed within the square inch. Since the area of a circle is found by multiplying the

square of its diameter by ·7854, therefore the area of a circular inch = ·7854 × 1^2 = ·7854, and hence

The number of square inches in any surface being given, to find the number of circular inches.

RULE LXXVIII.

***Divide the number of square inches by* ·7854, *and the result will be* the number of circular inches.**

EXAMPLES.

Ex. 1. If a valve contain 55 square inches, express its area in circular inches.

$\frac{55}{.7854}$ = 70 circular inches, nearly. *Ans.*

```
·7854)55·0000(70 circular inches.
      54978
      -----
        220
```

Ex. 2. A rivet hole is half an inch in diameter: express its area in circular inches.

$$\cdot5^2 \times \cdot7854 = \text{square inches.}$$

$$\therefore \frac{\cdot5^2 \times \cdot7854}{\cdot7854} = \text{circular inches.}$$

$$= \cdot5^2 = \cdot25 \text{ circular inches.}$$

Ex. 3. A valve contains 100 square inches: how many circular inches does it contain?

$\frac{100}{.7854}$ = 127·3.

```
·7854)100·0000(127·3
      7854
      -----
      21460
      15708
      -----
       57520
       54978
       -----
       25420
       23562
       -----
```

EXAMPLES FOR PRACTICE.

1. In 201 square inches, how many circular inches?
2. If the pressure on a valve be 15 lbs. on each square inch, what is the pressure on the circular inch?
3. If the pressure on a valve be 10 lbs. on each square inch, what is the pressure on each circular inch?
4. A valve is 6 inches in diameter: show that the pressure on it is 141·372 lbs. whether it be considered to be loaded to 5 lbs. per square inch, or 3·927 lbs. per circular inch.*

228. **To find the area of the circular ring, or space included between the circumferences of two concentric circles.**

RULE LXXIX.

1°. ***Multiply the square of the greater diameter by* ·7854 *for the area.***

2°. ***Multiply the square of the inner diameter by* ·7854 *for the area.***

3°. ***From the area answering to greater diameter, subtract the area of inner circle,* the result is the area required.**

Or, *multiply the sum of the diameters by their difference, and this product multiplied by* ·7854 *will give the* required area.

---

* But that we have become habituated to the square inch in practice, the circular inch would be less troublesome than the square inch for many calculations. The constant reduction of circular areas into square areas involves an amount of labour which is inconvenient without tables of reference; and whenever comparisions of circles are concerned, as for example, in proportioning the cylinders of compound engines, circular inches are better than square, and they give ratios which are accurate without the use of decimals.

EXAMPLE.

Ex. What is the area of an annular valve whose greater and less diameters are 15 and 5 inches respectively?

$$\begin{array}{lcl} \text{Area of whole circle} & = \cdot7854 \times 15^2 = & 176\cdot7150 \text{ sq. inch.} \\ \text{Area of less circle} & = \cdot7854 \times 5^2 = & \underline{19\cdot6350} \text{ sq. inch.} \\ \text{Area of annular valve} & = & 157\cdot0800 \text{ sq. inch.} \end{array}$$

$$\begin{array}{r} \cdot7854 \\ 15^2 = \quad 225 \\ \hline 39270 \\ 15708 \\ 15708 \\ \hline 176\cdot7150 \text{ square inches.} \\ 19\cdot6350 \text{ square inches.} \\ \hline 157\cdot0800 \text{ square inches} = \text{area of annular valve.} \end{array} \qquad \begin{array}{r} \cdot7854 \\ 5^2 = \quad 25 \\ \hline 39270 \\ 15708 \\ \hline 19\cdot6350 \text{ sq. inches.} \end{array}$$

Otherwise,

$$15^2 - 5^2 = 225 - 25 = 200.$$

$$\begin{array}{r} \cdot7854 \times 200 = 157\cdot0800 \\ 200 \\ \hline 157\cdot0800 \text{ square inches.} \end{array}$$

229. To find the area of an Ellipse or Oval.

An Ellipse or Oval is like a circle flattened in one direction and elongated in the other.

RULE LXXX.

Multiply the greater *and* lesser *diameters together, and that product by* ·7854, and the result will be the area.

NOTE.—The square of the mean diameter is equal to the product of the two diameters.

EXAMPLE.

Ex. The greater diameter of an oval or elliptical funnel is 4 ft. 10 in., and the lesser diameter 4 ft. 4 in., what space will it occupy on the deck?

$$\text{Area} = \cdot7854 \times 58 \times 52 = 2368\cdot7664 \text{ square inches} = \frac{2368\cdot7664}{144} = 16\cdot449 \text{ sq. feet.}$$

$$\begin{array}{r} 4 \text{ ft. } 10 \text{ in.} = \quad 58 \text{ in.} \\ 4 \text{ ft. } 4 \text{ in.} = \quad \underline{52} \text{ in.} \\ 116\phantom{ \text{ in.}} \\ 290\phantom{0 \text{ in.}} \\ \hline 3016 \text{ in.} \end{array} \qquad \begin{array}{rl} & \cdot7854 \\ & 3016 \\ \hline & 47124 \\ & 7854 \\ & 235620 \\ \hline 144 \left\{ \begin{array}{r} 12)2368\cdot7664 \\ \hline 12)197\cdot3972 \end{array} \right. & \text{sq. inches.} \\ \hline \textit{Answer,} \quad 16\cdot449 & \text{sq. feet.} \end{array}$$

EXAMPLES FOR PRACTICE.

1. The longer diameter of an ellipse is 70, and the shorter 50, what is the area?
2. The greater and lesser diameters of an oval man-hole door are 2 ft. 9 in. and 2 ft. 6 in.: find the area.
3. What is the area of an oval door whose diameters are 12 inches and 9 inches?
4. The greater diameter of an oval funnel is 4 ft. 6 in., and the lesser diameter 4 ft.: what space will it occupy on the deck?

230. To find the circumference of an Ellipse.

RULE LXXXI.

Multiply half the sum of the two diameters by 3·1416, *and the result will give the circumference* sufficiently accurate for practice.

Examples.

Ex. 1. What is the circumference of an ellipse whose diameters are 58 and 52 inches respectively?

$\frac{1}{2}$ sum diameters $= \frac{58 + 52}{2} = 55.$

Circumference $= 3{\cdot}1416 \times 55 = 172{\cdot}7880.$

Ex. 2. What is the circumference of an ellipse whose diameters are 24 and 18 inches respectively?

$\frac{1}{2}$ sum diameters $= \frac{24 + 18}{2} = 21.$

Circumference $= 3{\cdot}1416 \times 21 = 65{\cdot}9736.$

Examples for Practice.

1. What is the circumference of an ellipse whose diameters are 30 and 20 respectively?
2. If the diameters of an ellipse were respectively 60 and 40, what is the circumference?
3. The diameters of an ellipse are 6 and 4 respectively, what is the circumference?
4. Given diameters of an ellipse 3 and 2, find the circumference.

To find the area of a parabola.

RULE LXXXII.

Multiply the base by the height, and take two-thirds of the product for the area.

MENSURATION OF SOLIDS.

It has been already said that a line has only one dimension, viz., length, and that a superficies has two dimensions, viz., length and breadth; hence, in calculating area, we always proceed by multiplying two dimensions together.

We now pass on to speak of solid figures which have three dimensions, viz., length, breadth, and thickness, and it will be noticed that in every case the *volume* or *solidity* is calculated by multiplying *three dimensions* together.

Every superficies is bounded by lines; every solid is bounded by superficies.

Def.—A cube or solid square is a solid figure contained by six equal squares.

Def.—A parallelopiped, or solid rectangle is a solid figure having six rectangular sides, every opposite two of which are equal and parallel.

The above definition really includes only what may be called *rectangular* parallelopipeds; the regular solid constructed, or a rhombus or rhomboid, is also a parallelopiped, and may be treated by the same rules as the rectangle, only remembering that the thickness of the solid must then be measured by the *perpendicular* distance of the sides.

A definition of the term as used in this wider sense is:—

A Parallelopiped is a solid figure contained by six quadrilateral figures, whereof every opposite two are parallel.

231. To find the volume or solidity of a cube, or parallelopiped.

RULE LXXXIII.

Multiply the length by the breadth and that product by the depth, and the result will be the solidity.

Let l = length of a side of the cube, S = its solidity, and s = its surface; then $S = l^3$, and $l = \sqrt[3]{s}$; also, $s = 6\,l^2$.

EXAMPLES.

Ex. 1. Find the solid content of a cube whose side is 24 inches.

The solid content $= 24 \times 24 \times 24 = 13824$ cubic inches, *Answer*.

Ex. 2. A cubical box contains 343 cubic feet: required the length of its side.

Length of side $= \sqrt[3]{343} = 7$ feet.

Ex. 3. Find the content of a block of stone 17 ft. 9 in. long., 14 ft. 3 in. broad, and 5 ft. 6 in. thick.

The cubic content = (17 ft. 9 in.) × (14 ft. 3 in.) × (5 ft. 6 in.)
= 213 in. × 171 in. × 66 in.
= 2403918 cub. in. = 1391 cub. ft. 270 cub. in. *Answer*.

Since the CUBIC CONTENT = LENGTH × BREADTH × HEIGHT, it follows that if any three of these be given the fourth may be found thus—

1. The *Cubic Content* may be found by multiplying together the *Length*, *Breadth*, and *Height*.

2. The *Length* may be found by dividing the *Cubic Content* by the product of the *Breadth* and *Height*.

3. The *Breadth* may be found by dividing the *Cubic Content* by the product of the *Length* and *Height*.

4. The *Height* may be found by dividing the *Cubic Content* by the product of the *Length* and *Breadth*.

EXAMPLES FOR PRACTICE.

1. A piece of timber is 25 ft. long., 3 ft. wide, and 5 ft. thick, what is its solid content?
2. What is the solidity of a block of stone 5 ft. 6 in. long, 3 ft. 2 in. wide, and 1 ft. 9 in. thick?
3. How many cubic feet of air are there in a room 12 ft. high, 18 ft. long, and 15 ft. 9 in. wide?
4. What is the length of the side of a cubic stone containing 13824 solid feet?
5. What must be the depth of a cubical box that shall contain 125000 cubic feet?
6. Find the content of a coal bunker 29 ft. 6 in. long, 9 ft. 3 in. broad, and 9 ft. 3 in. high?

232. To find the volume of a cylinder, having given the diameter and length.

RULE LXXXIV.

1°. *Multiply the square of the diameter by* ·7854, the product is the area of the section.

2°. *Then multiply this area by the length.*

EXAMPLES.

Ex. 1. Required the volume of a cylinder whose diameter of base is 30 inches, and the length 25 inches.

Area of base $= 30^2 \times \cdot 7854 = 706 \cdot 86$; $\therefore$ volume $= 706 \cdot 86 \times 25 = 17671 \cdot 5$ cubic in.

Ex. 2. Find the volume and weight of a brass cylinder, the diameter of which is 12 inches, and height 9 inches; a cubic foot of brass weighing 525 lbs.

$$\text{Volume} = \cdot 7854 \times 12^2 \times 9 = 1017 \cdot 8784 \text{ cubic inches.}$$

$$\text{Hence weight} = \frac{\overset{\text{cubic inches.}}{1017 \cdot 8784} \times \overset{\text{lbs.}}{525}}{1728} = 309 \cdot 25125 \text{ lbs.} = 309 \text{ lbs. } 4 \text{ oz.}$$

233. **To find the volume of a cone.**

RULE LXXXV.

Proceed exactly as in the foregoing Rule, and divide the result by 3.

EXAMPLE.

Required the volume of a cone, the diameter of the base being 10 inches, and height 15 inches.

$$\text{Volume} = \text{area of base} \times \tfrac{1}{3} \text{ of height} = \frac{\cdot 7854 \times 10^2 \times 15}{3} = 392 \cdot 70 \text{ cubic inches.}$$

234. **Def.—The frustum of a cone is the part that is left after the top or end is cut off by a plane parallel to the base.**

To find the volume of the frustum of a cone.

RULE LXXXVI.

Add the squares of the diameters of the two ends to their product; this sum multiplied by one-third the perpendicular height, and again by ·7854 will give the solidity of the frustum.

235. **To find the volume of a sphere.**

Solidity of sphere whose diameter is D.

$$S = \tfrac{1}{6} D^3 \pi \text{ and D, and } D = \sqrt[3]{\frac{6\,S}{\pi}} \text{ hence Rule.}$$

RULE LXXXVII.

Cube the diameter and multiply by ·5236 $\left(= \frac{3 \cdot 1416}{6}\right)$; the product will be the solidity.

EXAMPLES.

Ex. 1. What is the solidity of a sphere whose diameter is 24 inches?

$$\begin{aligned} \text{Solidity} &= \text{diam.}^3 \times \cdot 5236 \\ &= 24^3 \times \cdot 5236 = 13824 \times \cdot 5236 \\ &= 7238 \cdot 2464 \text{ cubic inches.} \end{aligned}$$

Ex. 2. What is the cubical content of a sphere whose diameter is 7 inches?

$$\begin{aligned} \text{Cubical content} &= \text{diam.}^3 \times \cdot 5236 \\ &= 7^3 \times \cdot 5236 = 343 \times \cdot 5236 \\ &= 179 \cdot 5948 \text{ cubic inches.} \end{aligned}$$

EXAMPLES FOR PRACTICE.

1. The radius of a sphere is 4¼ inches: find its solidity.
2. If the diameter of a balloon is 6 ft. 2 in.: how many cubic feet of gas will it contain?
3. A spherical body contains 2·80957 cubic feet: what is its surface?
4. The diameter of the sun is 112 times greater than that of the earth: how many times greater is its volume?
5. A sphere contains 5946 cubic inches: find its radius.
6. If the cubical content of the earth is taken as 236855164967 cubic feet: what is its circumference.

To find the volume of the segment of a sphere.

RULE LXXXVIII.

To three times the square of the radius of its base add the square of its height; then multiply the sum by the height, and the product by ·5236.

To find the volume of a cylindrical ring.

RULE LXXXIX.

Add the inner diameter to the thickness of the ring, multiply the sum by the square of the thickness, and the product by 2·4674 ($= \frac{3·1416^2}{4}$), and it will give the solidity.

EXAMPLE.

Ex. The thickness of a cylindrical ring is 2 inches, and the inner diameter 5 inches: required the solidity?

$(2 + 5) \times 2^2 = 7 \times 4 = 28$; then $28 \times 2·4674 = 69·0872$ cubic inches, *Ans.*

EXAMPLES FOR PRACTICE.

1. Required the solidity of an iron ring whose axis forms the circumference of a circle; the diameter of a section of the ring 2 inches, and the inner diameter, from side to side, 18 inches.
2. The thickness of a cylindrical ring is 7 inches, and the inner diameter 20 inches: required its solidity.
3. What is the solidity of a circular ring whose thickness is 2 inches, and its diameter 12 inches?

SUPERFICIAL AREAS OF SOLID BODIES.

236. To find the superficies of a sphere.

RULE XC.

Multiply the diameter by the circumference, or multiply 3·1416 *by the square of the diameter*, and the product will be the surface.

Or, *multiply the square of the diameter by* 3·1416.

EXAMPLE.

Ex. What is the surface of a globe whose diameter is 24 inches?

Circumference $= 24 \times 3·1416 = 75·3984$ inches.

Surface $= 75·3984 \times 24 = 1809·5616$ square inches. *Ans.*

Or, $24^2 \times 3·1416 = 576 \times 3·1416 = 1809·5616$ inches. *Ans.*

EXAMPLES FOR PRACTICE.

1. What is the surface of the earth, its diameter being 7957¾ miles, and circumference 25000 miles? also find it from each of these data taken separately.
2. The surface of a sphere contains 6734 square feet: find its radius.
3. The radius of a sphere is 1 ft. 4½ in.: what is its surface?
4. Find the surface of a sphere whose diameter is 7.

237. To find the convex surface of a cylinder, having given the diameter and length.

RULE XCI.

1°. *Multiply the circumference by the length of the cylinder, the product is the convex surface.*

NOTE.—If the surface of the entire solid be required, the area of the two ends must be found and added to the convex surface, the sum is the entire surface required.

EXAMPLES.

Ex. 1. Find the rounded surface of a cylindrical vessel whose diameter is 20 inches, and height 14 inches.

1. Circumference of base $= 3{\cdot}1416 \times 20 = 62{\cdot}832$,

2. Surface required $=$ circumference $\times$ height $= 62{\cdot}832 \times 14 = 879{\cdot}648$ sq. inches.

$$\begin{array}{r} 3{\cdot}1416 \\ 20 \\ \hline 62{\cdot}8320 \\ 14 \\ \hline 251328 \\ 62832 \\ \hline 879{\cdot}648 \text{ sq. inches.} \end{array}$$

Ex. 2. How many square feet of felt will be required to coat a cylinder which is 66 inches in diameter, and height 5 ft. 3 in.

Circum. of base $= 3{\cdot}1416 \times 66 = 207{\cdot}3456$

Surf. required $= 207{\cdot}3456 \times 63$

$= 13082{\cdot}7728$ sq. inches,

$90{\cdot}571$ sq. feet.

Ex. 3. What is the entire surface of a cylinder whose length is 10 feet and its diameter 5 feet?

Circumference $= 3{\cdot}1416 \times 5 = 15{\cdot}7080$

Convex surface $= 15{\cdot}7080 \times 10 = 157{\cdot}08$

$5 \times 5 \times {\cdot}7854 =$ area of base; then $\cdot 2 \times 5 \times 5 \times {\cdot}7854 = 50 \times {\cdot}7854 = 39{\cdot}277$ area of both bases,

whence $157{\cdot}08 + 39{\cdot}277 = 190{\cdot}357$, *Ans.*

EXAMPLES FOR PRACTICE.

1. What is the number of square inches in the outside surface of a cylinder whose outer diameter is 15 inches, and height 20 inches?
2. What is the convex surface of a cylinder whose circumference is 76 inches and length 5 ft. 4 in.?
3. What is the surface of a cylinder, including the ends, its girth being 12 ft. and length 15 ft.?
4. Required the interior surface of a cylinder the length being 12 ft. and the radius of the bore 23 inches.
5. Required the superficial content of a cylinder whose diameter is 21·5 in. and height 16 ft.?

238. To find the convex surface of a right cone.

RULE XCII.

Multiply the circumference of the base by the slant height or side of the cone, and take half the product for the area.

239. To find the convex surface of a cylindrical ring.

RULE XCIII.

Multiply the extreme diameter by the thickness of the ring, and the product by $9{\cdot}8696$ $(= 3{\cdot}1416^2)$ the result is the area.

SIMPSON'S RULE FOR AREAS.

When more than ordinary accuracy is required in finding the area of a space lying between a curve and a straight line.

Divide the base of the figure (suppose it to be the following) into any number of equal parts, say eight (the more we take the greater accuracy we attain), by drawing lines at right-angles to the base; this will give an odd number of points of division. The measurements of the length of these perpendiculars, or *ordinates*, must be taken from a scale of equal parts, and used as in the following, which is called Simpson's Rule:—

Let S = the sum of the lengths of the first and last ordinates.
E = the sum of the lengths of all the other *even* ordinates.
O = the sum of the remaining or *odd* ordinates.
d = the common distance between the ordinates.

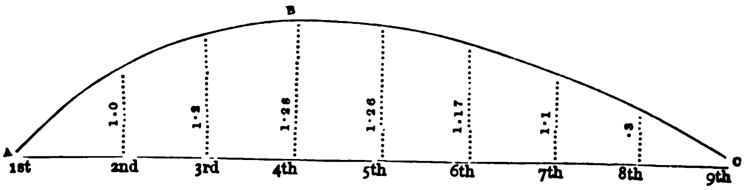
$$\text{Then the area} = \frac{(S + 4E + 2O)}{3} \times d.$$

Whence the following

RULE XCIV.

To the sum of the lengths of the first and last ordinates add four times the sum of the lengths of all the other even ordinates, and twice the sum of the remaining, or odd ordinates; then this sum multiplied by one-third the common distance between the ordinates is equal to the area.

For example: given the following figure, ABC: required the area.



$$S = 0$$
$$4E = 4(1·0 + 1·28 + 1·17 + ·3) = 4(3·75) = 15.$$
$$2O = 2(1·2 + 1·26 + 1·1) = 2(3·56) = 7·12.$$
$$d = 1.$$
$$\text{Then } \frac{S(+4E+2O)\times d}{3} = \frac{0 + 15 + 7·12}{3} = \frac{22·12}{3} = 7·37.$$

Or thus,

	Evens.		Odds.		
2nd	1·0	3rd	1·2	first	0
4th	1·28	5th	1·26	last	0
6th	1·17	7th	1·1	evens	15·00
8th	·3		3·56	odds	7·12
	3·75		2		22·12
	4		7·12	com. dist.	1
	15·00				3)22·12
				Area	7·37

This rule is much used by shipbuilders in calculating displacements, water lines, &c., and is on the general assumption that the curve is parabolic; it may also be applied to the "working off" the indicator diagram.

EXAMPLES FOR PRACTICE.

1. The length of a rectangle is three times its breadth, and its area is 5808 yards: what is the length in feet?

2. The largest possible circle is cut out of a piece of paper in the form of a square whose side is 7 inches: what is the area of the remnants?

3. A circle of 2 inches diameter is cut out from one whose diameter is 3 inches: what is the area of the remainder?

4. Show that a circle of 5 inches diameter is as large as two others of 4 inches and 3 inches diameter together.

5. The area of the coal field of South Wales is 100 square miles, and the average thickness of the coal is 60 feet. If a cubic yard of coal weighs 1 ton, and the annual consumption of coal in Great Britain be 70,000,000 tons: find the number of years for which this coal field alone would supply Great Britain with coal at the present rate of consumption.

If the coal annually consumed in this country were piled up into a pyramid, having for its base the great court of Trinity College, the dimensions of which are 110 by 90 yards: find the height of the pyramid.

N.B.—The volume of pyramid is equal to the area of the base multiplied into one-third of the height.

6. The sides of a rectangle contain 12·19 feet and 37·5 feet: find its area.

If the length of the sides were only known to be correct within ·06 of an inch, how much might the true area exceed the approximate one?

7. What length must be cut off a board $6\frac{3}{4}$ inches wide, that the area may be a square foot?

8. What fraction of a square foot is a rectangle whose sides are $\frac{3}{8}$ of a foot and $\frac{15}{16}$ of a foot?

9. A ship's hold is a 100 feet long, 50 feet broad, and 4 feet deep: how many bales of goods, each 2 feet 4 inches long, 2 feet 1 inch broad, and 2 feet 1 inch deep, can be stowed into it, leaving a gangway of 3 feet broad, and of the same length and depth as the hold?

10. What is the area contained between two concentric circles whose radii are 2 feet 2 inches and 4 feet?

11. Find the area of an annular valve of which the greater and less diameters are 9 and 5 inches respectively.

12. The diameters of the circles are 10 and 6: required the area of the circular ring.

13. The area of an oval varies as its length × breadth. When the length and breadth are each 1 inch, the area is $\frac{11}{14}$ square inch: what is the area of an oval whose length is 7 inches and breadth 5 inches?

14. The volume of a circular cylinder varies as the height × square of diameter. When the diameter and height are each 1 inch, the volume is $\frac{11}{14}$ cubic inch: what is the volume of a cylinder whose diameter is 4 inches and height 21 inches?

15. Find the surface and volume of a sphere whose radius is 6 feet 9 inches.

16. Of two cylinders, one contains 154 lbs. of water, and its dimensions are half those of the other: how many cubic feet of water will the latter cylinder hold? A cubic foot of water weighing 1000 ounces nearly.

17. If the diameter of the earth is 8000 miles, and the interior to the depth of 5 miles is known: how much of the earth's interior would still be unknown?

18. Admitting the height of the atmosphere to be 45 miles, what would be its solid contents?

19. If 405 million square inches of gilding can be made of a cubic foot of gold: find its thickness.

20. A valve is 6 inches in diameter: show that the pressure on it is 141·372 lbs., whether it be considered to be loaded to 5 lbs. per square inch or 3·927 per circular inch.

21. A tank which is circular, and 16 feet in depth, contains 20,000 gallons: what is its diameter?
22. How many 3 inch cubes can be cut out of a 12 inch cube?
23. If the diameter of the earth be 7930 miles, and that of the moon 2160 miles: required the ratio of the surfaces, and also of their volumes, supposing them both to be spherical.
24. The circumference of a ring is 161 inches, and its width 1 inch: required its internal diameter and area.
25. Required the surface and solidity of a sphere 17 inches in diameter.
26. If a cubic foot of gold may be made in gilding to cover 402600000 square miles: find the thickness of the coating of gold.
27. A cube contains 568 solid feet 568 solid inches: find its edge.
28. The edge of a rectangular chest, which contains 64 cubic feet, are in the proportion of 1, 2, 4: find the actual length of its edges.
29. What will the diameter of a globe be, when the solidity and superficial content are expressed by the same number?
30. Divide a cone into three equal parts by sections parallel to the base, and find the height of the three parts, that of the whole cone being 20 inches.
31. It is required to find the thickness of the lead in a pipe of an inch-and-a-quarter bore, which weighs 14 lbs. per yard in length, the cubic foot of lead weighing 11325 ozs.
32. A cable which is 3 feet long and 9 inches in compass weighs 22 lbs.: required the weight of a fathom of a cable which measures a foot round.
33. Compute the total surface and the solidity of a cylinder 4 inches in diameter and 5 inches long.
34. From a solid cylinder 6 inches long and 3 inches in diameter a hole is bored out 1·8 in. diameter: required the solidity of the tube thus made.
35. Required the bulk of the metal in a tube 12 inches long, 2·7 inside diameter, the metal being ·05 thick.
36. How many cubic inches of metal are needed to make a hollow spherical shell 13 in. in outer diameter and ·47 of an inch thick?
37. Compute the radius of a circle which shall contain exactly 1 square foot of surface.
38. What is the smallest round out of which a triangle, with the sides 89, 82, 57, may be cut, and what is the waste?
39. A three-cornered scrap of wood has its sides 8·7, 6·5, 4·4: what is the largest round that can be made out of it, and how much waste is there?
40. Describe a circle which shall have as much surface as an equilateral triangle on the base 52.
41. In order to get the diameter of a cylinder, a strip of paper was wrapped round it, and the overlapped portion cut through both folds; the length of the strip was then measured and found to be 23·69 inches: what is the diameter?
42. What are in the outer and inner circumferences of a ring included between two concentric circles whose radii are 13 and 17 inches?
43. The radius of a circle being 1 foot, how many square inches are in its area, and what is the side of a square containing the same area?
44. Assuming the earth a sphere whose radius is 4000 miles, what decimal part of the surface is occupied by England and Wales, the area of which is 37324915 acres?

SPECIFIC GRAVITY.

240. This is, in fact, a ratio by means of which the relative heaviness of different substances is considered without reference to actual weight or bulk. This ratio is obtained by comparing the weights of similar bulks of two substances. If, for instance, a cubic inch of one substance weighs 10 ozs., and a cubic inch of another substance weighs 20 ozs., the latter, being the same in bulk, is said to have twice the specific gravity of the former.

For convenience, it is usual, instead of comparing each substance in common use with each other, to compare all of them separately with one or two standard substances. We generally take distilled water at the temperature of 60° as the standard of weight for all solids and liquids, and dry atmospheric air with the barometer at 30 inches as the standard of weight for gases. Water, however, seldom differs so much in specific gravity as to cause any appreciable error in practice. **The specific gravity, then, of a solid or liquid is the weight of any unit of bulk of that substance divided by the weight of the same unit of bulk of water.** For instance, the weight of a cubic foot of a particular specimen of iron is found to be 467 lbs. 6 ozs. = 7478 ozs.

Now, the weight of a cubic foot of water is 997 ozs. The specific gravity of iron is therefore said to be 7·5. The practical meaning of this is, that any given bulk of this iron weighs 7½ times as much as an equal bulk of water. The specific gravity is not the most convenient form in which to record the weight of materials for the engineer. The weight per cubic foot is better for his purpose. But the specific gravity has the advantage, being independent of the unit of bulk, being the same for all countries and all languages, however the standards may vary.

241. **Specific Gravity of Liquids.**—The specific gravity of liquids is generally obtained by means of a bottle which is ground, so as to contain exactly 1000 grains (or other units of weight) of water; being carefully dried, it is then filled with the given liquid and weighed. The known weight of the bottle being subtracted gives the nett weight of the liquid in grains, and we have only to mark off three figures of decimals to get the specific gravity of the liquid.

242. **Specific Gravity of a Solid heavier than Water.**—The specific gravity of a solid heavier than water and not soluble in it is found by weighing it, first in air and then in water. The difference between the two weights is evidently the weight of the water which has been displaced by the solid, that is to say, of an equal bulk of water. Hence *the specific gravity is got by dividing the weight in air by this difference, or the loss in water*. For instance, suppose a substance is first weighed in the ordinary method, and its weight is found to be 250 grains. The substance is then suspended by means of a horse-hair or fine thread under one scale, so that it dips into

water, the weight in water being also found to be 150 grains. Then according to the preceding rule we have

Weight of body in air	250 grains
" " water	150 "
	100)250(2·5 = specific gravity.

The rule is obtained thus:—

Let a = weight of substance in air
w = " " water
s = specific gravity of the water
G = " " substance
Then $(a - w) : a :: s : G$;

$$\text{or } G = \frac{a \times s}{a - w} = \text{from the above } \frac{250 \times 1}{250 - 150} = \frac{250}{100} = \frac{\text{weight in air}}{\text{loss in water}} = 2{\cdot}5.$$

243. **Specific Gravity of a Solid lighter than Water.**—Attach to the lighter body another that is much heavier than the fluid, so that the compound mass may sink in the fluid. Weigh the heavier body and the compound mass separately, both in water and out of it; then find how much each loses in water, by subtracting its weight in water from its weight in air; and subtract the less of these remainders from the greater.

Then, as this last remainder $= a - b$,
: the weight of the light body in air $= c$
:: the specific gravity of the fluid $= s$
: the specific gravity of that body $= g$

EXAMPLE.

Ex. Required the specific gravity of a piece of beeswax (using a piece of gold as a *sinker*).

	Grains.	
Weight of beeswax in air	42·0	$= c$
" sinker in air	77·2	
" thread (to tie them) in air	·65	
Weight of compound in air	119·85	
" " water	68·70	
Loss of compound in water	51·15	$= a$
Weight of gold in air	75·5	
" " in water	70·2	
Loss of sinker in water	5·3	$= b$
Difference of loss	45·85	

Then $(a - b) : c :: s : g$

$$\text{Or, specific gravity} = \frac{c \times s}{a - b} = \frac{42{\cdot}0 \times 1}{51{\cdot}15 - 5{\cdot}3} = \frac{42{\cdot}0}{45{\cdot}85} = {\cdot}916.$$

Again, "Suppose a pound of poplar wood to be sunk by the help of two pounds of iron, specific gravity 7·2, and that the total weight in water is ·0556 lbs., we know that the bulk of the iron and wood together will be the same as that of (3 — ·0556 =) 2·9444 lbs. of water. But the bulk of the iron is that of $\frac{2}{7{\cdot}2}$ lbs. = ·2778 lbs. of water. The bulk of the pound of wood is consequently that of 2·9444 — ·2778 lbs. = 2·6666 lbs. of water. Its specific gravity is therefore $\frac{1}{2{\cdot}666} = {\cdot}375$.

To obtain accuracy many precautions are necessary, such as varnishing the wood, and so on. In some cases it is more convenient to ascertain the bulk by actual measurement and then weighing."

For the sake of simplicity the weight of the air is neglected. The consequence is that the specific gravity thus obtained is incorrect, since the weight of the water and of the substance are each diminished by that of an equal bulk of air. "Thus, if a cubic foot of limestone weighs 172 lbs. in air, and a cubic foot of water 62·365 lbs. in air, if we neglect the weight of the air we get—

$$\text{Specific gravity} = \frac{172}{62{\cdot}365} = 2{\cdot}757957.$$

But a cubic foot of air weighs ·08 lb. at 40° Fahrenheit and atmospheric pressure. The true specific gravity is therefore

$$\text{Specific gravity} = \frac{172 + {\cdot}08}{62{\cdot}365 + {\cdot}08} = \frac{172{\cdot}08}{62{\cdot}445} = 2{\cdot}734424.$$

The difference of these is ·003533, or about $\frac{1}{1000}$. This shows very well what is the proportionate error made by neglecting the weight of the air. For substances very much lighter than water the error is greatest. For those about the same weight as water this error vanishes, and for very heavy substances the error is also small."

The operation of finding the specific gravity of the materials with which he may have to operate should be practised by every engineer; it is exceedingly simple, and requires only the use of an ordinary good balance with small weights. At the end of the volume a table is given of the specific gravity of substances in common use.

A Table of specific gravities shows the relative weights of the various substances. In order, thence, to determine the actual weight per cubic inch or cubic foot of any substance, we must multiply the weight of a cubic inch or cubic foot of water by the specific gravity: thus, platinum having the specific gravity 22·07, is rather more than twenty-two times as heavy as water, and a cubic foot of it would weigh (62·5 × 22·07) 1379 pounds.

When the dimensions of a solid body are given we are now able to compute its weight, by multiplying the number of cubic units by the weight of a cubic unit of water, and again by the specific gravity of the substance.

EXAMPLES.

Ex. 1. What is the weight of a cubic foot of sea water?

Weight of cub. foot of sea water = weight of cub. foot of pure water × specific gravity of sea water.

Weight of cub. ft. of pure water = 62·5 lbs.
Specific gravity of sea water = 1·025 (Table)

3125
1250
625

Answer 64·0625 lbs.

In practice, 64 lbs. is taken as the weight of a cubic foot of sea water.

Ex. 2. What is the weight of a cubic foot of wrought iron?

62·5 lbs.
(See Table) 7·7 specific gravity.

4375
4375

481·25 lbs.

In practice the weight of a cubic foot of wrought iron is taken as 480 lbs.

Ex. 3. What is the weight of a solid cast copper cylinder 4½ inches diameter and 6 inches high?

Specific gravity of cast copper 8·788.

Specific gravity 8·788
62·5

43940
17576
52728

549·2500

or 549 lbs. per cub. foot.

Dia. 4·5
4·5

225
180

20·25

·7854
20·25

39270
15708
15708

Area of base = 15·904350
6

Volume 95·42620 cub. ins.

Cubic in. Cubic in. lbs.
1728 : 95·4 :: 549 : x
549

8586
3816
4770

1728 { 12)52374·6
12) 4364·55
12) 363·7125

30·3093 lbs. *Answer*, 30¼ lbs. (over).

EXAMPLES FOR PRACTICE.

1. Required the weight of a block of marble 6 ft. 7 in. long, 4 ft. 2 in. broad by 3 ft 10 in. thick, in pounds and tons.
2. What is the weight of water in a cistern 43 by 32 by 23 inches?
3. Required the weight of a bar of steel 1½ in. square and 3 ft. 6 in. long.
4. A piece of lead is in the form of a pyramid, on a square base whose side is 5 in., the height of the pyramid being 6 in.: required the weight.
5. What is the weight of a round rod of iron 1¾ in. in diameter and 11 ft. long?
6. Required the weight of a silver disc 1·16 in. in diameter and ·063 thick.
7. What is the weight of a gold wire ·0163 in. thick and 18·27 in. long?
8. A cylindric pitcher is 4·37 in. in diameter and 5·23 in. deep: what weight of water does it hold?

245. As we discover the weight from the known bulk, so we can determine the bulk when the weight is known. Thus, if we wish to know the volume of an irregular mass, the measurement of which geometrically might be impossible, we may weighit in air and in water (Nos. 243—244), and so ascertain the weight of its bulk of water; then allowing at the rate of 252·458 grains per cubic inch we obtain its volume.

Similarly, we are able to compute the diameter of a body which shall have a proposed weight. Thus, we may propose to construct a cylindrical measure to hold one gallon of water, that is, 10 pounds or 7000 grains. A cylindrical inch of water weighs 198·28 grains, therefore a gallon measure must contain 353·031 cylindrical inches, and if D be the diameter and H the height of the cylindric gallon measure, we must have

$$D^2 H = 353·031$$

If the diameter be given we may compute the height by formula,

$$H = \frac{353·031}{D^2}$$

or, if the height be prescribed the diameter is

$$D = \sqrt{\left\{\frac{353·031}{H}\right\}}$$

And if it were desired to make such a measure with the height equal to the diameter we should have

$$H = D = \sqrt[3]{\left\{353·031\right\}}$$

Again, the question may arise, "what must be the diameter of a cast-iron ball that shall weigh so many pounds?"

In such a case we have the formula

$$W = \tfrac{1}{6}\,\pi\, D^3 \times 252·458 \times \text{specific gravity},$$

$$\text{Whence } D = \sqrt[3]{\left\{\frac{6\,W}{\pi \times 252·458 \times \text{specific gravity}}\right\}}$$

when the weight is taken in grains and the diameter in inches.

246. The other problems relating to specific gravity are the following:—Given the quantities and specific gravities of two or more substances separately to find the specific gravity of the mixture. For this it is sufficient to multiply each quantity by its specific gravity, add the products, and divide by the total quantity. Thus, if we melt together 15 ozs. of copper, specific gravity 8·7, 5 ozs. of tin, specific gravity 7·4, and 2 ozs. of lead, specific gravity 11·4, the specific gravity of the bronze will be

$$\frac{15 \times 8·7 + 5 \times 7·4 + 2 \times 11·4}{15 + 5 + 2} = \frac{190·3}{22} = 8·65,$$

supposing that no dilatation or contraction takes place. But if there is a contraction from the original bulk, say in the ratio of 1 : 99, the specific gravity will be increased in the ratio $\frac{100}{99}$, and will therefore be 8·74.*

The other practical question is: Having given the specific gravity of a mixture in unknown proportions of two substances, the specific gravities of which are known, to find the proportions in which they are mixed. This requires the solution of an algebraic equation, of which only the result is here given. Suppose, to fix our ideas, we have a lump of gold quartz, the specific gravity of which we call W, and the specific gravities of gold and quartz we call G and Q respectively. The quantity of gold in an unit of weight of the mixture will be $\frac{W - Q}{G - Q}$, and the quantity of quartz will be $\frac{G - W}{G - Q}$. Here again it is supposed that there is no dilatation or contraction.

---

* *Technical Arithmetic and Mensuration*, by Charles W. Merrifield, F.R.S. London: Longmans & Co. This work is earnestly recommended to the attention of the reader.

ON BUNKERS, TANKS, &c.

247. Given the length, width, and height of a coal bunker, and the number of cubic feet to a ton, to find the quantity of coals it will contain.

RULE XCV.

1°. *Reduce the inches to the decimal of a foot* (see Rule LIII, § 159, pages 117—118).

2°. *Multiply the length, width, and height together* (see Rule XLVI, page 102), the product is the contents of the bunker in cubic feet.

3°. *Divide this last* (2°) *by the number of cubic feet to a ton;* the quotient is the quantity of coal the bunker will contain, *expressed* in tons and decimals of a ton.

4°. *Find the value of the decimals of a ton in the inferior denominations* (see Rule LV, § 161, page 120).

EXAMPLES.

Ex. 1. A coal bunker is 18 feet 9 inches long, 7 feet 6 inches wide, and 8 feet 3 inches high: required the quantity of coals it will contain at 38·6 cubic feet per ton (by decimals).

12)9·0 = ·75 12)6·0 = ·5 12)3·0 = ·25

18 ft. 9 in. = 18·75 ft.; 7 ft. 6 in. = 7·5 ft.; 8 ft. 3 in. = 8·25 ft.

Contents of bunker = 18·75 × 7·5 × 8·25 = 1160·15625 cubic feet.

```
   18·75
    7·5
  ------
   9375
  13125
  ------
  140·625
    8·25
  -------
   703125
  281250
 1125000
 ---------
 1160·15625 cubic feet.
```

Cubic feet in 1 ton = 38·6

.˙. tons in bunker = $\frac{1160·15625}{38·6}$

```
38·6)1160·15625(30·0559 nearly.
     1158             20
     ----          -------
        2156       1·1189 cwt.
        1930            4
        ----          ---
         2262      0·472 qrs.
         1930          28
         ----         ----
          3325        3776
          3474        944
          ----       ------
                    13·216 lbs.
```

Ans.: 30 tons 1 cwt. 0 qrs. 13 lbs.

Ex. 2. A coal bunker is 20 feet 4 inches long, 8 feet 7 inches broad, 4 feet 3 inches in depth: required the quantity of coal it will contain, 44 cubic feet being equal to a ton (to be worked by decimals).

4 in. = $\frac{4}{12}$)4·0 = ·33$\dot{3}$ 7 in. = $\frac{7}{12}$)7·0 = ·583$\dot{3}$ 3 in. = $\frac{3}{12}$)3·0 = ·25

20 ft. 4 in. = 20·333; 8 ft. 7 in. = 8·5833; 4 ft. 3 in. = 4·25,

20·333 × 8·5833 (= 174·5242389 × 4·25) = 741·728015325 cubic feet.

Cubic feet in 1 ton = 44.

.˙. tons in bunker = $\frac{741·728015325}{44}$ = 16·85745 tons = 16 tons 17 cwt. 0 qrs. 16 lbs.

Ex. 3. A cross bunker is 20 feet deep, and 18 feet long, and to find its width three measurements are made, the upper one is 29 feet, the middle one 23 feet, and the lowest one 18 feet: how many tons of coal will it hold at 45 cubic feet to the ton?

To find the mean width, add together the first and third, and 4 times the middle width, and divide by 6.

1st 29 feet
4 times 2nd = 92
3rd 18

6)139·000

23·167 mean width
20

463·340
18 length

3706720
463340

8340·120 cubic feet

$$45\begin{cases}5)8340·120 \text{ cubic feet}\\ 9)1668·024\end{cases}$$

185·336 tons
Ans. 185·336 tons.

Ex. 4. A coal bunker is 6 feet 6 inches at the top and 4 feet 6 inches at the bottom, 16 feet long and 8 feet 6 inches high: what weight of coals will it contain?

State how many cubic feet per ton you will allow.

For the mean width add width at top and at bottom together and divide by 2: thus,

6 feet 6 inches + 4 feet 6 inches = 11 feet 0 inches,
then 11 feet 0 inches ÷ 2 = 5 feet 6 inches, or 5·5 feet mean width.
Allow 45 cubic feet to a ton.
Then 16 × 5·5 × 8·5 ÷ 45 = 16·622 tons = 16 tons 12½ cwt. nearly.

Ex. 5. Two bunkers are to be partitioned off to hold 54 tons of coal at 45 cubic feet per ton: the bunker is to be 12 feet and 5 feet.

Then since *Length* = cubic contents divided by product of *Breadth* and *Height* (Rule LXXXIII, page 190).

Cubic contents of 1 bunker = 54 × 45 ÷ 2 = 12·15 cubic feet.
Area of section = 12 × 5 = 60 square feet,
∴ length = $\frac{1215}{60}$ = 20·25 feet, or 20 feet 3 inches, *Answer*.

Ex. 6. A coal bunker is 28 feet 9 inches long, by 12 feet 6 inches wide, and 8 feet high; and 2 side bunkers each 29 feet 9 inches by 9 feet 6 inches, and the same height as the cross one, how many tons of coal will each hold at 45 cubic feet to the ton? And how many baskets of coal will there be altogether if 20 baskets weigh 16 cwts.?

Contents of cross bunker = 28·75 × 12·5 × 8 = 2875 cubic feet.
" 2 side " = 29·5 × 9 × 5 × 8 × 2 = 4484 "

$$45\begin{cases}5)7359 \text{ "}\\ 9)1471·8\end{cases}$$

163·87 nearly.

Cwts.		Tons.		Baskets.		
16	:	163·87	::	20	:	x

20

3277·40 cwt.
20

$$16\begin{cases}4)65548·0\\ 4)16387\end{cases}$$

4096¾ baskets.

∴ total number of baskets = 4096¾, *Answer*.

EXAMPLES FOR PRACTICE.

NO.	LENGTH.	BREADTH.	HEIGHT.	CUBIC FEET PER TON.
1.	15 feet 0 inches.	5 feet 0 inches.	9 feet 0 inches.	44
2.	24 „ 0 „	8 „ 0 „	3 „ 6 „	43
3.	32 „ 3 „	8 „ 6 „	8 „ 0 „	42
4.	50 „ 8 „	11 „ 10 „	12 „ 5 „	42
5.	33 „ 9 „	8 „ 9 „	10 „ 9 „	44
6.	29 „ 6 „	9 „ 3 „	9 „ 3 „	42

248. Any two dimensions of a square or rectangular tank being given, to find the third so that the tank shall hold any number of imperial gallons.

RULE XCVI.

1°. **Multiply** ***the*** **length** ***by the*** **breadth** ***(both being expressed in feet and decimals of a foot)***, **the product will be the square feet.**

2°. **Multiply** ***the*** **number** ***of*** **gallons** ***the*** **tank** ***is*** **required to contain** ***by*** **·16,** ***or divide by*** **$6\frac{1}{4}$ (6·25),** ***the result in either case is the contents in cubic feet.***

NOTE.—1 gallon of water = ·16 cubic feet, or $6\frac{1}{4}$ gallons = 1 cubic foot.

3°. **Divide** ***the*** **result** ***obtained by No. 1, by the*** **product** ***found by No*** **2°; the quotient** ***is the*** **height** ***in feet and decimals of a foot.***

$$\text{Or, height} = \frac{\cdot16 \times \text{gallons}}{\text{length} \times \text{breadth}} = \frac{\text{gallons} \div 6\cdot25}{\text{length} \times \text{breadth}}$$

EXAMPLES.

Ex. 1. A tank is 7 ft. long by 3 feet 4 inches broad, what height must it be to contain 900 gallons?

$$\text{Height} = \frac{\cdot16 \times 900}{7 \times 3\cdot3333} = \frac{900 \div 6\cdot25}{7 \times 3\cdot3333}$$

$$\therefore 900 = 900 \times \cdot16 = 144\cdot00 \text{ cubic feet.}$$

$$\begin{array}{r} 900 \\ \cdot16 \\ \hline 144\cdot00 \text{ cubic feet.} \end{array} \qquad\qquad \begin{array}{r} 6\cdot25)900(144 \\ 625 \\ \hline \&c. \end{array}$$

$$3 \text{ ft. } 4 \text{ in.} \times 7 \text{ ft.} = 3\cdot333\dot{3} \times 7 = 23\cdot3333.$$

$$\begin{array}{rr} 23\cdot3333)144\cdot000000(6\cdot171 & \text{ft.} \\ 1399998 & 12\phantom{\text{ft.}} \\ \hline 400020 & 2\cdot052 \text{ in.} \\ 233333 & \\ \hline 1666870 & \\ 1633331 & \\ \hline 335390 & \\ 233333 & \\ \hline \end{array}$$

Answer: **6 feet 2 inches.**

Ex. 2. A tank is 8·75 feet long by 4¼ feet broad: what height must it be to contain 1210 gallons?

$$\text{Height} = \frac{\cdot 16 \times 1210}{8\cdot 75 \times 4\cdot 25} \qquad \text{Height} = \frac{1210 \div 6\cdot 25}{8\cdot 75 \times 425}$$

1210 gallons.
·16
7260
1210
193·60 cubic feet.

8·75
4·25
4375
1750
3500
37·1875

37·1875)193·6000(5·206 ft.
1859375
766250
743750
2250000
2231250

5·206 ft.
12
2·472 in.

∴ height = 5 ft. 2½ in., nearly.

In each of the following examples it is required to find the height of the tank, having given the following quantities.

Examples for Practice.

	Length.	Breadth.	Gallons.		Length.	Breadth.	Gallons.
1.	6 ft. 4 in.	4 ft. 8 in.	660	5.	14 ft. 3 in.	7 ft. 9 in.	3766·3
2.	11 3	3 9	1252½	6.	12 8	9 4	5280
3.	9 3	3 6	657½	6.	33 9	11 3	16232·4
4.	18 6	7 0	5260	8.	49 9	23 6	132971

249. If we have the three dimensions of a square or rectangular tank given, to find the number of gallons it would contain we may proceed thus:—

RULE XCVII.

1°. *Take the* continued product *of the length, breadth, and depth in feet and decimals of a foot.*

2°. *Next, multiply this product by the number of gallons to a cubic foot, viz.,* 6¼ (*or* 6·25), *the result is the number of gallons required to fill the tank.*

Examples.

Ex. 1. A tank is 13 feet 6 inches long, 5 feet 6 inches wide, and 5 feet deep: how many cubic feet will it contain, also the number of gallons required to fill it?

13 ft. 6 in. = 13·5 feet. 5 ft. 6 in. = 5·5 feet.

6¼ gallons = 1 cubic foot.

13·5 × 5·5 × 5 = 371·25 cubic feet.

371·25 × 6¼ = 2320·3125 gallons.

Length 13·5 feet.
Width 5·5 „
675
675
74·25
Depth 5
Contents 371·25

371·25 cubic feet.
6·25
185625
74250
222750
2320·3125 gallons.

Again, it is required to know what weight of fresh water would fill the above tank.

A cubic foot of fresh water = 62·5 lbs.

∴ contents of tank = 371·25 cubic feet × 62·5 = 23203·125 lbs. of fresh water.
= 23203·125 lbs. ÷ 2240 = 10·3585 tons.
= 10 tons 7 cwt. 0 qr. 19 lbs., *Answer*.

```
                         371·25       2240)23203·125(     10·3585 tons.
                           62·5            2240                20
                         ------            ----            -------
                         185625            8031            7·17[illegible] cwts.
                         74250             6720                4
                        222750             ----             -----
                        -------            13112            0·68 qrs.
Weight of fresh water = 23203·125 lbs.     11200             28
                                           -----            ---
                                            19125           544
                                            17920          136
                                            -----          ---
   Answer 10 tons 7 cwt. 0 qr. 19 lbs.      ·12050         19·04 lbs.
```

If it be required to know what weight of sea water this tank would hold:

A cubic foot of sea water = 64 lbs.

∴ contents of tank = 317·25 cubic feet × 64 = 23760 lbs. of sea water.
= 23760 lbs. ÷ 2240 = 10·6071 tons.
= 10 tons 12 cwt. 0 qr. 16 lbs.

```
                         371·25          2240)23760(      10·6071 tons.
                           64                 2240              20
                         ------               ----          -------
                         148500               13600         12·142[illegible] cwt.
                        222750                13440              4
                        -------               -----         -------
Weight of sea water = 23760·00 lbs.            16000        0·568 qrs.
                                               15680           28
                                               -----          ---
                                                3200         4544
                                                2240        1136
                                                ----        -----
                                                            15·904
```

Or, since 35 cubic feet of sea water make a ton, we may proceed to find what weight of salt water the tank will hold as follows:—

Contents 371·25 cubic feet.

```
   ⎧ 5)371·25
35 ⎨ -------
   ⎩ 7)74·25
     --------
     10·6071 tons.
          20
     --------
     12·142[illegible] cwt.
          4
        ----
       0·568 qrs.
         28
        ----
       4544
      1136
      -----
      15·904 lbs.
```

Answer, 10 tons 12 cwt. 0 qr. 16 lbs. nearly.

Ex. 2. A tank is 12 feet long, 6 feet wide, and 5 feet deep: how many cubic feet of water will it contain, and how many gallons of fresh water will fill it?

Contents = 12 × 6 × 5 = 360 cubic feet.

A gallon of fresh water = 6¼ (= 6·25) cubic feet,

∴ 360 cubic feet × 6·25 = 2250 gallons of fresh water.

What weight of fresh water would fill it?

A cubic foot of fresh water is equal to 62·5 lbs.,

∴ 360 cubic feet × 62·5 = 22500 lbs.

22500 lbs. ÷ 2240 = 10·04464 tons

= 10 tons 0 cwt. 3 qrs. 16 lbs.

What weight of salt water will it contain?

A cubic foot of sea water equals 64 lbs.

∴ 360 cubic feet × 64 = 23040 lbs.

23040 lbs. ÷ 2240 = 10 tons 5 cwt. 2 qrs. 24 lbs.

Or thus:—

A ton of sea water = 35 cubic feet.

$$35\left\{\begin{array}{l}5)\underline{360}\\7)\underline{72}\end{array}\right.$$

$$\quad 10\ \ 5\ \ 2\ \ 24$$

Ex. 3. A tank is 10 feet 9 inches long, 4 feet 9 inches wide, and 7 feet 6 inches high: find the number of gallons, and the weight in tons, of fresh water it contains?

10·75 × 4·75 × 7·5 = 382·96875 cubic feet,

∴ 382·96875 × 6·25 = 2393·5546875 gallons.

A cubic foot of fresh water weighs 62½ lbs.

∴ 382·96875 cubic feet × 62·5 = 23935·546875 lbs.

23935·546875 lbs. ÷ 2240 = 10·685512 tons,

= 10 tons 13 cwt. 2 qrs. 23½ lbs.

Ex. 4. A tank is 4 feet long, 4 feet wide, and 1 foot deep: how many gallons of water will it contain?

4 × 4 × 1 = 16 cubic feet.

16 cubic feet × 6·25 = 100 gallons.

Whence it is evident that tanks 4 feet long by 4 feet wide contain, for every foot of depth, 100 gallons of water, which makes it very easy to ascertain the quantity of water remaining in these tanks at any time during a voyage.

Ex. 5. A four feet square tank is sounded, and the rod shows 4¾ feet of water: how many gallons are there in it?

Since in a four feet square tank for every foot of depth of tank there are 100 gallons of water, then for 4¾ feet depth there will be

100 gallons × 4¾ = 475 gallons.

Ex. 6. How many gallons of water will 14 tanks hold, each tank 6 ft. by 5 ft. by 3 ft. 6 in., and what quantity must be allowed per day per man to serve 262 persons for 12 days?

Contents of each tank = 6 ft. × 5 ft. × 3·5 ft. = 105·0 cubic feet, ∴ contents of 14 tanks = 105·0 × 14 = 1470 cubic feet; and 1470 × 6·25 = 9187·5 gallons.

$$9187{\cdot}5 \div 262 = 35{\cdot}0668 \div 24 = 1{\cdot}4611 \text{ gallons},$$

$$\text{or, } 9187{\cdot}5 \div (262 \times 24) = 9187{\cdot}5 \div 6288 = 1{\cdot}4611 \text{ gallons.}$$

```
     6                          Cubic contents of 14 tanks  1470 feet.        262
     5                          1 cubic foot             =  6·25 gallons.      24
   ---                                                      ----             ----
    30                                                      7350             1048
   3·5                                                     2940              524
   ---                                                     8820              ----
 105·0 cubic feet 1 tank.                                  ------            6288
   14  number of tanks.         Contents in gallons        9187·50
 -----
  420
 105
 -----
 1470 cubic contents of tanks.
```

```
6288)9187·5(1·4611 gallons.    262)9187·5(35·0668          { 4)35·0668 each for 12 days.
     6288                          786                  24 {   -------
     ----                          ----                    { 6) 8·7667
     28995                         1327                        -------
     25152                         1310                        1·4611 gallons.
     -----                         ----                             4
      38430                         1750                       -------
      37728                         1572                       1·8444 quarts.
      -----                         ----                            2
        7020                         1780                      -------
        6288                         1572                      1·6888 pints.
        ----                         ----                           4
         7320                         2080                     -------
                                      2096                     2·7552 gills.
                                      ----
                                                       1 gal. 1 qt. 1 pt. 2¾ gls.
```

EXAMPLES FOR PRACTICE.

Ex. 1. How many gallons will 32 tanks hold, each tank 7 ft. by 6 ft. by 3¾ ft., and what quantity must be allowed per man per day to serve 262 persons for 12 days?

Contents of 32 tanks = 31500 gallons; allowance for each person per day 1·727 or 1 gal. 2 quarts 1 pint 3·26 gills.

Ex. 2. How many gallons will 12 tanks hold, each tank 7½ ft. by 8 ft. by 4½ ft., and what quantity must be allowed per day to serve 360 persons for 54 days?

Contents = 20250 gallons; allowance per day for each person 1 gallon 1·33 gills.

Ex. 3. 22 tanks, each 4½ ft. by 4 ft. by 2½ ft., and 145 persons for 8 days.

Ans. 2·667 gals., or 2 gals. 2 qts. 1 pt. 1·34 gls.

Ex. 4. 24 tanks, each 3½ ft. by 3½ ft. by 9 ft., 200 persons for 36 days.

Ans. 2·297 gals., or 2 gals. 1 qt. 0 pt. 1½ gls.

Ex. 5. 8 tanks, each 4¼ ft. by 4¼ ft. by 6 ft., 91 men for 24 days.

Ans. 2·481 gals., or 2 gals. 1 qt. 1 pt. 3½ gls.

250. Given the dimensions of an oil tank, the number of gallons in the tank, to find how far from the top the oil will be when the quantity stated is in the tank.

Note.—1 gallon = ·16 cubic feet, or 1 cubic foot = 6¼ gallons.

$$\frac{\text{Gallons} \times \cdot 16}{\text{length} \times \text{width}} = \text{height of oil},$$

∴ height of tank — height of oil = distance of oil from top of tank.

Examples.

Ex. 1. An oil tank is 3 feet 5 inches deep, 4 feet 6 inches long, and 2 feet 6 inches wide, and has 95 gallons of oil in it, how far down from the top of the tank is the oil.

Multiply the 95 gallons by ·16 in order to ascertain the number of cubic feet of oil in the tank; or, divide the gallons by 6·25.

95 × ·16 = 15·20 cubic feet of oil in tank.
or, 95 ÷ 6·25 = 15·20 „ „

The tank is 4 feet 6 inches by 2 feet 6 inches base. Reducing inches to decimals of a foot, 4 feet 6 inches = 4·5 feet, and 2 feet 6 inches = 2·5 feet, then 4·5 feet × 2·5 feet = 11·25 square feet of base.

Next, cubic feet of oil in the tank divided by the square feet in base gives the height of the oil in the tank in feet.

15·20 ÷ 11·25 = 1·351 feet, or 1 foot 4·2 inches,

and the height of oil subtracted from the height of tank, the remainder is the distance (in feet) the oil is from the top of the tank.

Height of tank — height of oil = 3 feet 5 inches — 1 foot 4·2 inches = 2 feet 0·8 inches.

95 gallons.
·16
570
95
15·20 gallons.

Length 4·5
Width 2·5
225
90
11·25 square feet in base.

11·25)15·20 (1·351 feet.
1125 12
3950 4·212 — 1 ft. 4·2 in.
3375
5750
5625
1250

Height of tank	3 ft.	5·0 in.
„ oil	1	4·2
Distance from top of oil to top of tank	2	0·8

Ex. 2. An oil tank 4 feet long, 2 feet deep, and 3 feet wide: 75 gallons of oil is the usual order at the commencement of the voyage, what is the distance of the top of the oil from the top of the tank?

4 feet × 3 feet = 12 square feet.
80 gallons × ·16 = 12·80 cubic feet,
or, 80 gallons ÷ 6·25 = 12·80 cubic feet.

Then 12·80 cubic feet ÷ 12 square feet = 1·066 feet, the height of the oil above the bottom of the tank.

∴ Distance of top of oil from the top of tank = 2 feet 0 inches — 1 foot 0·8 inches
= 0 feet 11·2 inches.

Ex. 3. An oil tank is 4 feet 6 inches high, and 3 feet by 2 feet base: what was the consumption per day in gallons, if in 12 days the oil in the tank is lowered 11 inches in the glass?

Cubic feet of oil used = 3 × 2 × $\frac{11}{12}$ = 5·5.
Gallons of oil used = 5·5 × 6·25 = 34·375.
∴ Consumption per day = 34·375 ÷ 12 = 2·864 gallons, *Answer.*

Ex. 4. How many gallons of olive oil will a tank hold which is 4 feet by 3 feet and 2 feet 6 inches deep, and what would be the weight of the oil?

Specific gravity of oil 0·908.

$$\text{Contents} = 4 \times 3 \times 2{\cdot}5 = 30 \text{ cubic feet.}$$

$$\text{No. of gallons} = 30 \times 6{\cdot}25 = 187{\cdot}5 \text{ gallons.}$$

$$\text{Weight of 1 cubic foot of oil} = {\cdot}908 \times 62{\cdot}5 = 56{\cdot}75.$$

$$\text{Total weight of oil} = 56{\cdot}75 \times 30 = 1702{\cdot}5 \text{ lbs.}$$

$$\begin{array}{r} 4 \\ \underline{3} \\ 12 \\ \underline{2{\cdot}5} \\ \text{Contents} = 30{\cdot}0 \text{ cubic feet.} \end{array} \qquad \begin{array}{r} 6{\cdot}25 \\ \underline{30} \\ 187{\cdot}50 \text{ gallons.} \end{array}$$

$$\begin{array}{r} \text{Specific gravity of oil } {\cdot}908 \\ \underline{62{\cdot}5} \\ 4540 \\ 1816 \\ \underline{5448} \\ \text{Weight of 1 cub. ft. of oil} = 56{\cdot}7500 \end{array} \qquad \begin{array}{l} 56{\cdot}75 \\ \underline{30 \text{ cubic feet.}} \\ 1702{\cdot}50 \text{ lbs.} \end{array}$$

Ex. 5. A circular tank is 6 feet 6 inches diameter: what height must it be to hold 360 gallons of oil?

$$\text{Content} = 360 \div 6{\cdot}25 = 57{\cdot}6 \text{ cubic feet.}$$

$$\text{Area of base} = 6{\cdot}5^2 \times {\cdot}7854 = 6{\cdot}5 \times 6{\cdot}5 \times {\cdot}7854 = 33{\cdot}18315 \text{ square feet,}$$

$$\therefore \text{the cubic contents of circular tank} = \text{height} \times (6{\cdot}5^2 \times {\cdot}7854),$$

$$\textit{i.e.}, 57{\cdot}6 \text{ cubic feet} = \text{height} \times 33{\cdot}18315 \text{ square feet,}$$

$$\therefore \text{height} = \tfrac{57{\cdot}6}{33{\cdot}183} = 1{\cdot}736 \text{ feet} = 1 \text{ foot } 8{\cdot}8 \text{ inches.}$$

Ex. 6. What must be the depth of a cylindrical tank to hold 50000 gallons, its diameter being 24 feet?

I. The gallon contains ·16 cubic inches,

$$\therefore 50000 \text{ gallons must contain } {\cdot}16 \times 50000 = 8000 \text{ cubic feet.}$$

II. The area of a circle = ·7854 × diameter².

$$\therefore \text{the area of the base of the cylindrical tank} = ({\cdot}7854 \times 24 \times 24) \text{ square feet.}$$

III. The cubic content of a cylinder = height × base,

$$\therefore \text{the content of the cylindrical tank} = \text{height} \times ({\cdot}7854 \times 24 \times 24),$$

$$\therefore 8000 \text{ cubic feet} = \text{height} \times ({\cdot}7854 \times 24 \times 24),$$

$$\therefore \text{height} = \frac{8000}{{\cdot}7854 \times 24 \times 24} = \tfrac{8000}{452{\cdot}3904} = 17{\cdot}683 \text{ feet} = 17 \text{ feet } 8{\cdot}12 \text{ inches.}$$

EXAMPLES FOR PRACTICE.

No.	Depth.	Length.	Width.	Quantity of oil.	*Ans.*: Distance of top of oil from top of tank.
1.	3·75 ft.	3 ft. 0 in.	2 ft.	45 gals.	2·58 ft. or 2 ft. 7 in.
2.	1 ft. 9 in.	4 „ 6 „	2·5 „	90 „	0·47 „ 0 „ 5·6 „
3.	2 „ 0 „	4 „ 0 „	3 „	80 „	0·924 „ 0 „ 11·2 „
4.	4·6 ft.	4·5 „	3·6 „	120 „	3·415 „ 3 „ 5·0 „
5.	4·5 „	4·5 „	4·5 „	150 „	3·315 „ 3 „ 3·8 „
6.	7·3 „	4·5 „	1·25 „	100 „	4·456 „ 4 „ 5·5 „

ON CONSUMPTION OF FUEL, &c.

251. **Given the quantity of coal on board on leaving port, and the consumption in a given number of days: to find what quantity remains on board.**

Example.

Ex. The quantity of coal on board on leaving port is 350 tons, the consumption in 17 days is 258 tons 11 cwt. 1 qr. 25 lbs.: what quantity remains on board?

	Tons.	cwt.	qrs.	lbs.
Quantity of coal on board	350	0	0	0
Consumption	258	11	1	25
Quantity remaining	91	8	2	3

252. **Given the daily consumption of coal and the speed per hour, to find what the consumption would be in a voyage of a given number of knots.**

RULE XCVIII.

1°. ***Multiply the speed per hour by* 24; the result is the number of knots the ship makes per day.**

2°. ***Next, say, as the knots sailed per day is to the knots in the voyage, so is the consumption per day to the consumption for the voyage.***

Examples.

Ex. 1. If the consumption of coal is 36 tons per day to produce a speed of 10 knots, what is the consumption on a voyage of 960 knots?

Speed per day = 10 knots × 24 = 240 knots.

Then as 240 knots : 960 knots :: 36 tons : x

$$\begin{array}{r} 36 \\ \hline 576 \\ 288 \\ \hline \end{array}$$

$$240 = \left\{ \begin{array}{r} 4,0)3456,0 \\ \hline 6)864 \\ \hline 144 \text{ tons.} \end{array} \right.$$

Or thus—

$$\frac{\overset{4}{\cancel{960}} \times 36}{\underset{1}{\cancel{240}}} = 144 \text{ tons.}$$

Ex. 2. If the consumption of coal is 20 tons per day to produce a speed of 12 knots, what would the consumption be in a voyage of 1200 knots?

Speed per day = 12 knots × 24 = 288.

Then as 288 knots : 1200 knots :: 20 tons : x

```
                  20
               ------
288)24000(83 tons 6 cwt. 2 qrs. 18⅔ lbs.
    2304
    -----
      960
      864
     ----
       96
       20
     ----
288)1920(6 cwt.
    1728
    ----
     192
       4
    ----
288)768(2 qrs.
    576
    ---
    192
     28
   ----
   1536
    384
   ----
288)5376(18⅔ lbs.
    288
   ----
   2496
   2304
   ----
   192/288 = 16/24 = 2/3
```

$$x = \frac{\overset{\overset{25}{\cancel{100}}}{\cancel{1200}} \times \overset{10}{\cancel{20}}}{\underset{\underset{\underset{3}{\cancel{6}}}{\cancel{24}}}{\cancel{288}}} = \frac{250}{3}\text{ tons}$$

= 83 tons 6 cwt. 2 qrs. 18⅔ lbs.

or thus:—

```
12)288 : 1200 :: 20 : x
 4) 24    100     10
 2)  6     25
     3     10
          ----
        3)250
          -----------------
           83   6   2  18⅔
```

Examples for Practice.

No.	Coals per day. tons.	Speed. knots.	Voyage. knots.
1.	16	12	2880
2.	10	10½	4350
3.	12	9	1380
4.	14	11½	1850
5.	12½	9	3590
6.	8	8½	1800
7.	21	13	1260
8.	18·5	10·4	4320
9.	24·6	12·5	4560
10.	36·5	18·4	7560

253. **Consumption of Coal and Speed of Vessel.**—The consumption of coal varies as the distance multiplied into the square of the speed, or as the cube of the speed itself.

Examples.

Ex. 1. Suppose a ship uses 200 tons of coal in a voyage of 500 miles, and steams at 10 knots an hour: what quantity would take her 1000 miles at 6 knots an hour?

$$(500 \times 10^2) \ : \ (1000 \times 6^2) \ :: \ 200\text{ tons} \ : \ x$$

$$x = \frac{1000 \times 6^2 \times 200}{500 \times 10^2} = \frac{36\cancel{000} \times 20\cancel{0}}{5\cancel{0000}} = \frac{720}{5} = 144\text{ tons, the consumption.}$$

$$500 \times 10^2 \;:\; 1000 \times 6^2 \;::\; 20\cancel{0} \;:\; x$$

$$\begin{array}{ll} \underline{\;100\;} & \underline{\;36\;} \\ 5\cancel{0000} & 36\cancel{000} \\ & \underline{\;20\;} \\ & 5)\underline{720} \end{array}$$

Consumption 144 tons.

Ex. 2. During a voyage of 1200 miles, performed at an average speed of 10 knots, the total consumption of fuel is 150 tons: it is required to know the consumption for a voyage of 2400 miles at a reduced speed of 8 knots.

$$(1200 \times 10^2) \;:\; (2400 \times 8^2) \;::\; 150 \text{ tons} \;:\; x$$

$$x = \frac{2400 \times 8^2 \times 150}{1200 \times 10^2} = \frac{\overset{2}{\cancel{2400}} \times \overset{32}{\cancel{64}} \times \overset{3}{\cancel{150}}}{\cancel{1200} \times \underset{2}{\cancel{100}}} = 192 \text{ tons, the consumption.}$$

Ex. 3. If the consumption of coal per day be 16 tons to produce a speed of 9 knots per hour, how many knots should she steam when the consumption is 12 tons?

$$\begin{array}{ccccccl} \text{Tons.} & & \text{Tons.} & & \text{Knots.} & & \\ \underline{16} & : & \underline{12} & :: & \underline{9^3} & : & \text{the cube of the required speed.} \\ \text{or, } 4 & : & 3 & :: & 729 & & \\ & & & & \underline{\;\;3} & & \\ & & & & 4)\underline{2187} & & \\ & & & & 546{\cdot}75 & & \end{array}$$

$$\begin{array}{r|r|l} 241 & 252 & 5\dot{4}6{\cdot}75\dot{0}(8{\cdot}1 \text{ knots.} \\ & \underline{241} & \underline{512} \\ & 25441 & \;\;34750 \\ & & \;\;\underline{25441} \end{array}$$

Ex. 4. Steaming at 11 knots an hour 4 tons of coal are used: find the consumption when steaming 9 knots.

$$11^3 \;:\; 9^3 \;::\; 4 \text{ tons} \;:\; x$$

$$\frac{9^3 \times 4}{11^3} = \frac{9 \times 9 \times 9 \times 4}{11 \times 11 \times 11} = \frac{2916}{1331} = 2{\cdot}19 \text{ tons.}$$

Ex. 5. The average rate of speed of a ship last voyage was 7 knots per hour, with a consumption of 11 tons per day; this voyage (her boilers having been renewed), she makes 8½ knots per hour: what is the present consumption?

$$7^3 \;:\; 8{\cdot}5^3 \;::\; \overset{\text{Tons.}}{11} \;:\; x$$

$$\frac{8{\cdot}5 \times 8{\cdot}5 \times 8{\cdot}5 \times 11}{7 \times 7 \times 7} = \frac{6755375}{343} = 196{\cdot}99 \text{ tons.}$$

Answer, 197 tons.

Ex. 6. If a ship of 350 I.H.P. steams 8 knots an hour, what should be the I.H.P. of a new engine so that she may steam 10 knots.

$$8^3 \;:\; 10^3 \;::\; 350$$

$$\frac{10^3 \times 350}{8^3} = \frac{10 \times 10 \times 10 \times 350}{8 \times 8 \times 8} = 683{\cdot}6 \text{ I.H.P.}$$

$$\begin{array}{lcccl} \text{Or,} & \underline{8^3} & \underline{:} & \underline{10^3} & :: \; 350 \\ & 512 & : & 1000 & :: \; 350 \\ & & & & \;\;\;\;\;\underline{1000} \end{array}$$

$$\begin{array}{l} 512)350000(683{\cdot}6 \\ \;\;\;\;\;\;\underline{3072} \\ \;\;\;\;\;\;\;4280 \\ \;\;\;\;\;\;\;\underline{4096} \\ \;\;\;\;\;\;\;\;\;1840 \\ \;\;\;\;\;\;\;\;\;\underline{1536} \\ \;\;\;\;\;\;\;\;\;\;3040 \\ \;\;\;\;\;\;\;\;\;\;\underline{3072} \end{array}$$

EXAMPLES FOR PRACTICE.

1. If 195 tons of coal suffice for a voyage of 1700 miles at the rate of 9 knots: how many tons will be required for a voyage of 2200 miles at a reduced speed of 8 knots?
2. Suppose a ship uses 160 tons of coal in a voyage of 2250 miles at the rate of 8 knots: what quantity will take her 3150 miles at a reduced speed of 6 knots?
3. If 100 tons of coal suffice for a voyage of 1000 miles at the rate of 10 knots: it is required to know the consumption for a voyage of 2400 miles at a reduced speed of 9 knots.
4. A ship made a voyage of 1900 miles at 10 knots per hour, with a total consumption of 100 tons of coal: what should be her consumption for a voyage of 2320 miles at a speed of 9 knots per hour?

254. Given the consumption of coal per day and the average indicated horse-power: to find the consumption per hour per indicated horse-power.

RULE XCIX.

1°. *Multiply* the tons of coal consumed per day by 2240 *(the number of pounds in a ton), and* divide *the* product *by* 24: *the* quotient *thus found is the* number of pounds *of coal consumed per hour.*

2°. *Divide this last quantity by the indicated horse-power* (I.H.P.): *the result is the* consumption per hour per indicated horse-power (I.H.P.)

EXAMPLES.

Ex. 1. The consumption of coal is 81 tons per day, the average horse-power indicated is 2760: what is the rate of consumption per indicated horse-power?

$$81 \times 2240 \div 24 = 7560 \text{ lbs. of coal per hour.}$$

$$7560 \div 2760 = 2{\cdot}74 \text{ lbs. of coal per hour per I.H.P.}$$

```
     2240 lbs.                 2760)7560(2·74 lbs. nearly, per I.H.P.
       81  tons.                    5520
     ------                         -----
      224                           20400
     1792                           19320
     ------                         -----
   ( 4)181440 lbs.                   1080
24 {  -------
   ( 6) 45360
     ------
     7560 lbs. of coal per hour.
```

∴ *Answer*, 2·74 lbs. nearly per I.H.P.

Ex. 2. The consumption of coal is 37½ tons per day, the average horse-power indicated is 960: what is the rate of consumption per indicated horse-power?

$$37\tfrac{1}{2} \times 2240 \div 24 = 3500 \text{ lbs. of coal per hour.}$$

$$3500 \div 960 = 3{\cdot}64 \text{ lbs. of coal per I.H.P.}$$

```
   2)2240 lbs. = 1 ton.            96,0)350,0(3·64 lbs. per I.H.P.
       37½ tons.                        288
     ------                             ---
     15680                              620
     6720                               576
      1120*                             ---
     ------                              440
   ( 4)84000 lbs.                        384
24 {  -----                              ---
   ( 6)21000                              56
     ------
     3500 lbs. of coal per hour.
```

* This line is got by dividing 2240 by 2, which is in fact multiplied by ½.

Examples for Practice.

In each of the following examples the I.H.P. and consumption per day are given, to find the consumption per I.H.P.

No.	I.H.P.	Con. per day.	No.	I.H.P.	Con. per day.
1.	436	12 tons.	7.	720	19½ tons.
2.	340	8 „	8.	450	12 „ 15 cwt.
3.	1750	50 „	9.	650	29 „ 10 „
4.	3000	65 „	10.	1480	33 „
5.	4550	96 „	11.	540	25 „
6.	840	21½ „	12.	1510	37 „

Examples.

Ex. 1. If the total consumption of coal be 35 tons per day, how many basketsful, each weighing 63 pounds, will be burnt in a watch of 4 hours?

Coals consumed per day 35 tons.

2240 lbs. = 1 ton.
35 tons.
11200
6720
63 { 7)78400 **lbs. per day.**
9)11200
1244·4 **baskets per day.**

As the watch is 4 hours, there will be 6 watches in the day, then:

6)1244·4
207·4 **basketsful per watch.**

Or thus:—
2240 lbs. = 1 ton.
35
11200
6720
$24^{h} = 1$ **day** | 4^{h} | $\frac{1}{6}^{d}$ | 78400
63 { 7)13066·66 **lbs. per watch.**
9)1866·66
207·4 **basketsful.**

Ex. 2. If the total consumption of coal be 14 tons per day, how many basketsful, each weighing 58 lbs., will be burnt in a watch of 4 hours?

Coals consumed per day 14 tons.

2240 lbs. = 1 ton.
14 tons.
8960
2240
31360 **lbs. per day.**

58)31360(540·7 **baskets per day.**
290
236
232
400
406

6)540·70
90·11 **basketsful.**

2240 lbs.
14 tons.
8960
2240
4^{h} | $\frac{1}{6}^{d}$ | 31360 **lbs. per day.**
5226·66 **lbs. per watch.**

58)5226·66(90·11 **basketsful.**
522
66
58
86
58
28

EXAMPLES FOR PRACTICE.

No.	Consumption of coal per day in tons.	Weight of basketful of coal.	Length of Watch.
1.	16 tons 0 cwt.	61 lbs.	4 hours.
2.	8 „ 0 „	66 „	6 „
3.	14 „ 0 „	64 „	4 „
4.	10 „ 0 „	65 „	4 „
5.	17 „ 0 „	60 „	4 „
6.	12 „ 0 „	64 „	4 „

255. Given the basketsful of coal consumed per day, and the weight of each basketful, to find what the consumption is in tons.

$$\text{Consumption per day (in tons)} = \frac{\text{No. of basketsful in the day} \times \text{lbs. in each basket}}{2240}$$

EXAMPLES.

Ex. 1. The consumption of coals during the day is 120, 135, 148, 130, 126, 127 basketsful, each basket weighing 68 lbs.: what is the consumption in tons?

120
135
148
130
126
127
———
786 basketsful.

No. of basketsful 786
Weight per basket 68 lbs.
———
6288
4716
———
Total consumption = 53448 lbs.

2240)53448(23·86 tons.
4480
———
8648
6720
———
19280
17920
———
13600

20
———
17·20
———

Ans.: 23 tons 17 cwt.

Ex. 2. The tally of baskets of coals for 24 hours is 220, 225, 242, 255, 254, and 244 basketsful, each basket weighing 62 lbs.: what is the consumption in tons per day?

$$220 + 225 + 242 + 255 + 254 + 244 = 1440 \text{ basketsful.}$$

$$\frac{1440 \times 62}{2240} = 39{\cdot}86 \text{ tons nearly, or 39 tons 17 cwt.}$$

EXAMPLES FOR PRACTICE.

No.	Tally of Baskets. 1st watch.	2nd watch.	3rd watch.	4th watch.	5th watch.	6th watch.	Weight. in lbs.
1.	240	263	254	232	258	240	68
2.	260	258	261	258	240	249	72½
3.	210	215	220	217	214	225	48
4.	225	230	246	258	260	252	63
5.	230	225	229	227	235	232	64
6.	223	232	246	225	230	250	67

EXAMPLE.

Ex. Fill up the tabular form given below from the following statement, and say whether you consider the quantity of coal remaining to be sufficient for the rest of the passage. The *S.S.* "Candidate" left port on the 5th day of last month, for a destination 3000 miles distant, with a stock of coal on board amounting to 160 tons: the distances run on the successive days are 192, 200, 212, 184, 190, 194, 210, 214, 189, and 213 miles. The coal burnt each day was 7½, 8, 7, 8½, 9, 7¼, 8, 10, 7½, and 8 tons.

The following is the tabulated form.

DAILY REPORT.				STOCK OF FUEL.		
Distance accomplished and coal consumed.				Remaining coal and distance to run.		
Date.	Miles run.	Miles per ton.	Coal consumed.	Tons remaining.	Miles to run.	Miles per ton.
5	192	25·6	7½	152½	2808	18·413
6	200	25·0	8	144½	2608	18·048
7	212	30·3	7	137½	2396	17·425
8	184	21·65	8½	129	2212	17·147
9	190	21·11	9	120	2022	16·850
10	194	26·76	7¼	112¾	1828	16·213
11	210	26·25	8	104¾	1618	15·446
12	214	21·4	10	94¾	1404	14·818
13	189	25·2	7½	87¼	1215	13·925
14	213	26·62	8	79¼	1002	12·643
	1998		80¾			
	3000		160			
	1002		79¼			

1st column.—Place one under another the dates of the ten day's run, beginning with the 5th.

2nd column.—Place each day's run opposite its date.

4th column.—Place each day's consumption opposite its date.

3rd column.—Divide the "miles run" by the coals consumed; thus:—

```
7·5)192·0(25·6      8)200       7)212       8·5)184·0(21·65
    150               ---         ---           170
    ----              25          30·3          ---
    420                                         140
    375                                          85
    ----                                        ---
    450                                         550
    450                                         510
    ----                                        ---
                                                400

9)190       7·25)194·00(26·76       8)210       10)214
  ----           1450                 ------       ------
  21·1           ----                 26·25        21·4
                 4900
                 4350
                 ----
                 5500
                 5075
                 ----
                 4250
                 4350
                 ----
```

```
7·5)189·00(25·2          8)213
    150                  ─────
    ───                  26·62
     390
     375
     ───
      150
```

5th column.—From the stock of coal at starting subtract the 1st day's consumption, and from the remainder subtract the 2nd day's consumption.

6th column.—From the whole distance at starting subtract the 1st day's run, and from the remainder subtract the 2nd day's run, and so on.

5th Column.		6th Column.	
Stock of coal =	160 tons.	Whole voyage	3000
1st day's con.	7½	1st day's run	192
Coals rem.	152½	Miles to run	2808
2nd day's con.	8	2nd day's run	200
	144½		2608
	7		212
	137½		2396
	8½		184
	129		2212
	9		190
	120		2022
	7¼		194
	112¾		1828
	8		210
	104¾		1618
	10		214
	94¾		1404
	7½		189
	87¼		1215
	8		213
	79¼		1002

Before finding the numbers for the 7th column, prove that the subtractions are right. Add up the 2nd and 4th columns; they come to 1998 miles, and 80¾ tons. Then,

Whole voyage	3000 miles.	Coals on board	160 tons.
Dist. run in 10 days	1998 „	Coals burnt	80¾ „
Distance still to go	1002	Coals remaining	79¼

These remainders should agree with the last numbers in columns 6 and 5. If they do not agree it will be necessary to go over the subtractions until the error is found.

To find column 7.—Divide "miles to run" in column 6 by "tons remaining" in column 5, gives "miles per ton" in column 7.

Lastly, to find if there be a sufficiency for voyage, make the proportion:—

Miles.		Miles.		Tons.		Tons.
As 1998 gone	:	1002 to go	: :	80·75 burnt	:	40·496 still to burn.

Then 1998 miles : 1002 miles :: 80·75 tons : 40·496 tons.

$$\begin{array}{r} 80{\cdot}75 \\ 1002 \\ \hline 16150 \\ 807500 \\ \hline \end{array}$$

$$\begin{array}{r} 1998)8091150(40{\cdot}496 \\ 7992 \\ \hline 9915 \\ 7992 \\ \hline 19230 \\ 17982 \\ \hline 12480 \\ 11988 \\ \hline \end{array}$$

$$\begin{array}{r} 79{\cdot}25 \\ 40{\cdot}496 \\ \hline 38{\cdot}754 \end{array}$$ tons

left at the finish of voyage.

Examples for Practice.

No.	Date.	Length of voyage. miles.	Coal on board. tons.	Distance run on the successive days.	The coal burnt each day.
1.	18th	4000	140	240, 242, 239, 238, 245, 246, 240, 243, 239, and 242 miles.	8½, 9, 9½, 10, 9, 9¼, 9½, 9, 8¾, and 9 tons.
2.	21st	3550	148	284, 275, 286, 279, 280, 281, 287, 300, 289, and 295 miles.	10, 11, 12, 10, 13, 12, 11½, 11, 12, and 13 tons.
3.	10th	5500	400	165, 180, 200, 200, 190, 190, 200, 200, 175, and 195 miles.	9, 8½, 7, 10, 8, 9, 7, 8, 8½, and 9 tons.
4.	12th	7000	480	216, 217, 214, 219, 220, 212, 200, 208, 206, and 221 miles.	14, 15, 16, 13, 14, 16, 14, 15, 12½, and 15 tons.
5.	26th	2400	210	190, 198, 199, 200, 201, 190, 189, 194, 200, and 201 miles.	18, 17, 18, 16½, 17, 18½, 17, 17½, 17¼, and 18 tons.
6.	25th	8400	450	192, 196, 199, 189, 196, 200, 206, 207, 190, and 188 miles.	7½, 8, 8½, 7¾, 8, 8½, 9½, 9, 10, and 10¼ tons.

Ex. For a voyage of 3000 knots we have a stock of 420 tons of coal; after steaming 1440 knots and consuming 192 tons: find how much coal she will have at the end of the voyage, steaming at the same rate.

Since the ship has steamed 1440 knots and consumed 192 tons, then by proportion we find how many tons will be consumed in going 3000 knots at the same rate.

144,0 knots : 300,0 knots :: 192 tons : x

$$\begin{array}{r} 192 \\ \hline 144\left\{\begin{array}{r} 12)57600 \\ \hline 12)\ \ 4800 \end{array}\right. \\ \hline 400 \text{ tons.} \end{array}$$

$$\text{Or, } x = \frac{\overset{25}{\cancel{3000}} \times \overset{16}{\cancel{192}}}{\underset{\cancel{12}}{\cancel{1440}}} = 400 \text{ tons.}$$

The stock at starting	420 tons.
Burnt during the voyage	400 „
Quantity left at end of voyage	20 „

EXAMPLES FOR PRACTICE.

No.	Length of Voyage.	Coal on Board.	Distance Steamed.	Coals Consumed.	*Ans.:* Quantity over or Deficient.
1.	3450	225	1500	90 tons.	18 tons over.
2.	2500	500	1600	320 „	none left.
3.	2730	165	1200	80 „	17 tons short.
4.	1750	120	500	32 „	8 „ left.

Ex. If 120 tons of Welsh coal fill the bunkers and is sufficient for a voyage of 2700 miles, and Newcastle coal is for space in the proportion of 45 : 40 and for quality as 7·73 : 9·06: what distance will the bunkers carry her if filled up with Newcastle coal?

Since Newcastle coal occupies a greater space per ton than Welsh coal (in the ratio 45 : 40), therefore a less number of tons can be put into the bunkers, and consequently the ship will be driven a less number of miles, upon the supposition that the coals are equally good.

But Newcastle coal is of an inferior quality (in the ratio of 7·73 : 9·06), therefore the ship will be driven a less number of miles (see Rule LVII, page 153); hence,

$$\begin{array}{rl} 45 & : 40 \quad :: \quad 2700 \\ 9{\cdot}06 & : 7{\cdot}73 \end{array}$$

$$\frac{40 \times 7{\cdot}73 \times \overset{60}{\cancel{2700}}}{\cancel{45} \times 7{\cdot}73} = \frac{18552}{7{\cdot}73} = 2400 \text{ miles.}$$

Ex. Two qualities of coal have been tried on different voyages: what should the second-class quality cost to be equally economically with the first-class, considering only their efficiency?

Cost of first quality 18s. 6d. per ton, and consumption per day 17 tons of first quality and 19 tons per day of the second quality.

$$\begin{array}{ccccccc} \text{Tons.} & & \text{Tons.} & & \text{s. d.} & & \\ 19 & : & 17 & :: & 18\ 6 & : & x \end{array}$$

Ans. £0 16s. 6·63d.

Ex. A coal bunker is 21 ft. 4 in. wide and 14 ft. 11 in. high; after steaming 12 days it is found that a section 16 ft. long has been burnt: what has been the daily consumption?

21 ft. 4 in. = 21·333 ft.; 14 ft. 11 in. = 14·9167

$$\frac{21{\cdot}333 \times 14{\cdot}9167 \times 16}{45 \times 12} = \frac{5091{\cdot}48736}{540} = 9{\cdot}4287 \text{ tons per day.}$$

Ex. If ·07 cubic feet of feed water is required per N.H.P. per minute, what quantity of coal is required per N.H.P. per hour, if 1 lb. of coal will evaporate 7·8 lbs. of water per hour?

$\cdot 07 \times 62{\cdot}5 = 4{\cdot}375$ lbs. weight of water required per N.H.P.

$\therefore 4{\cdot}375 \times 60$ minutes $\div 7{\cdot}8 = 33{\cdot}65$ lbs. of coal per hour.

And, if the consumption is 12 lbs. of coal per N.H.P. per hour, what will be the weight of water used as steam per minute?

$$\frac{12 \times 7{\cdot}8}{60} = 1{\cdot}76 \text{ lbs. of water per minute.}$$

Ex. At noon on Monday a ship sailed from a port, and went 228, 236, 240, 250, and 230 miles per day; supposing the voyage to be 2640 miles: what time and date will the ship arrive at her port of destination, and what coals will she burn, supposing 13 cwt. per hour to be consumed?

Distance steamed = 228 + 236 + 240 + 250 + 232 = 1186 miles.

Distance to steam = 2640 — 1186 = 1464 miles.

Distance steamed (on an average) per day = 1186 ÷ 5 = 237·2 miles.

Time to finish the voyage 1464 ÷ 2372 = $6{\cdot}172^{d}$, or $6^{d}\ 4^{h}\ 7{\cdot}7^{m}$.

Coals consumed per day = 13 cwt. × 24^{h} = 15 tons 12 cwt.

Coals to finish voyage = 15·6 tons × 6·172 = 96·2832 tons, or 96 tons 5·66 cwt.

Date of arrival will be Sunday, $6^{h}\ 4^{m}$ P.M.

QUESTIONS RELATING TO BOILERS, &c.

256. Given the number of feet the bottom of a boiler is below the water-line: to find what pressure of steam is requisite to blow off the brine through the blow-off cock.

RULE C.

Divide the number of feet and decimals of a foot, if any, which the bottom of the boiler is below the water-line by 2·305;* the quotient is the required pressure, expressed in pounds.

Example.

Ex. What pressure of steam in a boiler will be requisite to blow off the brine through the blow-off cock, the bottom of the boiler being 18 feet 6 inches below the water-line of the ship, allowing a column of water 2·305 feet to equal 1 lb.?

```
6 in. = ·5   12)60        18 ft. 6 in. = 18·5 ft.       2·305 ft. = 1 lb.
                ——
                ·5

                 2·305)18·500(8·026 lbs. pressure.
                       18440
                       ——————
                          6000
                          4610
                          —————
                          13900
                          13830
                          —————
```

Examples for Practice.

1.	18 feet 6 inches.	Height of a column of water always the same, viz., 2·305	4.	16 feet 5 inches.	Height of a column of water always the same, viz., 2·305
2.	12 " 6 "		5.	13 " 11 "	
3.	15 " 4 "		6.	16 " 8 "	

257. Given the diameter of the tubes, their length and number, also the dimensions of the plates, to find the amount of heating surface.

RULE CI.

1°. *Find the circumference of tubes* (see Rule LXXIV, page 183), *which multiply by their length in inches, and the product again by the number of tubes;* the result is the heating surface of the tubes.

2°. *Find the area of the plate in square inches, by multiplying the length by the breadth, (both in inches).*

---

* The position of the water level in the boiler often differs from that outside the ship; this would affect considerably the process of blowing out if the boiler be placed low in the ship; hence, calculations of the following nature are at times requisite. It has been noticed that atmospheric pressure at 14·75 lbs. of mercury, sustains a column of water, in a vertical tube, to the height of about 34 feet. We have then this analogy:—

$$14·75 \; : \; 34 \; :: \; \left\{\begin{array}{c}\text{any given pressure}\\ \text{of mercury}\end{array}\right\} \; : \; \left\{\begin{array}{c}\text{its corresponding}\\ \text{vertical column}\end{array}\right\}$$

whence it appears that 2·305 feet of water is balanced by 1 lb.

3°. *Find the area of the mouth of one tube, and multiply it by the number of tubes.*

NOTE.—There are two plates, but they are not entire, the openings of the tubes (found by No. 3°) must be deducted.

4°. *Deduct the area of the tube mouths* (No. 3°) *from the area of the plate* (No. 2°); the remainder is the heating surface of one plate, *this multiplied by* 2 is the heating surface of both plates.

5°. *Add together the heating surface of the two plates* (No 4°) *and the heating surface of the tubes* (No. 1°); the sum *divided* by 144 is the heating surface, in square feet, of the tube and tube-plates as required.

EXAMPLE.

Ex. Required the heating surface in the tubes and tube-plates of a boiler, the tubes being 2½ inches in diameter, 8 feet 3 inches in length, and 462 in number, and the plates 19 feet 8 inches long, by 8 feet 7 inches wide.

Heating surface of tubes = 2·5 inch × 3·1416 × 99 inch × 462.

3·1416
2·5 inches diameter.
157080
62832
7·854,00 circumference.
8 ft. 3 in. = 99 length of tubes (8 feet 3 inches)
12
— 70686
99 70686
777·546 heating surface of 1 tube.
462 number of tubes.
1555092
4665276
3110184
359226·252 heating surface in tubes.

To find the area of plate.

Length 19 ft. 8 in. = 236
Width 8 ft. 7 in. = 103
236
103
708
2360
Area of plate 24308
Area of tube mouths 2267·8225
Heating surface 1 plate 22040·1575
2
Heating surface 2 plates 44080·3150

To find the area of tube mouths.

·7854
2½² = 6·25
2·5
2·5 39270
15708
125 47124
50
4·90875,0
6·25 462
981750
2945250
1963500
Area of tube mouths 2267·8425,0

Heating surface in tubes 359226·252
Heating surface in plates 44080·315

$$144\left\{\begin{array}{l} 12)\overline{403306{\cdot}567}\text{ square inches.} \\ 12)\overline{33608{\cdot}880} \end{array}\right.$$

2800·740 square feet.

EXAMPLES FOR PRACTICE.

	Tubes.			Plates.	
No.	Diameter.	Length.	No.	Length.	Width.
1.	2½ in.	6 ft. 4 in.	130	6 ft. 5 in.	5 ft. 9 in.
2.	3¼	7 3	183	10 3	7 6
3.	2	6 6	145	6 9	5 0
4.	3½	7 2	126	6 4	4 10
5.	2¾	6 9	148	7 0	6 4
6.	3	6 0	241	12 0	7 3

258. Given the length and width of boiler, the pressure of steam in the boiler, and the height of the water in the boiler, to find the total pressure on the bottom of the boiler. It is also required to find the number of stays of given diameter.

RULE CII.

1°. *Divide the height of the water in the boiler by* 2·305 *feet* (See Rule C and Note); the result is the pressure of water per square inch in the boiler.

2°. *To the pressure of water* (No. 1°) *add the pressure of steam;* this gives the total pressure per square inch on the boiler.

3°. *Multiply the length of the bottom of the boiler by its breadth (both in inches),* and thus obtain the area of the bottom of the boiler.

4°. *Multiply the area by the total pressure per square inch;* the product is the total pressure on the bottom of the boiler.

5°. *Multiply the square of the diameter of the stays by* ·7854; the result is the sectional area (see Rule LXXVI, page 184).

6°. *Multiply this last result* (No. 5°) *by the strain that each square inch of section shall bear;* the product is the strain each stay bears.

7°. *Divide the total pressure* (No. 4°) *by the strain each stay bears* (No. 6°); the quotient is the number of stays required.

EXAMPLE.

Ex. What is the total pressure on the flat bottom of a boiler 10 feet 6 inches long, 4 feet 3 inches wide, the pressure of steam in the boiler being 35 lbs. per square inch, and the height of the water in the boiler 3 feet 9 inches? Also, the number of stays 2 inches in diameter required, so that each square inch of section of the stays shall bear 4500 lbs.?

(1) Height of water 3 feet 9 inches = 3·75 feet.

$\frac{3\cdot75}{2\cdot305}$ = 1·63 nearly, pressure of water per square inch.

2·305)3·750(1·63 nearly.
2·305

14450
13830

6200

(2) Pressure of water per square inch = 1·63 lbs.
" steam " = 35 lbs.

Total pressure per square inch = 36·63 lbs.

(3) Area = length × breadth in inches.

Length 10 feet 6 inches = 126 inches, breadth 4 feet 3 inches = 51 inches.

Area = 126 × 51 = 6424 square inches.

126
51

126
630

Area = 6426 square inches.

6426 square inches.
36·63 total pressure.

19278
38556
38556
19278

Total pressure on bottom of boiler = 235384·38 lbs.

Each stay bears $2^2 \times \cdot7854 \times 4500$ lbs.

·7854
2^2 = 4

3·1416 sectional area.
4500

157080
125664

14137·2,000 lbs. strain each stay bears.

No. of stays = $\frac{235384\cdot38}{14137\cdot2}$ = 17 nearly.

14137·2)235384·38(16·6
141372

940123
848232

918918
848232

The number of stays must evidently be 17, since 16 stays will not completely stay the boiler.

Examples for Practice.

No.	Length.		Width.		Pressure.	Depth water.		Stay.	lbs.
1.	9 ft.	6 in.	7 ft.	6 in.	20 lbs.	6 ft.	0 in.	1¼ in.	11200
2.	12	0	7	0	20	4	0	1½	6000
3.	17	6	11	3	32	3	8	2¼	4500
4.	19	7	7	8	18	6	3	2	5500
5.	23	2	18	9	30	9	5	1½	5400
6.	24	7	23	3	26	8	3	1·7	6000

Ex. A boiler measures 10½ feet long, and 15½ feet broad, rectangular; the water is 9 feet high, and one-third of that space is furnaces and tubes, the rest is water. The furnaces consume 10 cwt. of coal per hour, and each pound of coal is equal to heating 12000 pounds of water one degree Fahrenheit. The difference between the temperature of 10 pounds steam and that of 25 pounds steam is 28 degrees: in what time will the pressure rise from the former to the latter pressure, if all the valves are shut and the fires in action? (In the calculation, neglect the heat absorbed by the steam, and suppose the circulation of the water to be perfect. It should, however, be remembered that practically, the circulation is always imperfect, and especially so under the above circumstances; the pressure will, therefore, in practice, increase more rapidly than will be shown by the answer.)

Water space = 10·5 feet × 15·5 feet × 9 feet = 1464·75 cubic feet.
Deduct $\frac{1}{3}$ for furnaces and tubes = 1464·75 ÷ 3 = 488·25.
∴ actual water space = 1464·75 − 488·25 = 976·50.

Taking the weight of cubic foot of fresh water at 62·5 lbs., then the
Weight of water in boilers = 976·50 × 62·5 = 61031·25 lbs.
Coal required to heat it 1° = 61031·25 ÷ 12000 = 5·086
" " 28° = 5·086 × 28 = 142·408 lbs.
lbs. of coal consumed per hour 10 × 112 = 1120 lbs.

Then, as 1120 : 142·408 :: 60^m (= 1^h) : x
$x = 7·63^m$ nearly, or 7^m $37·8^s$.

Or, thus— Weight of water in the boiler = 61031·25 lbs.
Coal consumed per hour = 10 × 112 = 1120 lbs.
The difference in temperature of the steam at 10 lbs. and 28 lbs. = 28°

Then if one lb. of coal heats 12000 lbs. of water 1° in 1^h, it will raise 61031·25 lbs. $\frac{12000}{61031}$° in one hour.

1120 lbs. of coal will raise it 1120 × $\frac{12}{61}$° in one hour, that is, 220·3°. Then

$$220·3° : 28° :: 1^h (= 60^m) : x$$

$$\begin{array}{rl} & 60 \\ \hline 220·3)1680·0 & (\ 7·626^h \\ 15421 & \quad 60 \\ \hline 13790 & 37·560 \\ 13218 & \\ \hline 5720 & \\ 4406 & \\ \hline 13140 & \end{array}$$

Ans.: 7^m $37^s·6$.

Again, taking the above example, and supposing it to be salt water, then

Taking the weight of a cubic foot of salt water at 64 lbs., the weight of water in boiler equals 62496 lbs., which multiplied by the difference of temperature 28° = 1749888 the number of units of heat to raise the water in the boiler to 25 lbs. pressure.

The number of units of heat given out by furnaces in one minute is

$$\frac{10 \times 112 \times 12000}{60} = 224000.$$

$$\frac{1749888}{224000} = 7·812^m, \text{ or } 7^m\ 48^s·7.$$

EXAMPLES FOR PRACTICE.

No.	Length. ft.	Breadth. ft.	Height. ft.	Consumption per hour.	Pressure lbs.	Pressure. lbs.	Diff. of Temperature.	*Ans.*: Time required to raise the Pressure Fresh water.
1.	12	10	8	18 cwt.	22	60	61·8	6·130
2.	12	12	8	2½ "	37	55	20	17·144
3.	14	9	9	3 "	37	56	26	18·280
4.	12	10	7	2¼ "	37	55	20	13·888
5.	10	10	6	2 "	20	40	39·3	21·484
6.	20	15	11	6 "	20	50	44·7	4·157

EXAMPLE.

The stays on a flat sided boiler are 12 inches apart, centre to centre; some of these have wasted away to $\frac{7}{8}''$ diameter at the smallest section; if the pressure of steam carried be 30 lbs. per square inch, what strain is there upon the square inch of the section of these stays? (What is the total pressure upon the area supported by the stay? Divide that by the sectional area of the stay.)

Area of section of stay = diameter² × ·7854.

lbs. held up by 1 stay = pitch × pressure.

Strain per square inch of section = lbs. held up by 1 stay ÷ by area.

$$\therefore \text{strain} = \frac{\text{pitch}^2 \times \text{pressure}}{\text{area of sec. of stay}}$$

$$\text{Strain} = \frac{12^2 \times 30}{{\cdot}875^2 \times {\cdot}7854} = 7184{\cdot}4 \text{ lbs. per square inch.}$$

Pitch of stay 12
12
Pitch² = 144 square inches.
Pressure = 30 lbs.
4320 lbs. held up by 1 stay.

$\frac{7}{8}$ = ·875

·875 diameter.
·875
4375
6125
7000
·765625 = dia.²
·7854
3062500
3828152
6125000
5359375
·601321 = area of section of stay.

·6013)4320·00000(7184·4 lbs.
42091
11090
6013
50770
48104
26660
24052
26080

Answer: 7184·4 lbs. per square inch.

Note.—The greatest strain per square inch of section of stay allowed is 5000 lbs.; therefore the above strain is half as much again as it should be; and this shows that the pressure used (30 lbs.) is half as much again as it should be, therefore the pressure must be reduced to 20 lbs.

EXAMPLES FOR PRACTICE.

No.	Pitch of stays from centre to centre.	Stays have corroded away.	Working pressure of boilers.	No.	Pitch of stays from centre to centre.	Stays have corroded away.	Working pressure of boilers.
1.	16 inches.	$\frac{13}{16}$ inch.	25 lbs.	3.	16 inches.	1¼ inch.	26 lbs.
2.	14 „	1 „	28 „	4.	15 „	1$\frac{1}{16}$ „	20 „

Answers.

1. Pitch 16² × 25 = 16 × 16 × 25 = 6400 lbs. held up by 1 stay; $\frac{13}{16}$ = ·8125, and ·8125 × ·8125 (= ·66015625) × ·7854 = ·518486718750 area of section of stay; ∴ 6400 ÷ 5185 = 12343 lbs. per square inch.

2. Pitch 14²″ × 28 = 14 × 14 (= 196) × 28 = 5488 lbs. held up by 1 stay; 1² × ·7854 = 1 × 1 (= 1) × ·7854 = ·7854, the area of section of stay; 5488 ÷ ·7854 = 6987·5 lbs. per square inch.

3. Pitch 16″ × 16″ (= 256) × 26 = 6656 lbs. held up by 1 stay; 1¼″ = 1·25 × 1·25 (= 1·5625) × ·7854 = '1·22718750 = area of section of stay; ∴ 6656 ÷ 1·2272 = 5424 lbs. (nearly) per square inch.

4. Pitch 15″ × 15″ (= 225) × 20 = 4500 lbs. held up by 1 stay; 1$\frac{1}{16}$ = 1·0625, and 1·0625 × 1·0625 (= 1·12890625) × ·7854 = ·886642968740 area of stay; ∴ 4500 ÷ ·8866 = 5075 lbs. per square inch.

EXAMPLE.

A flat boiler bottom is 16 ft. by 12 ft., pressure 30 lbs. per square inch, 9 rows of stays and 7 stays in each row, allowing 5000 lbs. per square inch of section on the stays, what must be the diameter of the stays?

Note.—The sides of the boiler act as a row of stays, not a row at each end, but only half a row at each end, so that M rows + 1, and N stays + 1, must be used as divisors.

Length (in inches) × breadth (in inches) × pressure = total pressure on boiler.

No. of rows × No. of stays × 5000 = total pressure allowed on stays.

$$\therefore \frac{\text{total pressure on boiler}}{\text{total pressure allowed on stays}} = \text{area of stays.}$$

$$\text{and } \sqrt{^2}\,\frac{\text{area of stays}}{\cdot 7854} = \text{diameter of stays.}$$

Length 16 ft. × 12 = 192 inches.

Breadth 12 ft. × 12 = 144 inches.

9 rows + 1 row = 10 rows.
7 stays + 1 stay = 8 stays.
80 stays.
5000
400000 = total pressure allowed on stays.

192
144
768
768
192
27648 square inches.
30
400000)829440 total press. on boiler.
2·0736 area of stays.

Otherwise:—
80)27648
345·6 sq. in. held up by each stay.
30 lbs.
5000)10368·0 lbs. held up by 1 stay.
2·0736 = area of stays.

·7854)2·07360(2·64 = diameter^2
15708
50280
47124
31560
31416

1	2·64(1·6 inches.
1	1
26	1·64
6	156
32	8

Ans: 1·6 inches, the diameter of stay.

Examples for Practice.

No.	Length. ft.	in.	Breadth. ft.	in.	No. of Rows of Stays.	No. of Stays in one row.	Pressure of steam. lbs.	Press. each Stays to bear. lbs.	*Ans.:* Dia. of stay. in.
1.	9	6	7	6	4	4	20	5000	1·446
2.	12	0	7	0	4	6	20	5000	1·32
3.	17	6	11	3	6	9	32	5000	1·82
4.	19	7	7	8	6	4	18	5000	1·68
5.	23	2	18	9	22	11	30	5000	1·31
6.	24	7	23	3	12	15	26	5000	1·62

To find the pressure that may be carried in a flat bottomed boiler.

Examples.

A flat bottomed boiler is 15 ft. 6 in. by 12 ft. 7 in., water 9 ft. 6 in., diameter of stays 1¼ in., and 72 in number: find the steam pressure per sq. inch the boiler may be worked at, the strain on the stays being 5000 lbs. per sq. inch of section.

(*a*). No. of inches held up by 1 stay $= \frac{\text{length} \times \text{breadth}}{\text{No. of stays}}$

(*b*). Lbs. pressure on each stay = diameter^2 × ·7854 × strain.

(*c*). Total pressure $= \frac{a}{b}$

15 ft. 6 in. × 12 ft. 7 in.
12 12
186 151
151
186
930
186
28086 area.

$$72 \left\{ \begin{array}{l} 9)28086 \\ \hline 8)3120{\cdot}6 \\ \hline \quad 390{\cdot}1 \end{array} \right.$$

Or 390 sq. inches held up by 1 stay (*a*).

(*b*). 1·25 × 1·25 × ·7854 × 5000 lbs. = 6135·9375 lbs. pressure held up by 1 stay.

390)6136(15·7 lbs. per sq. inch.
390
2236
1950
2860

feet. feet.
2·305)9·500(4·1 lbs.
9220
2800
2305

∴ Total pressure 15·7 lbs.
Water „ 4·1 „
Steam „ 11·6 „

The stays on the flat sides of a steam boiler are 1½ inches diameter, and they are spaced 12 inches apart, centre to centre: what should the safety-valve be loaded at if the strain per square inch of stay section is not to exceed 5000 lbs. per square inch?

$$\text{Load} = \frac{1\tfrac{1}{2}^2 \times {\cdot}7854 \times 5000}{12^2}$$

Dia. 1½″ = 1·5
1·5
75
15
Dia.² = 2·25

2·25
·7854
900
1125
1800
1575
1·767150 area of stay
5000 = strain
8835·750000 lbs. press. on 1 stay

Pitch 12″
12
144 sq. in. of surface held up by 1 stay

$$144 \left\{ \begin{array}{l} 12)8835{\cdot}75 \\ \hline 12)736{\cdot}312 \\ \hline \end{array} \right.$$

61·359 lbs. press. per sq. in.

The stays of a flat sided boiler are 1⅞ inches diameter, and the pressure carried is 50 lbs. per square inch: how many square inches of the side of the boiler is one of these stays sufficient for, allowing a strain of 5000 lbs. per square inch of section?

Area of stay = 1·875 × 1·875 × ·7854 = 2761171875.

Strain on stay = 2·761171875 × 5000 = 13805·859375.

Space held up by a stay = 13805·859375 ÷ 50 = 276·117 square inches.

Examples for Practice.

No.	Diameter of Stays.	Pitch of Stays.	Strain of sq. inch of Section.	*Ans:* Load on Safety valve.
1.	1¼ inches.	18 inches.	5000 lbs.	18·938 lbs.
2.	1 „	10 „	5000 „	39·27 „
3.	1½ „	15 „	5000 „	39·27 „
4.	2½ „	20 „	5000 „	61·359 „
5.	1¼ „	13½ „	5000 „	33·66 „
6.	1¾ „	14½ „	5000 „	57·20 „
7.	2⅛ „	19 „	5000 „	49·121 „
8.	1⅞ „	17½ „	5000 „	45·08 „

To find the pressure that may be carried in a circular boiler.

The pressure that may be carried in a circular boiler depends upon the tensile strength, or bursting strain, of the plates, their thickness, and the diameter of the boiler. The tensile strength of best

Yorkshire plates is 26 tons,
Staffordshire „ 20 „
2)46
Mean 23 tons.

The mean of these is 23 tons per sq. inch of sectional area, which is the allowance made by the Board of Trade.

Given the diameter of a circular boiler and thickness of plate, &c., to find what pressure is to be carried.

RULE CIII.

Multiply the given strain by twice the thickness of plates in inches, and divide the product by diameter of boiler in inches.

Or, $P = \frac{s \times 2t''}{d''}$ where P = pressure,
s = strain,
t'' = thickness of plates,
d'' = diameter of boiler in inches.

EXAMPLE.

Required the pressure to be carried in a circular boiler, the diameter of which is 12 feet, and thickness of plates $\frac{7}{8}$ of an inch, seams double rivetted.

1 ton = 2240 lbs,

Strain in lbs. = 23 × 2240 = 51520 lbs.

$$\therefore \text{Pressure} = \frac{23 \times 2240 \times \frac{7}{8} \times 2}{12 \times 12} = 626 \text{ lbs.}$$

2240 lbs.
23 tons
672
448
51520 lbs.

51520
7
8)360640
45080
2
144 { 12)90160
12)7513
626 lbs.

Supposing there are no seams the above is the bursting pressure; but since the boiler is formed of plates joined by rivets, and that pieces of the plates have been drilled out to allow the rivets to enter, the plates cannot necessarily be as strong as before, as they have a less sectional area in consequence of these rivet holes.

It is usual to allow 70 per cent. of the full pressure to represent the strength at the seams when they are double rivetted, and 56 per cent. when they are single rivetted.

In the preceding example the seams, it will be observed, are double rivetted.

Then, since full bursting pressure = 626 lbs.,

∴ 70 per cent. of this will equal the bursting pressure at seams,

i.e., 626 × 70 ÷ 100 = 438·2 lbs. the bursting pressure at seams.

The working pressure is between $\frac{1}{6}$ and $\frac{1}{8}$ of the bursting, depending upon the nature of the iron, workmanship, &c. Then

$$6\overline{)438{\cdot}2}$$
$$73{\cdot}0 = \text{working pressure.}$$

THE STRENGTH OF SEAMS.

In the above example we have taken the allowance for seams to be 70 and 56 per cent. This is not always correct, as the real strength will depend upon the proper proportions being maintained between the thickness of the plate and the diameter, and the pitch of the rivets.

The strength of the joints is found by the following method:—

$$\frac{\text{Pitch} - \text{dia. of rivet}) \times 100}{\text{pitch}} = \text{per centage of plate at joint compared with solid plate.}$$

$$\frac{(\text{Area of rivet} \times \text{No. of rows of rivets} \times 100}{\text{pitch} \times \text{thickness of plate.}} = \text{per centage of strength of rivets compared with}$$
solid plate.

NOTE.—If the rivets are exposed to double shear, multiply the last per centage by 1·5. Then the smallest of the two per centages is to be taken as the strength of the joint.

EXAMPLES.

EX. 1. In a double-rivetted seam the rivets are $\frac{13}{16}$ inch diameter, and placed at $2\frac{1}{4}$ inch pitch, the plates are $\frac{1}{2}$ inch thick: what per centage of strength has the seam as compared with the strength of the solid plate? (Neglect friction and take material of rivets and of plates as of equal strength).

$$\frac{(\text{Pitch} - \text{dia. of rivets}) \times 100}{\text{pitch}} = \frac{(2{\cdot}25 - {\cdot}8125) \times 100}{2{\cdot}25} = 63{\cdot}89 \text{ per cent. of strength of solid plate.}$$

$$\begin{array}{r} 2{\cdot}25 \\ -\ {\cdot}8125 \\ \hline 1{\cdot}4375 \\ 100 \\ \hline 143{\cdot}7500 \end{array} \qquad\qquad \begin{array}{l} 2{\cdot}25)143{\cdot}7500(63{\cdot}89 \text{ nearly.} \\ \quad\ 1350 \\ \quad\ \overline{875} \\ \quad\ 675 \\ \quad\ \overline{2000} \\ \quad\ 1800 \\ \quad\ \overline{2000} \\ \quad\ 2025 \end{array}$$

Then the lesser of these, viz., 63·89 per cent. will be the strength of the joint.

Diameter of rivets $\frac{13}{16}$.

$$\frac{13}{16}\overline{)13{\cdot}0}$$
$$16\begin{cases} 4\overline{)13{\cdot}0} \\ 4\overline{)3{\cdot}25} \end{cases}$$
$${\cdot}8125$$

Pitch $2\frac{1}{4}$ inches.

$$\tfrac{1}{4}\overline{)10}$$
$${\cdot}25$$
$$\therefore\ 2\tfrac{1}{4} = 2{\cdot}25 \text{ inches.}$$

Area of rivet, &c. $= \frac{\cdot 8125 \times \cdot 8125 \times \cdot 7854 \times 2 \times 100}{2\cdot 25 \times \cdot 5} =$ 92·17 per centage of strength of seam.

```
Diameter of rivet 13/16 = ·8125                  Pitch  2·25
                          ·8125       Thickness of plate  ·5
                         ——————                        —————
                         40625                         1·125
                        16250
                        8125               1·125)103·6972(92·17
                       65000                      10125
                      ————————                   ——————
                      ·66015625                    2447
                        ·7854                      2250
                     —————————                   ——————
                     264062500                     1972
                    330078125                      1125
                   528125000                     ——————
                  462109375                        8470
                 ——————————————                    7875
                 ·518486,71875                   ——————
                   200 (= 2 × 100)                  595
                 ——————————————
                 103·6972              Ans.: 92·17 per cent.
```

Otherwise, expressed pitch and diameter of rivets each in sixteenths, we have—

$$\frac{\text{pitch} - \text{dia. of rivets}}{\text{pitch}} = \frac{(2\frac{1}{4} - \frac{13}{16}) \times 100}{2\frac{1}{4}} = \frac{(\frac{36}{16} - \frac{13}{16}) \times 100}{\frac{36}{16}} = \frac{\frac{23}{16} \times 100}{\frac{36}{16}}$$

$$= \frac{23000}{36} = 63\cdot 89 \text{ nearly.}$$

$$\frac{\text{Area of rivet} \times \text{No. of rows} \times 100}{\text{pitch} \times \text{thickness of plate}} = \frac{\frac{13}{16} \times \frac{13}{16} \times \cdot 7854 \times 2 \times 100}{2\frac{1}{4} \times \frac{1}{2}} = \frac{\frac{13}{16} \times \frac{13}{16} \times \cdot 7854 \times 2 \times 100}{\frac{36}{16} \times \frac{8}{16}}$$

$$= \frac{13 \times 13 \times \cdot 7854 \times 2 \times 100}{36 \times 8} = \frac{26546\cdot 52}{288} = 92\cdot 17 \text{ per cent.}$$

Ex. 2. In a treble rivetted boiler the rivets are $\frac{3}{4}$ inch diameter and placed at $2\frac{5}{16}$ inches pitch, the plates are $\frac{11}{16}$ inch thick: what per centage of strength has the seam as compared with the strength of the solid plate? (Neglect friction and take the materials of rivets and of plates as of equal strength).

$$\text{Plate} = \frac{(\text{pitch} - \text{dia. of rivets}) \times 100}{\text{pitch}} = \frac{(2\frac{5}{16} - \frac{12}{16}) \times 100}{2\frac{5}{16}} = \frac{(\frac{37}{16} - \frac{12}{16}) \times 100}{\frac{37}{16}}$$

$$= \frac{\frac{25}{16} \times 100}{\frac{37}{16}} = \frac{2500}{37} = 67\cdot 57 \text{ per cent.}$$

$$\text{Rivets} = \frac{\text{area} \times \text{No. of rows}}{\text{pitch} \times \text{thickness of plate}} = \frac{\frac{12}{16} \times \frac{12}{16} \times \cdot 7854 \times 3 \times 100}{\frac{37}{16} \times \frac{11}{16}}$$

$$= \frac{12 \times 12 \times \cdot 7854 \times 3 \times 100}{37 \times 11} = \frac{33929\cdot 28}{407} = 83\cdot 36 \text{ per cent.}$$

EXAMPLES FOR PRACTICE.

No.	Dia. of rivet Inch.	Pitch. Inch.	Thickness of plate. Inch.	*Answer:* Per cent. of plate.	Per cent. of rivet.
1.	$\frac{7}{8}$	$2\frac{1}{2}$	$\frac{9}{16}$	65·	85·52
2.	$1\frac{1}{4}$	3	$\frac{7}{8}$	58·33	93·5
3.	$1\frac{7}{16}$	$3\frac{1}{4}$	$1\frac{1}{8}$	55·77	88·77
4.	1	$3\frac{1}{2}$	$\frac{11}{16}$	71·43	65·28

259. To find the number of rivets required for a single rivetted cylindrical boiler, giving the courses of plates, the number of plates in each course, the number of holes along the side of each plate, and also along each end.

RULE CIV.

1°. Multiply *the number of holes along the side by the number of* plates, *and from the product* deduct *the number of* plates *(that is, deduct one hole for each landing), and this* remainder *being* multiplied *by the* number of courses, increased *by one*, the product is the number of rivets in circular seams.

2°. Multiply *the* number *of end holes by the* number *of plates, and* from *the* product take twice *the* number *of plates, the* remainder *being* multiplied *by the* number *of courses,* gives *the number of rivets in* longitudinal seams.

3°. Add *the results obtained in* Nos. 1° and 2° *together, that is,* add *together the* rivets in circular seams, *and the* rivets in longitudinal seams, *the* sum *of these is* the total number of rivets required for all the holes.

Or thus:—

1°. *Multiply the number of courses by* one less *than the* number *of* end holes, *and* add 1 *to the product, the* sum *multiplied by the* number of plates *gives the number of rivets in horizontal seams.*

2°. *Next, multiply the number of plates by* one more *than the* number of courses, *and this again by* two less *than the number of* side seams, *the product is the number of circular seams.*

3°. *Add together the results obtained by* No. 1° *and* No. 2°, *the sum is the total number of rivets required.*

EXAMPLE.

The shell of single rivetted cylindrical boilers consists of 9 courses of plates with 4 plates in each course; the number of holes along the side of each plate is 42, the number along each end of each plate is 18, how many rivets will be required for all these plates?

$42 \times 4 - 4 \times 10 = 1640$, the number of rivets in circular seam.

$18 \times 4 - 8 \times 9 = 576$, the number of rivets in longitudinal seam.

$\therefore 1640 + 576 = 2216$ rivets, the total number required.

Side holes	42	End holes	18
Multiply No. of plates	4	Multiply by No. of plates	4
	168		72
Deduct No. of plates	− 4	Deduct twice No. of plates	− 8
	164		64
Multiply by No. of courses + 1	10	Multiply by No. of courses	9
	1640 rivets in circular seam.		576 rivets in longitudinal seam.

Rivets in circular seam	= 1640
„ longitudinal seam	576
Total number required	2216

Or thus:—

If there are 9 courses there must be 10 circular seams; then 42 holes × 4 plates — 4 (deducting one hole for each plate) × 10 seams = 168 — 4 × 10 = 164 × 10 = 1640 rivets for circular seams; and since there are 4 plates to each course, and there are 9 courses, there must be 4 × 9 = 36 cross seams altogether. Then 36 seams × 18 holes = 648 rivets in cross seams, but the corner rivets in these cross seams are also counted in the circular seams; therefore, for every cross seam 2 rivets must be left out in the calculation, that is, 36 cross seams × 2 = 72 rivets, which must be subtracted from the 648 cross seams = 648 — 72 = 576: then, 1640 rivets for circular seams + 576 rivets in cross seams = 2216 rivets, the total number required.

Or thus:—

$$\begin{array}{lcl}
\text{Horizontal seams} & = 4\left\{(9 \times 17) + 1\right\} = & 616 \\
\text{Vertical seams} & = 4 \times 10 \times 40 & = 1600 \\
\hline
\text{Total number of rivets} & & = 2216
\end{array}$$

$$\begin{array}{lr}
\text{Courses} & 9 \\
\text{End holes, less } 1 = & 17 \\
\hline
 & 153 \\
 & +\ 1 \\
\hline
 & 154 \\
\text{No. of plates} & 4 \\
\hline
\text{Horizontal seams} & 616 \text{ rivets.}
\end{array}
\qquad
\begin{array}{lr}
\text{No. of plates} & 4 \\
\text{Course} + 1 & 10 \\
\hline
 & 40 \\
\text{Side seam, less } 2 = & 40 \\
\hline
\text{Vertical seams} & 1600 \text{ rivets.}
\end{array}$$

$$\begin{array}{lr}
\text{Horizontal seams} & 616 \text{ rivets.} \\
\text{Vertical seams} & 1600 \quad ,, \\
\hline
\text{Total number} & 2216 \quad ,,
\end{array}$$

EXAMPLES FOR PRACTICE.

No.	No. of courses.	No. of Plates in each course.	No. of side holes.	No. of end holes.	*Ans.:* No. of rivets.
1.	7	6	36	14	2184
2.	8	8	44	20	4248
3.	10	10	46	27	7450
4.	9	4	39	21	2204
5.	8	6	28	17	2178
6.	12	4	26	12	1784

Ex. What is the collapsing pressure on a furnace tube whose thickness is $\frac{7}{16}$ of an inch, diameter 2 ft. 11 in., length 10 ft. 3 in.?

RULE CV.

1°. *Square the thickness and multiply by* 806300.

2°. *Multiply length in feet by diameter in inches.*

3°. *Divide the first product by the second.*

$$\frac{\frac{7}{16}^2 \times 806300}{10\frac{1}{4} \text{ ft.} \times 35 \text{ in}} = \frac{\cdot4375^2 \times 806300}{10\cdot25 \times 35} = \frac{154330\cdot859375}{358\cdot75} = 430\cdot19 \text{ lbs.}$$

QUESTIONS RELATING TO PUMPS, &c.

260. Given the diameter and length of the stroke of a brine-pump, also the number of strokes per minute, and the ratio of the fulness of the pump, to find how many cubic feet of water it will extract in an hour.

RULE CVI.

1°. *Find the area of a horizontal section of the pump by multiplying the square of the diameter by* ·7854.

2°. *Multiply this by the inches in the stroke; the product is the capacity of the pump.*

3°. *Next, multiply this product* (No 2°) *by the fraction it is full;* the result is the quantity delivered at one stroke.

4°. *Next, find the quantity delivered in an hour by multiplying the last* result (No. 3°) *by the strokes per minute, and by* (60) *the number of minutes in an hour.*

5°. *Divide the cubic inches delivered in an hour as found above* (No. 4°) *by the number of cubic inches in a cubic foot* (1728); the result is the cubic feet of water discharged by the pump in an hour.

EXAMPLES.

Ex. 1. How many cubic feet of water will be extracted in an hour by a brine-pump 3 inches in diameter and 10 inches stroke, making 18 strokes per minute; the pump being ⅔ full each stroke?

1. Capacity of pump $= \cdot 7854 \times 3^2 \times 10 = 70\cdot 686$ cubic inches.
2. $\therefore$ quantity delivered at each stroke $= \frac{2}{3} \times 70\cdot 686 = 47\cdot 124$ cubic inches.
3. And quantity delivered in an hour $= 18 \times 60 \times 47\cdot 124 = 50893\cdot 92$.
4. But a cubic foot is equal to 1728 cubic inches $\therefore$ cubic feet discharged $= \frac{50893\cdot 92}{1728}$.

1.
$$\begin{array}{r} \cdot 7854 \\ 9 \\ \hline 7\cdot 0686 \\ 10 \\ \hline 70\cdot 6860 \end{array}$$
cubic inches.

2.
$$\begin{array}{r} 70\cdot 686 \\ 2 \\ \hline 3)141\cdot 372 \\ \hline 47\cdot 124 \end{array}$$
cubic inches.

3.
$$\begin{array}{r} 47\cdot 124 \\ 60 \\ \hline 282\cdot 744{,}0 \\ 18 \\ \hline 2261952 \\ 282744 \\ \hline 50893\cdot 92 \end{array}$$
cubic inches.

4.
$$1728 \left\{ \begin{array}{r} 12)50893\cdot 92 \\ \hline 12)4241\cdot 16 \\ \hline 12)353\cdot 43 \\ \hline 29\cdot 45 \end{array} \right.$$
cubic feet.

An examination of the foregoing solution will show that the quantities ·7854 and 60 occur as constant multipliers, and that 1728 is a constant divisor. Now, if we divide the product of ·7854 × 60 by 1728, the constant multiplier ·02727 is obtained, and the work will be somewhat abridged. The work will stand thus:—

$3^2 \times 10 \times \frac{2}{3} \times 18 \times \cdot 02727 = 9 \times 10 = (90) \times \frac{2}{3} = (60) \times 18 = (1080) \times \cdot 02727 =$ 29·4516 cubic feet.

Ex. 2. How many cubic feet of water will be extracted in an hour by a brine-pump $3\frac{1}{4}$ inches in diameter and 12 inches stroke, making 15 strokes a minute; the pump being $\frac{3}{4}$ full at each stroke?

$3\frac{1}{4}$ inches $= 3{\cdot}25$ and $3{\cdot}25^2 = 10{\cdot}5625$.

1. Capacity of pump $= {\cdot}7854 \times 10{\cdot}5625 \times 12 = 99{\cdot}54945$.
2. $\therefore$ quantity delivered at each stroke $= \frac{3}{4} \times 99{\cdot}54945 = 74{\cdot}66209$.
3. And quantity delivered in an hour $= 15 \times 60 \times 74{\cdot}6621 = 67195{\cdot}8900$.
4. But a cubic foot equals 1728 cubic inches $\therefore$ cubic feet discharged $= \frac{67195{\cdot}89}{1728}$.

Examples for Practice.

No.	Diameter. of Pump.	Stroke.	No.	Ratio.	No.	Diameter. of Pump.	Stroke.	No.	Ratio.
1.	$3\frac{1}{4}$ inches	12 inches	15	$\frac{2}{3}$	7.	4 inches	40 inches	25	$\frac{4}{5}$
2.	$3\frac{1}{2}$ „	14 „	12	$\frac{3}{4}$	8.	$5\frac{1}{2}$ „	20 „	30	$\frac{6}{7}$
3.	31 „	30 „	36	$\frac{1}{3}$	9.	$7\frac{3}{8}$ „	17 „	27	$\frac{4}{5}$
4.	$2\frac{1}{2}$ „	30 „	50	$\frac{5}{8}$	10.	$6\frac{5}{8}$ „	18 „	12	full
5.	10 „	21 „	18	$\frac{1}{2}$	11.	$5\frac{1}{2}$ „	$15\frac{1}{2}$ „	35	$\frac{3}{4}$
6.	$3\frac{5}{8}$ „	$12\frac{1}{2}$ „	15	$\frac{2}{3}$	12.	$12\frac{3}{4}$ „	30 „	21	$\frac{5}{8}$

261. **Given the diameter and the length of the stroke of a pump, also the number of revolutions per minute, and the ratio of the fulness of the pump, to find how long it will take to pump a ballast tank out containing a given number of tons of sea water.**

RULE CVII.

1°. ***Proceed according to*** **Rule CVI, Nos. 1° to 5°, inclusive.**

2°. ***Reduce the tons of water contained in the tank to cubic feet by multiplying them by the number of cubic feet*** **(35)** ***in a ton.***

3°. ***Divide this last product by the cubic feet of water discharged by the pump in an hour*** **(No. 2°); the quotient is the time required to discharge the tank.**

Examples.

Ex. 1. How long will it take to pump a ballast tank out containing 150 tons of sea water, the diameter of the ballast donkey pump is 5 inches, and length of stroke 10 inches (double-acting), making 100 strokes per minute, $\frac{1}{4}$ of the pump being empty space?

Note.—The question as worded at the examinations is 100 strokes, but it is properly 100 revolutions, and as it is double-acting it must be doubled.

1. Capacity of pump $= {\cdot}7854 \times 5^2 \times 10 = 196{\cdot}35$ cubic inches.
2. $\therefore$ quantity delivered at each stroke $= \frac{3}{4} \times 196{\cdot}35 = 147{\cdot}2625$.
3. And quantity delivered in an hour $= (100 \times 2) \times 60 \times 147{\cdot}2625 = 1767150$.
4. But a cubic foot is equal to 1728 cubic inches, $\therefore$ cubic feet discharged in an hour $= \frac{1767150}{1728} = 1022{\cdot}656$.
5. Cubic feet contained in tank $= 150 \times 35 = 5250$ cubic feet.
6. Time required to discharge water in tank $= \frac{5250{\cdot}000}{1022{\cdot}656} = 5^h\,8^m\,1^s$.

$\frac{1}{4}$ pump empty leaves $\frac{3}{4}$ pump full.

$$\begin{array}{lrr} 1. & & {\cdot}7854 \\ & 5^2 & 25 \\ \hline & & 39270 \\ & & 15708 \\ \hline & & 19{\cdot}6350 \\ & & 10 \\ \hline & & 196{\cdot}3500 \text{ cubic inches.} \end{array} \qquad \begin{array}{lr} 2. & 196{\cdot}35 \\ & 3 \\ \hline & 4)58905 \\ \hline & 147{\cdot}2625 \end{array}$$

$147{\cdot}2625 =$ cubic inches discharged each stroke.

3\.
$$\begin{array}{r} 147{\cdot}2625 \\ (100 \times 2) = \quad 200 \\ \hline 29452{\cdot}5{,}000 \\ 60 \\ \hline 1767150{\cdot}0 \end{array} = \text{cubic inches discharged.}$$

4\.
$$1728 \left\{ \begin{array}{r} 12)1767150 \\ \hline 12)147262{\cdot}5 \\ \hline 12)12271{\cdot}875 \\ \hline 1022{\cdot}656 \end{array} \right. = \text{cubic feet discharged.}$$

5. Tank contains 150 tons.
1 ton = 35 cubic feet.

$$\begin{array}{r} 750 \\ 450 \\ \hline \end{array}$$

Tank contains 5250 cubic feet.

Ans.: $5^h\ 8^m\ 1^s$.

$$\begin{array}{r} 1022{\cdot}656)5250{\cdot}000(5^h \\ 5113280 \\ \hline 136720 \\ 60 \\ \hline 8203200(8^m \\ 8181248 \\ \hline 21952 \\ 60 \\ \hline 1317120(1^s \\ 1022656 \\ \hline \end{array}$$

Ex. 2. How long will it take to pump a ballast tank out containing 165 tons of water the diameter of the ballast donkey-pump is 6 inches, and length of stroke 12 inches (double acting), making 120 strokes per minute, $\frac{1}{8}$ of the pump being empty space.

$$\frac{6^2 \times {\cdot}7854 \times 12 \times \frac{7}{8} \times 60 \times 120 \times 2}{1728 \times 35} = 70{\cdot}686 \text{ tons pumped per hour.}$$

$$\therefore \frac{165}{70{\cdot}686} = 2^h\ 20^m\ 3^s, \text{ time required to pump the tank out.}$$

Ans.: $6 \times 6 \times {\cdot}7854\ (= 28{\cdot}2744) \times 12 \times \frac{7}{8}\ (= 296{\cdot}8812) \times 60 \times 120\ (= 2137544{\cdot}64) \times 2 = 4275089{\cdot}28 \div 1728 \div 35 = 70{\cdot}686$ tons pumped per hour.

165 tons $\div 70{\cdot}686 = 2^h\ 20^m\ 3^s$, the time required to pump the tank out.

EXAMPLES FOR PRACTICE.

No.	Diameter of pumps.	Length of stroke. (Double-acting).	No. of strokes per minute.	Empty space in pumps.	Ballast tank contains.	*Ans.*: Time required to pump the tank out.
1.	9 inches.	10 inches.	70	$\frac{1}{8}$	240 tons.	$3^h\ 37^m\ 18^s$
2.	9 "	14 "	120	$\frac{1}{8}$	231·43 "	1 27 18

EXAMPLES.

Ex. 1. The water ballast donkey working alone can discharge the water ballast in 2 hours; the boiler donkey working alone can discharge it in 8 hours: in what time will it be discharged if both pumps are working?

In 1 hour 1st empties $\frac{1}{2}$ of tank.
" 2nd " $\frac{1}{8}$ "

Hence in 1 hour both together discharge

$$\tfrac{1}{2} + \tfrac{1}{8} = \tfrac{4}{8} + \tfrac{1}{8} = \tfrac{5}{8} \text{ of tank,}$$

and the whole work 1 divided by $\frac{5}{8}$ gives the hours, &c., they will require to do it jointly; thus:—

$$1 \div \tfrac{5}{8} = \frac{1}{\tfrac{5}{8}} = 1 \times \tfrac{8}{5} = 5)8$$

$$\begin{array}{r} \hline 1^h\ 3 \\ 60 \\ \hline 5)180 \\ \hline 36^m \end{array}$$

Ans.: $1^h\ 36^m$

NOTE.—To divide by a fraction, invert the divisor and multiply.

Or, stating the question directly we should say

$$\tfrac{5}{8} : 1 :: \text{time required} : 1^h$$

But the greater the fraction representing the hourly work done, the smaller must be the time required for any given quantity of work.

Hence $\frac{5}{8} : 1 :: 1^h$: time required $= \frac{8}{5}$ of 1^h, and $\frac{8}{5}$ of $1^h = 1^h\ 36^m$.

Or again, here 1^h in which the fraction $\frac{5}{8}$ is done is obviously to be increased in that ratio which will turn $\frac{5}{8}$ into 1, or the whole, and this ratio is $\frac{8}{5}$, for $\frac{8}{5} \times \frac{5}{8} = 1$.

The following rule will perhaps be preferred by some as being simpler than the preceding methods.

RULE CVIII.

1°. *Take the* sum *of the times in which they* separately *discharge the tank.*

2°. *Find the* product *of these* times.

3°. Divide *the* product *found by* No. 2° *by the* sum *found in* No. 1°; the quotient is the time they will require to do it jointly.

Then (ex. above) since they separately discharge the tank in 2 hours and 8 hours respectively, we have

$2^h + 8^h = 10^h$

and $2^h \times 8^h = 16^h$ Then 10)16

$1^h\ 6$

60

10)360

36^m *Ans.:* $1^h\ 36^m$

Ex. 2. The water ballast donkey working alone can discharge the water ballast in 2 hours, the boiler donkey working alone can discharge it in 9 hours, in what time will it be discharged if both pumps are working?

In 1 hour the 1st discharges $\frac{1}{2}$ tank.

" 2nd " $\frac{1}{9}$ "

Hence they both together discharge

$$\frac{1}{2} + \frac{1}{9} = \frac{9}{18} + \frac{2}{18} = \frac{11}{18} \text{ of tank.}$$

Then since they both (together) discharge $\frac{11}{18}$ of the tank in 1 hour, the whole work 1 divided by $\frac{11}{18}$ gives the hours, &c., they will require to do it jointly. Here, again

$\frac{1}{\frac{11}{18}} = \frac{18}{11}$)18

$1^h\ 7$

60

11)420 *Ans.:* $1^h\ 38^m{\cdot}2$

$38^m{\cdot}2$ nearly.

Or, since they separately discharge the tank in 2 hours and 9 hours respectively, we have

$2^h + 9^h = 11^h$

$2^h \times 9^h = 18^h$ 11)18

$1^h\ 7$

60

11)420 *Ans.:* $1^h\ 38{\cdot}2^m$

$38^m{\cdot}2$

EXAMPLES FOR PRACTICE.

In each of the following examples given the times in which the ballast donkey and boiler donkey can separately discharge the water ballast tank, to find the time in which both can do the work.

No.	Ballast Donkey. h	m	Boiler Donkey. h	m	*Ans.:* Time both can pump it out. h	m
1.	3	0	7	0	2	6
2.	4	30	6	30	2	39·5
3.	2	15	9	0	1	48
4.	2	40	6	50	1	55·10
5.	2	10	4	15	1	26·1
6.	3	30	5	45	2	10

262. Given the diameter of cylinder, number of revolutions, length of stroke, and the cut-off of a pair of engines, to find the cubic feet of steam used.

RULE CIX.

1°. *Square the diameter of cylinder, multiply this by* ·7854, *by the number of revolutions per minute, by* 60, *by* double *the stroke in inches, by the cut-off, and by* 2 *(being a pair of engines)*, the result is cubic inches of steam used.

2°. *Divide this last by* 1728, the quotient is the cubic feet of steam used.

EXAMPLES.

Ex. 1. How many cubic feet of steam will be used per hour by a pair of engines making 47 revolutions per minute, the diameter of the cylinders being 42 inches, the stroke 42 inches, the cut-off being at ½ of the stroke from the beginning?

$$\text{Cubic feet of steam} = \frac{42^2 \times \cdot7854 \times 47 \times 60 \times 84 \times \frac{1}{2} \times 2}{1728}$$

42″ diameter.
42
84
168
1764 = diameter²
·7854
7056
8820
14112
12348
1385·4456 = area of base.

1385·4456
47 revolutions.
96981192
55417824
65115·9432
60ᵐ
3906956·5920
84 twice the stroke.
156278263680
312556527360
½)328184353·728,0
164092176·864
2
1728 { 12)328184353·738 cubic inches.
12)27348696·144
12)2279058·012

Stroke 42″
2
84

Ex. 2. How many cubic feet of steam will be used per hour by a pair of engines making 74 revolutions per minute, the diameter of the cylinders being 63¾ inches, the stroke 36 inches, the cut-of being at ⅚ of the stroke from the beginning?

$$\text{Cubic feet of steam} = \frac{63\tfrac{3}{4}^2 \times {\cdot}7854 \times (36 \times 2) \times 74 \times \tfrac{5}{6} \times 60 \times 2}{1728}$$

Ans.: 63·75 × 63·75 (= 4064·0625) × ·7854 (= 3191·91468750) × twice stroke in inches, viz., 72 (= 229817·8575) × 74 (= 17006521·455) × 5 ÷ 6 (= 14172101·2125) × 60 (= 850326072·75) × 2 = 1700652145·5 cubic inches, and this last divided by 1728 (the number of cubic inches in a cubic foot) gives 984173·6953125 cubic feet of steam.

Ex. 3. How many cubic feet of steam will be used per hour by a pair of engines making 54 revolutions per minute, the diameter of the cylinders being 42 inches, the stroke 36 inches, the cut-off being at ⅔ of the stroke from the beginning?

$$\text{Cubic feet of steam} = \frac{42^2 \times {\cdot}7854 \times (36 \times 2) \times 54 \times \tfrac{2}{3} \times 60 \times 2}{1728}$$

Ans.: 42 × 42 (= 1764) × ·7854 (= 1385·4456) × 72 (= 99752·0832) × 54 (= 5386612·4928) and ⅔ (= 3591074·9952) × 60 (= 215464499·712) × 2 = 430928999·424 cubic inches, which divided by 1728 gives 249380·208 cubic feet of steam.

EXAMPLES FOR PRACTICE.

No.	Diameter of Cylinder.	Length of Stroke.	Revolutions per minute.	Steam cut off.	*Ans.*: Cubic feet of steam used per H.P.
1.	76 inches	42 inches	51	2/7 of the stroke	385599·984
2.	92 „	48 „	49	¾ „	1628668·272
3.	94 „	54 „	62·4	⅝ „	2029889·862
4.	54 „	72 „	26·5	5/7 „	433507·14
5.	42 „	42 „	47	½ „	189921·501
6.	45 „	36 „	51	⅝ „	253475·578
7.	36 „	50 „	50	7/16 „	154625·627
8.	62 „	48 „	47½	¾ „	717030·93

263. Given the area at water level of boiler, to find how many tons of water will have to be pumped into it to raise the water in the glass a given number of inches.

In this question the following figures should be used:—

One cubic foot of fresh water weighs 62½ lbs. = 1000 ounces.
One cubic foot of sea water weighs 64 lbs.
Thirty-five cubic feet of sea water weighs one ton.
35·84 cubic feet of fresh water weighs one ton.

EXAMPLES.

Ex. 1. How many tons of water have to be pumped into a boiler whose area at water level is 20 feet by 13 to raise the water 8 inches in the glass?

$$\text{Weight of salt water} = \overset{(a)}{\frac{20\text{ ft.} \times 13\text{ ft.} \times \tfrac{2}{3}\text{ ft.}}{35}} \text{ or } \overset{(b)}{\frac{240'' \times 156'' \times 8''}{1728'' \times 35}}$$

(a)
13
20
———
260
8 in. = ⅔ ft. 2
———
3)520
———
35 { 7)173·3333 cubic feet.
———
5)24·7619
———
4·95238 tons.
20
———
19·04760 cwt.

Ans.: 4 tons 19 cwt.

(b)
156″
240
———
6240
312
———
37440
8
———
1728 { 12)299520 cubic inches.
———
12)24960
———
12)2080
———
35 { 7)173·3333 cubic feet.
———
5)24·7619
———
4·95238 tons.
20
———
19·04760 cwt.

(a) In this solution the dimensions, &c., are taken in feet; the height to which the water has to be raised, viz., 8 inches, is also expressed as a fraction of a foot—8 inches being equal to ⅔ of a foot.

(b) The dimensions are here taken in inches, and the result is cubic inches, which must be divided by 1728, in order to obtain cubic feet.

Or thus, by cancelling—

$$\frac{\overset{48}{\cancel{240}} \times \overset{13}{\cancel{156}} \times 8}{\underset{\underset{3}{\cancel{144}}}{\cancel{1728}} \times \underset{9}{\cancel{35}}} = \frac{104}{21} = 4{\cdot}9523 \text{ tons, or 4 tons } 19{\cdot}0 \text{ cwt.}$$

Same question worked out for fresh water.

Weight of fresh water $= \frac{240'' \times 156'' \times 8''}{1728 \times 35{\cdot}84} = \frac{299520}{61931{\cdot}52} = 4{\cdot}836309$ tons $=$ 4 tons 16·72 cwt.

$$\frac{\overset{\overset{5}{\cancel{20}}}{\cancel{240}} \times \overset{13}{\cancel{156}} \times \overset{2}{\cancel{8}}}{\underset{\underset{\underset{3}{\cancel{12}}}{\cancel{144}}}{\cancel{1728}} \times \underset{8{\cdot}96}{\cancel{35{\cdot}84}}} = \frac{130}{26{\cdot}88} = 4{\cdot}8363 \text{ tons, or 4 tons } 16\tfrac{3}{4} \text{ cwt., nearly.}$$

240 × 156 (= 3740) × 8 = 299520 ÷ 1728 (= 173·3333) ÷ 35·84 = 4·836309 tons,
or 4 tons 16·72 cwt.

Ans.: Fresh water, 4 tons 16¾ cwt., nearly.

Ex. 2. The area of the boiler level is 144 square feet; a single-acting donkey pumps 10 inches into the glass per hour, diameter of pump 3 inches, stroke 6 inches, revolutions per minute 120: what is the efficient stroke of the pump?

Work actually done by pump per hour =

$$144 \times \tfrac{10}{12} = 120 \text{ cubic feet.}$$

Work done by pump, if full, per hour =

$$\frac{3^2 \times {\cdot}7854 \times 6 \times 110 \times 60}{1728} = \frac{279916{\cdot}56}{1728}.$$

3 × 3 (= 9) × ·7854 (= 7·0686) × 6 × 110 (= 4665·2760) × 60 = 279916·56
÷ 1728 = 161·99 cubic feet (nearly).

161·99 : 120 :: 6 : x

$\frac{120 \times 6}{161{\cdot}99}$ = 4·44 inches of the stroke are efficient, or, ·74 of the stroke is efficient.

EXAMPLE.

The plunger of a bilge-pump is 5 inches diameter, and has a travel of 200 feet per minute, the diameter of the delivery valve is $3\frac{1}{2}$ inches, and has a lift of ·25 of an inch: what extra pressure is required to discharge the water?

Excess of pressure $= \frac{F^2\,D^2}{2{,}000{,}000\,d^2\,b^2}$

$F =$ feed per minute of plunger.
$D =$ diameter of plunger in inches.
$d =$ diameter of valve.
$b =$ breadth of opening of valve.

In small valves with a deep cone the breadth of opening bears the proportion of ·706 to the lift.

$$\frac{200^2 \times 5^4}{2{,}000{,}000 \times 3\frac{1}{2}^2 \times {\cdot}25^2} = \frac{200 \times 200 \times 5 \times 5 \times 5 \times 5}{2{,}000{,}000 \times 3{\cdot}5 \times 3{\cdot}5 \times {\cdot}25 \times {\cdot}25} = 16{\cdot}32 \text{ lbs.}$$

```
d = 3·5              b = ·25          F = 200          D = 5
    3·5                  ·25               200              5
    ———                  ———          ————————             —
    175                  125          F² = 40000           25
   105                  50                                  5
   ————                 ———                                —
d² = 12·25           b² = ·0625                            125
b² = ·0625                                                   5
   ——————                                                ————
    6125                                               D⁴ = 625
   2450                                                   40000
  7350                                                ——————————
  ——————                                          F² D⁴ = 25000000
 ·765625                  1531250)25000000(16·32 lbs. pressure.
   2000000                        1531250
 ———————————                      ———————
 1531250,000000                   9687500
                                  9187500
                                  ———————
                                   5000000
                                   4593750
                                   ———————
                                    4062500
                                    3062500
                                    ———————
                                    1000000
```

EXAMPLES FOR PRACTICE.

No.	Travel ft. per minute.	Diameter of Plunger.	Diameter of Valve.	Opening of Valve.	*Ans.:* Excess of Pressure.
1.	180	$4\frac{1}{2}$	$2\frac{1}{2}$	$\frac{1}{4}$	17·006
2.	250	$6\frac{1}{4}$	$4\frac{1}{2}$	·5	9·419
3.	300	$6\frac{1}{2}$	$4\frac{3}{4}$	·75	6·329
4.	280	$5\frac{1}{4}$	$4\frac{1}{4}$	·25	26·37
5.	195	$4\frac{3}{4}$	$3\frac{1}{2}$	$\frac{3}{8}$	5·62

EXAMPLES.

Ex. 1. A plunger 13 inches diameter has a travel of 181 feet per minute: with what velocity will the water travel through a discharge pipe 6 inches diameter?

The velocities are in inverse proportion to the areas, or what amounts to the same thing, as the squares of the diameters. Hence,

$$6^2 \; : \; 13^2 \; :: \; 181 \text{ ft.} \; : \; x$$

$$x = \frac{13^2 \times 181}{6} = \frac{13 \times 13 \times 181}{6 \times 6} = \frac{30589}{36} = 849{\cdot}7 \text{ feet, the velocity per minute.}$$

Ex. 2. An air-pump is 14 inches diameter and 17 inches stroke, the cylinder is 29 inches diameter and 30 inches stroke. The Board of Trade used to allow $\frac{1}{8}$th the capacity of cylinder for the capacity of air-pump single acting. The air-pump in this instance is double acting: is the capacity more or less than the Board of Trade allows?

$$\text{Capacity of cylinder} = 29^2 \times \cdot7854 \times 30 = 19815\cdot642$$

$$\text{Capacity of air-pump according to Board of Trade allowance} = \frac{19815\cdot642}{8} = 2476\cdot955.$$

$$\text{Capacity of pump} = 14^2 \times \cdot7854 \times 17 = 2616\cdot9528 \therefore \frac{2476\cdot955}{2616\cdot9528} = \cdot946$$

whence the capacity of the pump is greater than the allowance of the Board of Trade in the ratio of 1 : ·946.

Ex. 3. Two bilge-pumps, one 3 inches the other $4\frac{1}{2}$ inches diameter: what weight of water in tons will be discharged per hour?

RULE CX.

Square each diameter, add the results and divide by 2; the quotient is the weight of water discharged in tons per hour.

		Tons.
3	4·5	One 9
3	4·5	The other 20·25
9	225	2)29·25
	180	
	20·25	*Ans.* 14·625 tons.

Ex. 4. A hole 1 square inch in section and $1\frac{1}{4}$ foot deep admits 5 tons of water per hour: how deep down is a leak to admit 34 tons of water per hour, the hole being 3 square inches in section?

RULE CXI.

20 × ***depth = (tons per hour per square inch) squared.***

sq. in. 3)34 tons per hour.

11·33 tons per hour per square inch.
11·33

3399
3399
12463

20)128·3689 = (tons per hour per sq. in.) squared.

6·41844 feet, the depth of the leak.

Ex. 1. A hole 4 square inches section admits 40 tons per hour; how deep down is the leak? *Ans.*, 5 feet.

Ex. 2. How deep down is a leak to admit 28 tons per hour, the hole being $3\frac{1}{2}$ square inches section? *Ans.*, 3·2 feet.

Ex. 3. A $\frac{7}{8}$ inch rivet has fallen out a ship's bottom 16 feet below the water line: how many tons of water would accumulate in an hour if it were not pumped out?

By the above rule:—the tons per hour per square inch of section are equal to $\sqrt{16 \text{ ft.} \times 20} = 17\cdot885$.

And the square inches in section of the hole $= \frac{7}{8} \times \frac{7}{8} \times \cdot7854 = \cdot6013$,

$\therefore 17\cdot885 \times \cdot6013 = 10\cdot75425$ tons, *Ans.*

HORSE-POWER, &c.

In order to estimate and compare the different kinds of work performed under different circumstances, it becomes necessary to have a distinct measure or *unit of work* by which the various results may be calculated and the amounts ascertained.

The English unit of work is the power necessary to raise one pound through a space of one foot.

Thus, if one pound be raised one foot by a machine, then one unit of work has been performed; if four pounds be raised six feet, then $4 \times 6 = 24$ units of work have been performed; and so on for other combinations. *Hence the units of work performed are measured by the product of the weight of the body in pounds, and the space or height in feet through which it is raised;* also pressures and resistances of every kind, in whatever direction they are exerted, may be expressed in pounds, and therefore measured by the unit of work here described. But for purposes of calculation in connection with steam engines this unit of work is referred to one minute, as the unit of time. Taken in this point of view, the unit of work will be represented as one pound raised one foot high in one minute. But as such an amount is in general inconveniently small for engines of any size, the standard power usually adopted in practical mechanics is that which is called a horse-power. It is assumed that a horse is able to raise 33,000 pounds one foot high in one minute; and that is therefore called a horse-power.* Hence to find the number of horse-powers consumed in any proposed work it will only be required to divide the units in that work by 33,000 times the minutes in which it is done.

To find the horse-powers of an engine.

RULE CXII.

1°. *Square the diameter of the cylinder expressed in inches, and multiply this by* ·7854; the result is the area of piston (Rule LXXVI, pages 184—185).

2°. *Multiply the area of piston (found above,* No. 1°*) by the pressure per square inch;* the result is the pressure in pounds on the piston.

3°. *Multiply twice the length of stroke in feet by the number of revolutions per minute;* the product is the space the piston moves per minute.

Note.—If there are inches in the length of stroke, we may bring the feet into inches, multiply them by 2 and by the number of revolutions, and divide by 12.

4°. *Multiply the pressure in pounds* (No. 2°) *by the space in feet* (No. 3°); the product is the units of work done by the engine per minute.

5°. *Divide the units of work* (No. 4°) *by* 33,000 (the units of work in 1 horse-power); the quotient is the horse-power sought.

---

* Watt defined this to be 33,000 pounds. Although this is not accurate, the number has been retained as an element in calculating the horse-power of work.

EXAMPLES.

☞ **Ex. 1.** What is the horse-power of an engine, the diameter of cylinder being 5 feet, length of stroke 4 feet 3 inches, revolutions per minute 23, and effective pressure per square inch of the piston 8 lbs.?

$$\text{Horse-power} = \frac{60^2 \times \cdot7854 \times 8 \times 8\cdot5 \times 23}{33000}$$

$$\begin{array}{rl}
60 & \\
60 & \\ \hline
3600 & \\
\cdot7854 & \\ \hline
47124 & \\
23562 & \\ \hline
2827\cdot44,00 & \text{area.} \\
8 & \text{pressure per square inch.} \\ \hline
22619\cdot52 & \text{pressure in lbs.} \\
8\cdot5 & \text{twice the length of stroke.} \\ \hline
11309760 & \\
18095616 & \\ \hline
192265\cdot92,0 & \\
23 & \\ \hline
57679776 & \\
38453184 & \\ \hline
\end{array}$$

$$33,000\left\{\begin{array}{r}
3,000)4422,116\cdot16 \\ \hline
11)1474 \\ \hline
134
\end{array}\right.$$

Explanation.—The area of a cylinder 60 inches diameter (= 60 × 60 × ·7854) = 2827·44 square inches, and as there are 8 lbs. pressure on each square inch, there will be a total pressure 8 times 2827·44 or 22619·52 lbs. urging the piston. Now, as the length of the stroke is 4 feet 3 inches, and the piston traverses it each way in each revolution, the piston will travel 8 ft. 6 in. or 8·5 ft. for each revolution, and 22619·52 multiplied by 8·5 will give 192265·92 as the number of foot-pounds exerted in each stroke; or, as there are 23 strokes each minute, there will be 23 times 192265·92 or 4422116·16 foot-pounds exerted each minute. As an actual horse-power is 33,000 foot-pounds per minute, we shall, by dividing by 33,000, get the actual horse-power exerted by the engine to be 134 horse-power.

Instead of multiplying by ·7854, and dividing by 33,000, as above, we may use the constant multiplier ·0000238, which is the quotient of ·7854 divided by 33,000; and thus somewhat abbreviate the work.

$$\begin{array}{rl}
60 & \text{diameter in inches.} \\
60 & \\ \hline
3600 & \\
8 & \text{pressure per square inch.} \\ \hline
288,00 & \\
8\cdot5 & \text{twice the length of stroke.} \\ \hline
1440 & \\
2304 & \\ \hline
2448,00\cdot0 & \\
23 & \text{revolutions per minute.} \\ \hline
7344 & \\
4896 & \\ \hline
56304,00 & \\
\cdot0000238 & \text{constant multiplier.} \\ \hline
450432 & \\
168912 & \\
112608 & \\ \hline
134\cdot00352,00 & \text{horse-power.}
\end{array}$$

Ex. 2. What is the horse-power of H.M.S.S. "*Sphynx*," the diameter of each piston (2 in number) being 82½ inches, length of stroke 6 feet, revolutions per minute 16½, and the mean pressure per square inch on piston 16·35 lbs.

Diameter of piston, inches	82·5
" "	× 82·5
Circular inches in piston	6806·25
	× ·7854
Square inches in ditto	5345·62875
Number of pistons	2
Area of the two pistons	10691·2575
Pressure per square inch	16·35
Total pressure on piston	174802·06
Velocity of piston, feet	198
	33000)34610807·88
	1048·8 H.P.

Ex. 3. What is the horse-power of an engine, the diameter of cylinder being 5 feet 1½ inch, length of stroke 3 feet 6 inches, number of revolutions per minute 59, and the pressure of steam 21 lbs.?

5 ft. 1½ in. = 61·5 in.; twice length of stroke = 3 ft. 6 in. × 2 = 7 ft.;

space in feet per minute = 7 ft. × 59 (No. of revolutions) = 413 ft.

61·5 × 61·5 (= 3782·25) × ·7854 = 2970·57915,0 area × 21 (= 62382·16215) pressure in lbs.

∴ 62382·16215 × 413 (= 25763832·96795) ÷ 33000 = 780·7 horse-power.

Examples for Practice.

No.	Cylinder.	Pressure.	Stroke.	Rev.	No.	Cylinder.	Pressure.	Stroke.	Rev.
1.	5 ft. 2 in.	15 lbs.	4 ft. 6 in.	26	6.	4 ft. 2 in.	12 lbs.	2 ft. 6 in.	58
2.	8 7	9	9 0	14	7.	5 2	10	5 0	26
3.	5 1½	10	5 9	18	8.	2 6	16	2 3	45
4.	6 6	12	6 6	17	9.	6 3	9	7 6	14½
5.	5 10	13½	5 0	28	10.	7 2	12	6 11	20

ON NOMINAL OR COMMERCIAL HORSE-POWER.

The horse-power of an engine found in the way described, Rule CXII, must not be confounded with what is termed the **Nominal or Commercial horse-power**, which is the work expected from an engine, founded on the assumed pressure of steam, amounting to 7 lbs. per square inch of piston surface, and as used in tenders for contracts for Her Majesty's Navy in London; while in the Clyde it is 7½ lbs. Therefore, 14 horse-power in London equals 15 on the Clyde.

The following is the Admiralty equation for *nominal* horse-power:—

$$1.\ \text{Nominal horse-power} = \frac{\text{area of piston in in.} \times 7 \times \text{twice the stroke} \times \text{No. of revol. per minute.}}{33000}$$

And supposing, for example, the diameter of the cylinder to be 60 inches, and the pressure 7 lbs., the length of stroke 3 feet, and the revolutions per minute to be 50 in number,

$$\begin{aligned}\text{the horse-power would} &= (60^2 \times {\cdot}7854) \times 7 \times (2 \times 3) \times 50 \\ \text{or} &= \quad 2827 \quad \times 7 \times \quad 6 \quad \times 50\end{aligned} = \frac{5936700}{33000} = 180$$

Or thus:—

$$2.\ \text{Nominal horse-power} = \frac{(\text{diameter of cylinder in inches})^2 \times 2\ (\text{stroke}) \times \text{revolutions per minute.}}{6000}$$

$$\text{or} = \frac{3600 \times 6 \times 50}{6000} = 180 \text{ as before.}$$

The foregoing may be expressed in words as follows:—

RULE CXIII.

Square the diameter of the cylinder in inches, which multiply by the speed of the piston in feet per minute, and divide by 6000; *the quotient is the power of the engine.*

Example.

What is the power of an engine of 42 inches diameter, $3\frac{1}{2}$ feet stroke, and making 85 strokes per minute?

The speed of the piston will be 7 (the length of a double stroke) × 85 = 595 feet per minute.

$$\text{Now } \frac{42 \times 42 \times 595}{6000} = \frac{1049580}{6000} = 175 \text{ horse-power.}$$

The mode of computing nominal horse-power, established among *civil* manufacturers of steam engines, by the practice of Messrs. Boulton and Watt, is as follows:—

Assume the velocity of the piston to be 128 feet per minute, multiply cube root of length of stroke in feet.

Assume the mean effective pressure to be 7 lbs. on the square inch;

Then compute the horse-power from these fictitious data, and the area of the piston; that is to say,

$$\text{Nominal horse-power} = 7 \times 128 \times \sqrt[3]{\text{stroke in feet.}}$$

$$= \frac{\sqrt[3]{\text{stroke in feet}} \times \text{area of piston in inches.}}{47 \text{ nearly.}}$$

$$= \frac{\sqrt[3]{\text{stroke in feet}} \times \text{diameter}^2 \text{ in inches}}{60}$$

Instead of dividing by 47 we may multiply by its reciprocal ·0213.

The speed of the piston varies nearly as the cube root of the length of the stroke, and therefore some multiple of the cube root of the stroke may be substituted for the velocity of the piston in determining the nominal power, and will accomplish some simplification, as the speed of the piston proper for the different lengths of stroke cannot always be remembered. The rule for the nominal power of condensing engines when thus arranged will be as follows:—

RULE CXIV.

Multiply the square of the diameter of the cylinder in inches by the cube root of the stroke in feet, and divide the product by 47; *the quotient is the number of nominal horse-power of the engine.*

EXAMPLE.

Suppose diameter of cylinder $48\frac{5}{8}$ inches, length of stroke 8 feet, number of revolutions 16: required the nominal horse-power of engine.

$$\frac{48{\cdot}625 \times 48{\cdot}625 \times \sqrt[3]{8}}{47} = \frac{2364{\cdot}390625 \times 2}{47} = \frac{4728{\cdot}781250}{47} = 100{\cdot}61 \text{ horse power.}$$

Nominal power, then, is a term used particularly for commercial purposes. Each maker has his private rule*; hence the difference in opinion and dimensions in engines of the same class and power.† Actual horse-power is defined by the indicator diagram, speed of piston, &c.; the ratio between the nominal and actual horse-power is in some cases low, in others high. Instances are known where the nominal power being 1·0, the actual was 6; and in others, nominal 1·0, actual 2·123; the average ratio at present is, nominal 1·0, actual 4·0 to 5·0.

There are other methods which scarcely need mention, such as—

One horse-power = the evaporation of 1 cubic foot of water per hour (not easily ascertained in practice).

The formula for the Indicated Horse-Power (I.H.P.) of an engine‡ is

$$\text{I.H.P.} = \frac{d^2 \times {\cdot}7854 \times P \times \text{twice the stroke} \times \text{number of revolutions}}{33{,}000}$$

(d = diameter of cylinder, P = mean pressure of pistons).

Therefore, $d^2 \times {\cdot}7854 \times P \times \text{twice the stroke} \times \text{revolutions} = 33{,}000 \times \text{I.H.P.}$

Then if all the terms but one in the formula be given, that one can be found by multiplying 33,000 by the given I.H.P., and dividing the result by the product of all the known quantities on the left hand side; the quotient

---

* The rule at present adopted for nominal horse-power is 30 circular inches of piston per nominal horse-power, including all cylinders. This is the commercial nominal horse-power on the Clyde and in the North of England. It represents the horse-power of 7 lbs. effective pressure per square inch, and a speed of piston 200 feet per minute.

But on the Thames the nominal horse-power is taken at the same pressure, and at the speed of piston contracted for. Eighteen or twenty circular inches of piston, sometimes less, is then a nominal horse-power.

Mr. McFARLANE GRAY, in an article on "Nominal Horse-power," in the Nautical Magazine for May, 1872, proposes to use as nominal horse-power of engine, ten circular inches of piston area, counting only the low pressure pistons in compound engines; this corresponds with 300 feet piston speed, and 14 lbs. pressure effective.

† The number of rules for nominal or commercial horse-power is almost bewildering. Most of these rules have been collected by Mr. D. T. CLARKE in his article "Steam Engine," in the last edition of Encyclopædia Britannica, vol. 20, p. 602.

‡ In a steam engine, the area of whose piston is A square inches, where the mean pressure of steam on the piston is P lbs. per square inch, the product A P expresses the whole pressure on the piston; this multiplied by the length of the stroke L in feet will give the amount of work done in one stroke of the piston, and this product multiplied by the number of strokes in one minute gives P L A N as the work done by the steam in one minute; hence the expression—

$$\frac{PLAN}{33{,}000}$$

gives an easily remembered formula for the H.P. of a steam engine.

will be the unknown (or required) quantity of the formula. In the above the mean pressure is the required term, hence—

$$\text{Mean pressure} = \frac{33{,}000 \times \text{I.H.P.}}{d^2 \times {\cdot}7854 \times \text{twice stroke} \times \text{revolutions}}$$

whence the following

RULE CXV.

1°. *Multiply the Indicated Horse-Power* (I.H.P.) *by* 33,000.

2°. *Square the diameter of cylinder, multiply by* ·7854, *by twice the length of stroke, by number of revolutions, and by* 2 (being a pair of engines).

3°. *Divide the product found by* No. 1° *by the result found by* No. 2°; *the quotient is the pressure per square inch of piston in lbs.*

EXAMPLES.

Ex. 1. The indicated power of a pair of engines is said to be 1900 horse-power, the cylinders are each $63\frac{3}{4}$ inches diameter, the length of stroke is 36 inches, and the number of revolutions per minute is 74: what is the average effective pressure per square inch on the piston?

$$\text{Pressure per square inch} = \frac{\text{I.H.P. } 1900 \times 33000}{63\frac{3}{4}^2 \times {\cdot}7854 \times (3 \times 2) \times 74 \times 2}$$

63·75 diameter of cylinder.
63·75

31875
44625
19125
38250

4064·0625 dia.²
·7854

162562500
203203125
325125000
284484375

3191·91468750
6 twice stroke in feet.

19151·488125,00
74 revolutions.

76605952500
134060416875

1417210·12125
2

2834420·24250

Indicated H.P. 1900
33000

5700000
5700

62700000

2834420·24250)62700000·0000(22·12 lbs. per sq. in. of piston
56688404850

60115951500
56688404850

34275466500
28344202425

59312640750
56688404850

2624235900

Ex. 2. The indicated power of a pair of engines is said to be 2288 horse-power, the cylinders are each 76 inches diameter, the length of stroke is 42 inches, and the number of revolutions per minute is 51: what is the average effective pressure per square inch on the piston?

$$\text{Pressure per square inch} = \frac{2288 \times 33000}{76^2 \times {\cdot}7854 \times (3\frac{1}{2} \times 2) \times 51 \times 2}$$

$$2288 \times 33000 = 75504000$$

$$76 \times 76\ (= 5776) \times \cdot 7854\ (= 4536 \cdot 5704) \times 7 \times 51\ (= 1619519 \cdot 9328) \times 2 = 3239039 \cdot 8658$$

$\therefore 75504000 \div 3239039 \cdot 8658 = 23 \cdot 31$ lbs. pressure per square inch of piston.

QUESTIONS RELATING TO PRESSURES—SAFETY-VALVES, &c.

Given the pressure per square inch on the piston, and the height of the barometer on the condenser, to find the effective pressure.

RULE CXVI.*

1°. *Divide the inches of mercury, as indicated by the barometer on the condenser, by 2.*

2°. *To this add the pressure of steam on the piston;* the sum is the effective pressure per square inch on the piston.

Were the barometer on the condenser to stand at 30 inches, a perfect vacuum would exist; but there never is a perfect vacuum. The vapour and air in the condenser has a power, shown by the fall of the mercury, which acts as an opposing force to the pressure of the steam on the piston.

EXAMPLES.

Ex. 1. If the pressure of steam on the piston is 19½ lbs. per square inch, and the barometer on the condenser shows 27 inches of mercury: required the effective pressure on the piston.

2 inches of mercury = 1 lb. pressure.

$\therefore$ 27 inches of mercury = 13½ lbs. pressure.
Pressure on piston = + 19½ lbs.

Total pressure = 33 lbs. per square inch.

Ex. 2. The pressure of steam on the piston is 18 lbs. per square inch above the atmosphere, and the barometer on the condenser shows 27·8 inches of mercury: required the effective pressure per square inch on the piston.

2 inches of mercury = 1 lb. pressure.

$\therefore$ 27·8 inches of mercury = 13·9 lbs. pressure.
Pressure on piston = + 18 lbs.

Total pressure = 31·9 lbs. pressure.

---

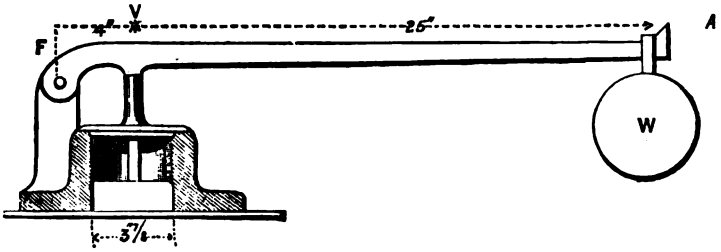
* *Atmospheric Pressure.*—A vacuum is a perfectly void space. Among engineers the term is more properly "partial" vacuum, for a "perfect" vacuum can scarcely exist on the surface of our earth. If it were possible to erect a tube (having a sectional area of one square inch, upon any part of the earth, supposing this tube to be long enough to reach up through the thickness of the atmosphere (which is said to be about forty-five or fifty miles), the air contained in such tube would, in round numbers, weigh about 14¾ lbs. If, further, we take a tube of the same diameter, and put water in it to the weight of 14¾ lbs., the level of the water in the tube (if standing upright) would be about 34 feet above the bottom of the tube. If, again, we take a similar tube, 14¾ lbs. of mercury would fill it up to about 30 inches. The accuracy of these numbers would be affected by temperature, &c.; therefore, for practical purposes, we assume atmospheric pressure to be 15 lbs. upon every square inch, so that *every pound of pressure equals two inches of mercury.*

EXAMPLES FOR PRACTICE.

No.	Pressure on piston.	Barometer.	No.	Pressure on piston.	Barometer.
1.	20 lbs.	28·6 inches.	3.	25 lbs.	25·0 inches.
2.	19 „	29·0 „	4.	17 „	26·0 „

264. Every boiler is tested to bear a certain amount of pressure per square inch of internal surface, but only a portion of this pressure is allowed in practice. To prevent explosions from the too great pressure in the boilers, an apparatus is attached to the steam chest, which consists in principle of an escape tube, which is loaded with a weight heavy enough to confine steam and prevent its waste, but not heavy enough to allow a dangerous quantity of steam to accumulate in the boiler. This apparatus is therefore called a "safety-valve," and is by law inaccessible to the engine driver, who, however, has the control over another safety-valve (there are frequently more than one) for the purpose of letting out surplus steam, &c.

The weight which closes the valve is usually kept in its place by a lever of the third kind.* The following figure is given in illustration.



FA is a graduated lever of the third kind, turning upon F as centre or fulcrum; V is the valve, opening or closing, as the case may be, the communication of the steam in the boiler with the external air; the lever FA rests upon a pin of this valve, and a sliding weight (W) is suspended from the lever, thereby enabling the engine-man to place any amount of pressure on the valve—this pressure measures the elasticity of the steam when it begins to escape.

The fundamental theorem of the calculation of the lever is "that to produce equilibrium the moment of power equals the moment of weight." The word moment being an expression for an amount which is the product of the mass and *velocity*; the "velocities" being in such cases represented by the lengths of arcs, described by the ends of the arms of the lever (as moving on its fulcrum) in the same period of time. Or, in other words:—The units of weight in the power V, multiplied by the units of its distance from the fulcrum F, will be equal to the units of weight in W multiplied by the units of distance from fulcrum F.

$$W \times FW = V \times FV;$$

$$\therefore W = \frac{V \times FV}{FW}$$

Hence,

To find the weight to be placed at the end of a safety-valve lever to be equal to a given number of pounds pressure per square inch on the valve, having given the diameter of valve, the distance from fulcrum to valve, and from valve to weight; also the effective weight of lever* and weight of valve are given.

---

* Levers are of three kinds:—The first kind have the fulcrum between the power and the weight. The second kind have the weight between the power and the fulcrum. The third kind have the power between the weight and the fulcrum. Or, in other words:—The first kind have the fulcrum in the middle. The second kind have the weight in the middle. The third kind have the power in the middle.

RULE CXVII.

1°. *Multiply the square of the diameter of the valve by* ·7854; the product will be the area of valve (see Rule LXXVI, page 184).

2°. *Multiply the area of valve by the pressure per square inch;* the product is the pressure on valve, *or* upward pressure.

3°. *From the product or upward pressure* (No. 2°) *subtract the sum of the weights of the lever and the valve or pressure downwards;* the remainder is the effective pressure on the valve (V of the formula).

4°. *Multiply this remainder* (No. 3°) *by the distance "from the fulcrum to the valve"* (FV), *and divide by the distance "from the fulcrum to the weight"* (FW) *found by* adding *"the distance from the fulcrum to the valve" to that "from the valve to the weight;"* the quotient is the weight required.

NOTE.—The distance from the fulcrum to the weight is obtained by adding the distance "from the fulcrum to the valve" to that "from the valve to the weight."

EXAMPLES.

EX. 1. Required the weight to be placed on the end of a safety-valve lever to be equal to 25 lbs. per square inch on the valve, the diameter of the valve being $3\frac{7}{8}$ inches, and the distance from the fulcrum to the valve 4 inches, and from the valve to the weight 25 inches, the effective weight of lever 24 lbs. 8 ozs., and the weight of the valve 3 lbs. 6 ozs. (see figure, page 250).

Diameter of valve $3\frac{7}{8}$ in. $=$ 3·875.

Weight of lever 24 lbs. 8 ozs. + weight of valve 3 lbs. 6 ozs. $=$ 27·875.

Downward pressure $=$ weight of lever 24 lbs. 8 ozs., and of valve 3 lbs. 6 ozs. $=$ 27 lbs. 14 ozs., or $27\frac{7}{8}$ lbs., which is 27·875 lbs. (see Rule L, page 110).

$3{\cdot}875^2 \times {\cdot}7854 \times 25 = 294{\cdot}83$ upward pressure

$$\begin{array}{r}
3{\cdot}875 \\
3{\cdot}875 \\
\hline
19375 \\
27125 \\
31000 \\
11625 \\
\hline
150156{,}25 = d^2 \\
{\cdot}7854 \\
\hline
600624 \\
750780 \\
1201248 \\
1051092 \\
\hline
11{\cdot}7932{,}5224 \text{ area of valve.} \\
25 \text{ lbs. pressure.} \\
\hline
589660\phantom{0 \text{ upward pressure on valve.}} \\
235864\phantom{00 \text{ upward pressure on valve.}} \\
\hline
294{\cdot}8300 \text{ upward pressure on valve.}
\end{array}$$

Leverage of weight FW $=$ 4 inch + 25 in. $=$ 29 in.; other FN, 4 in.

$$\therefore \text{weight} = \frac{294{\cdot}83 - 27{\cdot}875 \times 4}{29} = 36{\cdot}821 \text{ lbs.}$$

$$\begin{array}{lr}
\text{Pressure on valve} & 294{\cdot}83 \\
\text{Weight of lever \& valve} & -\ 27{\cdot}875 \\
\hline
\text{Effective press. on valve, V} & 266{\cdot}955 \\
\text{Distance from F to V} & 4 \\
\hline
\end{array}$$

$$\begin{array}{l}
29)1067{\cdot}820(36{\cdot}821 \text{ lbs.} \\
87 \\
\overline{197} \\
174 \\
\overline{238} \\
232 \\
\overline{62} \\
58 \\
\overline{40} \\
29
\end{array}$$

---

* The effective weight of lever means its effect at the valve.

Ex. 2. Required the weight to be placed on the end of a safety-valve lever to be equal to 12 lbs. per square inch on the valve, the diameter of the valve being 5 inches, and the distance from the fulcrum to the valve 5 inches, and from the valve to the weight 26 inches, the effective weight of the lever being 6 lbs., and the weight of the valve 6 lbs.

$$\text{Upward pressure} = 12 \times 5^2 \times \cdot 7854 = 235 \cdot 620.$$

Downward pressure = weight of lever 6 lbs., and of valve 6 lbs.

$$FW = 5 + 26 = 31 \text{ inches, others} = 5 \text{ inches.}$$

$$\therefore \text{Weight} = \frac{235 \cdot 62 - (6 + 6) \times 5}{31} = \frac{223 \cdot 62 \times 5}{31} = 36 \cdot 07 \text{ nearly.}$$

Examples for Practice.

No.	Pressure.	Diameter of valve.	Fulcrum to valve.	Valve to weight.	Weight of lever.	Weight of valve.
1.	18 lbs.	4·6 inch	6 inch	12 inch	63 lbs.	10 lbs.
2.	25 „	$3\frac{5}{8}$ „	3 „	26 „	$23\frac{1}{2}$ „	$3\frac{1}{2}$
3.	23 „	4 „	$2\frac{1}{4}$ „	$16\frac{3}{4}$ „	9 „	3
4.	18 „	12 „	$3\frac{1}{2}$ „	$17\frac{1}{2}$ „	$6\frac{1}{2}$ „	4
5.	37 „	$13\frac{1}{2}$ „	$2\frac{1}{2}$ „	$25\frac{1}{2}$ „	$17\frac{3}{4}$ „	6
6.	26 „	5 „	$5\frac{1}{2}$ „	$20\frac{1}{2}$ „	60 „	9
7.	17 „	4·8 „	5 „	17 „	77 „	10
8.	29 „	$6\frac{1}{4}$ „	$2\frac{3}{4}$ „	$21\frac{3}{4}$ „	8 „	$7\frac{1}{2}$
9.	42 „	$11\frac{3}{4}$ „	$3\frac{1}{4}$ „	$16\frac{1}{2}$ „	10 „	$4\frac{1}{4}$
10.	19 „	$5\frac{1}{2}$ „	3 „	$13\frac{3}{8}$ „	$10\frac{1}{2}$ „	6

265. To find the weight to be placed on a double-beat valve to be equal to a given number of pounds pressure, having the diameters of the valve given.

RULE CXVIII.

1°. *Find the square of each diameter and take the difference.*

2°. *Multiply this difference* (No. 1°) *by* ·7854 *and by the pressure per square inch;* the result is the required weight to be placed on the valve.

Otherwise:—

Because the difference of the square of two numbers is equal to the product of their sum and difference, we have the following

RULE CXIX.

1°. *Multiply the* sum *of the diameter by their* difference; the product is equal to the difference of the squares of the diameters.

2°. *Multiply this product* (No. 1°) *by* ·7854 *and by the pressure per square inch;* the result is the weight required to be placed on the valve.

Examples.

Ex. 1. Required the weight to be placed on a double-beat valve to be equal to 32 lbs. per square inch, the diameter of the valve being 8 and 6 inches.

(1).

$$\begin{aligned} \text{Weight} &= (8^2 - 6^2) \times \cdot 7854 \times 32 \\ \text{,,} \quad &= (64 - 36) \times \cdot 7854 \times 32 \\ \text{,,} \quad &= 28 \times \cdot 7854 \times 32 \end{aligned}$$

(2).

$$\begin{aligned} \text{Weight} &= (8 + 6) \times (8 - 6) \times \cdot 7854 \times 28 \\ \text{,,} \quad &= 14 \times 2 \times \cdot 7854 \times 28 \\ \text{,,} \quad &= 28 \times \cdot 7854 \times 28 \end{aligned}$$

·7854
28
62832
15708
21·9912
32 lbs. pressure.
439824
659736
703·7184 lbs.

$8 + 6 = 14$
$8 - 6 = 2$
28
·7854
28
62832
15708
21·9912
32 lbs. pressure.
439824
659736
703·7184 lbs.

Ex. 2. Required the weight to be placed on a double-beat valve to be equal to 26 lbs. per square inch, the diameters of the valve being 12½ and 9¼.

Weight $= (12{\cdot}5^2 - 9{\cdot}25^2) \times {\cdot}7854 \times 26$.
„ $= 156{\cdot}25 - 85{\cdot}5625 = 70{\cdot}6875 \times {\cdot}7854\ (= 55{\cdot}5179625) \times 26 = 144{\cdot}3468$ lbs.

Examples for Practice.

No.	Diameters.	Pressure.	No.	Diameters.	Pressure.
1.	9 and 10 inches	20 lbs.	5.	6 and 4 inches	24 lbs.
2.	13½ „ 12 „	25 „	6.	12 „ 9½ „	35
3.	9 „ 6 „	20 „	7.	13½ „ 11¾ „	36
4.	8 „ 7 „	26 „	8.	6 „ 5·5 „	24

266. A safety-valve is loaded to a certain pressure; it is required to find a new weight so as to reduce the pressure to a certain amount.

RULE CXX.

Say as the old pressure : new pressure :: old weight : new weight.

If the question be "How much would be required to be taken off?"

RULE CXXI.

As old pressure : difference of pressure :: old weight : weight to be taken off; or subtract answer, found by* Rule CXX, *from old pressure.

Example.

A safety-valve is loaded to 36 lbs. per square inch by a direct weight of 750 lbs.: how much would be required to reduce the pressure to 24 lbs. per square inch?

	Old pressure.		New pressure.		Old weight.
12)	36	:	24	::	750
	3		2		

Ans. $= \dfrac{750 \times 2}{3}$

750
2
3)1500
500 lbs. = new weight.

Explanation.—The first and second terms are each divisible by 12; whence we have 3 as the first term and 2 as the second; then the second (2) and third (750) terms are multiplied together, and we divide by the first (3); whence the answer 500 lbs.

If it were required to know "How much should be taken off?" we have only to subtract the new weight (500 lbs.) from the old weight (750 lbs.); thus:—

$$\begin{array}{r l} \text{Old weight} & = 750 \text{ lbs.} \\ \text{New weight} & = \underline{500} \text{ ,,} \\ \text{Weight to be taken off} & = 250 \text{ ,,} \end{array}$$

EXAMPLES FOR PRACTICE.

In the first six following examples the total new weight is required; in the remaining four the amount to be taken off.

No.	O. pressure.	N. pressure.	Old weight.	No.	O. pressure.	N. pressure.	Old weight.
1.	40 lbs.	32 lbs.	450 lbs.	6.	38 lbs.	34 lbs.	368 lbs.
2.	36 ,,	24 ,,	980 ,,	7.	27 ,,	24 ,,	775 ,,
3.	38 ,,	25 ,,	724 ,,	8.	24 ,,	20 ,,	666 ,,
4.	47 ,,	39 ,,	964 ,,	9.	25 ,,	15 ,,	425 ,,
5.	30 ,,	20 ,,	750 ,,	10.	29 ,,	24 ,,	298 ,,

267. Given the diameter of a safety-valve, to find what height it must be raised to allow a free escape of steam equal to the area of valve.

The valve when close forms a circle. If we suppose the valve to be gradually raised to the required height, during the process it will have formed an infinite number of circles, one piled on another, and forming a cylinder of the same height and dimensions as the valve; consequently, the *area* of this cylinder is equal to the *area* of the valve.

The area of a cylinder is found by multiplying the height by the circumference (Rule XCI, page 193).

$$\text{But since the area of cylinder} = \text{area of valve}$$

$$\therefore \text{height} \times \text{circumference} = \text{area of valve}$$

$$\text{and} \therefore \text{height} = \frac{\text{area of valve}}{\text{circumference}}$$

But, area of valve = square of diameter × ·7854 (Rule LXXVI, page 184), and circumference = diameter × 3·1416 (Rule LXXIV, page 183).

$$\therefore \text{height} = \frac{d^2 \times \cdot 7854}{d \times 3{\cdot}1416} = \frac{d \times d \times \cdot 7854}{d \times \cdot 7854 \times 4} = \frac{d}{4}$$

Hence

RULE CXXII.

1°. *Find the area of valve* (Rule LXXVI), *also the circumference* (Rule LXXIV), *and divide the former by the latter;* the quotient is the height required.

Or (2°), divide the diameter of the valve by 4; the quotient is the height which the valve must be raised.

EXAMPLES.

Ex. 1. To what height must a safety-valve $6\frac{1}{2}$ inches diameter be raised to allow a free escape of steam equal to the area of the valve?

(1°) $$\text{Height} = \frac{\text{area}}{\text{circumference}} = \frac{6{\cdot}5 \times 6{\cdot}5 \times \cdot 7854}{6{\cdot}5 \times 4 \times \cdot 7854} = \frac{6{\cdot}5}{4} = 1{\cdot}625$$

Explanation.—We cancel the quantities that occur above and below the line, viz., 6·5 above and 6·5 below: also ·7854 above and ·7854 below, which leaves 6·5 above and 4 below the line; then 6·5 divided by gives 1·625 inches.

(2°) $$4\overline{)6{\cdot}5}$$

1·625 inches. *Ans.*

Ex. 2. What height should a safety-valve be lifted, of 3¾ inches diameter, to equal the area of the valve?

3¾ inches = 3·75

4)3·75

·9375

EXAMPLES FOR PRACTICE.

No.	Diameter.	No.	Diameter.	No.	Diameter.
1.	6½ inches	4.	6·8 inches	7.	5 3/16 inches
2.	4¾ „	5.	8·4 „	8.	4⅝ „
3.	6⅞ „	6.	7·6 „	9.	8¼ „

EXAMPLES.

Ex. 1. For steam above 12 lbs. pressure above the atmosphere, the weight of steam that will escape into the atmosphere through an orifice one square inch in area is 70 seconds, just equal to the pounds in the gross pressure of the steam per square inch. To what breadth of opening must a 9 inch wide safety-valve rise to allow the escape of 12500 pounds of steam per hour, the pressure carried being 70 lbs. per square inch?

Gauge pressure	70 lbs.
Atmospheric pressure	15 „
Gross pressure	85 „

To find the escape in one hour (3600^s) per sq. in.

$70^s : 3600^s :: 85$ lbs. $(70 + 15) : x$

3600

51000

255

7,0)30600,0

4371·4

To find the inches of escape.

4371 lbs. : 12500 lbs. :: 1 in. : x

1

4371·4)12500·0(2·8595 nearly.

87428

375720

349712

260080

218570

415100

393426

216740

Circumference of valve = diameter 9″ × 3·1416 = 28·2744.

28·2744)2·859500(·101 lift of safety-valve.

282744

320600

282744

Bring this to 8ths.

·101

8

0·808

Ans.: Lift of safety-valve ·101, or 1/10 of an inch.

Ex. 2. For steam above 12 lbs. pressure above the atmosphere, the weight of steam that will escape into the atmosphere through an orifice one square inch in area is 70 seconds, just equal to the pounds in the gross pressure of the steam per square inch. To what breadth of opening must a 6½ inch wide safety-valve rise to allow the escape of 11400 pounds of steam per hour, the pressure carried being 75 lbs. per square inch?

Gauge pressure	75 lbs.
Atmospheric pressure	15 „
Gross pressure	90 „

To find the escape in one hour per sq. inch.

70^s : 3600^s :: 90 lbs. (75 × 15)
90
7,0)32400,0
4628·58

To find the inches of escape.

4628·6 lbs. : 11400 lbs. :: 1 in.
1
4628·6)11400·0(2·463 nearly.
92572
214280
185144
291360
277716
136440
92572
43868

Dia. of valve 6½ × 3·1416
6½
188496
15708
Circumference = 20·4204

20·4204)2·46300(·1206
204204
420960
408408
1255200
1225224

Ans.: Lift of safety-valve ·1206, or ⅛ of an inch nearly.

EXAMPLES FOR PRACTICE.

No.	Width of Safety-valve.	Escape of steam per hour.	Press. carried per sq. inch.	*Ans.:* Lift of safety-valve.
1.	9 inches.	24,000 lbs.	70 lbs.	·194 inch, or ⅕ inch nearly.
2.	8 „	19,000 „	65 „	·183 „ ⅙ „ „
3.	6 „	10,500 „	70 „	·127 „ ⅛ „ „
4.	8½ „	20,000 „	60 „	·194 „ ⅕ „ „
5.	9½ „	18,900 „	70 „	·145 „ ⅐ „ „
6.	6½ „	9,600 „	60 „	·122 „ ⅛ „ „

EXAMPLES.

Ex. 1. If the compression of a spring that loads a safety-valve be 1¼ inch, the pressure to which the valve is loaded being 65 lbs. per square inch, what will be the load per square inch on the valve when it has lifted to give an area of escape equal to one-sixth of the area of the valve? The face of the valve is flat. Valve 6 inches diameter.

A valve will lift one-fourth of its diameter to give an escape of steam equal to the area of the valve.

Dia. of valve $\frac{5}{4}$ = 1·25
Area $\frac{1·5}{6}$ = ·25
4)6·00 diameter of valve.
6)1·50 lift for escape = area of valve.
·25 lift for escape = ⅙ area of valve.

1·25 in. : ·25 in. :: 65 lbs.
·25
325
130
1·25)16·25(13 lbs.
125
375
375

Working load 65 lbs.
Extra 13
Total load 78 lbs.

Ex. 2. If the compression of a spring that loads a spring valve be 3 inches, the pressure to which a valve is loaded being 70 lbs. per square inch, what will be the load per square inch on the valve when it has lifted to give an area of escape equal to one-sixth of the area of the valve? The face of the valvo is flat. Valve 3½ inches diameter.

Dia. of valve $\frac{3·5}{4}$ = ·875
Area $\frac{·875}{6}$ = ·146
4)3·5 diameter of valve.
6)0·875 lift for escape = area of valve.
·146 lift for escape = $\frac{1}{16}$ area of valve.

3 in. : ·146 in. :: 70 lbs.
70
3)10·220
3·407

Working load 70 lbs.
Extra 3·407 lbs.
Load on valve 73·407 lbs.

EXAMPLES FOR PRACTICE.

No.	Compression of spring that loads safety-valve. inches.	Press. per square inch on valve. lbs.	Diameter of valve. inches.	*Ans.*: Load per sq. in. equal to 1-6th area of valve. lbs.
1.	1½	65	4½	73·125
2.	2	60	5	66·299
3.	2½	70	6	77·778
4.	1½	75	6½	88·541

268. When a safety-valve has the area required by the Board of Trade, and is spring loaded, the increase of pressure due to the increased compression of the spring in blowing off steam is, with bright fires and engine stopped, about equal to the quotient found by dividing the diameter of the valve by its original compression.

Question.—The spring has 5 inches compression, there are 70 square feet of fire-grate to one valve, the loaded pressure is 70 lbs., the increase, irrespective of the spring, would be, say 10 per cent.: to what pressure would the steam rise, including the effect of the spring?

The first step here is to find the diameter of the valve.

Its area is found by dividing the 70 square feet of fire-grate by 2. Thus—

$$\text{Area} = 70 \div 2 = 35 \text{ square feet.}$$

Then $\sqrt{35 \div \cdot7854} = \sqrt{44\cdot563279} = 6\cdot675$ inches, the diameter of valve.

Next, dividing diameter of valve by original pressure of the spring, we have—

$$\begin{array}{rrl} & 5)6\cdot675 & \\ \hline & 1\cdot335 & = \text{extra pressure due to spring.} \\ & 70 & = \text{original pressure.} \\ \frac{1}{10} = & 7 & = 10 \text{ per cent.} \\ \hline & 78\cdot335 & = \text{total pressure.} \end{array}$$

Ans.: 78·335 lbs.

Ex. 1. A safety-valve 6″ diameter has a spiral spring ⅞″ thickness of steel, what is the pressure per square inch?

Rule, $\frac{8000\ S^3}{d}$ Where S is the thickness of steel, and

d = diameter of spring from centre of steel to centre of steel.

$$\frac{8000 \times \frac{7}{8}^3}{(5 - \frac{7}{8}) = 4\frac{1}{8}} = \frac{8000 \times \cdot875^3}{4\cdot125.} = \frac{5359\cdot375}{4\cdot125} = 1299\cdot\dot{2}\dot{4}.$$

$$\cdot875 \times \cdot875\ (= \cdot765625) \times \cdot875\ (= \cdot669921875) \times 8000 = 5359\cdot375 \div 4\cdot125 = 1299\cdot24.$$

$$\text{Area of valve} = 6^2 \times \cdot7854 = 28\cdot2744.$$

$$\therefore 1299\cdot24 \div 28\cdot2744 = 45\cdot95 \text{ lbs. per square inch.}$$

Ex. 2. The topmast of a ship is 80 feet above the level of the sea when the steamer is upright, but when heeling over it is 72 feet above the sea level. The valve is 5 inches diameter, and is loaded with a direct weight of 600 lbs.: what will she blow-off at when heeling over to this angle?

$$\text{Pressure on valve when upright} = \frac{600}{5^2 \times \cdot7854}$$

$$\begin{array}{rrr} \text{Dia. } 5 & \cdot7854 & 19\cdot635)600\cdot00(30\cdot55 \\ \underline{5} & \underline{25} & \underline{58905} \\ d^2 = 25 & 39270 & 109500 \\ & \underline{15708} & \underline{98175} \\ & 19\cdot6350 & 113250 \end{array}$$

$$\begin{array}{rcccccc} & \text{Feet.} & & \text{Feet.} & & \text{lbs.} & \\ \text{Then} & 80 & : & 72 & :: & 30\cdot55 & : x \\ & 10 & & 9 & & \underline{9} & \\ & & & & & 27\cdot495 & \text{lbs. } \textit{Ans.} \end{array}$$

Ex. 3. **A waste steam-pipe of a spring loaded safety-valve is $23\frac{3}{4}$ feet high, what extra pressure per square inch is on the valve when the pipe gets filled with water?**

$$23\tfrac{3}{4} = 23{\cdot}75 \text{ feet}, \therefore \frac{23{\cdot}75}{2{\cdot}305} = 10{\cdot}30 \text{ lbs.}$$

Note.—2·305 = 1 lb. pressure.

$$\begin{array}{r} 2{\cdot}305)23{\cdot}75(10{\cdot}30 \text{ lbs.} \\ 2305\phantom{(10{\cdot}30 \text{ lbs.}} \\ \hline 7000\phantom{(10{\cdot}30 \text{ lbs.}} \\ 6915\phantom{(10{\cdot}30 \text{ lbs.}} \\ \hline 850\phantom{(10{\cdot}30 \text{ lbs.}} \end{array}$$

Ex. 4. **A 6-inch safety-valve is fitted to a boiler, how many square feet of grate surface will there be at $\frac{1}{2}$ a square inch of valve surface to 1 square foot of grate surface?**

Area of valve $= 6^2 \times {\cdot}7854 = 28{\cdot}2744$.

$\frac{1}{2}$ square inch of valve surface to 1 square foot of grate surface = 2 square feet of grate surface for 1 square inch of valve surface.

$\therefore$ 28·2744 inches of valve surface $\times$ 2 = 56·5488 square feet of grate surface.

Ex. 5. **A safety-valve 6 inches in diameter is loaded by dead weights consisting of three 72 and four 65 lbs. weights; the weight of the spindle is 18 lbs., and the valve 10 lbs.: find the effective pressure per square inch on valve.**

$$\text{Pressure} = \frac{(72 \times 3) + (65 \times 4) + 10 + 18}{6^2 \times {\cdot}7854} = \frac{504}{28{\cdot}2744} = 17{\cdot}82 \text{ lbs.}$$

$$\begin{array}{lr} & 72 \times 3 = 216 \text{ lbs.} \\ & 65 \times 4 = 260 \quad ,, \\ \text{Weight of valve} & 10 \quad ,, \\ \text{Weight of spindle} & 18 \quad ,, \\ \cline{2-2} \text{Total weight} & 504 \end{array} \qquad \begin{array}{r} \text{Dia. } 6 \\ 6 \\ \hline 36 \end{array} \qquad \begin{array}{r} {\cdot}7854 \\ 36 \\ \hline 47124 \\ 23562 \\ \hline 28{\cdot}2744 \end{array} \qquad \begin{array}{r} 28{\cdot}2744)504{\cdot}000(17{\cdot}82 \text{ lbs.} \\ 282744\phantom{(17{\cdot}82 \text{ lbs.}} \\ \hline 2212560\phantom{(17{\cdot}82 \text{ lbs.}} \\ 1979208\phantom{(17{\cdot}82 \text{ lbs.}} \\ \hline \text{\&c.}\phantom{(17{\cdot}82 \text{ lbs.}} \end{array}$$

Ex. 6. **A safety-valve $4\frac{3}{4}$ inches diameter is loaded by a dead weight consisting of eight 56 lbs., and six 90 lbs. weights, the weight of the spindle is 24 lbs., and the valve 12 lbs.: find the effective pressure per square inch on the valve.**

$$\text{Total weight} = (56 \times 8) + (90 \times 6) + 24 + 12 = 448 + 540 + 24 + 12 = 1024 \text{ lbs.}$$

$$\text{Area of valve} = 4{\cdot}75^2 \times {\cdot}7854 = 22{\cdot}5625 \times {\cdot}7854 = 17{\cdot}7205875.$$

$$\therefore \text{effective pressure per square inch} = 1024 \div 17{\cdot}7206 = 57{\cdot}7 \text{ lbs. } \textit{Ans.}$$

Ex. 7. **A safety-valve 6 inches diameter is weighted by 6 cast iron weights, each $1\frac{1}{4}$ inches thick and 11 inches diameter, and 7 weights each 2 inches thick and 12 inches diameter: what is the pressure per square inch on the valve?**

Short Rule. **$d^2 \times h = 5$ times the weight in lbs., and this will allow for the holes in the weights.**

$$\begin{array}{r} {\cdot}7854 \\ d^2 = 36 \\ \hline 47124 \\ 23562 \\ \hline 28{\cdot}2744 = \text{area of valve.} \end{array} \qquad \begin{array}{r} d = 11 \\ 11 \\ \hline d^2 = 121 \\ 1\tfrac{1}{4} = \text{thickness.} \\ \hline 121 \\ 30{\cdot}25 \\ \hline 151{\cdot}25 \\ 6 \\ \hline 907{\cdot}50 \end{array} \qquad \begin{array}{r} d = 12 \\ 12 \\ \hline d^2 = 144 \\ 2 = \text{thickness.} \\ \hline 288 \\ 7 \\ \hline 2016 \end{array} \qquad \begin{array}{r} 2016 \\ 907{\cdot}50 \\ \hline 5)2923{\cdot}50 \\ \hline 584{\cdot}7 \end{array}$$

$$\begin{array}{r} 28{\cdot}2744)584{\cdot}7000(20{\cdot}6 \text{ lbs. } \textit{Ans.} \\ 565488\phantom{0(20{\cdot}6 \text{ lbs. } \textit{Ans.}} \\ \hline 1921200\phantom{(20{\cdot}6 \text{ lbs. } \textit{Ans.}} \\ 1696464\phantom{(20{\cdot}6 \text{ lbs. } \textit{Ans.}} \\ \hline 224736\phantom{(20{\cdot}6 \text{ lbs. } \textit{Ans.}} \end{array}$$

EXAMPLES FOR PRACTICE.

No.	Diameter of safety-valve.	Weights	Weight of spindle.	Weight of valve.	*Ans.*: Eff. press. per sq. in. on valve.
1.	7 inches.	Fifteen 72 and three 45 lbs.	21 lbs.	12 lbs.	32·43 lbs.
2.	5 "	Four 72 and seven 90 "	16 "	10 "	47·67 "
3.	5½ "	Three 80 and eight 90 "	18 "	14 "	41·75 "

Ex. 8. A safety-valve is 5¼″ diameter, its bearing face is quite flat: what is the lift when the opening is ⅑ of the area of valve?

In Rule CXXII it is shown that the height a safety-valve must be raised to allow an escape of steam equal to the area of valve may be found thus:—

$$\text{Height} = \frac{\text{area of valve}}{\text{circumference}}$$

$$\therefore \text{Height} = \frac{5\tfrac{1}{4}^2 \times \cdot 7854}{5\tfrac{1}{4} \times 3\cdot 1416} = \frac{5\cdot 25^2 \times \cdot 7854}{5\cdot 25 \times 3\cdot 1416} = \frac{21\cdot 6475875}{16\cdot 4934} = 1\cdot 3125$$

$$\text{Or, since } \frac{5\cdot 25^2 \times \cdot 7854}{5\cdot 25 \times 3\cdot 1416} = \frac{\cancel{5\cdot 25} \times 5\cdot 25 \times \cancel{\cdot 7854}}{\cancel{5\cdot 25} \times \underset{4}{\cancel{3\cdot 1416}}} = \frac{5\cdot 25}{4} = 1\cdot 3125,$$

$\therefore$ 1·3125 ÷ 9 = ·1458, the lift when the opening is ⅑ of the area of valve.

Ex. 9. What boiler pressure should be allowed for a shaft 14 inches diameter; the diameter of cylinder 57 inches, and length of stroke 36 inches?

Rule. $B = \frac{2880\, d^3}{D^2\, S}$ where B = boiler pressure
d = diameter of shaft.
D = " cylinder.
S = length of stroke.

$$B = \frac{2880 \times 14^3}{57^2 \times 36} = \frac{7902720}{116964} = 67\cdot 5 \text{ lbs.}$$

Ex. 10. What pressure may be allowed on a plate in the combustion chamber, if the thickness of plate be ½″, and the stays 15″ apart?

Rule. $B = \frac{60\,(T+1)^2}{S-6}$ where T = thickness in sixteenths,
and S = surface held by 1 stay.

In this instance $\frac{1}{2} = \frac{8}{16}$; hence, T = 8,

$$\therefore B = \frac{60 \times (8+1)^2}{15^2 - 6} = \frac{60 \times 81}{225 - 6} = \frac{4860}{219} = 22\cdot 19 \text{ lbs. } \textit{Ans.}$$

Ex. 11. What would be the smallest diameter of stay in this case?

Pressure held up by 1 stay 225 × 22·19 lbs. = 4992·75 lbs.

Area of stay = 4992·75 ÷ 5000 = 99855 square inches.

$$\text{Diameter of stay} = \sqrt{\frac{99855}{\cdot 7854}} = \sqrt{1\cdot 27} = 1\cdot 1 \text{ inch. } \textit{Ans.}$$

Ex. 12. For tunnel shaft $\frac{3200\, d^3}{S}$ = whole pressure on the piston.

The diameter of the cylinder is 60 inches, stroke 40 inches, diameter of shaft 9 inches: find the effective pressure per square inch on the piston.

$$\frac{3200 \times 9^3}{40} = \frac{2332800}{40} = 58320 \text{ lbs. pressure on whole piston.}$$

$$\therefore \frac{58320}{60^2 \times \cdot 7854} = \frac{58320}{2827\cdot 44} = 20\cdot 62 \text{ lbs. per square inch.}$$

Ex. 13. Find the boiler pressure, the diameter of the two cylinders being 64″ and 41″, length of stroke 32″, and diameter of shaft 11″.

$B = \frac{f\, d^3 - 15\, S\, D^2}{S\, H^2}$ B = boiler pressure.
S = length of stroke in inches.
D = diameter of low pressure cylinder.
H = " high pressure cylinder.
f = 4936 for crank shaft.
d = diameter of shaft.

$$B = \frac{(4936 \times 11^3) - 15\,(32 \times 64^2)}{32 \times 41^2} = \frac{(4936 \times 1331) - 15\,(32 \times 4096)}{32 \times 1681} = \frac{6569816 - 1966080}{53792}$$

$$= \frac{4603736}{53792} = 85\cdot 5 \text{ lbs. } \textit{Answer.}$$

ON CALCULATION OF WEIGHT OF DIFFERENT PARTS OF MACHINERY.

269. **Wrought iron varies in specific gravity from 7·8 to 7·6. Taking the mean at 7·7 a cubic foot will weigh 479·8721664 lbs. exactly. This is so near 480 lbs. that we may conveniently reckon a square foot one inch thick to weigh 40 lbs., every quarter inch of thickness giving 10 lbs. per square inch, and every eighth of an inch of thickness giving 5 lbs. per square inch. Thus we may readily find the weight of boiler plates by multiplying the eighths of thickness by 5 for every square foot.**

EXAMPLES.

Ex. 1. A boiler plate 6 feet 9 inches long, 4 feet 6 inches wide, and $\frac{3}{8}$ of inch thick: find the weight.

Length 6 ft. 9 in. × width 4 ft. 6 in. × thickness
12 12 $\frac{3}{8}$ or ·375
81 54

Length 81 inches.
Width 54 „
324
405
4374 4374
3 $\frac{3}{8}$ = ·375
8)13122 21870
1640·25 30618
13122
1640·250

or

$3·6 \begin{cases} 6)1640·25 \\ ·6)273·375 \end{cases}$

$28 \begin{cases} 4)455·625 \text{ lbs.} \\ 7)113—3 \\ 4)16—1 \end{cases}$ 7

4 0 7·625
cwt. qr. lbs.

Since the plate in question contains 1640·25 cubic inches, and 3·6 inches of wrought iron weighs 1 lb., on dividing the former quantity by the latter, the weight in pounds (lbs.) of the plate is found as above.

Since 1 square foot of wrought iron weighs 5 lbs. for every $\frac{1}{8}$ of an inch of thickness: then a $\frac{3}{8}$ inch plate will weigh 15 lbs. per square foot; hence by multiplying the square feet of plate by 15 we obtain the weight in pounds.

9 inches = $\frac{9}{12}$)9·0
·75
∴ 6 ft. 9 in. = 6·75 feet.
4·5 feet.
3375
2700
30·375 sq. feet
15
151875
30375
Weight of plate = 455·625 lbs.

6 inches = $\frac{1}{2}$)1·0
·5
4 ft. 6 in. = 4·5 feet.

$28 \begin{cases} 4)455·625 \text{ lbs.} = \text{weight of plate.} \\ 7)113—3 \\ 4)16—1 \end{cases}$ 7 lbs.

4 0 7·625
cwt. qr. lbs.

Ex. 2. A boiler plate is 8 feet 4½ inches long, 3 feet 2 inches wide, and ⅝ thick: find the weight.

Length 8 ft. 4½ in. × width 3 ft. 2 in. × thickness
12 12 ⅝″ or ·625

100·5 inches. 38 inches.

$$\text{Weight} = \frac{100{\cdot}5 \times 38 \times \frac{5}{8}\ (\text{or } {\cdot}625)}{3{\cdot}6 \times 28 \times 4} = 2386{\cdot}875.$$

The plate in question contains 2386·875 cubic inches, and 3·6 cubic inches of wrought iron weighs 1 lb., therefore, on dividing the cubic inches in plate by 3·6 gives the weight of plate in lbs. Reduce the lbs. to qrs. by dividing by 28 (28 lbs. equals 1 qr.) and 4 (4 qrs. make 1 cwt.)

$$4\tfrac{1}{2} \text{ inches} = \frac{4\frac{1}{2}}{12})4{\cdot}5 \quad \overline{{\cdot}375} \qquad\qquad 2 \text{ inches} = \tfrac{2}{12})2{\cdot}0 \quad \overline{{\cdot}166}$$

$$\therefore 8 \text{ ft. } 4\tfrac{1}{2} \text{ in.} = 8{\cdot}375. \qquad\qquad 3 \text{ ft. } 2 \text{ in.} = 3{\cdot}166.$$

Since 1 square foot of wrought iron weighs 5 lbs. for every ⅛ of an inch thickness, then a ⅝ inch plate will weigh 25 lbs. per square foot.

$$8{\cdot}375 \times 3{\cdot}166\ (= 26{\cdot}51525 \text{ sq. ft.}) \times 25 = 662{\cdot}88125 \text{ lbs., weight of plate,}$$

or 5 cwt. 3 qrs. 19 lbs. nearly, *Ans.*

Ex. 3. A piston is 76 inches diameter, and supposing it to be 4¼ inches thick throughout: what would it weigh?

Note.—Cast iron is from 3·8 to 3·9 cubic inches per lb.

$$\frac{76^2 \times {\cdot}7854 \times 4{\cdot}25}{3{\cdot}8} = 76 \times 76\ (= 4536{\cdot}4704) \times 4{\cdot}25 = \frac{19279{\cdot}9992}{3{\cdot}8} = 5073{\cdot}684 \text{ lbs.,}$$

or 2 tons 5 cwt. 1 qr. 5 lbs.

At the examinations there is a short rule given for working this and similar questions, it is

$$d^2 \times h \times {\cdot}206 = \text{weight in pounds,}$$

$\therefore 76^2 \times 4{\cdot}25 \times {\cdot}206 = 5056{\cdot}888$ lbs., which differs from the former by about 17 lbs.

270. Given the length over all and diameter of an iron screw-shaft, also the diameter and thickness of coupling flanges: to find the weight.

RULE CXXIII.

1°. From *the length over-all of screw-shaft* subtract *the* thickness *of the* two flanges, *and reduce the remainder into inches by multiplying by* 12.

2°. *Square the diameter of* shaft and multiply by ·7854 *and by* the length *in inches of screw-shaft; the result is the contents of shaft in cubic inches.*

3°. *Next* square *the diameter of the flange,* multiply *by* ·7854, *and by the* thickness *in* inches *of the* two *flanges, the result is the cubic contents of the two flanges.*

4°. Add *together the cubic* contents *of the* screw-shaft *found by No.* 2, *and* that *of the* two flanges *found by No.* 3°, *the* sum *is the* contents *of the* whole screw-shaft *in cubic inches.*

5°. Divide *this last result by* 3·6 *(the number of cubic inches equal* 1 *lb.), the* quotient *is the* weight *of screw-shaft in lbs.*

NOTE.—We may, instead of dividing by 3·6, multiply by ·28, the weight of a cubic inch of iron, the product is the weight in lbs. of screw-shaft.

6°. *Lastly,* divide *the weight in lbs. found in No.* 5° *by* 2240, *the result is the weight of screw-shaft in tons, &c.*

NOTE.—Instead of dividing by 2240 at once, we may reduce the lbs. into qrs. by dividing by 28 (4 × 7 = 28), 28 lbs. being equal to a quarter; divide qrs. by 4, the result is cwt.; and finally divide cwt. by 20 (20 cwt. equals 1 ton).

EXAMPLES.

EX. 1. A screw-shaft of iron is 17 feet 6 inches long over all, and 13 inches diameter, the coupling flanges are 25 inches diameter and $4\frac{1}{2}$ inches thick: what is the weight of the shaft?

Suppose the two flanges cut off, then there would remain a bar 13 inches diameter, and 16 feet 9 inches = 201 inches long.

Cubic inches in bar = $13^2 \times ·7854 \times 201 =$ 26679·2526
" 2 flanges = $25^2 \times ·7854 \times 4·5 \times 2 =$ 4417·8750
" in screw-shaft 31097·1276

Then, $\frac{31097·1276}{3·6}$ = 8638·091 lbs. = 3 tons 17 cwt. 0 qrs. 14 lbs.

Length of shaft over all 17 ft. 6 in.
Thickness of two flanges 9
16 9
12
201

Diameter of shaft 13 inches
13
39
13
Diameter^2 = 169
·7854
676
845
1352
1183
132·7326 sectional area.
Length 201 inches.
1327326
26546520
Con. of shaft 26679·2526 cubic inches.

Diameter of flange 25 inches
25
125
50
625
·7854
2500
3125
5000
4375
490·875,0 sectional area.
4·5 thickness.
2454375
1963500
2208·9375
2
Contents of 2 flanges 4417·8750 cubic inches.

Contents of shaft = 26679·2526 cubic inches.
" of 2 flanges = 4417·8750

$$
3{\cdot}6\left\{\begin{array}{l} {\cdot}6)\underline{31097{\cdot}1276}\text{ cubic inches.}\\ \;\;6)\underline{51828{\cdot}546} \end{array}\right.
$$

$$
28\left\{\begin{array}{l} 4)\underline{8638{\cdot}091}\text{ lbs.}\\ 7)\underline{2159-2} \end{array}\right.\left.\begin{array}{l}\\ \\ \;4)\underline{308-3}\end{array}\right\}14\text{ lbs.}
$$

$$
\begin{array}{l} 2,0)\underline{7,7,0}\\ \quad 3\;\;17\;\;0\;\;14 \end{array}
$$

Ans.: 3 tons 17 cwt. 0 qr. 14·091 lbs.

Ex. 2. A screw-shaft of iron is 18 feet 9 inches long over all, 14 inches diameter, the coupling flanges are 27 inches diameter, and 5½ inches thick: what is the weight of the shaft?

Length of shaft over all	18 ft. 9 in.	Thickness of flanges	5½ in.
Thickness of 2 flanges	— 11		2
Length of shaft without flanges	17 10	Thickness of 2 flanges	11 in.
	12		
	214 in.		

Contents of shaft $= 14^2 \times {\cdot}7854 \times 214 = 14 \times 14\ (= 196) \times {\cdot}7854\ (= 153{\cdot}9384) \times 214 = 32942{\cdot}8176$ cubic inches.

Content of 2 flanges $= 27^2 \times {\cdot}7854 \times 11 = 27 \times 27\ (= 729) \times {\cdot}7854\ (= 572{\cdot}5566) \times 11 = 6298{\cdot}1226$ cubic inches.

∴ Contents of shaft + contents of 2 flanges = 32942·8176 + 6298·1226 =

39240·9402 cubic inches = 4 tons 17 cwt. 1 qr. 8·2432 lbs. *Answer.*

EXAMPLES FOR PRACTICE.

No.	Length of Shaft over all. ft.	in.	Dia. of Shaft.	Dia. of Flanges.	Thickness of Flanges.	*Ans.*—Weight. Tons	cwt.	qrs.	lbs.
1.	20	6	13 inches	24 inches	4 inches	4	7	1	8·288
2.	17	4	9 "	18 "	3 "	1	15	2	17·741
3.	19	6	14½ "	25¾ "	5 "	5	4	2	16·8
4.	20	4	12⅜ "	22 "	3½ "	3	17	1	5·35
5.	25	6	12½ "	25 "	3¼ "	4	19	0	7·14

Ex. 3. Find the weight of a shaft 16 ft. 6 in. over all, diameter of shaft 10½ inches, the flanges are unequal; 1st diameter 20½ inches, length 3¾ inches; 2nd diameter 20 inches, length 2⅜ inches. Give all the answers according to the following rules:—

This question may be solved by Rule CXXIII, pages 261-262, with this difference, that the flanges being unequal the contents of each is to be found separately.

Again, the multiplier ·7854 and the divisor 3·6 are constant numbers, and instead of multiplying by ·7854 and dividing by 3·6 we may multiply by ·218, the quotient of ·7854 by 3·6, the result is the weight of screw-shaft in lbs.

To find the weight of round iron, square the diameter in inches, multiply by length in inches, and multiply by ·218 equals the weight in lbs.

To find the net weight of a rough shaft, square the diameter in inches, multiply by length in inches, point off the last 4 figures, and the remainder is the weight in tons; should the shaft be smooth all over or turned up deduct ½ cwt. for each ton, hence,

Conceive the two flanges cut off, then there would remain a bar 10½ inches diameter, and 16 feet 0⅜ inch = 192·375 long.

```
                                              1st Collar.                 2nd Collar.
Length of shaft over all      16 ft. 6 in.   Flanges = 20·5 in.              20 in.
Length of flanges (2¾ + 2⅞) —       5⅝                 20·5                  20
                              ----------               ----                  --
  ⅝)3·0                        16    0⅜                1025                 400 = D²
  -----                        12                      4100          Length 2·875
   ·375                        --                     ------                -------
            Shaft = 192⅜                              420·25               1150·000
               or, 192·375                   Length   2·75
                                                      ------         Length 192·375
     Dia. of shaft  10·5″                            210125                 110·25 1st
                    10·5                             294175                --------
                    ----                             84050                  961875
                     525                            --------               384750
                    1050                           1155·6875              1923750
                    ------                                               192375
                    110·25 = D²                                          -----------
                                                             Shaft       21209·34375
                                    23515·03125              1st flange   1155·6875
                                   ·7854/3·6 = ·218          2nd  do.     1150·0000
                                   ----------------                      -----------
                                    18812025000                  Total   23515·03125
                                    2351503125                           = d² × l
                                   4703006250
                                   ----------------
                                   5126·27681250 Answer.
```

Rules.—$D^2 \times L \times \cdot218$ = weight in pounds.

$D^2 \times L$ and four figures marked off as decimals is equal to the weight in tons when the shaft is in a rough state.

Again:—

Cubic contents of shaft and flanges as above, is 23515·03125 inches, and pointing off 4 figures to the left = tons of a rough shaft.

```
Tons 2·3515
         20
     ------
Cwt.  7·0300
          4
      ----
Qrs.  0·12
         28
      ----
Lbs.  3·36 Ans.: 2 ton 7 cwt. 0 qr. 3·36 lbs.
```

For a smooth shaft subtract ½ cwt. for each ton the shaft weighs, thus:—

```
2)2·3515 tons
  -------
  1·17575 cwt.
        4
  -------
  0·70300 qrs.
       28
   -----
   5624
  1406
  ------
  19·684 lbs.
```

	tons	cwt.	qrs.	lbs.
Weight of rough shaft	2	7	0	3·36
Subtract for smooth shaft	—	1	0	19·68
Weight of smooth shaft	2	5	3	11·68*

* *Answers*
1st = 5126·27681 lbs.
2nd = 2 ton 7 cwt. 0 qr. 3·36 lbs.
3rd = 2 ton 5 cwt. 3 qr. 11·68 lbs.

EXAMPLES FOR PRACTICE.

No.	Length over all of Shaft. ft.	in.	Diameter of Shaft. inches.	Diameter of Flanges. inches.	Thickness of Flanges. inches.	*Answers.*
1.	20	6	13	23 and 25	$3\frac{1}{2}$ and $4\frac{1}{2}$	1st, 9785·148 lbs. 2nd, 4 tons 9 cwt. 3 qrs. 2 lbs. 3rd, 4 ,, 7 ,, 2 ,, 3 ,,
2.	17	4	9	17 ,, 19	$2\frac{1}{2}$,, $3\frac{1}{2}$	1st, 3983·864 lbs. 2nd, 1 ton 16 cwt. 2 qrs. 22 lbs. 3rd, 1 ,, 15 ,, 3 ,, 3 ,,
3.	19	6	$14\frac{1}{2}$	$24\frac{3}{4}$,, $26\frac{3}{4}$	4 ,, 5	1st, 11626·88 lbs. 2nd, 5 tons 6 cwt. 2 qrs. 19 lbs. 3rd, 5 ,, 4 ,, 0 ,, 0 ,,
4.	20	4	$12\frac{3}{8}$	21 ,, 23	3 ,, 4	1st, 8661·864 lbs. 2nd, 3 tons 19 cwt. 1 qr. 24 lbs. 3rd, 3 ,, 17 ,, 1 ,, 26 ,,

EXAMPLES.

Ex. 1. A shaft is 29 ft. 6 in. long over all, and 11 in. diameter; there are two flanges each 3 in. thick and 20 in. diameter, five bolts through each coupling $2\frac{1}{2}$ in. diameter: find the weight and make a sketch of it.

Length of shaft over all	29 ft. 6 in.	Thickness of flanges	3 in.
Thickness of 2 flanges	6 ,,		2
Length of shaft without flanges	29 ft. 0 in.	Thickness of 2 flanges	6 in.
	12		
	348 in.		

(1)

Contents of shaft $= 11^2 \times \cdot 7854 \times 348 =$ 33071·6232 cubic inches.

Contents of 2 flanges $20^2 \times \cdot 7854 \times 6 =$ 1884·96 ,,

34956·5832 ,,

Contents of holes $= 2\cdot 5^2 \times \cdot 7854 \times 3 \times 5 \times 2 =$ − 147·2625 ,,

$$3\cdot 6 \left\{ \begin{array}{r} \cdot 6)\,34809\cdot 3207 \\ \hline 6)\,\underline{58015\cdot 534} \end{array} \right.$$

9669·255 lbs.

(2)

Shaft $11^2 \times 348 = 42108$

Flanges $20^2 \times 3 \times 2 = 2400$

44508

Holes $2\cdot 5^2 \times 3 \times 5 \times 2 = -\ 187\cdot 5$

44320·5

$\frac{\cdot 7854}{3\cdot 6}$ ·218

3545640

443205

886410

9661·8690 lbs.

(3)

$D^2 \times L$ and four figures marked off to left gives weight in tons in rough shaft, thus:—

4,4320·5 tons.

20

8·6405 cwt.

4

2·5620 qrs.

28

4496

1124

15·736 lbs.

(4) For a smooth shaft, subtracting $\frac{1}{2}$ cwt. for each ton the shaft weighs, we have

2)4·4320

2·2160 cwt.

4

0·864 qrs.

28

6912

1728

24·192 lbs.

	tons.	cwt.	qrs.	lbs.
Weight of rough shaft	4	8	2	15·7
Subtract for smooth shaft	0	2	0	24·2
Weight of smooth shaft	4	6	1	19·5

Answers
1st, = 9669·255 lbs.
2nd, = 9661·869 lbs.
3rd, = 4 tons 8 cwt. 2 qrs. 15·74 lbs.
4th = 4 tons 6 cwt. 1 qr. 19·5 lbs.

Ex. 2. The diameter of a safety-valve is 6″ and is loaded with cast iron weights, what should be the height of these weights to press the valve equal to 11 lbs. per square inch, the weights being in a case 14″ diameter and $\frac{7}{8}$″ clearance all around?

Short Rule.—$d^2 \times h = 5$ times the weight in lbs.

Valve
·7854
$d_4^2 =$ 36

28·2744
11 lbs.

Weight of valve 311·0184 lbs.
5

1555·0920 = 5 times weight of valve.

Diameter of box 14 inches.
Clearance both sides $1\frac{3}{4}$ „

Diameter of weights $12\frac{1}{4}$ inches.

or, 12·25
12·25

6125
2450
14700

$d^2 =$ 150·0625

And, by rule, 150·0625 × h = 1555·0920,

$\therefore \frac{1555·0920}{150·0625} =$ 10·36 inches, the height of weights.

Ex. 3. A safety-valve is $5\frac{1}{2}$″ diameter, what height should the weights be to press the valve to 20 lbs. per square inch, the weights being in a box 15″ diameter, and $\frac{1}{2}$″ clearance all around?

Diameter of box 15″ — ($\frac{1}{2}$″ × 2) 1″ = 14″. $d^2 = 14''^2 = 196$.

·7854 × 5·5^2 (= 23·75835) × 20 (= 475·167) × 5 = 2375·835 = five times weight of valve.

$\therefore$ 2375·835 ÷ 196 = 12·12 inches, the height of weights.

Ex. 4. What is the weight of gun metal in a screw-shaft liner whose length is 79 inches, outer diameter 13 inches, and thickness $\frac{15}{16}$ of an inch, if a cubic inch weighs ·3 of a lb.

It is evident that the inner diameter will be equal to the outer diameter diminished by twice the thickness.

Thickness $\frac{15}{16} \times 2 = \frac{30}{16} = 1\frac{7}{8}$.

$\therefore$ inner diameter $13 - 1\frac{7}{8} = 11\frac{1}{8} =$ 11·125 inches.

Having now given the outer and inner diameters of shaft, proceed by No. 228, Rule LXXIX, to find the area of a section of shaft, thus:—

Area of section = outer d^2 × ·7854 — inner d^2 × ·7854
= 13^2 × ·7854 — 11·125^2 × ·7854
= 169 × ·7854 — 123·765625 × ·7854
= 132·7326 — 97·205521875
= 35·527078125.

Or thus, since the difference of the squares of two quantities is equal to the sum of the two quantities multiplied by their difference, we have,

Area of section = (outer d + inner d) × (outer d — inner d) × ·7854
= (13 + 11·125) × (13 — 11·125) × ·7854
= 24·125 × 1·875 × ·7854
= 45·234375 × ·7854
= 35·527078125.

Cubic inches in screw-shaft = 35·5271 × 79 = 2806·6409,
∴ 2806·6409 × ·3 = 841·99227 lbs. *Ans.*

271. To find the weight of a thrust-block.

Ex. Sketch a thrust block from the following sizes, and find the weight when 1 cubic inch of iron weighs ·3 of a lb.

There are 9 collars, each 1¼ inches thick, with spaces 1⅛ inches between them; diameter of shaft 9 inches, of collars 11 inches, of seat 13 inches, and of flanges 15 inches; the flanges are each 1¼ inches thick.

9 collars each 1¼ inches thick = 11¼ inches total.
10 spaces each 1⅛ inches wide = 11¼ „ „

Total length of seat = 22½ „

Shaft.	Collar.	Seat.	Flanges.
·7854	·7854	·7854	·7854
$d^2 (= 9^2) = 81$	$d^2 (= 11^2) = 121$	$d^2 (= 13^2) = 169$	$d^2 (= 15^2) = 225$
7854	7854	70686	39270
62832	94248	47124	15708
		7854	15708
63·6174	95·0334	132·7326	176·7150
	— 63·6174	— 63·6174	— 63·6174
	31·4160	69·1152	113·0976
	Length 11¼	Length 22·5	Length 2·5
	345576	3455760	5654880
	7854	1382304	2261952
		1382304	
	353·430	1555·09200	282·74400
		— 353·43	
		1201·662	
		282·744	

Total = 1484·406 cubic inches.
·3

445·3218 lbs. *Answer.*

Or we may proceed thus:—Instead of taking the difference of the squares of the two quantities, we multiply the sum of the two quantities by their difference, Rule LXXIX, and the work will then stand as below:—

Seat $(13 \pm 9) \times 22·5 = 22 \times 4 \times 22·5$ = 1980
Flanges $(15 \pm 9) \times 2·5 = 24 \times 6 \times 2·5$ = 360

2340
Collars (11 ± 9) $20 \times 2 \times 11·25 = 20 \times 2 \times 11·25$ = —450

1890

Total contents = 1890 × ·7854 = 1484·406 cubic inches.
∴ 1484·406 × ·3 = 445·3218 lbs. *Answer.*

272. To find weight of crank for paddle-shaft.

Ex. Make a sketch of a crank for a paddle-shaft, and find its weight. Pin-boss; pin 5 inches diameter, boss 10¼ inches diameter and 10½ inches across. Shaft-boss; shaft 8 inches diameter, boss 13½ inches diameter and 11¼ inches across, bar is 8 inches wide at pin-boss, and 12 inches at shaft-boss, and 4½ inches across. Add ⅙ the diameter of each boss to the length of bar for the fillets, the length from centre to centre being 26 inches.

Length of bar from boss to boss = length from centre to centre − ½ diameter pin-boss − ½ diameter shaft-boss

$$= 26 - \frac{10{\cdot}25}{2} - \frac{13{\cdot}5}{2} = 26 - (5{\cdot}125 + 6{\cdot}75)$$
$$= 26 - 11{\cdot}875 = 14{\cdot}125 \text{ inches.}$$

$$\text{Increased for fillets} = 14{\cdot}125 + \frac{10{\cdot}25}{6} + \frac{13{\cdot}5}{6} = 14{\cdot}125 + 1{\cdot}7083 + 2{\cdot}25$$
$$= 14{\cdot}125 + 3{\cdot}9583 = 18{\cdot}0833.$$

$$\text{Mean width of bar} = \frac{8+12}{2} = 10.$$

$$\text{Content of pin-boss } (10{\cdot}25^2 - 5^2) \times {\cdot}7854 \times 10{\cdot}5 = (105{\cdot}0625 - 25) \times {\cdot}7854 \times 10{\cdot}5$$
$$= 80{\cdot}0625 \times {\cdot}7854 \times 10{\cdot}5 = 660{\cdot}25141875.$$

$$\text{Shaft-boss } (13{\cdot}5^2 - 8^2) \times {\cdot}7854 \times 11{\cdot}25 = (182{\cdot}25 - 64) \times {\cdot}7854 \times 11\tfrac{1}{4}$$
$$= 118{\cdot}25 \times {\cdot}7854 \times 11\tfrac{1}{4} = 1044{\cdot}8274375.$$

Contents of bar = 18·0833 × 4·5 × 10 = 813·74850 cubic inches.

∴ total cubic contents = contents of pin-boss + contents of shaft-boss + contents of bar = 660·25141875 + 1044·8274375 + 813·74850 = 2518·82735625.

And as this is cast iron, 2518·82735 ÷ 3·6 = 699·5076 lbs., the weight of crank.

Or thus:— Pin-boss $(10{\cdot}25 \pm 5) \times 10{\cdot}5 = 15{\cdot}25 \times 5{\cdot}25 \times 10{\cdot}5 = 840{\cdot}65625$

Shaft-boss $(13{\cdot}5 \pm 8) \times 11{\cdot}25 = 21{\cdot}5 \times 5{\cdot}5 \times 11{\cdot}25 = 1330{\cdot}3125$

2170·96875

2170·96875 × ·7854 = 1705·07885

18·0833 × 4·5 × 10 = 813·7485

2518·82735

&c., &c.

273. To find weight of slide-valve.

Ex. A slide-valve is worked by a hooped spindle, the hoop surrounding the body of the valve. The port of the valve 20″ wide, exhaust port is 6″ deep, steam ports 2½″ deep, bars 1½″, steam lap 3″, internal cover ⅛″, the valve is 1¼″ on each side of the ports, face of valve is ⅞″ thick, while the rest is ⅝″ thick, the port of the valve is rectangular and 3¼″ deep: find the weight, allowing 3·9 cubic inches = 1 lb.

Whole depth of face of valve = 3″ + 2½ + ⅛ = 5⅝″.
Whole width of face = 20 + 1¼ + 1¼ = 22½″.
Depth of valve port = whole depth of valve − the 2 faces.

= 20 − (5⅝ + 5⅝) 11¼ = 8¾.

Height of walls above the face = depth of port − thickness of back and face of valve = 5¼ − ⅞ + ⅝ = 3″.

Total length of 4 walls = 2 (20″ + ⅝ + ⅝) + 2 (8¾) = 60½.

Cubic inches in face	22½ × 5⅝ × ⅞ × 2 = 22·5 × 5·625 × ·875 × 2	= 221·484375
„ sides	8¾ × 1¼ × ⅞ × 2 = 8·75 × 1·25 × ·875 × 2	= 19·140625
„ back	20 × 8⅞ × ⅝ = 20 × 8·875 × ·625	= 110·9375
„ 4 walls	60½ × 3 × ⅝ = 60·25 × 3 × ·625	= 112·96875
		464·531250

464·53125 ÷ 3·9 = 119·11, &c., lbs.

274. To find the weight of a crank shaft.

Make a sketch of crank-shaft with 6 collars turned on it, the length over all is 11′ 4½″, from centre to centre of engines 4′ 11″, fore end of shaft 1′ 2″ long, diameter of shaft 9″, diameter of crank-pin 8½″, length of pin 9″, web of cranks 29″ by 10½″ by 7″, one flange 18″ diameter, 2¼″ thick, with 8 holes in flange 2″ diameter, the collars are 1⅜″ thick and 1⅛″ spaces: find the weight.

Shaft $9''$ (dia.) $\times \cdot 7854 \times 88 \cdot 25$ (length) $= 5614 \cdot 23555$
Webs $29'' \times 10\frac{1}{2} \times 7 \times 4 = 8526 \cdot 00000$
Pins $8 \cdot 5^2 \times \cdot 7854 \times 9 \times 2 = 1021 \cdot 41270$
Flanges $(18 \pm 9) = 27 \times 9 \times \cdot 7854 = 429 \cdot 41745$
Collars $(11 \pm 9) = 20 \times 2 \times \cdot 7854 = 259 \cdot 18200$

$15850 \cdot 24770$
Holes $2^2 \times \cdot 7854 \times 8 = - 25 \cdot 13280$

$3 \cdot 6 \begin{cases} \cdot 6)15825 \cdot 11490 \\ 6)26375 \cdot 1915 \end{cases}$

$28 \begin{cases} 7)4395 \cdot 8651 \\ 4)627-6 \end{cases}$
$\left.\begin{array}{l} 4)627-6 \\ 4)156-3 \end{array}\right\} 27$ lbs.
$2{,}0)39-0$
$1 \quad 19 \quad 0 \quad 27 \cdot 865.$

Tons	cwt.	qr.	lbs.
1	19	0	27·865

275. To find the weight of a pair of brasses.

Ex. What is the weight of a pair of brasses 20 inches long, diameter of journal 11 inches, diameter of seat 13½ inches, diameter over flanges 16½ inches, and thickness of flanges 1¼ inch?

Length of seat = length of brasses — thickness of flanges =
$20'' - 2\,(1\frac{1}{4}'') = 20'' - 2\frac{1}{2} = 17\frac{1}{2}''.$

Sectional area of journal $= 11^2 \times \cdot 7854 = 95 \cdot 0334$
„ seat $= 13 \cdot 5^2 \times \cdot 7854 = 143 \cdot 13915$
„ flanges $= 16 \cdot 5^2 \times \cdot 7854 = 213 \cdot 82575$

Then $(143 \cdot 13915 - 95 \cdot 0334) \times \cdot 7854 = 841 \cdot 850625$ cubic inches in seat,
and $(213 \cdot 8215 - 95 \cdot 0334) \times 2 \cdot 5 = 296 \cdot 979375$ cubic inches in flanges.

$1138 \cdot 8300$

$\therefore$ weight $= 1138 \cdot 83 \times \cdot 3 = 341 \cdot 649$ lbs. *Answer.*

QUESTIONS RELATING TO RATE AND SLIP OF SCREW.

276. Given the number at which the engine counter stands at the beginning of a voyage, also what it indicates at end of the same, together with the time it has been working: to find how many revolutions per minute it has made on an average during the voyage.

Examples.

Ex. 1. An engine counter stood at 1587 at the beginning of a voyage, and it now stands at 772087, having been working for 11 days 15 hours 10 minutes: how many revolutions per minute has the engine made on the voyage?

At the end of the voyage counter stood at 772087 revolutions.
" beginning " " 1587 "

During the voyage the counter made 770500 "

The time the counter has been working is $11^d\ 15^h\ 10^m$, which is 16750 minutes.

$11^d\ 15^h\ 10^m$
24

59
22

279 hours.
60

16750 minutes.

Then $\frac{770500}{16750} = 46$ revolutions per minute.

1675,0)77050,0(46 revolutions.
6700

10050
10050

Ex. 2. An engine counter stood at 1001 at the beginning of a voyage, and it now stands at 796001, having been working for 10 days 10 hours: how many revolutions per minute has the engine made on the voyage?

At the end of the voyage counter stood at 796001 revolutions.
" beginning " " 1001 "

During voyage engine has made 795000 "

Then 10 days 10 hours = 15000 minutes.

$10^d\ 10^h$
24

250 hours.
60

15000

$\frac{795000}{15000} = 53$ revolutions per minute.

15,000)795,000(53
75

45
45

Ex. 3. A steamer goes at the rate of 10·2 knots per hour when the engines are making 65 revolutions per minute; the counter indicated 18908 at starting and is now at 214817: how many nautical miles has the vessel made in that time, supposing the slip to be the same throughout?

At starting counter stood at 18908
Now counter stands at 214817

Revolutions made 195909

Revolutions per minute 65
60

Revolutions per hour 3900

∴ 195909 ÷ 3900 = 50·23 hours on voyage,

And 50·23 × 10·2 = 512·346 nautical miles. *Answer.*

EXAMPLES FOR PRACTICE.

In each of the following exercises it is required to find the number of revolutions per minute.

	End of voyage.	Be. of voyage.	Interval.		End of voyage.	Be. of voyage.	Interval.
1.	981,260	5,040	$8^d 20^h 5^m$	4.	1,000,000	17,344	$14^d 5^h 12^m$
2.	888,963	28,563	9 23 0	5.	989,969	143,298	11 23 59
3.	1,126,617	69,243	12 0 54	6.	935,879	142,319	10 19 20

Answers.

1. 981260 — 5040 = 976220; $8^d 20^h 5^m$ = 12725^m; then 976220 ÷ 12725 = 76·71 revolutions per minute.

2. 888963 — 28563 = 860400; $9^d 23^h$ = 14340^m; then 860400 ÷ 14340 = 60 revolutions.

3. 1126617 — 69243 = 1057374; $12^d 0^h 54^m$ = 17334^m; then 1057374 ÷ 17334 = 61 revolutions.

4. 1000000 — 17344 = 982656; $14^d 5^h 12^m$ = 20472^m; then 982656 ÷ 20472 = 48 revolutions.

5. 989969 — 143298 = 846671; $11^d 23^h 59^m$ = 17279^m; then 846671 ÷ 17279 = 49 revolutions.

6. 935879 — 142319 = 793560; $10^d 19^h 20^m$ = 15560; then 793560 ÷ 15560 = 51 revolutions.

277. **Given the diameter of a paddle-wheel and revolutions per minute; to find the speed of the ship.**

EXAMPLE.

The paddle-wheel of a steamer is 27½ feet in diameter, and the revolutions 1 per minute: find the speed of ship.

3·1416
27·5
157080
219912
62832
86·39400
780
691152
604758
6080)6738·732,000(11·08 knots per hour.
6080
5587
6080
50732
48640

Revolutions per minmte 13
60
„ hour 780

NOTE.—The circumference of a paddle-wheel multiplied by the revolutions per minute, and two figures cut off, gives the speed in knots nearly. It is too great in the exact proportion of 304 to 300.

EXAMPLES FOR PRACTICE.

No.	Diameter of Paddle-wheel.	Revolutions per minute.	*Answer:* Knots per hour.
1.	16	23	11·4
2.	26	20½	16·52
3.	20	21	13·02
4.	30	10½	9·766

278. Given the diameter of a paddle-wheel, to find the number of revolutions an engine must make to drive the wheel at a given rate.

RULE CXXIV.

1°. *Find the circumference of the paddle-wheel by multiplying the diameter by* 3·1416 (Rule LXXIV, page 183); this is equal to the distance moved in one revolution.

2°. *Find the number of* feet *in the* knots *the paddle-wheel is driven per hour, by multiplying the number of knots by* 6080 *feet.*

3°. *Next find how often this last* (No. 2°) contains *the distance moved in one revolution* (No. 1°); the result is the number of revolutions required.

EXAMPLE.

Ex. How many revolutions must an engine make to drive a paddle-wheel 10½ knots per hour, the diameter of the wheel being 30 feet, and the knot being equal to 6080 feet?

Circumference of wheel = 30 × 3·1416 = 94·2480 (Rule LXXIV, page 183).

3·1416
30

Circumference = 94·2480 ft. = distance moved in 1 revolution.

10½ knots = 6080 × 10½ = 63840 feet.

6080
10½

60800
3040

63840 feet.

Number of revolutions = $\frac{63840}{94{\cdot}2480}$.

94·248)638400(677·3 revolutions.
565488

729120
659736

693840
659736

341040

279. Given the pitch* of a screw, it is required to find the number of revolutions it must make to advance at a given rate.

RULE CXXV.

1°. *Find the number of* feet *in the* knots *per hour, by multiplying the knots by* 6080 *feet.*

2°. *Divide the product* (No. 1°) *by the pitch of the screw or distance moved in one revolution;* the quotient is the number of revolutions required.

---

* The pitch of a screw may be explained to be the distance in the line of the shaft from one convolution to the next, supposing the screw to consist of two or more convolutions, [illegible] in other words, the axial length of a whole turn of the thread.

EXAMPLES.

Ex. 1. The pitch of a screw is 18 feet: how many revolutions must it make to advance 12 knots per hour?

12 knots = 6080 × 12 = 72960 feet.

Distance moved in 1 revolution = pitch = 18 feet.

Number of revolutions = $\frac{72960}{18}$ = $4053\frac{1}{3}$ revolutions.

$$\begin{array}{r} 6080 \\ 12 \\ \hline 72960 \text{ feet.} \end{array} \qquad 18\left\{\begin{array}{r} 3)72960 \\ \hline 6)24320 \end{array}\right. \\ \hline \qquad\qquad\qquad 4053\tfrac{1}{3} \text{ revolutions.}$$

Ex. 2. The pitch of a screw is 11 feet 6 inches: how many revolutions must it make to advance 9·5 knots per hour?

$9\frac{1}{2}$ knots = 6080 × 9·5 = 57760 feet.

Distance moved in one revolution = pitch of screw = 11·5 feet.

Number of revolutions = $\frac{57760\cdot0}{11\cdot5}$ = 50226 revolutions.

EXAMPLES FOR PRACTICE.

No.	Pitch.	Knots.	No.	Pitch.	Knots.
1.	19 feet 0 inches.	12	4.	11 feet 6 inches.	11·8
2.	17 „ 0 „	14·8	5.	6 „ 8 „	12
3.	15 „ 0 „	10	6.	$21\frac{4}{7}$ „ „	12

280. The screw propeller, if working in a medium that could not yield, would advance a distance equal to the pitch during each revolution. But such is not the case in the yielding medium in which it actually does rotate; and therefore it is found that the knots per hour passed over by the ship are less than those which ought to have been passed over, on the supposition that the screw was working in an unyielding medium. If, then, the former be taken from the latter, the difference is called the *slip of the screw in knots;* and if this difference be divided by the speed of the screw in knots, we shall have the fraction of the speed lost; this, again, if multiplied by 100, will give the per centage of loss.

281. Given the pitch of the propeller, and the number of revolutions per minute, to find the progression of the propeller, and the rate of the ship.

RULE CXXVI.

1°. *Multiply the pitch of propeller by the number of revolutions per minute, and this product by* 60; *this last result divided by* 6080, is the progression of propeller per hour in knots.

2°. *Take the amount of slip per cent. from* 100; *then say, as* 100 *is to remainder, just found, so is rate of progression of propeller (*1°*) to the rate of the ship.*

EXAMPLES.

Ex. 1. The pitch of the propeller is 16 feet, and it makes 65 revolutions per minute; what will be the progression of the propeller in knots per hour, and the rate of the ship, allowing 20 per cent. slip?

$$16 \times 65 \times 60 = 63840 = \text{advance of screw per hour.}$$

$$\therefore \tfrac{63840}{6080} = 10{\cdot}2 \text{ advance in knots per hour.}$$

$$\text{Or, Progression of propeller per hour} = \frac{16 \times 65 \times 60}{6080} = 10{\cdot}2 \text{ knots.}$$

$$\begin{array}{r}
16 \\
\underline{65} \\
80 \\
\underline{96} \\
1040 \\
\underline{60} \\
608{,}0)6240{,}0(10{\cdot}2 + \\
\underline{608} \\
1600 \\
\underline{1216} \\
384
\end{array}$$

$$100 : 80 \; (100 - 20 = 80) :: 10{\cdot}2 : \text{rate of ship.}$$

$$\therefore \text{rate of ship} = \frac{10{\cdot}2 \times 80}{100^{*}} = 8{\cdot}16 \text{ knots.}$$

Ex. 2. The pitch of a propeller is 11 feet 6 inches, and it makes 104 revolutions per minute: what will be the progression of the propeller in knots per hour, and the rate of the ship, allowing 19·2 per cent. slip?

$$11{\cdot}5 \times 104 \times 60 = 71760 \text{ feet} = \text{advance of screw per hour.}$$

$$\therefore \tfrac{71760}{6080} = 11{\cdot}8 = \text{advance in knots per hour.}$$

$$\therefore 100 : 80{\cdot}8 = (100 - 19{\cdot}2) :: 11{\cdot}8 : \text{speed of ship.}$$

$$\text{Hence speed of ship} = \frac{11{\cdot}8 \times 80{\cdot}8}{100} = 9{\cdot}5 \text{ knots.}$$

NOTE.—Deduct from the pitch 1 inch for every 6 feet in the pitch, and multiply the reduced pitch by the number of revolutions per minute and divide by 100, the result will be the speed of the screw propeller in knots per hour.

Ex. (Ex. 1 above). A screw is 16 feet pitch, and makes 65 revolutions per minute.

$$\tfrac{16}{6} - 2''{\cdot}66 = {\cdot}2$$

$$16 - {\cdot}2 = 15{\cdot}8$$

$$\begin{array}{r}
15{\cdot}8 \\
\underline{65} \\
790 \\
\underline{948} \\
\underline{1027{\cdot}0}
\end{array}$$

Speed = 10·27 knots.

---

[*] To divide by 100 we have simply to remove the decimal point *two places to the left hand* (see page 98, No. 126).

$$\begin{array}{r}
10{\cdot}2 \\
\underline{80} \\
8{\cdot}160
\end{array}$$

Ex. 3. A propeller of 21 feet pitch is altered to 18 feet pitch with the same pressure of steam; if the ship make 9 knots with the first pitch, what should she make now?

The square of the rates are in indirect proportion to the pitches, that is:—

$$18 : 21 :: 9^2 : x^2$$

$$x^2 = \frac{21 \times 9 \times \cancel{9}}{\underset{2}{\cancel{18}}} = \frac{189}{2} = 94·5.$$

$$\sqrt{94·5} = 9·72 \text{ knots.}$$

$$\begin{array}{r|l} 9 & 94·50(9·72 \text{ knots.} \\ 9 & 81 \\ \hline 187 & 1350 \\ 7 & 1309 \\ \hline 1942 & 4100 \end{array}$$

Examples for Practice.

No.	Pitch.	Rev.	Slip.	No.	Pitch.	Rev.	Slip.
1.	19 ft.	69	25	4.	6 ft. 8 in.	84	13
2.	17·1	74	20	5.	11·9	108	16
3.	14½	126	15·5	6.	11·3	95	11·3

Examples.

Ex. 1. If the pitch of a propeller is 21 feet and the revolutions per minute 64, and the ship makes 11·29 knots per hour: if the pitch be altered to 23 feet with 76 revolutions per minute, what should now be the rate of the ship, allowing the same per centage of slip?

21 ft. × 64 rev. : 23 ft. × 76 rev. :: 11·29 knots : x *Ans.* 14·68 knots.

Ex. 2. If the mean pressure was 29 lbs. per square inch, the ship running 10 miles per hour, then taking the mean pressure to be 32 lbs. per square inch: what is the speed of the ship?

$$\begin{array}{ccccc} \text{Press. lbs.} & & \text{Press. lbs.} & & \text{Speed.} \\ 29 & : & 32 & :: & 10^2 \\ & & 100 & & \end{array}$$

$$\begin{array}{l} 29)3200(110·35 \\ 29 \\ \hline 30 \\ 29 \\ \hline 100 \\ 87 \\ \hline 130 \\ 145 \\ \hline \end{array} \qquad \begin{array}{r|l} 1 & 110·35(10·5 \\ 1 & 1 \\ \hline 205 & 1035 \\ & 1025 \\ \hline & 10 \end{array}$$

Speed required 10·5 knots.

Ex. 3. A shaft in a marine engine was making 20 revolutions, and the speed was 8 knots: what will be the speed if the revolutions be increased to 25?

The revolutions of the crank vary as the cube of the speed.

Let V be the required speed.

$$\frac{V^3}{8^3} = \frac{25}{20}$$

$$\therefore V^3 = \frac{25}{20} \times 8^3 = \frac{25}{20} \times 512 = 640,$$

$$\therefore V = \sqrt[3]{640} = 8·617 \text{ knots.}$$

Ex. 4. The revolutions of the crank of a marine engine are 24 per minute, and the speed 10 knots: the revolutions are increased to 30: find the increase of speed. *Ans.* ·77 knot.

Ex. 5. A steamer from the Tyne to Flambro' Head, a distance of 72 miles, the counter registers 25671 revolutions, the pitch is 18 feet: how much is this per cent. slip?

25671 rev.
18 pitch.
205368
25671
462078 feet gone.

6080)462078(76 engines.
42560
36478
36480

Distance run by engines 76
„ „ ship 72
4

As 76 miles : 100 :: 4
4
·76)400(5·26 per cent. of slip.
380
200
152
480
456

Ex. 6. The speed of a ship is 8 knots per hour with a consumption of 10 tons per day; but the speed is increased to 9 knots per hour, and the consumption to 13 tons per day: what is the rate of cost per day saved?

Given formula $\frac{C_1 K - C K_1}{K_1 - K}$

$C_1 = 13$ tons
$K = 8$ knots
104
− 90
14

$C = 10$ tons
$K_1 = 9$ knots
90

$K_1 = 9$
$K = 8$
1

Then 1)14
14 *Ans.*

Ex. 7. Verify the above formula by working at full length under the two conditions on a voyage of 2400 miles.

$$\text{Days on voyage at the 1st rate} = \frac{2400}{24 \text{ hours} \times 8 \text{ knots}} = 12{\cdot}5 \text{ days.}$$

$$\text{„ „ 2nd „} = \frac{2400}{24 \times 9} = 11{\cdot}\dot{4}444 \text{ \&c.}$$

Tons burnt per day at 1st rate $= 12{\cdot}5 \times 10 = 125$

„ „ 2nd „ $= 11\frac{1}{9} \times 13 = 144\frac{4}{9}$

Difference of consumption $= 14{\cdot}4444 - 125 = 19{\cdot}4444$.

Difference of days $= 12{\cdot}5 - 11{\cdot}1111 = 1{\cdot}3889$.

Then $19{\cdot}4444 \div 1{\cdot}3889 = 14$ *Answer* as before.

Ex. A ship has a voyage of 2640 knots to go; during 5 days she steamed 228, 236, 240, 250, and 232 knots respectively, up to Friday at 9^h A.M. On what day and hour will she arrive at her port, supposing she steams $10\frac{1}{4}$ knots per hour the rest of the voyage?

	228
	236
	240
	250
	232
Knots steamed	1186
Knots in voyage	2650
Knots to steam	1464
Knots per hour	10·25

Time $142^h\ 49^m\ 45^s$, or
„ $5^d\ 22^h\ 49^m\ 45^s$.

Ans.: Thursday $7^h\ 49^m\ 45^s$ A.M.

10·25)1464·00(142 hours.
1025
4390
4100
2900
2050
850
60
10·25)51000(49
4100
10000
9225
775
60
10·25)46500(45
4100
5500
5125
375

$$24\left\{\begin{array}{l}4)142^h\\ 6)35-2\end{array}\right.$$
$5^d\ 22^h$

Examples for Practice.

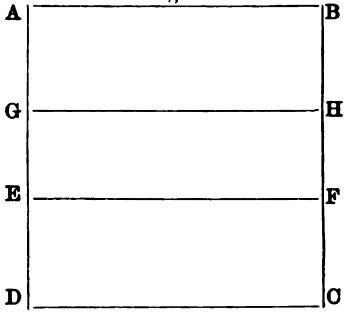
No.	Length of Voyage.	Knots steamed per day, respectively.	Time.	Knots steamed per hour.	*Ans.*: Time of arrival at port.
1.	1880 knots.	6 days, 184, 192, 208, 196, 188, and 192 knots.	Wednesday, at midnight.	$7\frac{1}{4}$ knots.	Sunday, $8^h\ 54^m\ 12^s$ P.M.
2.	2200 „	5 days, 216, 232, 240, 228 and 224 knots.	Sunday, at noon.	$9\frac{1}{2}$ „	Friday, $3^h\ 34^m\ 44^s$ A.M.
3.	1500 „	4 days, 149, 150, 147, and 160 knots.	Friday, 8^h A.M.	8 „	Tuesday, $11^h\ 45^m\ 0^s$ P.M.
4.	3000 „	9 days, 246, 225, 259, 260, 249, 230, 269, 271, and 282 knots.	Monday, 3^h P.M.	$9\frac{1}{2}$ „	Thursday, $2^h\ 28^m\ 25^s$ P.M.

TO FIND THE MEAN PRESSURE THROUGHOUT THE STROKE WHEN WORKING EXPANSIVELY.

282. Given the pressure at which steam is admitted into the cylinder, and the portion of the stroke at which it is cut off; it is required to find the mean pressure throughout the stroke.

We now proceed to calculate the work done by the expansion of steam into a larger volume by virtue of its elasticity. As the steam expands, its elastic force and temperature gradually diminish; in order, therefore, to conduct the calculation, it will be necessary to know beforehand the law according to which the diminution takes place. We shall assume, for the sake of simplicity, that Boyle and Marriotte's law applies to the expansion of steam; it may be thus enunciated:—If a given weight of steam be made to vary its volume without changing its temperature, the *elastic force* of the steam will vary in the *inverse ratio of the volume* it is made to occupy; that is, if its volume is increased two times, its pressure will be about one-half of what it was at first, and so on.

Fig. 2.



Thus, let ABCD be a cylinder, EF and GH any two positions of the piston; then the pressure in the position EF is the pressure in the position GH, as the space GHCD is to the space EFCD.

283. In this question four different methods of solution are given. Methods I and II being those mostly used. Method III is performed by using a short table of given multipliers, and is a very short and easy method. Method IV is a solution of the question given by Mr. M'Farlane Gray, the Examiner in Steam.

RULE CXXVII.—(METHOD I).

This is Simpson's Rule for finding the areas of irregular curvilinear figures.

1°. *Divide the cylinder into* any *number of* equal *parts.* (*a*)—*If the steam be cut off at ½, ⅓, ¼, ⅙, or any* measure *of* 12, divide *the cylinder into* 12 parts. (*b*)—*If* cut off at 5ths, divide *it into* 10 *equal parts.* (*c*)—*If* cut off at 7ths, divide *it into* 14 *equal parts.* (*d*)—*If* cut off at 8ths, divide *the cylinder into* 16 *parts.*

2°. *Divide that* part *of the* stroke through *which* expansion *takes place into any* even *number of* equal *parts*, and *calculate the pressure per square inch upon the piston at* each division *of the stroke*, by Marriotte's law.

3°. *Take the* sum *of the* extreme *pressures in pounds per square inch,* four times *the* sum *of the* even *pressures, and* twice *the* sum *of the* odd *pressures;* then, this sum multiplied by one-third of the distance between the consecutive points at which the pressures are taken, will give the work done expansively per square inch of the area of the piston in one stroke, *and* one-third *this* sum *is the* total pressure during expansion.*

4°. *To this* add *the* total *pressure* before *expansion and* divide *the* sum *by the* number of parts *into which the* cylinder is divided; *the result is the* whole work done during a single stroke.

NOTE.—The work done before the expansion begins is evidently equal to the pressure per square inch multiplied by the distance described before expansion. The whole work done is equal to the sum of the works done *before* and *after* expansion.

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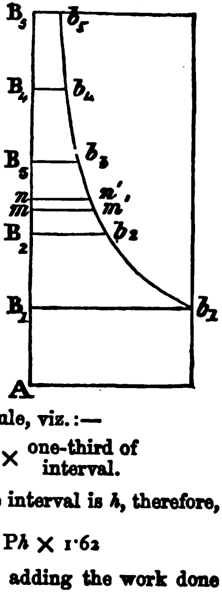
* Calculation of work done, by Simpson's rule. The following method of investigating the question, has the advantage of depending on elementary principles.

In the adjoining figure, let A B represent the volume when the steam is cut off; then, if $A B_1 = B_1 B_2 = B_2 B_3$, &c.; $A B_2$, $A B_3$, $A B_4$, and $A B_5$, will represent the volumes when the steam has expanded into double, treble, &c., its original bulk. Let the perpendicular $B_1 b_1$ represent the original pressure P, then $B_2 b_2$, $B_3 b_3$, $B_4 b_4$, and $B_5 b_5$, which are respectively $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, of $B_1 b_1$, will represent the pressures corresponding to the several volumes. A continuous curve which, in this case, is a hyperbolic, passing through b_1, b_2, b_3, b_4, and b_5, will represent, by its ordinates $m\,m'$, at any point m of the expansion, the pressure corresponding to the volume A m.

At the point m the pressure is represented by $m\,m'$, the work done therefore by the steam on one square inch while the load is moved upward through the indefinitely small space $m\,m'$, will be the product of $m\,n'$ and half the sum of $m\,n'$ and $n\,n'$, or the mean pressure $m\,n$; but $m\,n \times \frac{1}{2}(m\,m' + n\,n')$ is equal to the trapezium $m\,n\,n'\,m'$, the whole work done, therefore, will be represented by the sum of all these elementary trapeziums, or the area of the curve $B_1 B_5 b_5 b_1$.

Fig. 3.



The area of the curve may be found very exactly by Simpson's rule, viz.:—

$$\text{Area} = \left\{ \begin{matrix} \text{sum of extreme ordinates} + 4 \text{ even ordinates} \\ + 2 \text{ odd ordinates} \end{matrix} \right\} \times \begin{matrix} \text{one-third of} \\ \text{interval.} \end{matrix}$$

In the figure the ordinates are P, $\frac{1}{5}$ P, $\frac{1}{2}$ P, $\frac{1}{4}$ P, and $\frac{1}{3}$ P, and the interval is h, therefore,

$$\text{Area} = \left\{ P + \frac{P}{5} + 4\left(\frac{P}{2} + \frac{P}{4}\right) + 2\,\frac{P}{3} \right\} \frac{h}{3} = Ph \times 1{\cdot}62$$

multiply by 144 a the number of square inches in the piston, and adding the work done before expansion,

$$W = 2{\cdot}62\, w,$$

a result almost identical with the second method here given.

As Simpson's rule requires an even number of intervals, it is necessary when the number of volumes is even, as for instance four, in which case there will be only three intervals, to bisect each interval, insert corresponding ordinates, and multiply by one-sixth of the interval h.

The ordinates standing at the middle points of the first, second, and third intervals are $\frac{2}{3}$ P, $\frac{2}{5}$ P, and $\frac{2}{7}$ P, taking account merely of the numerical coefficients, the total coefficient may be found as follows:—

$$\left\{ 1 + \tfrac{1}{4}\; 4\left(\tfrac{2}{3} + \tfrac{2}{5} + \tfrac{2}{7}\right) + 2\left(\tfrac{1}{2} + \tfrac{1}{3}\right) \right\} \times \tfrac{1}{6} = 1{\cdot}38.$$

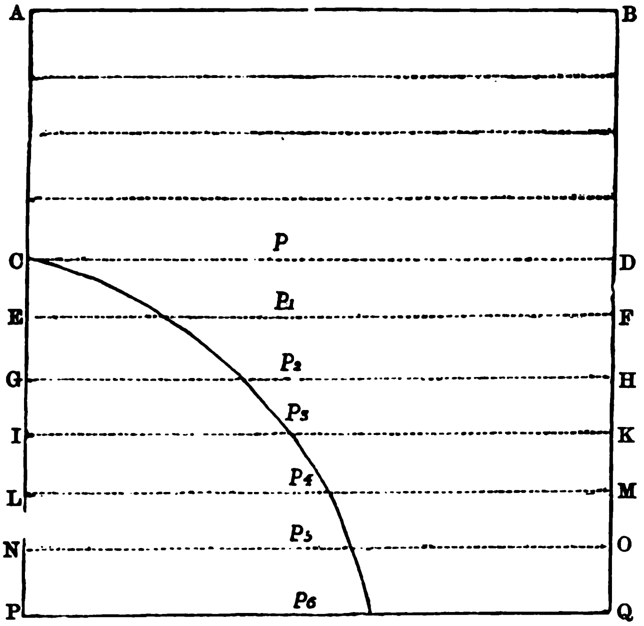
to which adding unity for the work done by expansion, the result 2·38.

Ex. 1. Steam is admitted into the cylinder at 25 lbs. per square inch above the atmosphere, and is cut off at $\frac{2}{5}$ths of the stroke: required the mean pressure on the piston throughout the stroke, supposing the vacuum to be perfect.

In the above example the pressure of the atmosphere is 15 lbs. per square inch, and the steam is admitted into the cylinder at 25 lbs. per square inch *above* the atmosphere, therefore, the effective pressure per square inch on the piston at first is 40 lbs. per square inch; but, when the piston has travelled $\frac{2}{5}$ the length of the cylinder the steam is cut off, and the piston travels the remaining $\frac{3}{5}$ by the expansive force of the steam already admitted.

Let the accompanying figure represent the cylinder; divide AP into 10 equal parts (see Rule CXXVII, 1° *b*); then $\frac{2}{5}$ of the cylinder is equal to four of these parts, and at the fourth line from the top mark the position of the piston when the steam is cut off, viz., CPD: and at all the other points draw horizontal lines to represent the position of the piston at the several instants chosen. Then the four parallelograms above the piston (above CPD) represent the pressure of the steam at the 1st, 2nd, 3rd, and 4th instants, and since the pressure is constant during this part of the stroke, 40 lbs. × 4 = 160 lbs. is the total pressure before the steam is cut off. The pressure of the steam upon the piston when it arrives at the different points of division after the steam is cut off, is ascertained by Marriotte's law (see fig. 2, p. 278), thus:—

Fig. 4.



Space AEFB : space ACDB :: P : P_1; or, which is the same, as AE : AC :: P : P_1.

Also AG : AC :: P : P_2
AI : AC :: P : P_3
AL : AC :: P : P_4
AN : AC :: P : P_5
AP : AC :: P : P_6

Where P is the full pressure; P_1 the pressure when the piston is in the position EF; P_2 the pressure when the piston is in the position GH; P_3 the pressure when the piston is in the position IK, &c.

Then as $5 : 4 :: 40 : P_1 = \frac{40 \times 4}{5} = 32$ lbs. (2) even

$6 : 4 :: 40 : P_2 = \frac{40 \times 4}{6} = 26{\cdot}66667$ (3) odd

$7 : 4 :: 40 : P_3 = \frac{40 \times 4}{7} = 22{\cdot}85714$ (4) even

$8 : 4 :: 40 : P_4 = \frac{40 \times 4}{8} = 20$ (5) odd

$9 : 4 :: 40 : P_5 = \frac{40 \times 4}{9} = 17{\cdot}77778$ (6) even

$10 : 4 :: 40 : P_6 = \frac{40 \times 4}{10} = 16$ (7) last

EVENS.
32
22·85714
17·77778
72·63492
4
290·53968

ODDS.
26·66667
20
46·66667
2
93·33334

First 40
Last 16
Evens 290·53968
Odds 93·33334
3)439·87302
146·62434

Total pressure during expansion 146·62434
Total pressure before expansion 160
10)306·62434
Mean pressure throughout the stroke = 30·6624

Ex. What is the mechanical efficiency gained in comparison to steam being admitted the whole length of the stroke?

The mechanical efficiency is found by dividing the mean pressure by the terminal pressure.

Here the mean pressure is 30·6624 lbs., and terminal pressure 16 lbs.

$$16 \begin{cases} 4)30{\cdot}6624 \\ 4)7{\cdot}6656 \end{cases}$$

1·9164 the mechanical efficiency.

284. The foregoing Method (Simpson's) of determining the mean pressure on the piston, requiring the divisions of the cylinder to be numerous, renders the operation tedious and lengthy. Fortunately, however, we can dispense with this part of the calculation altogether, for the Naperian or hyperbolic logarithms as set forth in the following Table furnish to our hand the ratios of pressures.

A TABLE OF HYPERBOLIC LOGARITHMS.

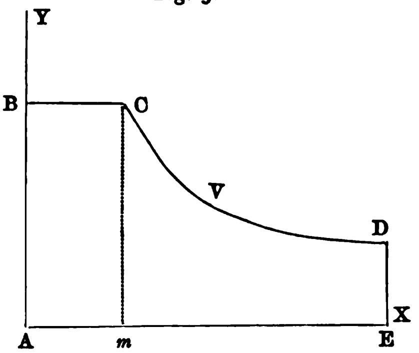
NO.	LOG.	NO.	LOG.	NO.	LOG.	NO.	LOG.
1·01	0·00995	2·25	0·81093	4·50	1·50408	7·25	1·98100
1·11	0·10436	2·50	0·91629	4·75	1·55814	7·50	2·01490
1·14	0·13103	2·66	0·97833	5·00	1·60944	7·75	2·04769
1·25	0·22314	2·75	1·01160	5·25	1·65823	8·00	2·97944
1·33	0·28518	3·00	1·09861	5·50	1·70475	8·25	2·11021
1·42	0·35066	3·25	1·17865	5·75	1·74920	8·50	2·14007
1·50	0·40547	3·33	1·20297	6·00	1·79176	8·75	2·16905
1·60	0·47000	3·50	1·25276	6·25	1·83258	9·00	2·19722
1·66	0·50682	3·75	1·32176	6·50	1·87180	9·25	2·22462
1·75	0·55962	4·00	1·38629	6·75	1·90954	9·50	2·25129
2·00	0·69315	4·25	1·44692	7·00	1·94591	9·75	2·27727
2·25	0·81093	4·50	1·50408	7·25	1·98100	10·00	2·30259

The theoretical curve of expansion is an hyperbola of which AX and AY, Fig. 5, are the asymptotes, or lines which the curve cuts at an infinite distance.

Now the rectangle ABC *m*, represents the mechanical effect of the steam before expansion, and the area CVDE *m*, represents the mechanical effect during expansion.

We call the area ABC *m* unity, and we will find the area CVDE *m*, by means of Naperian logarithms. These have the number 2·7182818 for a base, and the Naperian logarithm of any number may be found from the common logarithm by multiplying the latter by 2·30258509.* They are frequently called hyperbolic logarithms, because the numbers express the areas between the asymptotes and curve of the hyperbola; these areas being limited by ordinates parallel to the asymptotes. But such areas may be made to denote any system of logarithms whatever. In the case of the hyperbolic logarithms, however, the asymptotes are at right-angles to each other, and hence we see that the Naperian, or hyperbolic logarithm of the asymptote of the hyperbola in question is $n \times A\,m = n$, where n represents the number of times the line A *m*, which is unity, is contained in AE.

Fig. 5.



Hence, in order to find the mechanical effect of the steam during expansion, we have only to divide the space passed over by the piston before expansion (which we call unity), by the number of such spaces in the whole length of the cylinder (which we call *n*), and take the Naperian logarithm of the reciprocal of this fraction. Adding the work performed before expansion we have the total mechanical effect of the steam during one stroke.

---

* Very extensive tables of hyperbolic logarithms are contained in Dr. Hutton's Mathematical Tables, and in many other Works.

But the mechanical effect is equal to the mean pressure multiplied by the distance over which it is exerted. So we can obtain the mean pressure by multiplying the mechanical effect by the initial pressure, and dividing by *n*, the distance through which the force of the steam is exerted; hence the following Rule.

RULE CXXVIII.—(METHOD II).

BY HYPERBOLIC LOGARITHMS.

1°. Divide *the whole* length *of the* stroke *by the* distance *through which* the piston *moves* before *the* steam *is cut off*, the quotient is the ratio of expansion.

2°. Divide *the* pressure *of the* steam *in pounds per square inch by the* ratio of expansion *found by* No. 1°.

3°. *Find the* hyperbolic logarithm (see preceding table) *of the* ratio of expansion, *to which* add 1, the result represents the increase of efficiency due to expansion.

Ex. 1. Suppose that the steam be cut off at $\frac{2}{15}$ths of the stroke: what is the increase of efficiency due to expansion?

Here it is plain that $\frac{2}{15}$ths of the whole stroke is the same as $\frac{1}{7.5}$ of the whole stroke. The hyperbolic logarithm of 7·5 is 2·015, which increased by 1, the value of the portion performed with full pressure gives 3·015 as the relative efficacy of the steam when expanded to this extent, instead of 1, which would have been the efficacy if there had been no expansion.

If the steam be cut off at $\frac{1}{8}$, the ratio of expansion will be 8; if cut off at $\frac{2}{8}$ ($= \frac{1}{4}$) the ratio will be 4; similarly, if the steam be cut off at $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, or $\frac{7}{8}$ths, the respective ratios of expansion will be 2·66, 2, 1·6, 1·33, and 1·14, which may be obtained by dividing the denominator of each by its numerator; and if the steam be cut off at $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{6}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, or $\frac{9}{10}$ths of the stroke, the respective ratios of expansion will be 10, 5, 3·33, 2·5, 1·66, 1·42, 1·25, and 1·11.

4°. *Finally*, multiply *this* result (found by No. 3°) *by the* quotient *obtained by* No. 2°, *which gives the* mean pressure throughout the stroke in lbs. per square inch.

NOTE (*a*)—Should the number representing the ratio of expansion not be found exactly in the table, find the proportionate part between the two hyperbolic logs. corresponding to next less and the next greater number, and *add* it to the hyperbolic log. corresponding to the less number; the sum is the hyperbolic log. sought for.

Solution of Ex. 1, page 280, by Method II.

$$40 \div 2{\cdot}5 = 16.$$

$$2)\underline{5}$$

2·5 ratio of expansion.

$$\begin{array}{r} 2{\cdot}5)40(16 \\ \underline{25} \\ 150 \\ \underline{150} \end{array}$$

$$\begin{array}{lr} \text{Ratio of expansion } 2{\cdot}5 \text{ hyperbolic logarithm} = & 91629 \\ & +1{\cdot} \\ \hline \text{Total mechanical effect} & 1{\cdot}91629 \end{array}$$

Mean pressure = 1·91629 × 16 = 30·66064 lbs.

$$\begin{array}{r} 1{\cdot}91629 \\ 16 \\ \hline 1149774 \\ 191629 \\ \hline \end{array}$$

Mean pressure 30·66064

Here it is plain that $\frac{2}{5}$ of the whole stroke is the same as $\frac{1}{2\cdot5}$ of the whole stroke. The hyperbolic logarithm of 2·5 (see table) is ·91629, which increased by 1, the value of the portion performed with full pressure gives 1·91629 as the relative efficacy of the steam when expanded to this extent, instead of 1, which would have been the efficacy if there had been no expansion.

We next divide the initial pressure of the steam, viz., 40 lbs. per square inch by the ratio of expansion, viz., 2·5, the quotient is 16. The hyperbolic logarithm, increased by 1, of the number representing the ratio of expansion, viz., 1·91629, multiplied by the quotient last found (16) gives 30·66064, the mean pressure throughout the stroke.

RULE CXXIX.—(METHOD III).

BY EXPANSION TABLE.

1°. Divide *the* length *of the* stroke *by the* distance *the piston moves* before *the steam* is cut off, *the* quotient *will express the relative* expansion it undergoes.

2°. *Take from the* annexed TABLE *the* multiplier *corresponding to this* number, *and* multiply *it by the full pressure of steam per square inch on entering the cylinder; the* product *will be the* mean pressure per square inch nearly.

TABLE OF MULTIPLIERS.

Relative Expansion.	Multiplier.	Relative Expansion.	Multiplier.	Relative Expansion.	Multiplier.	Relative Expansion.	Multiplier.	Relative Expansion.	Multiplier.
1·0	1·00000	2·0	·84657	3·0	·69954	4·0	·59657	5·0	·52189
1·1	0·99574	2·1	·82949	3·1	·68755	4·1	·58805	5·1	·51554
1·2	0·98527	2·2	·81294	3·2	·67598	4·2	·57978	5·2	·50936
1·3	0·97105	2·3	·79692	3·3	·66482	4·3	·57177	5·3	·50334
1·4	0·95462	2·4	·78145	3·4	·65405	4·4	·56400	5·4	·49748
1·5	0·93698	2·5	·76652	3·5	·64365	4·5	·55646	5·5	·49177
1·6	0·91875	2·6	·75212	3·6	·63359	4·6	·54914	5·6	·48621
1·7	0·90037	2·7	·73824	3·7	·62387	4·7	·54203	5·7	·48078
1·8	0·88210	2·8	·72486	3·8	·61447	4·8	·53513	5·8	·47549
1·9	0·86413	2·9	·71197	3·9	·60538	4·9	·52842	5·9	·47033
2·0	0·84657	3·0	·69954	4·0	·59657	5·0	·52189	6·0	·46529

The multipliers in this table are found by adding one to each of the hyperbolic logarithms of the relative expansions, and then dividing the sums by the relative expansions.

NOTE.—If the number be not found exactly in the Table, find the proportional part between the multipliers corresponding to the next *greater* and the next *less* number, which, being subtracted from the first of the two multipliers, will give the one required.

Resuming the question on page 280.

$$\begin{array}{l} 2)5{\cdot}0 \\ \hline 2{\cdot}5 \text{ ratio of expansion.} \end{array}$$

2·5 gives in table the multiplier = ·76652
Initial pressure = 40 lbs.

Mean pressure 30·66080 lbs.

RULE CXXX.—(METHOD IV).

BY ARITHMETIC.

1°. *Take* eleven *ordinates, marking as below the numbers* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10, *always under one another* (see Example following).

2°. *Reduce the* ratio of expansion *to a* decimal *if it is not already so expressed.*

3°. *Note the* first figure *of the* decimal, *and draw a line to mark the* "cut off" below *the* same figure *in the* column of ordinates.

4°. Opposite *each number (except the first)* above *the line* (see No. 3°) *put* ·1, *but for the* first *number put* ·05, *which is half of* ·1.

5°. *Divide the* decimal *of the ratio of expansion, found as above* (No. 2°), *by each of the other numbers below,* in succession; *but for the* last *write down* half *an ordinate.*

6°. *Find the* sum *of* all *these lines, and* multiply *it by the* given pressure; the result is the mean pressure on the piston throughout the stroke.

Resuming the question on page 280.

⅕)2·0

·4 ratio of expansion.

0	(½ ordinate)	·05
1		·1
2		·1
3		·1
4		·1
5		·08
6		·066666
7		·057143
8		·05
9		·044444
10	(½ ordinate)	·02
		·768253
		40

Mean pressure 30·730120 lbs.

The value of each ordinate *below* the cut off is found by dividing the decimal of ratio of expansion by the number, thus, the first below the cut-off is 5; divide ·4 by 5, which gives ·08; next divide ·4 by 6 — ·066666; then divide ·4 by 7 = ·057143 nearly, × ·4 by 8 = ·05; ·4 by 9 = ·044444 &c.; and last ·4 by 10 = ·04; but then since the last ordinate has always to be halved, it is therefore ·02. Arranging all these as in the margin, and adding them together, multiply the sum by the effective pressure, the product is the mean pressure throughout the stroke.

Ex. 2. Steam is admitted into the cylinder at 65 lbs. per square inch above the atmosphere, and cut off ⅕ of the stroke: required the mean pressure on the piston throughout the stroke, supposing the vacuum to be perfect.

The pressure of the atmosphere is 15 lbs. and in this example the steam flows from the boiler through the steam pipe into the cylinder with a pressure of 65 lbs. per square inch *above* the atmosphere; therefore, the effective pressure on the piston is 80 lbs. per square inch; but the flow of the steam is stopped when the piston has performed ⅕ of the stroke, leaving the rest of the stroke (⅘) to be completed by the expanding force of the steam.

We divide the cylinder into *ten* equal parts (see Rule CXXVII, 1° *b*), then ⅕ of the cylinder equals *two* of these parts; and at the second from the top mark the position of the piston CPD, and at all the other points of division draw horizontal lines which will represent the position of the piston at the different instants chosen. The two rectangles above CPD represent the pressure of the steam at the 1st and 2nd instants, and since the pressure of steam, during this portion of the stroke, is constant, 80 lbs. × 2 = 160 lbs. = the pressure of steam before the flow of the steam is shut off from the cylinder.

The pressure of the expanding steam during the remaining portion of the stroke, as calculated by Marriotte's law (fig. 2, p. 278), is as follows:

$$3 : 2 :: 30 : P_1 = \frac{2 \times 80}{3} = 53{\cdot}33333 \quad \text{pressure even}$$

$$4 : 2 :: 30 : P_2 = \frac{2 \times 80}{4} = 40 \quad \text{odd}$$

$$5 : 2 :: 30 : P_3 = \frac{2 \times 80}{5} = 32 \quad \text{even}$$

$$6 : 2 :: 30 : P_4 = \frac{2 \times 80}{6} = 26{\cdot}66666 \quad \text{odd}$$

$$7 : 2 :: 30 : P_5 = \frac{2 \times 80}{7} = 22{\cdot}857143 \quad \text{even}$$

$$8 : 2 :: 30 : P_6 = \frac{2 \times 80}{8} = 20 \quad \text{odd}$$

$$9 : 2 :: 30 : P_7 = \frac{2 \times 80}{9} = 17{\cdot}77777 \quad \text{even}$$

$$10 : 2 :: 30 : P_8 = \frac{2 \times 80}{10} = 16 \quad \text{last}$$

$$\begin{array}{r} \text{EVENS.} \\ 53{\cdot}333333 \\ 32 \\ 22{\cdot}857143 \\ 17{\cdot}777777 \\ \hline 125{\cdot}968253 \\ 4 \\ \hline 503{\cdot}873012 \end{array} \qquad \begin{array}{r} \text{ODDS.} \\ 40 \\ 26{\cdot}66666 \\ 20 \\ \hline 86{\cdot}66666 \\ 2 \\ \hline 173{\cdot}33332 \end{array} \qquad \begin{array}{lr} \text{First} & 80 \\ \text{Last} & 16 \\ \text{Evens} & 503{\cdot}873012 \\ \text{Odds} & 173{\cdot}333333 \\ \hline & 3)773{\cdot}206345 \\ \hline \end{array}$$

Total pressure during expansion 257·735448 lbs.

$$\begin{array}{lr} \text{Total pressure during expansion} & 257{\cdot}735448 \text{ lbs.} \\ \text{Total pressure before expansion } (80 \times 2) & 160 \\ \hline & 10)417{\cdot}735448 \\ \hline \text{Mean pressure throughout the stroke} & 41{\cdot}773 \quad \text{lbs.} \end{array}$$

METHOD II.—(BY HYPERBOLIC LOGARITHMS).

$$\begin{array}{l} 1)5 \\ \hline 5 \text{ the ratio of expansion.} \end{array} \qquad \frac{80 \text{ lbs.}}{5} = 16 \text{ lbs.}$$

$$\begin{array}{lr} \text{Ratio of expansion 5, its hyperbolic log.} = & 1{\cdot}60944 \\ + & 1 \\ \hline \text{Increase of efficiency due to expansion} & 2{\cdot}60944 \end{array}$$

Mean pressure $2{\cdot}60944 \times 16 = 41{\cdot}75104$ lbs.

$$\begin{array}{r} 2{\cdot}60944 \\ 16 \\ \hline 1565664 \\ 260944 \\ \hline \end{array}$$

Mean pressure throughout the stroke 41·75104 lbs.

The denominator of the fraction (⅕) expressing the part of the stroke at which the steam is cut off, being 1 we merely take the hyperbolic log. of the denominator 5, which is 1·60944; to this we add 1, the value of the portion performed by the full pressure, which gives 2·60944 as the relative efficiency of the steam when expanded to this extent, instead of 1, which would have been the efficiency had there been no expansion.

We multiply this last result by 16, the quotient obtained by dividing the initial pressure 80 lbs. by the ratio of expansion 5: the product = 41·78104 lbs. = the mean pressure throughout the stroke.

METHOD III.—(BY EXPANSION TABLE).

The ratio of expansion as before is 5.

5 in the table gives the multiplier	·52189
Multiplied by the initial pressure	80 lbs.
Mean pressure throughout the stroke	41·7512 lbs.

METHOD IV.—(BY MR. M'FARLANE GRAY).

$$\tfrac{1}{5} = 5)\overline{1{\cdot}0}$$

$$\cdot 2 = \text{ratio of expansion.}$$

0	(½ ordinate)	·05
1		·1
2		·1
3		·06667
4		·05
5		·04
6		·03333
7		·02857
8		·025
9		·02222
10	(½ ordinate)	·01
		·52579
		80
Mean pressure throughout the stroke		42·0632 lbs.

Ex. 3. Steam is admitted into the cylinder at 35 lbs. pressure per square inch above the atmosphere, and is cut off at ⅙ the stroke; required the mean pressure throughout the stroke, supposing the vacuum to be perfect.

The pressure of the atmosphere is 15 lbs. per square inch, and the steam admitted into the cylinder is 35 lbs. per square inch *above* the atmosphere, therefore the effective pressure per square inch on the piston is 50 lbs.; but, when the piston has travelled ⅙ the length of the cylinder the steam is cut off, and the piston performs the remaining ⅚ by the expansive force of the steam already admitted.

Draw a figure ABPQ (see fig. 4, p. 280) to represent the cylinder, which divide into *twelve* equal parts (this is according to Rule CXXVII, 1° *a*); the ⅙ of the cylinder will be equal to *two* of these parts, then at the 2nd line from the top draw a line CPD to represent the position of the piston when the steam is cut off, at the other points draw lines to denote the position of the piston at the different instants chosen. Then the two rectangles above CPD represents the pressure at the *first* and *second* instants; and, since the pressure is constant, 50 lbs. × 2 = 100 lbs. is the total pressure before the steam is cut off; and the pressure of the piston when it arrives at the different lines forming the divisions of the remaining part of the stroke is determined by Marriotte's law as follows:

50 × 2 ÷ 3 = 33·33333 even, 50 × 2 ÷ 4 = 25 odd, 50 × 2 ÷ 5 = 20 even, 50 × 2 ÷ 6 = 16·66667 odd, 50 × 2 ÷ 7 = 14·28571 even, 50 × 2 ÷ 8 = 12·5 odd, 50 × 2 ÷ 9 = 11·11111 even, 50 × 2 ÷ 10 = 10 odd, 50 × 2 ÷ 11 = 9·09091 even, 50 × 2 ÷ 12 = 8·33333 last.

Then by Simpson's rule we have—

50 lbs. + 8·33333 lbs. = 58·33333 lbs. = sum of extreme pressure.

EVENS.	ODDS.
33·33333	25
20	16·66667
14·28571	12·5
11·11111	10
9·09091	64·16667 = sum of odd pressure.
87·82106 = sum of even pressure.	2
4	128·33334 = twice sum of odd pressure.
351·28424 = 4 times sum of even pressure.	

Sum of extreme pressures = 58·33333 lbs.
Four times sum of even pressures = 351·28424 lbs.
Twice sum of odd pressures = 128·33334 lbs.

3)537·95091

Total pressure during expansion 179·31697
Total pressure before expansion 100

12)279·31697

Mean pressure throughout the stroke 23·2764 lbs.

METHOD II.—(BY HYPERBOLIC LOGARITHMS).

1)6
6 the ratio of expansion.

50
6)50
8·3333

6, its hyperbolic log. 1·79176 + 1 = 2·79176 × 8·3333 = 23·264573 lbs., the mean pressure throughout the stroke.

NOTE.—When it is stated that the steam is cut off at ⅙, ⅓, ¼, &c., of the stroke, there is no necessity for dividing the whole distance moved by the piston by the distance it moves before expansion; we need only find the hyperbolic logarithm of the denominator of the fraction, and add unity to it. In this instance, the denominator of the fraction (⅙) is 6; the hyperbolic logarithm of this is found in the table to be 1·79176; to this we add 1, which gives 2·79176, the increase of efficiency due to expansion. This is next multiplied by 8·3333, the quotient obtained by dividing the pressure of steam 50 lbs. by the ratio of expansion 6; the result is the mean pressure throughout the stroke.

METHOD III.—(BY THE EXPANSION TABLE).

1)6
6 ratio of expansion = 6.

6 gives in the table the multiplier = ·46529
Multiply by initial pressure 50
23·2645 lbs.

METHOD IV.—(BY MR. M'FARLANE GRAY).

⅙)1·0

·16667 where the ratio of expansion ⅙ expressed decimally is ·16667.

0	·16667, (½ ordinate)	·05
1		·1
2		·08333
3		·05556
4		·04167
5		·03333
6		·02778
7		·02381
8		·02083
9		·01852
10	(½ ordinate)	·00833
		·46316
Multiplied by initial pressure		50
Mean pressure		23·1580 lbs.

Examples for Practice.

In each of the following examples, the pressure of the initial pressure of the steam and the ratio of expansion are given, to find the mean pressure throughout the stroke.

No.	Pressure.			Ratio.	No.	Pressure.			Ratio.
1.	26 lbs.	+	15	⅜	4.	20 lbs.	+	15	½
2.	28			⅓	5.	23			⅖
3.	16			½	6.	20			⅔

Examples.

Ex. 1. Steam by the gauge is 20 lbs., and is cut off at 11 inches of the stroke, which is 30 inches: find the mean pressure throughout the stroke.

30)11

·3667 decimal of cut-off.

½ of 0	......	·05
1	......	·1
2	......	·1
3	......	·1
4	......	·0917
5	......	·0733
6	......	·0611
7	......	·0524
8	......	·0458
9	......	·0407
½ of 10	......	·0183
		·7333
		35
		36665
		21999
		25·6655 mean pressure throughout the stroke.

Gauge	20 lbs.
Atmosphere	15 lbs.
Gross	35 lbs.

Note.—Observe that if the steam had been given at 20 lbs. gross, then 20 would be the multiplier, not 35 (20 + 15).

Ex. 2. An engine has a cylinder 56 inches diameter, the length of stroke 54 inches, making 60 revolutions per minute, the steam is cut off at 32 inches, the pressure of steam maintained to the cut-off was 42 lbs. per square inch above the atmosphere, and the vacuum was 9·4 lbs.: what is the indicated horse-power?

$$54\left\{\begin{array}{r} 6)32 \\ \hline 9)5{\cdot}3333 \\ \hline {\cdot}5926 \end{array}\right.$$

$$\frac{\text{Cut-off } 32''}{\text{Stroke } 54} = {\cdot}5926 \text{ decimal of cut-off.}$$

0	½ ordinate	·05
1		·1
2		·1
3		·1
4		·1
5		·1
6		·09877
7		·08466
8		·07407
9		·06584
10	½ ordinate	·02963

$$\begin{array}{lr} \text{Steam} & 42 \text{ lbs.} \\ \text{Atmosphere} & 15 \\ \hline \text{Gross} & 57 \end{array} \qquad \begin{array}{lr} \text{Full vacuum} & 15 \\ \text{Working do.} & 9{\cdot}4 \\ \hline \text{Back pressure} & 5{\cdot}6 \end{array}$$

$$\begin{array}{rr} \text{Diameter of cylinder} & 56'' \\ & 56 \\ \hline & 336 \\ & 280 \\ \hline 12 = & 3136 \\ & {\cdot}7854 \\ \hline & 12544 \\ & 15680 \\ & 25088 \\ & 21952 \\ \hline \text{Area} = & 2463{\cdot}0144 \\ \text{Twice stroke in feet} & 9 \\ \hline & 22167{\cdot}1296 \\ & 60 \text{ revolutions.} \\ \hline & 1330027{\cdot}7760 \end{array}$$

$$\begin{array}{rr} & {\cdot}90297 \\ & 57 \\ \hline & 632079 \\ & 451485 \\ \hline & 51{\cdot}46929 \\ \text{Back pressure} & 5{\cdot}6 \\ \hline \text{Mean effective pressure} = & 45{\cdot}87 \end{array}$$

$$\begin{array}{r} 1330027{\cdot}776 \\ 45{\cdot}87 \\ \hline 9310194432 \\ 10640222208 \\ 6650138880 \\ 5320111104 \\ \hline 33000)61008374{\cdot}08512 \\ \hline 11)20336{\cdot}124 \\ \hline 1848{\cdot}738 \end{array}$$

Ex. 3. If P is the gross pressure of the steam at the beginning of the stroke, and R the number of times the steam is expanded,

$$\text{Then F} \times \left(\frac{18-\text{R}}{40} + \frac{{\cdot}85}{\text{R}}\right) = \text{mean pressure.}$$

Question. What is the H.P. if the diameter of cylinder is 52″, stroke 36″, steam gauge showed 60 lbs., steam cut off at 12 inches, revolutions per minute 70, and back pressure 2·5 lbs.

$$\text{Here P} = 60 \text{ lbs.} + 15 \text{ lbs.} = 75 \text{ lbs.}$$

$$\text{R} = 36 \text{ lbs.} \div 12 \text{ lbs.} = 3 \text{ lbs.}$$

$$\therefore 75\left(\frac{18-3}{40} + \frac{{\cdot}85}{3}\right) = 75 \times {\cdot}658 = 49{\cdot}35 \text{ lbs. gross mean pressure,}$$

$$\therefore \text{mean effective pressure} = 49{\cdot}35 - 2{\cdot}5 = 46{\cdot}85 \text{ lbs.,}$$

$$\text{Then } \frac{52^2 \times {\cdot}7854 \times 46{\cdot}85 \times 3 \times 140}{33000} = 1266{\cdot}317 \text{ H.P.}$$

Ex. 4. The steam gauge shows 60 lbs., and this pressure is carried for 11″ of the stroke, length of stroke 33″, and mean effective pressure 52½ lbs.: what is the co-efficient of efficiency?

$$\text{Rule, } C = \frac{\text{mean pressure} \times \text{length of stroke}}{\text{gross pressure} \times (\text{length of stroke} + \text{clearance})} = \frac{52\tfrac{1}{2} \text{ lbs.} \times 33}{75 \times 11} = 2{\cdot}1 \textit{ Ans.}$$

EXAMPLES FOR PRACTICE.

No.	Dia. Cylinder. in.	Length of Stroke. in.	Revolutions per minute.	Steam cut off. in.	Press. per sq. inch. lbs.	Vacuum. lbs.	*Ans.* Horse-power.
1.	56	54	60	32	42	9·4	1848·738
2.	64	56	58	28	50	12·5	2784·953
3.	74	48	64	25	65	13·	3696·692

CUTTING OFF STEAM.

Let S = stroke of engine in inches.
T = travel of valve in inches.
L = lap of valve in inches.
l = lead of valve in inches.
D = distance of piston from end of stroke.
x = cut-off.

Then

$$D = \left(\frac{L \times \frac{1}{2} l}{\frac{1}{2} T}\right)^2 \times S$$

$$\therefore x = S \left\{ 1 \left(\frac{2L + l}{T}\right)^2 \right.$$

RULE CXXXI.

Multiply the lap or cover by 2 and add the lead, divide the sum by the travel, **square *the quotient, and multiply it by the length of stroke in inches,* the product is the distance of piston from the end of stroke when the steam is cut off.**

NOTE.—The length of stroke, minus the last product, equals the distance the piston will have travelled when the steam is cut off.

EXAMPLES.

Ex. 1. The stroke of an engine is 24 inches, the slide-valve travel is 8 inches, the cover on the steam port is 1½ inches, and the lead is ⅛ inch: at what distance from the end of the stroke will the piston be when the steam is cut off? (Neglecting the obliquity of the rods.)

The distance the piston is from end of stroke $= \frac{(1{\cdot}5 \times 2 + {\cdot}125)^2}{8} \times 24$

Lap 1½ inches = 1·5 inches.
2
3·0
Lead ⅛ inch = ·125
8)3·125
Divided by travel = ·3906
·3906
23436
35154
11718
·15256836
Length of stroke 24
61027344
30513672

or, Lap = 1·5 inches.
½ lead + ·0625
Divide by ½ travel 4)1·5625
·3906
·3906
23436
351540
11718
·15256836
24
61027344
30513672
3·66164064

Distance from end of stroke 3·66164064 inches. ***Ans.:*** **nearly 3⅔ inches.**

Ex. 2. Given as above to find what part of the stroke has been completed when the steam has been cut off.

Then stroke = 24 inches — 3·66 = 20·34 inches = part completed.

Ex. 3. Given as above to find at what part of the stroke the steam is cut off.

Proceeding as above, then $\frac{22}{24}$ nearly $\frac{7}{8}$ of stroke, that is, the part completed divided by the stroke is the fraction of cut-off required.

Ex. 4. The stroke of an engine is 27 inches, the slide-valve travel is 8 inches, the cover on the steam port is $1\frac{1}{4}$ inch, and the lead is $\frac{1}{16}$ inch: at what distance from the end of the stroke will the piston be when the steam is cut off? (Neglecting the obliquity of the rods.)

$$\text{The distance the piston is from end of stroke} = \frac{(1{\cdot}25 \times 2 + {\cdot}0625)^2}{8} \times 24$$

Lap $1\frac{1}{4} = 1{\cdot}25$
2

2·50
$+ {\cdot}0625 = \frac{1}{16}$″ lead.

Travel 8)2·5625

·3203
·3203

9609
64060
9609

·10259209
27 stroke.

71814463
20518418

Distance from end of stroke 2·76998643 or $2\frac{3}{4}$ inches.

Examples for Practice.

No.	Stroke of Engine. in.	Travel of Slide Valve. in.	Cover on Steam Port. in.	Lead of Valve. in.	*Ans*: The distance piston is from end of stroke. in.
1.	30	5	$1\frac{1}{2}$	$\frac{1}{16}$	11·2546
2.	34	8	3	$\frac{1}{8}$	21·0786
3.	36	$6\frac{1}{2}$	$2\frac{3}{4}$	$\frac{1}{8}$	26·9361
4.	42	$9\frac{1}{4}$	$3\frac{1}{4}$	$\frac{1}{16}$	21·1364

285. **Various questions in practice arise as to working expansively, but the following will furnish ample instructions to the practical engineer.**

If we let a = length of stroke (in inches), then $a = \frac{b\,c}{d}$ (1.)

b = the part of stroke cut off before expansion (in inches) then $b = \frac{d\,a}{c} = \frac{h\,i}{c}$ (2.)

c = whole pressure in boiler, then $c = \frac{a\,d}{b} = \frac{h\,i}{b}$ (3.)

d = terminal pressure in cylinder, then $d = \frac{b\,c}{a} = \frac{b \times (h + 15)}{a}$ (4.)

e = mean pressure in cylinder, then $e = c - d = \frac{a}{b\,c} - d$ (5.)

h = load in lbs. on square inch of piston then $h = \frac{b\,c}{i}$ (6.)

i = point of stroke when velocity of piston is greatest then $i = \frac{b\,c}{h}$ (7.)

p = expansion . . . then $p = \frac{a}{b}$ (8.)

The method of using these is the following:—

Ex. 1. Suppose we have given the pressure in the boiler 40 lbs.; length of stroke 10 feet, and steam cut off at 3 feet: required the terminal pressure; then (see 4 above)

$$\left.\begin{array}{lcl} \text{Pressure} & = c = & 40 \text{ lbs.} \\ \text{Length of stroke} & = a = & 120 \text{ inches} \\ \text{Part of stroke cut off before expansion} & = b = & 36 \text{ inches} \end{array}\right\} \text{To find the terminal pressure } d.$$

Reference to the equations above for an expression of the value of d would give the equation

$$d = \frac{b\,c}{a} \text{ or } d = \frac{36 \times 40}{120} = 12 \text{ lbs.}$$

Ex. 2. The given pressure is 60 lbs.; length of stroke 3 ft. 7 in.; steam cut off at 1 ft. 5·2 in.: required the terminal pressure.

$$\left.\begin{array}{lcl} \text{Pressure} & = c = & 60 \text{ lbs.} \\ \text{Length of stroke} & = a = & 43 \text{ inches} \\ \text{Part of stroke cut off before expansion} & = b = & 17{\cdot}2 \text{ inches} \end{array}\right\} \text{To find the terminal pressure } d$$

Then, by equation (4),

$$d = \frac{b\,c}{a} \text{ or } d = \frac{17{\cdot}2 \times 60}{43} = 24 \text{ lbs., the terminal pressure.}$$

Ex. 3. The initial pressure of steam in the cylinder is 30 lbs., length of stroke 4 feet 6 inches, and the steam is cut off at 3 feet 2 inches: find the terminal pressure.

By referring to the equation above, we have,

$$d = \frac{b\,c}{a} \text{ or } d = \frac{38 \times 30}{54} = 21\tfrac{1}{9} \text{ lbs.,}$$

$$\begin{array}{llll} \text{Otherwise, } 4 \text{ ft. } 6 \text{ in.} & = 4\tfrac{1}{2} \text{ ft.} & = \tfrac{9}{2} \\ \quad 3 \quad\; 2 & = 3\tfrac{1}{6} & = \tfrac{19}{6} \end{array}$$

$$\therefore \text{ power required} = \frac{\frac{9}{2}}{\frac{19}{6}} \times 30 \text{ lbs.} = \frac{19}{27} \times 30 = 21\tfrac{1}{9} \text{ lbs.}$$

Ex. 4. The initial pressure of steam is 45 lbs., the length of stroke 10 feet 9 inches, and the steam is cut off at 3 feet 6 inches: find the terminal pressure.

By equation (4) above, we have,

$$d = \frac{b\,c}{a} \text{ or } d = \frac{42 \times 45}{129} = 14{\cdot}6 \text{ lbs.}$$

$$\begin{array}{llll} \text{Otherwise, } 10 \text{ ft. } 9 \text{ in.} & = 10\tfrac{3}{4} \text{ ft.} & = \tfrac{43}{4} \\ \quad 3 \quad\; 6 & = 3\tfrac{1}{2} & = \tfrac{7}{2} \end{array}$$

$$\therefore \text{ the pressure required} = \frac{\frac{43}{4}}{\frac{7}{2}} \times 45 = \frac{14}{43} + 45 = 14{\cdot}6 \text{ lbs.}$$

And again—

Ex. 1. Given pressure 50 lbs., terminal pressure 20 lbs., stroke 3 feet 10 inches: required to know at what point expansion commenced.

Then, by equation (2),

$$\left.\begin{array}{lcl} \text{Pressure} & = c = & 50 \text{ lbs.} \\ \text{Terminal pressure} & = d = & 20 \text{ lbs.} \\ \text{Stroke} & = a = & 46 \text{ inches.} \end{array}\right\} \text{To find point of ``cut-off''} = b.$$

$$\text{By formula, } b = \frac{d\,a}{c} = \frac{20 \times 46}{50} = 18{\cdot}4 \text{ in.} = 1 \text{ ft. } 6{\cdot}4 \text{ in.}$$

$\frac{18{\cdot}4}{46} = {\cdot}4$; that is, the expansion commenced at $\frac{4}{10}$ or $\frac{2}{5}$ of the stroke.

If it be required to know at what point of the stroke it is most beneficial to cut off steam, such will depend entirely upon the force of steam in the boiler, as estimated in inches of mercury; for this force, divided by 10, gives an easy approximation to the maximum power of work: thus, suppose the force of steam in the boiler to be 42 inches, then $\frac{42}{10} = 4{\cdot}2$, or $\frac{1}{4{\cdot}2}$, or $\frac{10}{42} = \frac{5}{21}$, the length of the stroke.

N.B.—The number 10 is the estimate by Tredgold for the friction of the machinery, and the force of the vapour in the condenser as equal to 5 lbs. per square inch, or 10 inches of mercury.

Ex. 2. The initial pressure of steam in the cylinder is 55 lbs. above the atmosphere, and the terminal pressure 28 lbs.; the length of the stroke is 6 feet: at what part of the stroke does expansion commence?

$$b = \frac{d\,a}{c} = \frac{28 \times 72}{70} = 28{\cdot}8 \text{ inches, the point of "cut off;"}$$

and $\frac{28{\cdot}8 \text{ inches}}{72 \text{ inches}} = {\cdot}4$; that is, the expansion commences at $\frac{4}{10}$ $(\frac{2}{5})$ of the stroke.

EXAMPLES FOR PRACTICE.

Ex. 1. The terminal pressure is 40 lbs.: what was the initial pressure if the stroke is 5 feet and steam is cut off at $\frac{1}{4}$ stroke? *Answer*, 160 lbs.

Ex. 2. The stroke is 8 feet long, the initial pressure 120 lbs., and terminal pressure 50 lbs.: at what point of the stroke was steam cut off? *Answer*, $3\frac{1}{5}$ feet.

Ex. 3. What was the length of stroke when initial pressure was 80 lbs., terminal pressure 45 lbs., and steam cut off at 2 feet? *Answer*, $3\frac{5}{9}$.

286. Before leaving the subject of expansion, we will take into consideration the gain in fuel derived by cutting off.

When we know the point of cutting off, and, consequently, what fraction of a cylinder full of steam is used, we can at once ascertain the per cent. of gain in fuel. We can also ascertain the mechanical effect of the steam. Now this mechanical effect will, of course, be less than it would be if the steam were not cut off.

We ascertain the difference between the mechanical effect at full stroke and the effect when the steam is cut off at a certain point, and then proceed to find the per cent. of fuel required to make the latter mechanical effect equal to the former. This we do by making a proportion, as follows:

The mechanical effect when the steam is cut off at a certain point : the difference between the mechanical effects of steam following full stroke, and steam cut off at the given point of stroke :: the per cent. of fuel used when the steam is cut off at a given point : the per cent. of fuel required to make up the difference in mechanical effects.

This latter quantity must then be subtracted from the per cent. of fuel gained by cutting off at the given point, and the remainder is the per cent. of gain in fuel derived by cutting off, when the same effect is produced as if the steam followed full stroke.

EXAMPLES.

Ex. 1. What is the per cent. of gain in fuel if the steam is cut off at half stroke and made to produce the same effect as if it followed full stroke?

By Hyperbolic Logs. the mechanical effect, if the steam were cut off at half stroke, would be 1·693, and as only half a cylinder full of steam would be used, the saving in fuel would be 50 per cent., so that only 50 per cent. of fuel would be used. But if the steam followed full stroke, the mechanical effect would be 2.

$2 - 1{\cdot}693 = {\cdot}307 =$ difference in mechanical effect.

Now to find the per cent. of fuel required to make up this difference, we make the proportion

$$1{\cdot}693 \;:\; {\cdot}307 \;::\; 50 \;:\; 9.$$

So that 9 per cent. more fuel must be expended to produce the same mechanical effect, cutting off at half-stroke, as if the steam followed full stroke. Hence $50 - 9 = 41$ per cent. gain in fuel, cutting off at half-stroke.

Ex. 2. What is the per cent. of gain in fuel, fulfilling the same conditions as in Example 1st, except that the steam is cut off at quarter stroke?

2·386 = mechanical effect, cutting off at quarter stroke.

75 per cent. = gain in fuel, cutting off at quarter stroke.

4 = mechanical effect, following full stroke.

25 = per cent. of fuel used, cutting off at quarter stroke.

4 — 2·386 = 1·614 = difference in mechanical effects.

2·386 : 1·614 :: 25 : 16·8.

16·8 = per cent. of fuel required to make up the difference in mechanical effects.

75 — 16·8 = 58·2 = per cent. of gain in fuel derived from cutting off at quarter stroke, the steam producing the same mechanical effect, as if it followed full stroke.

Now, it is evident that to make the mechanical effect of the steam, when it is cut off, equal to the effect when the steam follows full stroke, the initial pressure must be increased, so as to make the mean pressure the same in both cases.

Suppose that, when we follow full stroke, the mean pressure of the steam is a given number of pounds; it is required to find the initial pressure of the steam that is cut off at a certain point, which in expanding will have the same mean pressure as in the first case. This is a problem which may be stated as follows:

Required to find a number, which, multiplied by the mechanical effect of the steam that is cut off, and divided by the distance over which this mechanical effect is exerted (this result being the mean pressure of the expanded steam), shall equal the mean pressure when the steam follows full stroke.

To find this number, we have only to make a proportion, thus:—

The mechanical effect of the steam that is cut off : the number of times the steam is expanded, or the distance over which the mechanical effect is exerted :: the mean pressure following full stroke : the mean pressure required.

EXAMPLES.

In the two preceding Examples, suppose the pressure of the steam when following full stroke to be 20 lbs.: required to find the initial pressure of the steam that is cut off.

Ex. 1. When the steam is cut off at half stroke: Here, 1·693 = mechanical effect.

2 = number of times the steam is expanded. Hence, 1·693 : 2 :: 20 : 24·09.

So that 24·09 is the initial pressure when the steam is cut off at half stroke, which will produce the same mechanical effect as steam of 20 lbs. pressure following full stroke.

Ex. 2. When the steam is cut off at quarter stroke.

2·386 = mechanical effect.

4 = number of times the steam is expanded.

Then, 2·386 : 4 :: 20 : 33·5.

So that 33·5 lbs. is the required initial pressure of steam.

It must be remembered, that though the initial pressure of the steam is greater when cutting off, to produce the same mechanical effect as when following full stroke, the gain in fuel is effected, from the fact that, in the first case, only half a cylinder full of steam is used, and in the second, only a quarter. Table II gives the gain in fuel, derived by cutting off at various points; also the relation between the initial pressure of steam when cutting off and following full stroke.

TABLE II.

Point of Cutting off.	Per cent. of Gain in fuel.	Initial pressure required.	
		Following Full Stroke.	Cutting off
$\frac{7}{8}$ stroke.	11·73	1	1·008
$\frac{3}{4}$ "	22·4	1	1·03
$\frac{5}{8}$ "	32·	1	1·09
$\frac{1}{2}$ "	41·	1	1·18
$\frac{3}{8}$ "	49·6	1	1·32
$\frac{1}{4}$ "	58·2	1	1·67
$\frac{1}{8}$ "	67·6	1	2·6

When the pressure of steam following full stroke is given, the initial pressure of the steam that is cut off, can be found by multiplying the number in the last column, by the pressure of the steam that follows full stroke.

It must be remembered that all the results given in the preceding articles are theoretical. They must not be confounded with the gains in actual practice.

Back pressure is a constant prejudicial resistance, caused, in the case of a condensing engine, by the impossibility of obtaining a perfect vacuum in the condenser; and in a high pressure engine, by the pressure of the atmosphere and resistance offered to the exhaustion of the steam by friction and tortuous passages. This back pressure and the friction of the engine, interpose a limit even to theoretical expansion. For the effective pressure at the end of the stroke, which is the difference between the actual pressure at the end of the stroke and the back pressure, must be at least equal to the pressure per square inch required to overcome friction. So, if we know the initial pressure of steam, the length of the stroke and the required terminal pressure, we can find the point of cutting off by the relation between the volume and pressure of steam.

Examples.

Suppose the initial pressure of the steam is 20 lbs. above a perfect vacuum; that the stroke is 4 feet; that the back pressure is 3 lbs. per square inch, and the pressure necessary to overcome friction, 2 lbs. per square inch: what is the shortest point of stroke at which the steam can be cut off?

$3 + 2 = 5$ lbs. a constant resistance. So that the least terminal pressure of the steam is 5 lbs.

Now, as the initial pressure of the steam is 20 lbs., if it is expanded to 5 lbs. its volume must be

$$20 \div 5 = 4$$

So that the steam is cut off at quarter stroke, or, as the stroke is 4 feet, at 1 foot from commencement.

The following example will serve as a good application of the principles enunciated in this article.

There are two engines, which we will designate as No. 1 and No. 2. No. 1 is a high pressure engine, uses steam of 105 lbs. pressure for gauge, and cuts off at quarter stroke. No. 2 is a condensing engine, uses steam of 20 lbs. pressure, and cuts off at half stroke. The feed water is 100° in both cases. The back pressure in the condensing engine is 2 lbs. per square inch, and the friction of the unloaded engine is 2·5 lbs. per square inch in each engine.

Which will develop the greatest mechanical effect from one cubic foot of water evaporated, and how much coal will be required for No. 1 engine to evaporate the water, if 10 lbs. are required for No. 2?

For No. 1.

$$1 \cdot 38 = \text{mechanical effect during expansion.}$$

$$(1 + 1 \cdot 38) \times 120 \div 4 = 71 \cdot 4 = \text{mean pressure.}$$

But as the steam exhausts into the atmosphere, and 2·5 lbs. per square inch are required to overcome friction,

$$71 \cdot 4 - 15 - 2 \cdot 5 = 53 \cdot 9 = \text{mean effective pressure.}$$

For No. 2.

$$\cdot 69 = \text{mechanical effect during expansion.}$$

$$(1 + \cdot 69) \times 35 \div 2 = 29 \cdot 575 = \text{mean pressure.}$$

Deducting the back pressure and friction,

$$29 \cdot 575 - 2 - 2 \cdot 5 = 25 \cdot 075 = \text{mean effective pressure.}$$

Now let us suppose that the steam acts in both cases in a cylinder of 1 square inch cross section, and a stroke of 1 foot. As the volume of steam of 105 lbs. pressure is 251 times as great as that of the water from which it is generated, 1 cubic foot, or 1728 cubic inches of water, will generate.

$$1728 \times 251 = 433728 \text{ cubic inches of steam.}$$

The steam being cut off at quarter stroke, 3 cubic inches of steam are used at each stroke, and the number of strokes made by No. 1 engine will be

$$433728 \div 3 = 144576.$$

The distance in feet travelled by the piston, multiplied by the mean pressure, gives the mechanical effect.

The distance in feet is

$$144576 \times 1 = 144576.$$

Hence the mechanical effect is

$$144576 \times 53 \cdot 9 = 7792646 \cdot 4.$$

For No. 2 the relative volume of the steam and water is 767, and one cubic foot of water generates

$$1728 \times 767 = 1325376 \text{ cubic inches of steam.}$$

As 6 inches of steam are used at each stroke,

$$1325376 \div 6 = 220896 = \text{number of strokes.}$$

Hence the mechanical effect is

$$220896 \times 25 \cdot 075 = 5538967 \cdot 2.$$

Calling the mechanical effect of No. 2 unity, that of No. 1 is

$$7792646 \cdot 4 \div 5538967 \cdot 2 = 1 \cdot 406.$$

So that the per cent. of gain of No. 1 is

$$\cdot 406 \times 100 = 40 \cdot 6.$$

Now let us find the coal required for No. 1. The total heat in the steam of No. 2 engine is 1193°·45, and since the temperature of the feed water is 100°,

$1193° \cdot 45 - 100° = 1093° \cdot 45$ = heat imparted to a cubic foot of water by the consumption of 10 pounds of coal.

The total heat in the steam of No. 2 engine is 1219°·36, and
1219°·36 — 100 = 1119°·36 = heat required to be imparted to a cubic foot of water.
If 10 lbs. of coal are needed in the first case, we shall now require*
(1119°·36 × 10) ÷ 1093°·45 = 10·237 lbs.)

RELATIONS BETWEEN THE LAP OF THE VALVE AND THE AMOUNT OF EXPANSION.

287. There are various expedients for stopping off the supply of steam to the engine at any part of the stroke, which will be fully explained further on. One mode is by the use of an expansion valve, and another mode is by extending the length of the face of the ordinary slide valve by which the steam is let into and out of the cylinder, which extension of the face is called *lap* or *cover*. Having shown the mode of ascertaining the mean pressure of the steam on the piston throughout the whole stroke, supposing the steam to be cut off at any given part of the stroke, we shall now proceed to explain what amount of lap answers to a given expansion, and what expansion follows the use of a given proportion of lap.

Supposing there is no lead, the point of the stroke at which the steam is cut off is determined by the proportion which the lap on the steam side bears to the stroke of the valve. Whatever the absolute dimensions of the lap may be, it will follow, therefore, that in every case in which it bears the same ratio to the stroke of the valve, the steam will be cut off at the same point of the stroke.

As the rules for determining the relations between the lap of the valve and the amount of expansion are difficult to be worked out by persons unacquainted with trigonometry, it will be convenient to collect the principal results into tables, which may be applied without difficulty to the solution of any particular example. This accordingly has been done in the three following tables, the mode of using which it will now be proper to explain.

PROPORTION OF LAP REQUIRED TO ACCOMPLISH VARIOUS DEGREES OF EXPANSION.

Distance of the piston from the termination of its stroke, when the steam is cut off, in parts of the length of its stroke............	$\frac{8}{24}$ or $\frac{1}{3}$	$\frac{7}{24}$	$\frac{6}{24}$ or $\frac{1}{4}$	$\frac{5}{24}$	$\frac{4}{24}$ or $\frac{1}{6}$	$\frac{3}{24}$ or $\frac{1}{8}$	$\frac{2}{24}$ or $\frac{1}{12}$	$\frac{1}{24}$
Lap on the steam side of the valve, in decimal parts of the length of its stroke	·289	·270	·250	·228	·204	·177	·144	·102

Example.—In the first line of the table will be found eight different parts of the stroke of the piston designated, and directly below each, in the second line, is given the quantity of lap requisite to cause the steam to be cut off at that particular part of the stroke. The different amounts of the lap are given in the second line in decimal parts of the length of the stroke of the valve; so that, to get the quantity of lap corresponding to any of the given degrees of expansion, it is only necessary to take the decimal in the second line, which stands under the fraction in the first, that denotes the degree of expansion, and multiply that decimal by the length we intend to make the stroke of the valve. Thus, suppose we have an engine in which we wish to have the steam cut off when the piston is a quarter of the length of its stroke from the end of it, we look in the first line of the table, and we shall find in the third column from the left, ¼. Directly under that, in the second line, we have the decimal ·250. Suppose that we consider 18 inches will be a convenient length for the stroke of the valve, we multiply the decimal ·250 by 18, which gives 4½. Hence we learn that, with an 18-inch stroke for the valve, 4½ inches of lap on the steam side will cause the steam to be cut off when the piston has still a quarter of its stroke to perform.

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* The Engineer Cadet, by Long and Buel.

Half the stroke of the valve should always be, *at least*, equal to the lap on the steam side added to the breadth* of the port; consequently, as the lap, in this case, must be 4½ inches, and as half the stroke of the valve is 9 inches, the efficient breadth of the port cannot be more than 9 — 4½ = 4½ inches, since half of it is covered over by the lap. If this breadth of port is not sufficient to give the required area to let the steam in and out, we must increase the stroke of the valve, by which means we shall get both the lap and the breadth of the port proportionally increased. Thus, if we make the length of the valve-stroke 20 inches, we shall have for the lap ·250 × 20 = 5 inches, and for the breadth of the port 10 — 5 = 5 inches.

This table is computed on the supposition that the valve is to have no lead; but if it is to have lead, all that is necessary is to subtract half the proposed lead from the lap found from the table, and the remainder will be the proper quantity of lap to give the valve. Suppose that in the last example, the valve was to have ¼-inch of lead, we should subtract ⅛-inch from the 5 inches found for the lap in the table. This would leave 4⅞ inches for the quantity of lap that the valve ought to have.

The following table is an extension of the first, for the purpose of obviating, in most cases, the necessity of even the very small degree of trouble required in multiplying the strokes of the valve by one of the decimals in the first table. The first line of the second table, as in the first table, consists of eight fractions indicating the various parts of the stroke at which the steam may be cut off. The first column on the left hand consists of various numbers that represent the different lengths that may be given to the stroke of the valve, diminishing by half-inches from 24 inches to 3 inches. Suppose that we wish the steam to be cut off at any of the eight parts of the stroke indicated in the first line of the table (say at $\frac{1}{6}$ from the end of the stroke),

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* By the breadth of the port is meant its dimensions in the direction of the valve's motion; in short, its perpendicular depth when the cylinder is upright.

TABLE V.—LAP IN INCHES REQUIRED ON THE STEAM SIDE OF THE VALVE TO CUT THE STEAM OFF AT ANY OF THE UNDERNOTED PARTS OF THE STROKE.

Length of the stroke of the valve in inches.	Proportion of the stroke at which the steam is cut off.							
	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
24	6·94	6·48	6·00	5·47	4·90	4·25	3·47	2·45
23½	6·79	6·34	5·88	5·36	4·79	4·16	3·39	2·39
23	6·65	6·21	5·75	5·24	4·69	4·07	3·32	2·34
22½	6·50	6·07	5·62	5·13	4·59	3·98	3·25	2·29
22	6.36	5·94	5·50	5·02	4·49	3·89	3·13	2·24
21½	6·21	5·80	5·38	4·90	4·39	3·80	3·10	2·19
21	6·07	5·67	5·25	4·79	4·28	3·72	3·03	2·14
20½	5·92	5·53	5·12	4·67	4·18	3·63	2·96	2·09
20	5·78	5·40	5·00	4·56	4·08	3·54	2·89	2·04
19½	5·64	5·26	4·87	4·45	3·98	3·45	2·82	1·99
19	5·49	5·13	4·75	4·33	3·88	3·36	2·74	1·94
18½	5·34	4·99	4·62	4·22	3·77	3·27	2·67	1·88
18	5·20	4·86	4·50	4·10	3·67	3·19	2·60	1·83
17½	5·06	4·72	4·37	3·99	3·57	3·10	2·53	1·78
17	4·91	4·59	4·25	3·88	3·47	3·01	2·45	1·73
16½	4·77	4·45	4·12	3·76	3·36	2·92	2·38	1·68
16	4·62	4·32	4·00	3·65	3·26	2·83	2·31	1·63
15½	4·48	4·18	3·87	3·53	3·16	2·74	2·24	1·58
15	4·33	4·05	3·75	3·42	3·06	2·65	2·16	1·53
14½	4·19	3·91	3·62	3·31	2·96	2·57	2·09	1·48
14	4·05	3·78	3·50	3·19	2·86	2·48	2·02	1·43
13½	3·90	3·64	3·37	3·08	2·75	2·39	1·95	1·37
13	3·76	3·51	3·25	2·96	2·65	2·30	1·88	1·32
12½	3·61	3·37	3·12	2·85	2·55	2·21	1·80	1·27
12	3·47	3·24	3·00	2·74	2·45	2·12	1·73	1·22
11½	3·32	3·10	2·87	2·61	2·35	2·03	1·66	1·17
11	3·18	2·97	2·75	2·51	2·24	1·95	1·58	1·12
10½	3·03	2·83	2·62	2·39	2·14	1·86	1·51	1·07
10	2·89	2·70	2·50	2·28	2·04	1·77	1·44	1·02
9½	2·65	2·56	2·37	2·17	1·94	1·68	1·37	·96
9	2·60	2·43	2·25	2·05	1·84	1·59	1·30	·92
8½	2·46	2·29	2·12	1·94	1·73	1·50	1·23	·86
8	2·31	2·16	2·00	1·82	1·63	1·42	1·15	·81
7½	2·16	2·02	1·87	1·71	1·53	1·33	1·08	·76
7	2·02	1·89	1·75	1·60	1·43	1·24	1·01	·71
6½	1·88	1·75	1·62	1·48	1·32	1·15	·94	·66
6	1·73	1·62	1·50	1·37	1·22	1·06	·86	·61
5½	1·58	1·48	1·37	1·25	1·12	·97	·79	·56
5	1·44	1·35	1·25	1·14	1·02	·88	·72	·51
4½	1·30	1·21	1·12	1·03	·92	·80	·65	·46
4	1·16	1·08	1·00	·91	·82	·71	·58	·41
3½	1·01	·94	·87	·80	·71	·62	·50	·35
3	·86	·81	·75	·68	·61	·53	·44	·30

we find $\frac{1}{6}$ at the top of the sixth column from the left. We next look for the proposed length of stroke of the valve (say 17 inches) in the first column on the left. From 17 in that column, run along the line towards the right; and, in the sixth column, and directly under the $\frac{1}{6}$ at the top, we find 3·47, which is the amount of lap required, in inches, to cause the steam to be cut off at $\frac{1}{6}$ from the end of the stroke, if the valve has no lead. If we wish to give it lead (say ¼ inch), we subtract the half of that, or ⅛ = ·125 inch, from

3·47, and we have 3·47 — ·125 = 3·345 inches, the quantity of lap that the valve should have.

To find the greatest efficient breadth that we can give to the port in this case, we have, as before, half the length of stroke, 8½ — 3·345 = 5·155 inches, which is the greatest efficient breadth we can give to the port with this length of stroke. It is scarcely necessary to observe that it is not at all essential that the port should be so broad as this; indeed, where great length of stroke in the valve is not inconvenient, it is always an advantage to make it travel further than is just necessary to make the port open fully; because, when it travels further, both the exhausting and steam ports are more quickly opened, so as to allow greater freedom to the steam.

The manner of using this table is so simple that we shall pass on to explain the use of the sixth table.

Suppose that the piston of a steam engine is making its downward stroke, that the steam is entering the upper part of the cylinder by the upper steam port, and escaping from below the piston by the lower exhausting port; if, as is generally the case, the slide valve has some lap on the steam side, the upper port will be closed before the piston gets to the bottom of the stroke, and the steam above them acts expansively, while the communication between the bottom of the cylinder and the condenser still continues open, to allow vapour from the condensed water in the cylinder, or any leakage past the piston, to escape into the condenser; but, before the piston gets to the bottom of the cylinder, this passage to the condenser will also be cut off by the valve closing the lower port. Soon after the lower port is thus closed, the upper port will be opened towards the condenser, so as to allow the steam that has been acting expansively to escape. Thus, before the piston has completed its stroke, the propelling power is removed from behind it, and a resisting power is opposed before it, arising from the vapour in the cylinder, which has no longer any passage open to the condenser. It is evident that, if there is no lap on the exhausting side of the valve, the exhausting port before the piston will be closed, and the one behind it opened, at the same time; but, if there is any lap on the exhausting side, the port before the piston will be closed before that behind it is opened; and the interval between the closing of the one and the opening of the other will depend on the quantity of lap on the exhausting side of the valve. Again, the position of the piston in the cylinder, when these ports are closed and opened respectively, will depend on the quantity of lap that the valve has on the steam side. If the lap is large enough to cut the steam off when the piston is yet a considerable distance from the end of its stroke, these ports will be closed and opened at a proportionably earlier part of the stroke; and in the case of engines moving at a high speed, it has been found that great benefit is obtained from allowing the steam to escape before the end of the stroke.

Table VI is intended to show the parts of the stroke where, under any given arrangement of slide valve, the eduction ports close and open respectively, so that thereby the engineer may be able to estimate how much, if any, of the efficiency he loses, while he is trying to add to the power of the steam by increasing the expansion. In this table there are eight columns marked A, standing over eight marked B; and, at the heads of these columns are eight fractions as before, representing as many different parts of the stroke at which the steam may be supposed to be cut off.

TABLE VI.—PROPORTION OF THE STROKE AT WHICH THE EDUCTION PORT IS SHUT AND OPENED.

Lap on the eduction side of the valve, in parts of the length of its stroke.	Proportion of the stroke at which the steam is cut off.							
	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
	A	A	A	A	A	A	A	A
1-8th	·178	·161	·143	·126	·109	·093	·074	·053
1-16th	·130	·118	·100	·085	·071	·058	·043	·027
1-32nd	·113	·101	·085	·069	·053	·043	·033	·024
0	·092	·082	·067	·055	·043	·033	·022	·011
	B	B	B	B	B	B	B	B
1-8th	·033	·026	·019	·012	·008	·004	·001	·001
1-16th	·060	·052	·040	·030	·022	·015	·008	·002
1-32nd	·073	·066	·051	·042	·033	·023	·013	·004
0	·092	·082	·067	·055	·043	·033	·022	·011

The columns marked A express *the distance of the piston—in parts of its stroke—from the end of the stroke when the eduction port before it is shut;* and the columns marked B, and which stand immediately under the columns marked A, express *the distance of the piston from the end of its stroke when the exhausting port behind it is opened*—also measured in parts of the stroke.*

Suppose we have an engine in which the slide valve is made to cut the steam off when the piston is at 1-3rd from the end of its stroke, and that the lap on the eduction or exhausting side of the valve is 1-8th of the whole length of its stroke. Suppose the stroke of the piston is 6 feet, or 72 inches. It is required to know when the exhausting port before the piston will be closed, and when the one behind it will be opened. At the top of the left hand column marked A, the given degree of expansion (1-3rd) is given; and in the extreme left column we have at the top the given amount of lap (1-8th). Opposite the 1-8th in the first column marked A, we have, ·178; and in the first column marked B, ·033, which decimals multiplied respectively by 72, the length of the stroke, give the required positions of the piston, thus 72 × ·178 = 12·8 inches = distance of the piston from the end of the stroke when the exhausting port *before* the piston is shut; and 72 × ·033 = 2·38 inches = distance of the piston from the end of its stroke when the exhausting port *behind* it is opened.

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* In engines moving at a slow rate, the same necessity for an early eduction does not exist as in quick moving engines, such as locomotives; and, in such engines, there will be a loss from opening the eduction much before the end of the stroke, as the moving pressure urging the piston is thus removed before the termination of the stroke. When the valve is closed before the piston previously to the end of the stroke, the attenuated vapour in the cylinder will be compressed, and sometimes the compression will be carried so far that the pressure resisting the piston at the end of the stroke will exceed the pressure of the steam in the boiler. The indicator diagram will, in such cases, appear with a loop at the upper corner, which shows that the pressure before the end of the stroke exceeds the pressure of the steam, and that the first effect of opening the communication between the cylinder and the boiler is to enable the cylinder to discharge its highly compressed vapour backward into the boiler. The act of compressing the steam is what is called *cushioning*, and in all ordinary [illegible]grams this action is more or less to be seen.

Again, let the stroke of the valve be 16 inches, the lap on the exhausting side ½-inch, the lap on the steam side 3¼ inches, and the length of the stroke of the piston 60 inches. It is required to ascertain all the particulars of the working of this valve. The lap on the exhausting side is evidently $\frac{1}{32}$ of the length of the valve stroke. Then, looking at 16 in the left hand column of the Table V in page 300, we find in the same horizontal line 3·26, or very nearly 3¼, under ⅙ at the head of the column, thus showing that the steam will be cut off at one-sixth from the end of the stroke. Again, under ⅙ at the head of the sixth column from the left in the Table VI, page 302, and in a line with $\frac{1}{32}$ in the left-hand column, we have ·053 under A, and ·033 under B. Hence, ·033 × 60 = 3·18 inches = distance of the piston from the end of its stroke when the exhausting port behind it is opened. If, in this valve, the lap on the exhausting side were increased, say to 2 inches or ⅛ of the stroke, the effect would be to cause the port before the valve to shut sooner in the proportion of ·109 to ·053, and the port behind it later in the proportion of ·008 to ·003. Whereas, if the lap on the exhausting side were removed entirely, the port before the piston would be shut, and that behind it opened at the same time. The distance of the piston from the end of its stroke at that time would be ·43 × 60 = 2·58 inches.

If we inspect the Table VI, it will be seen that the effect of increasing the expansion by the slide valve is augmenting the loss of power occasioned by the imperfect action of the eduction passages. On referring to the bottom line of the Table, it will be seen that the eduction passage before the piston is closed, and that behind it is opened, thus destroying the whole moving power of the engine when the piston is ·092 from the end of its stroke, the steam being cut off at ⅓ from the end. Whereas, if the steam is only cut off at $\frac{1}{24}$ from the end of the stroke, the moving power is not withdrawn till ·011 of the stroke remains uncompleted. It will also be observed that increasing the lap on the exhausting side has the effect of retaining the action of the steam longer *behind* the piston, but it, at the same time, causes the eduction port *before* it to close sooner.

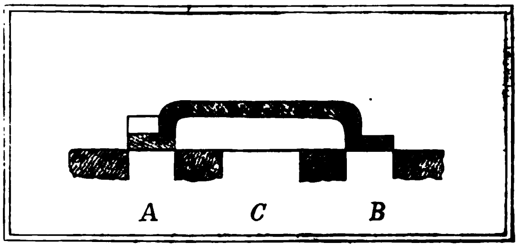
A very cursory examination of the action of the slide valve is sufficient to show that the lap on the steam side should always be greater than that on the eduction side. If they were equal, the steam would be admitted on one side of the piston at the same time that it was allowed to escape from the other; but universal experience has shown that when this is the case, a very considerable part of the power of the engine is destroyed by the resistance opposed to the piston, by the escaping steam not getting away to the condenser quickly enough. Hence, will be seen the necessity of the lap of the eduction side being always less than the lap on the steam side; and the difference should be the greater the higher the velocity of the piston is intended to be, because the quicker the piston moves, the passage for the waste steam requires to be the larger, so as to admit of its getting away to the condenser with as great rapidity as possible. In engines where it is not wished to expand the steam in the cylinder at all, the slide valve is sometimes made with very little lap on the steam side; and under these circumstances, in order to obtain a sufficient difference between the lap on the steam and the eduction sides of the valve, it may be necessary not only to take away all the lap on the eduction side, but to take off still more, so as to cause both eduction passages to be, in some degree, open, when the valve is at the middle of its stroke. But when there is a considerable amount of lap on the steam side, this plan of taking *more than all* the lap on the eduction side ought never to be resorted to, since it can serve no good purpose, and will materially increase an evil we have already alluded to, viz.:—the opening of the eduction port behind the piston, before the stroke is nearly completed. In the case of engines moving rapidly, as in locomotives for instance, it conduces much to efficiency to begin the eduction before the end of the stroke, as the piston moves slowly at that time; and at that point a very small amount of travel of the piston corresponds to a considerable additional time given for the accomplishment of the eduction. The tables apply equally to the common short slide three ported valves, and to the long D valves. The extent to which expansion can be conveniently carried by means of lap upon the valve is about one-third of the stroke; that is, the valve may be made with so much lap that the steam will be cut off when one-third of the stroke has been performed, leaving the rest to be performed by the expanding steam; but if much more lap be put on than answers to this amount of expansion, a distorted action of the valve will be produced, and the efficiency of the engine will be impaired. By the use of the link motion, however, much of this distorted action can be compensated. If a further amount of expansion than this is wanted, where the link motion is not used, it may be attained by wire-drawing the steam, or by so constructing the steam passage that the pressure within the cylinder must decline when the speed of the piston is accelerated, as it is about the middle of the stroke. Thus, for example, if the valve be so made as to shut off the steam by the time two-thirds of the stroke has been performed, and the steam be at the same time throttled in the steam pipe, the full pressure of the steam within the cylinder cannot be maintained except near the beginning of the stroke, where the piston travels slowly, for as the speed of the piston increases, the pressure necessarily subsides, until the piston approaches the other end of the cylinder, where the pressure would rise again; but the operation of the lap on the valve by this time has had the effect of closing the communication between the cylinder and steam pipe, so as to prevent more steam entering. By throttling the steam, therefore, in the manner here indicated, the amount of expansion due to the lap may be doubled, so that an engine with lap enough upon the valve to cut off the steam at two-thirds of the stroke, may, by means of wire-drawing, be virtually rendered capable of cutting off the steam at one-third the stroke.

GEOMETRICAL DEMONSTRATION OF THE SLIDE-VALVE.

This subject has been well digested by many authors, both English and Foreign. Of the latter the best authority is Dr. Gustav. Zeuner, of the Swiss Federal Polytechnical School, but his researches and expositions pertain, however, more to the locomotive than the marine type, but in the main his views are correct for either, under relative circumstances. In this country, Mr. Watt, Professor Rankine, Mr. D. K. Clarke, C.E., and Mr. M. F. Gray, claim attention for their productions. The information now given is taken from the Engineer Cadet, by Messrs. Buell and Long, of the United States Navy.

A three-ported slide-valve in its simplest form is represented in Fig. 6. The valve has the position shown in the sketch, when the piston is at one end of the stroke. When the piston commences to move, the valve also receives motion from the eccentric. If it moves to the right, the port A will be uncovered and steam will be admitted into one end of the cylinder, while the port B will be open to the exhaust port C, and the steam in the other end of the cylinder will be exhausting. When the valve has moved far enough to uncover the port A (at which time the piston will have made half a stroke), it commences to move to the left, and at the termination of the stroke, the port A will again be closed. On the return stroke of the piston, the valve will continue to move to the left until the port B is fully open, when it will again move to the right, closing the port B at the termination of the stroke. It is evident that the motions of the valve and

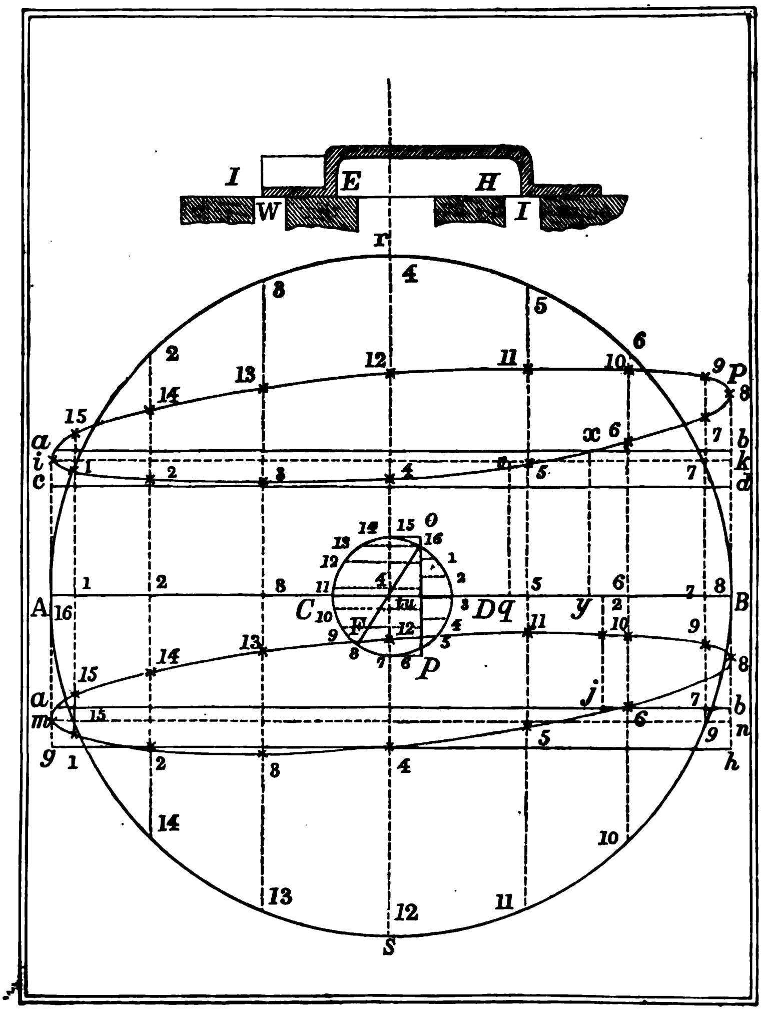
Fig. 6.



piston are at right angles; for the valve is at half stroke when the piston has completed its stroke, and vice versa. So to set the eccentric of this valve, we have only to make the line joining the centre of eccentric and the centre of the shaft, perpendicular to the line joining the centre of crank-pin and the centre of the shaft. The throw of the eccentric, or the travel of the valve (supposing the eccentric rod to be directly connected to the valve stem), is evidently equal to twice the breadth of the steam port, and the face of the valve is just as wide as the steam port.

With the valve set in this manner, the steam is not admitted to the cylinder until the crank is on the centre. But it is found desirable to admit steam a little before the end of the stroke; and this can readily be accom-

Fig. 7.



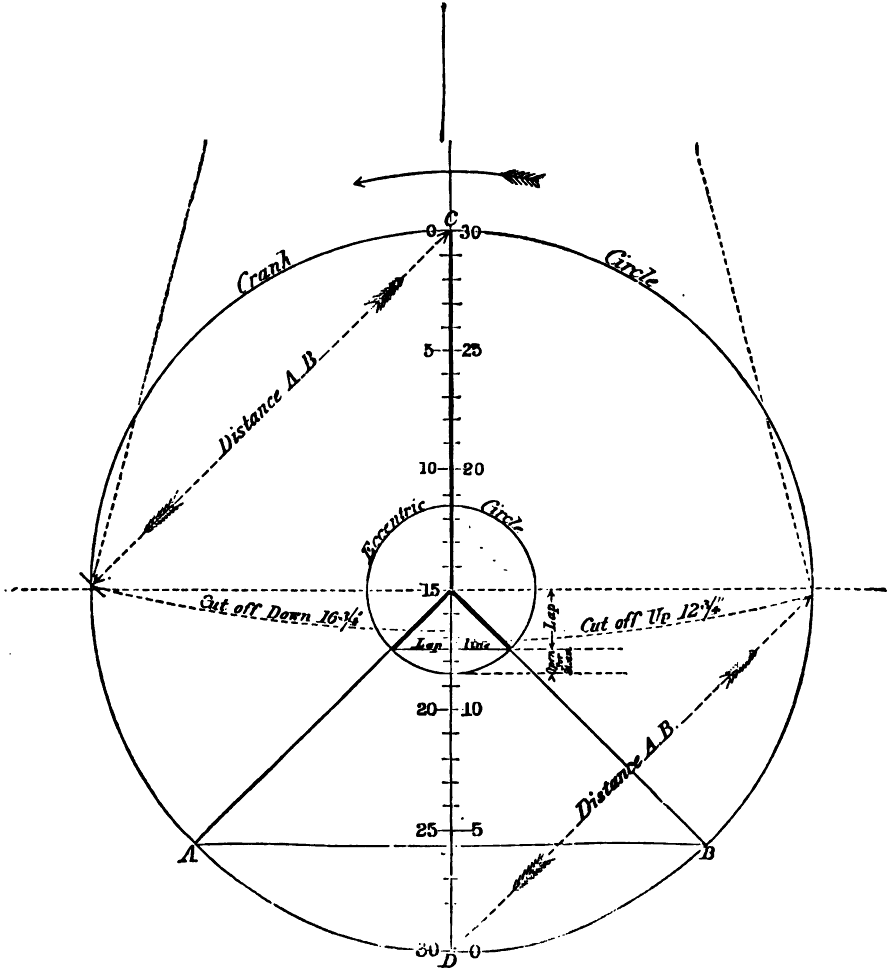
plished by turning the eccentric on the shaft. This we call given the valve *lead*. Thus, if the steam port is open a sixteenth of an inch, when the crank is on the centre, we say that the valve has a sixteenth of an inch lead. But if we open the steam port A before the end of the stroke, we also open the port B to the exhaust, and the amount of this opening at the termination of the stroke, we term the *exhaust lead*.

Now, if we wish the port to be closed before the termination of the stroke, we make the face of the valve longer, or put on *lap*. In this case the throw of the valve must be increased by an amount equal to twice the lap. But if excessive lap be put on, it is evident that the travel will be so much increased as to permit the steam to exhaust at an early part of the stroke. To obviate this, we must put lap on the exhaust side of the valve. This has a bad effect in causing the exhaust valve to close too soon. This will be seen clearly in the illustration of the geometrical action of the slide valve, Fig. 7. Let AB equal the length of stroke of our engine drawn to any scale. We will give the valve an amount of lap on the steam side equal to half the breadth of the steam port. The travel of the valve will then be equal to three times the breadth of the steam port. On AB, as a diameter, describe a circle which will represent the path described by the centre of the crank-pin, while the piston is travelling twice the distance AB. Divide this circle into any number of equal parts, and draw perpendiculars to AB from every point of division. We shall thus determine the position of the piston corresponding to that of the crank at various points. With the same centre *t*, as that of the circle A*r*B*s*, describe a circle C*o*D*p*, having the travel of the valve for its diameter. This will represent the path of the centre of the eccentric, during a revolution of the crank. When the crank is on the centre, the line connecting the centre of the crank-pin and the centre of the shaft will be A*t*; so that if the valve had neither lap nor lead, the line connecting the centre of the eccentric and the centre of the shaft should take the direction *tr*, perpendicular to A*t*. But in the present case, when we have both lap and lead, we make *tu* equal to the sum of the lap and lead, and through *u*, draw a line parallel to *tr*. Connect the point *o* where this line cuts the circle with the centre, and *ot* will be the proper position for the line connecting the centre of the eccentric and the centre of the shaft, when the crank is on one centre. When the crank is on the other centre, this line will appear at *t*F. Divide the circle C*o*D*p* into the same number of equal parts as we divided the circle A*r*B*s*, and draw perpendiculars to *op* from every point of division. The lengths of these perpendiculars show the distances travelled by the valve at various points. Now, let AB represent the centre of the exhaust port. Then draw *ab* and *od* to represent the width of one steam port, and *ef* and *gh* for the other. Make *ai* equal to *lw*, the steam lead, and draw a line *ik* parallel to *ab*. This is the line to which all our measurements must be referred since the valve commences to move from this position. Thus, when the crank has moved the distance A 1, the centre of the eccentric has moved the distance *o* 1, and the perpendicular distance of this point 1 from *op* will be the distance the valve has travelled. Lay off this distance below the line *ik* on the first perpendicular, and the point so determined will represent one position of the valve. Similarly, when the crank has travelled the distance A 2, the centre of the eccentric has passed over *o* 2, and the valve has travelled the perpendicular distance between 2 and *op*. Lay off this distance on the second perpendicular below *ik*, and we determine the position of the valve at another point of the stroke. Find the position of the valve in this manner at every point of division, and through the points so found draw a curve which will represent all the positions of the valve during one revolution of the crank. It must be observed, in laying off perpendicular distances from *op*, that all points to the right of *op* are positive, and are laid off below *ik*, while all points to the left of *op* are negative, and must be laid off above *ik*. We have not yet determined the amount of exhaust lap, but this can readily be fixed, now that the motion of the valve is represented. When the piston has made one stroke, the crank is at B and the valve is at P. Now, if the face of the valve was only 1½ times the width of the port, the whole port would be open for the steam to exhaust. So we must put lap on the exhaust side of the valve, and we put on enough to have the exhaust lead equal to H I. This gives us the width, *w* E, of the valve face.

From the curve A*i*P, we can readily find the position of the valve, corresponding to any position of the piston. Thus, when the piston has travelled the distance A*q*, the valve is at *v*, and the distance the port is open is equal to the perpendicular distance between *v* and the top of the port *ab*. At *x*, where the curve cuts *ab*, the port is closed, and the steam expands during the remainder of the stroke from *y* to B.

We can readily lay down the curve described by the lower extremity of the valve. The distance *em*, between *ef*, the top of the port, and *mn*, is the exhaust lead, and we have only to transfer the distances of the various points in the first curve from *ik* to their respective distances from *mn*, on the same perpendicular on which they were first laid down. By this curve we see that the exhaust opening is closed at *j*, when the piston has travelled the distance A*z*. The steam being pent up in the cylinder, from *z* to B, occasions excessive cushioning, and that is the reason why expansion by means of lap on a slide valve cannot be carried to any great extent. About two-thirds of the stroke is the earliest point at which a slide valve should close the steam port.

Slide-valve Motion.—Stroke of engine, 30 in.; travel of eccentrics, 7 in.; lap on end of valve, 2½ in.; length of connecting rod, 4 cranks or 5 ft.


C
0
30
Crank
Circle
Distance A B
5
25
10
20
Eccentric
Circle
15
Cut off Down 16·¼
Cut off Up 12·¾"
Lap
Lap
line
Open for Lead
20
10
Distance A B
25
5
A
B
30
D
0

Proceed as follows:—Draw to scale, say 1½ inches to a foot, a circle, having its diameter equal to the travel of the valve or eccentrics, marked eccentric circle; from the same centre draw a circle, having its diameter equal to the stroke of the engine, marked crank circle; draw lines through the centre at right-angles to each other, continue one of the lines out, representing the centre line through the cylinder, for the purpose of marking the length of the connecting rod on; divide the part of this line within the crank circle into 30 equal spaces, each of these spaces will represent one inch of the stroke. Now, imagine the crank to be at the top centre marked C, and during the down stroke, the steam to be cut off at some distance from the top. The question, therefore is, "a valve having 7 inches of travel, and 2½ inches lap, at what part of the stroke is the steam cut off?" Mark off 2½ inches from the centre of eccentric circle downwards; draw a line through this point (parallel to the centre line) until it meets the eccentric circle marked lap-line. This distance is the lap; from this lap-line downwards to the circumference of the eccentric circle is the maximum opening for steam; in this case, one inch. Now, from the centre draw lines through the points where the lap-line meets the eccentric circle, and continue them outwards until they intersect the crank circle marked A and B; these lines are the centre lines of the "go-ahead and go-astern eccentrics." Measure or mark off the distance A B *downwards* from C, or the top centre to the crank circle, and with the same distance measure *upwards* from D to crank circle on the opposite side. These points are the positions of the crank when the steam is cut off, but we want the position of the piston; therefore, with the length of the connecting rod from these points, mark on the centre of the engine line a point (this is the centre of the cross-head) where the top end of connecting rod would be, and see what distance on the divided or stroke-line this corresponds to, the number of inches is at once obtained; in this case it is 16¾ inches down, and 12¾ up when cut off. The divided or stroke-line should be numbered from the top, downwards for the down stroke, and from the bottom upwards for the up stroke (see plate). It is at once apparent that if the lap is increased, or the "valves pieced," the distance from A to B will be less, consequently, the steam will sooner be cut off; if the lap is decreased, the distance will be greater, and the steam later in being cut off.

To determine the position of the eccentrics on the shaft, or where the key-way should be cut (on the supposition that the key-way is in the centre of the sheave). From the centre of the eccentric circle, draw a circle, having its diameter equal to the shaft; the lap in one case is 2½ inches, and say, we give an ⅛th of lead, the lap-line will, therefore, be drawn 2⅝ths downwards, and where the lap-line meets the eccentric circle, draw lines from the centre through these points, and where these lines intersect the shaft circle, is the centre of the key-way.

THE INDICATOR.

288. **The Indicator** is a small instrument temporarily applied either at the top or bottom of the cylinder, for the purpose of ascertaining the pressure of steam at each point of the stroke of the piston. It not only, however, discovers to us the *actual power* at work in the engine, but also what proportion of the boiler-pressure is obtained in the cylinder—at what part of the stroke the highest pressure is reached—how it is maintained—at what point, and at what pressure, the steam is cut off—whether it is cut off sharply, or in what degree it is wire-drawn—at what point, and at what pressure it is released—in a non-condensing engine, whether it is freely discharged, or what proportion of it remains to exert a counter-pressure—in a condensing engine, the amount of the vacuum, and how quickly, or how gradually, it is obtained—and in both classes of engines, whether before the commencement of the stroke, there is any compression of the vapour remaining in the cylinder, and if so, at what point it commences, and to what height of pressure it rises. From the diagram, the mean pressure exerted during the stroke, to produce and to resist the motion of the piston may be ascertained, and thus may be known accurately the power required to resist the whole aggregate resistance on the engine, and, also, by taking separate diagrams for each, the power required by each of the several resistances or classes of resistance separately. The *causes* of the various features presented in the diagram may be ascertained, and thus may be learned the effect produced by this or that form of arrangement of parts, and any imperfection in their construction or action may be detected. In fact, in the hands of a skilful engineer, the indicator is as the stethescope of the physician, revealing the inner workings of the inner system, and detecting minute derangements in parts obscurely situated.*

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* It must be borne in mind, that the Indicator shows only the pressure at each point of the stroke—to portray this faithfully is its sole use. "It tells nothing about the causes which have determined the form of the figure which it describes; the engineer concludes what these are, as the result of a process of reasoning, and this is the point where errors are liable to be committed. Conclusions which seem obvious sometimes turn out to have been wrong, and the ability to form an accurate judgment, as to the causes of the peculiarities presented in a diagram, is one of the highest attainments of an engineer."

"The variety of diagrams given by different engines, and by the same engine under different circumstances, is endless; and there is perhaps nothing more instructive to the student of engineering, as there is nothing more interesting to the accomplished engineer, than their careful and comprehensive study, with a knowledge of the modifying circumstances under which each was taken. Lines at first meaningless become full of meaning; that which scarcely arrested his attention comes to possess an absorbing interest; he becomes acquainted with the innumerable variety of vicious forms and learns the points and degrees, as well as the causes, of their departure from the single perfect form; he becomes familiar with the effects produced by different constructions and movements of parts, and competent to judge correctly as to the performance of engines, and to advise changes by which it may be improved."

Principle on which the Indicator works.—Putting aside for the present the mechanical arrangements as the Indicator is now made, and considering it in its simplest form, it is merely a small piston working in a cylinder with considerable clearance, carrying a pencil at the end of its piston-rod. One end of this small cylinder is placed at pleasure in connection with either end of the main cylinder (that is the cylinder of which it is desired to know the horse-power), by means of a cock and pipes, and the other end of the Indicator cylinder is in free communication with the air by means of the loose stuffing-box through which the piston-rod of the indicator moves, and further, by means of a hole drilled in the cylinder cover of the indicator, so that if steam goes into the main cylinder, a portion of it may be admitted directly to the bottom side of the indicator piston, while upon the other side the air presses continually with whatever the barometric pressure may be at the time.

A spiral spring is attached to the cover of the indicator cylinder at one end, and to the indicator piston itself at the other. This spring regulates the movements of the piston, and as the steam is of a greater or less pressure, so the spring is more or less compressed.

Suppose now that the piston of the engine is at one end of the stroke, and about to begin a fresh stroke, and that the steam is admitted to the cylinder for this purpose, the indicator spring will be compressed by the steam pressure under it, and the amount to which the indicator piston rises is a measure of the steam pressure; for example, supposing that the spring compresses ⅛th inch for every pound on it, then, if the steam pressure is 20 lbs., the piston will rise 2½ inches. As the piston of the engine travels forward on its stroke, the steam pressure begins to diminish, and becomes less and less able to compress the indicator spring, and consequently the indicator spring continually falls. In order to register these continually varying pressures, a piece of paper is held on a small cylinder or barrel* in front of the pencil on the indicator piston, and as the main piston moves backwards and forwards, the barrel of the indicator partially rotates backwards and forwards, and the curved line traced by the pencil moving up and down, on the paper moving at right-angles to the up and down movement of the pencil, is called the indicator card or diagram. The diagram is nothing more than a register of the varying pressures in the cylinder as the piston moves to and fro.

289. **Richards' Indicator.**—Figure 8 is a drawing of Richards' Indicator, which is the best form of indicator now in use, and is becoming generally adopted, as it is in some respects better fitted for obtaining accurate results from engines working with steam of high pressure, and having a high speed of piston, and great range of expansion. Its peculiarity, as distinguishing it

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* In the original form of the indicator invented by JAMES WATT, a flat reciprocating plate carries the sheet of paper, and modern indicators differ chiefly from the original construction by the substitution of a reciprocating cylinder for the flat plate.

from older forms (as McNaught's, for example), consists in the arrangement by which the motion of the piston is transmitted to the pencil, the object being to enable stronger springs to be used, so as to lessen the tendency to vibration of the pencil. The drum or brass barrel A, having the split graduated scale B attached, is called the paper cylinder, and on it the paper for the diagram is stretched; this paper cylinder can be detached from the instrument by pulling it upwards; on being detached another barrel is seen, having the horizontal pulley L on its bottom end, and a small coiled spring contained in a drum at the top. It will be observed that on pulling the cord, the pulley barrel will revolve until it comes against a stop, on slackening the cord the coiled spring will bring it back; at the bottom of this pulley barrel will be seen a small projection C. When the paper cylinder is replaced, and the notch in it fitting this projection, the revolving and return motions of the paper cylinder will be clearly understood.* On the upper end of the barrel D the lever arm M is carried; these levers EE_1 are connected by the link F—this link is the pencil carrier; † this metallic pencil is held by a screw which regulates its distance from the paper cylinder; the lever E_1 is attached by a swivel at J to the piston-rod G. This arrangement constitutes a simple straight line motion, and moves just four times the travel of the indicator piston. On unscrewing the top milled cap H the lever arm can be detached from the barrel D, bringing with it the piston-rod and also the spring used. Within the barrel D is contained the small cylinder in which the piston I works; to shift the spring, unscrew the rounded milled nut J on the end of the piston-rod, then unscrew the milled cap H from the nut on the end of the spring, the piston-rod is then free, and the spring can then be unscrewed from the piston I. It will be observed that the springs have a brass nut on each end, on one nut is marked the limit pressures, thus, $\frac{47}{15}$—47 lbs. steam, 15 lbs. vacuum—on the other the scale strength of the spring, thus, $\frac{1}{20}$. On putting the small piston into the cylinder it will be seen that it is an accurate fit, great care must therefore be taken to keep the cylinder perfectly clean. On the bottom end of the barrel D is the conical stem having the tightening nut K attached, this nut has two small arms on it; the conical stem is a ground fit into the top end of the cock supplied with each indicator, this cock has its

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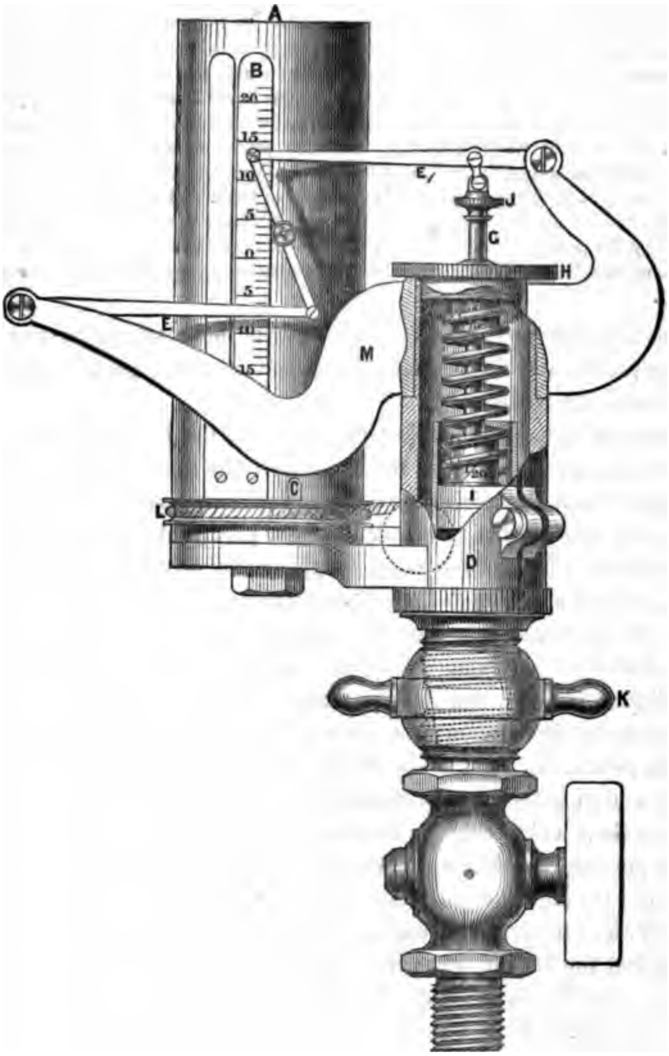
* All the RICHARDS' Indicators now supplied by the makers are fitted with DARKE'S "Patent Detent" and "Cord Adjuster," which is an arrangement for instantaneously stopping and starting the paper-drum, thus avoiding the inconvenience and delay incedental to the present method of connecting and disconnecting the cord which gives motion to the paper-drum. It is exceedingly simple; a pawl is made to fall or rise by the movement of a spring, so as to engage or liberate the paper-drum by means of a small segment of a ratchet placed at its base. It will be found most convenient in all cases; especially useful in trial trips, and where it is desirous to take a number of diagrams in rapid succession. It is certain and smooth in its action. The CORD ADJUSTER will be found of great service as a means of adjusting and re-adjusting the length of the cord of the paper-drum without the use of knots, which are generally difficult (and indeed sometimes through the presence of grease *almost impossible*) to untie.

† In McNaught's instrument the pencil is attached to the piston-rod.

Fig. 8.

RICHARD'S PATENT INDICATOR,

—— Half Size. ——



conical seat protected by a brass cap; unscrew this cap and insert the conical stem, screw down the tightening nut, and as this nut has its threads of different pitches, it firmly secures the indicator to the cock. The communication between the top and bottom of the engine cylinder is obtained by means of a pipe attached to each end, and coupled in the centre to a cock, this cock has a projection on it which is tapped to suit the indicator cock.

NOTE.—In order to reduce the weight of the moving parts as far as possible, with a view of lessening the vibration, Mr. THOMPSON exhibited an indicator at the Philadelphia Exhibition. A single lever only is used, one end being caused to move in a straight line by a parallel motion. The lever is pivoted to a bracket which has its lower end fixed so that its upper end is free to move. A light and short radius bar is attached to a rigid standard, and also to the lever aforesaid, at such a point that its effect is to counteract the arc which the free end of the lever would describe. The line produced is practically straight, though beyond certain limits the lines produced by this and Mr. RICHARDS' indicator, begin to curve, and the length of the straight portion is the same in each. The joint is brought down near the piston, and is a modified form of ball and socket, which allows both the lateral movement required by the parallel motion and the rotative movement involved, in swinging the lever to and from the drum. This joint is made compensating, so that any looseness may be taken up.

There are thus two classes of indicators: those where the piston travels with the pencil, and those where the pencil moves a greater distance than the piston of the indicator.

The reason adduced for giving preference to the latter system consists, as before stated, in the lessened momentum of the moving parts, owing to the small space through which they have to pass in a given period, and thus for quick speed engines the indicator diagram produced has a more steady and reliable form. But it does not yet appear that any indicator has been constructed which shall, under all circumstances, give a reliable diagram for engines which run at very high rates of speed.

The defect of irregular movement is generally most perceptible at the beginning of a stroke, and this is commonly supposed to be due to causes in the instrument itself, though some consideration of the subject leads to a doubt as to whether such is always the case. A sudden admission of steam, often at a high pressure, may probably cause a movement in the piston and its connections which sets the indicator piston into a synchronous vibration; and the yet only half-known effects of condensation in the cylinder may give an impulse through the indicator cocks, which results in a more or less rapid blow delivered immediately underneath the indicator piston; and it may so happen that the instrument forms a convenient thing to blame for a vibrating appearance as the stroke begins, in cases where a current of water actually causes the evil. Such a defect might perhaps be obviated by having a crooked passage or a guard under the indicator piston, so that steam could pass easily underneath it while water is intercepted.

Care must always be taken to have the indicator cleaned after every time of using; and the best easy test to discover that its piston works frictionless, is to draw it up and let it gently descend to a fixed point, which ought to bring the pencil to the atmospheric line, and then to press the piston down and let it slowly ascend. If the pencil reaches the same position with both tests, an easy, frictionless motion can be relied upon.

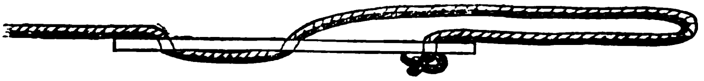
When applied to purposes where the temperature is not more than that of the atmosphere, a corrector is necessary, because springs are stronger when cold than when hot; and there will have to be 1 lb. added to the apparent indication for every 40 lbs. shown by the diagram—the springs being all set for a temperature of 212° Fahrenheit.

289. **How to affix and use the Indicator.**—It must be borne in mind that it is the action of the steam inside the cylinder that we wish to portray; therefore, the motion for the indicating paper, or card, *must* be derived from the engine piston-rod. Starting then with this rule, without exception, we must next note the correct position of the indicator on the engine cylinder. The indicator should be secured to the *end* of the cylinder (front and back ends of the cylinders), *never* on the *side;* for this reason, "the steam that is impelling the piston is also rushing along with it, and when exhaustion ensues, the volume has a retrograde motion, but still in a line with the cylinder's length." The position of the indicator beyond the nozzle *must* be *close* to it to ensure a truthful indication of the steam in the engine cylinder. If a branch pipe is requisite, let it be as straight as possible.

A common plan of fixing indicators, especially for marine purposes, is to have pipes from the top to the bottom of the cylinder, leading to the same indicator cock, so that both may be indicated without loss of time, and both figures be produced on the same indicator card. If such pipes are employed it is necessary to have them continuous from the cylinder to the indicator without any length being open past the indicator cock itself. Such a *cul-de-sac* has the effect of distorting the apparent pressure altogether. All bends and turns in such pipes should be gradual, and of the largest convenient radius, and narrow crooked passages avoided. Having supposed the indicator to be fixed, and in connection with the cylinder, the next point for consideration is the best mode to give motion to the paper or card. In the case of a marine engine, there is more trouble and difficulty in applying the indicator than in that of a land engine, as the proper amount of travel for the string is not so easily to be obtained. The most convenient arrangement would seem to be to add a small wheel and axle, or wheel and pinion, to the indicator, so that a string attached to the piston-rod and wound on the wheel will give the right motion to the cylinder on which the paper is fixed; and, the string being tied on the top of the piston-rod, the motion will be reduced, at once, to that for the paper cylinder, by means of the wheel and axle. It will do, however, when the proposed wheel and axle is not in readiness, to attach the string to any part of the parallel motion, or any part of the side lever which partakes of the motion of the piston. The parallel bar and parallel motion shaft are often selected. If the motion be taken from the side lever, the string must be attached in the central line of the beam, for if attached on the top of the beam near the main centre, the motion may not be the same as that of the piston, and a distorted figure will be obtained.

290. **To fix the paper.**—Detach the paper cylinder from the indicator, and enter one corner of the paper under the split scale, leaving about half-an-inch projecting, turn the paper round on the cylinder and enter the other corner; take hold of the two corners and pull the paper gently down for about one inch, with the finger and thumb, make the two ends of the paper assume, as near as possible, the form of the part entered on the cylinder; this will tend to prevent the ends of the scale cutting the paper. Now draw the paper down to the bottom of the scale, and have it lying even and tight on the cylinder; the paper being on, replace the cylinder, taking care that the projection C on the pulley barrel is in the notch on the paper cylinder. Before screwing the indicator cock on to the pipe cock,* open the pipe cock to each end of the cylinder so as to blow out any grease or dirt that may be in the pipe; having done so, screw on the indicator cock, then fix the indicator by means of the tightening nut K. In order to regulate the length of cord correctly, a slip noose is found very useful. This consists of a strip of brass, say 1½ inch long, ½ inch broad, by ⅛ inch thick. Three fine holes are perforated through the brass and a piece of cord (this cord should be hard and have no stretch)† about 2 feet long, tied to one end. The other end of the string, after passing through the remaining two holes, thus:—

Fig. 9.



has a hook attached, and a ring may be conveniently placed on the opposite loop. By this means a ready adjustment of the cord is obtained. Ascertain if the hook on the lower end has a travel of about five inches (this should be about the length of the diagram), now put the loop end of the cord on to the lever hook, and put the other (or adjusting loop) on to the hook of the pulley barrel cord; adjust the cord by the brass strip until the paper cylinder revolves free; *there must be no slack in the cord, and the paper cylinder must not touch the stop.* Having got the motion regulated, open the pipe cock to either end of the cylinder, and then open the indicator cock, allowing the indicator to work a few revolutions before putting the pencil on to the paper, for the double purpose of heating the indicator and ascertaining if its piston and spring work freely. Then close the stop-cock; when this is done the atmosphere acts equally on both sides of the indicator piston. If the pencil be now put on the paper, the cylinder revolving, a horizontal line will be traced, called the *atmospheric line*, and if, when the engine is at the top of its stroke, the indicator stop-cock be opened, it will commence its registration, and a line will be traced upon the paper, which rises higher up on the cylinder as the pressure of the steam increases, and comes lower upon it as the steam pressure subsides. The area of the curve traced out by the pencil, therefore, represents the pressure on the piston through all its variations.

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* Before fixing any indicator its permanent stop-cock must be opened for several revolutions of the engine, to clear away any foreign matter that is sure to accumulate therein. When engines are indicated for the first time, this precaution is more especially necessary.

† String for driving the indicator should not be used quite new as it continually stretches, and when any great length has to be used, fine brass wire is preferable.

If it is intended to take a diagram from each end of the cylinder, open the pipe-cock to either end of the cylinder; turn the lever arm towards the paper so that the pencil will mark a distinct line, and having made the diagram for that end of the cylinder, turn the pipe-cock for the other end of the cylinder and make its diagram, then shut the indicator cock; when this is done, the atmosphere acts equally on both sides of the indicator piston, put the pencil on to the paper again and draw the *atmospheric line*. After taking the paper from the barrel it is completed by writing on it the day of the month, the name of the ship, that of the engine (whether starboard or port, high or low pressure), top or bottom of the cylinder (as the case may be), the number of revolutions, the pressure of steam by steam gauge, and of condensation by barometer gauge.

NOTE.—To a beginner this manipulation may appear very difficult and tedious, but it is not so; he should repeat the operation until he is thoroughly "up" in putting on the paper, adjusting the cord, and tracing the diagram; a few trials is quite sufficient to make him an expert at it; this must always be remembered when taking diagrams. "*Any inaccuracies in the adjustment affects the diagram most seriously.*"

291. **Best form of Diagrams.**—Were all engines constructed in the best possible proportions of valves, steam passages, &c., and run at the proper speed for such proportions, and were the valves and pistons always tight, the same allowance made for clearance of piston, and no condensation of steam allowed in the cylinder, general rules might be given for the shape of diagrams from condensing and non-condensing engines, for each different pressure of steam and gradation of expansion.

292. **Diagram from High Pressure Engine.**—Thus, if steam were admitted freely from the boiler to the cylinder of a high pressure engine during the whole stroke of the piston, the diagrams should be nearly a parallelogram, the upper line of which would represent the pressure of the steam on one side of the piston, and the lower that of the exhaust steam on the other side, and the ends of it would be described while the piston of the indicator was passing from one end of the cylinder to the other. Three of the corners of such a diagram might, as a general thing, be quite well defined angles; but the fourth—that which is produced while the piston of the indicator is

passing from the steam line to the exhaust—must be more or less curved, unless the engine was going very slow, or the ports extraordinarily large, as the contents of the cylinder of the engine must be discharged during the time this line is being described.

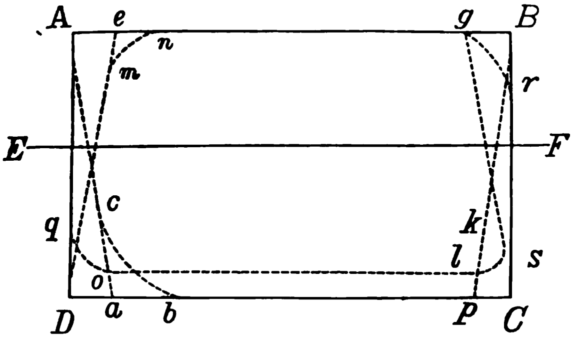
A diagram in such a form would indicate that the engine was receiving a power equal to the pressure of steam acting upon the piston during the whole length of the stroke.

293. **Diagram from Condensing Engine.**—A diagram from a condensing engine differs from one produced by a non-condensing, in the fact that, in the former, the lower line, instead of being a little above atmospheric pressure, approaches nearly to that of perfect vacuum; and, as the steam has to be condensed while the pencil is tracing the exhaust line, it is still more difficult, with such engines, to produce in the lines well-defined angles. There is also more or less of a curve produced at the termination of the vacuum line, owing either to the lead of the valves, or the compression of vapour, as the valve approaches its seat. But nearly the whole of the area of the diagram lost from want of sharpness in the several parts, or their departure from right-angles, except when it is the result of expansion, represents so much capacity of the engine, for which there has been an expenditure of steam, and for which there has been no other consideration exchanged than the relief to the engine, from the effects of the greater percussion produced by a more instantaneous charge of the force acting upon the piston. To produce these with well-defined angles, it is evident that the pressure upon the piston of the engine must be changed from one side to the other, nearly or so far as the usual diagram would show, quite instantaneously—a usage that few large engines would be able to stand. While economy of fuel, therefore, requires well-defined angles, the stability of the engine, or economy of repairs, must direct how nearly they may be allowed to approach them.

294. If we suppose the valve to have neither lap nor lead, and the engine to be perfect in its action (which is of course only a theoretical case), the diagram will be the rectangle ABCD, Fig. 10. EF is the atmospheric line, traced by the pencil, when the stop cock is closed, and D C is the vacuum line. Now, when the crank is on the centre, steam is admitted, the piston rises a distance proportionate to the amount of pressure, and the pencil traces the line DA, called sometimes the *receiving line*, also called the *admission line*. As the main piston makes a stroke, the paper cylinder revolves, and the pencil traces the horizontal line AB, called the *steam line*, the indicator piston being kept on a level with the point A by the pressure of the main cylinder. When the main piston arrives at the other end of the stroke, the *exhaust valve* is opened, and the pressure falls to the perfect vacuum line, causing the pencil to describe the line BC, called the *exhaust line*. On the

return stroke of the main piston, there will be a perfect vacuum in the cylinder, and the pencil will trace the line CD, the *vacuum line*.

Fig. 10.



If the steam valve was set with lead, or so as to open before the end of the stroke, the steam line would have the direction *a* A. If the exhaust valve had lead, *g* C would represent the exhaust line, instead of B C. The exhaust valve is often made to close before the end of the stroke, so as to produce cushioning, and this would appear in our theoretical card as the hyperbolic *b c*. If the steam valve opened too late, the admission line would take the position D *e*.

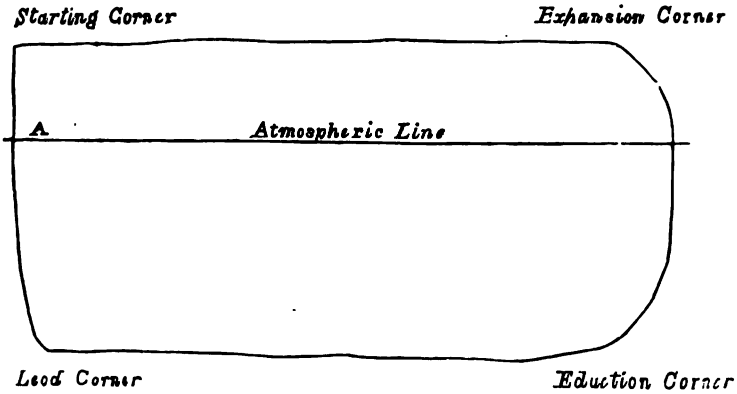
It will be understood that these are extreme cases, as represented in the diagram, and made so as to aid in showing the principle clearly.

If the exhaust valve opened too late, the exhaust line would appear as B*p*. By accustoming himself to the meaning of the various directions taken by these several lines, an engineer can tell at once, on seeing a diagram, whether the engine from which it is taken is in good running order.

In the practical case of a diagram from an engine, the vacuum line would appear above the perfect vacuum line, as at *o s*, and this perpendicular distance between the line is the *back pressure*. The corner at A, sometimes called the *starting corner*, will be square if the valve is rightly set, and the ports are of sufficient size. The corner at B, sometimes called the *expansion corner*, has a slight rounding from where the exhaust valve opens at *g*. The corner *s*, called the eduction corner, should be square, or nearly so. The corner D, called the lead corner, has also a slight rounding *o q*. A *g r s o q* will then represent what is called a good diagram.

The following figure is a cylinder diagram of a steamer. The atmospheric line EF running through the centre of it shows the point at which the pencil of the indicator stands when there is neither pressure nor vacuum below the indicator piston, and this line is traced by causing the pencil to make a line upon the paper wound round the drum before the cock of the indicator is opened. From the point E, near the left hand top corner of the figure, the pencil starts on its course; and, on the cock being opened, it immediately flies up and traces the upper line of the figure, which represents the pressure of the steam; and for every tenth of an inch—or other distance which, in the graduation of the instrument, is taken to represent a pound of pressure —that the pencil rises, the pressure within the cylinder is that number of pounds above the atmosphere. By the time the paper cylinder has reached the extremity of its horizontal travel, the slide-valve has shut the steam off and opened the passage to the condenser, when the pencil descends to a position below the atmospheric line corresponding to the degree of exhaustion existing in the cylinder. The quality of the vacuum is, therefore, represented by the botton line, and by the time the paper has travelled back to the point from which it started, the slide-valve has closed the eduction passage and again let the steam into the cylinder, when the pencil springs up to the position due to the pressure of the steam.

Fig. 11.

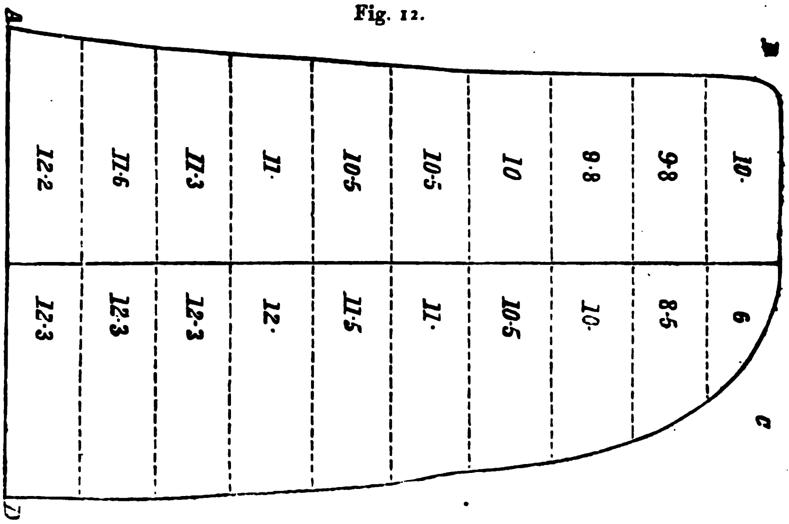


A diagram, then, with the steam or starting corner A slanted off, shows the steam does not gain admission sufficiently early; indeed, any rounding of it is a defect from want of lead, or rather from lead on the wrong side—the steam valve not opening until some part of the stroke has been performed. The eduction corner should be as square as possible, and if it be much rounded or slanted away, then the eduction passages are too small. It is found advantageous to give some lead to the valve, but if the "lead corner" be much slanted off, then the lead given to the engine is great, or in other words, the steam side of the slide is opened before the end of the stroke, or the eduction passage is shut too soon, which causes the vapour within the cylinder to assume considerable elasticity from compression. In some cases this vapour may be compressed to a greater tension than the steam possesses, and the line of the diagram may, therefore, run up above the steam line before the end of the stroke, and a loop will be formed at the "starting corner" A. (See Fig. 11.)

295. To enable the reader to understand more clearly the instructions just given, he is here presented with a diagram, showing the defects and advantages alluded to in the foregoing description. By attentively examining its form at the points marked, the reader will soon be able to comprehend the subject sufficiently well to "read off" any diagram taken from an engine.

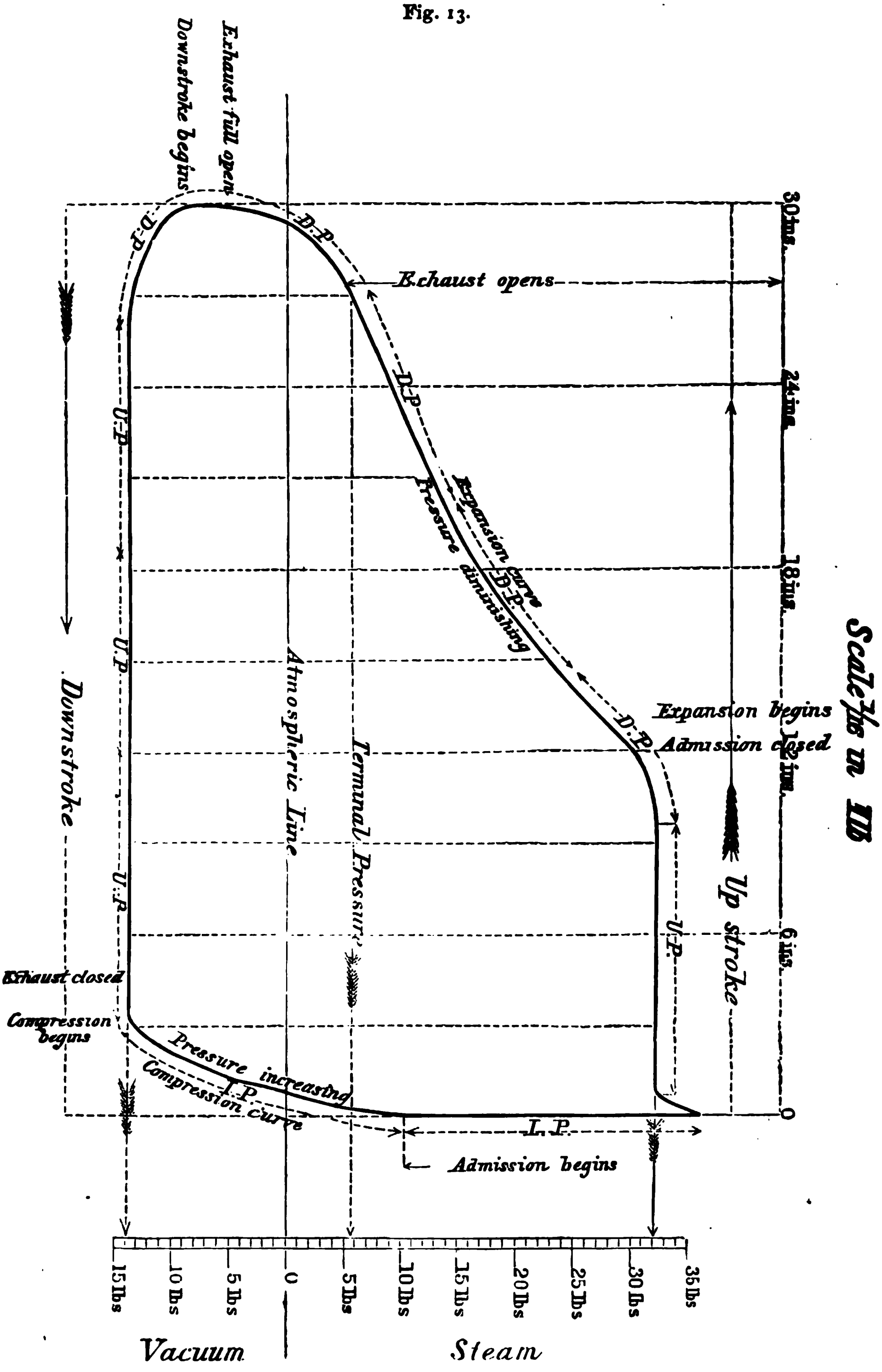
Figure 12. Taken from the top of the cylinder (not a marine engine). The steam corner A shows the valve at the commencement of the stroke to be rather too forward. It would be better were it a little later. Then the diagram would be a little rounded at the corner A, thus preventing the steam from pressing too much upon the piston, when the crank is the "plum centre." The engine is supposed to be working without expansion. The steam continues to the end of the stroke, as shown at B, nearly at the same pressure the whole length, showing that the valve on the steam side has been open the full length of the stroke. The commencement of the exhaust corner C being rounded, shows that the valve is too late. It would be better had the exhaust "more lead"; that is, were the valve on the exhaust side to open a little sooner. This would allow the steam to escape into the condenser earlier, and give more power to the steam at the commencement of the stroke. The termination of the exhaust corner D being square, shows the valve also to be too late in closing the exhaust. If closed sooner, as before explained, the engine would work much better. Should the valve be a D or a common slide three-port valve, add a little less to the steam side, and set forward the valve by the eccentric. The valve, in that case, would not open full port. The exhaust would be opened sooner and closed sooner, which is required. It would then show a rounding at the corner D, in proportion to the distance the valve had been moved forward. If tapit valves, they can be regulated independent of each other, by the moving of the tapits.

Fig. 12.



296. **Explanatory Diagram,** Fig. 13.—In practice the steam is generally cut off sometime before the end of the stroke, and the pencil always stops in in its descent before reaching the line of no pressure, CD, fig. 10. The diagram, fig. 13, shows the usual shape of those taken from marine engines, and under the letters U.P. uniform pressure, I.P. increasing pressure, D.P. decreasing pressure, a correct opinion can be formed of the varying pressures in the cylinder. The engine is supposed to have 30 inches stroke, the pressure of steam in the boiler—say 35 lbs.; in order to make the explanation as lucid as possible, we will suppose the piston to be at the bottom of the cylinder receiving steam; this steam to be cut off when the piston has travelled a certain distance upwards; the valve to open to the exhaust a certain distance before the piston reaches the top, and to continue open until the piston has nearly come to the bottom again; this diagram will therefore tell us what has been going on *on the under side of the piston.* For the better understanding of the diagram, it is drawn to a scale of $\frac{1}{16}$ths (the spring used) the pressure carried by the boiler, 35 lbs.; and, also, as the length of the diagram always corresponds to the length of the stroke, it is also divided into five equal spaces, each space will therefore be equal to 6 inches of travel of piston. At the point on the diagram where the steam is admitted, it will be observed that the line shoots straight upwards, this is owing to the indicator-piston being acted upon by the steam, marked IP, while the piston of the engine is for a moment stationary on turning the centre, but, on the engine-piston commencing to move, this line falls quickly, forming a loop, the piston is moving upwards, the pressure of the steam continues uniform, marking the straight line UP; but it will be observed that a little before the point marked "admission closed" the line begins to curve downwards, this is owing to the port being *gradually* closing, and at the point marked "admission closed" (corresponding, as will be seen, to a travel of 12 inches of the piston) the steam is "cut off," and by reason of the expansion of the steam and consequent decrease of pressure, the line falls rapidly, marked DP; this line assumes a curved form called the expansion curve. At the point marked "exhaust opens" (corresponding to a travel of 27 inches of the piston, or 3 inches from the end of the stroke) the line falls suddenly, consequent on the release of the steam to the condenser. At the point marked "exhaust full open" the piston is almost at the end of its upstroke, and the steam that forced it upwards has been condensed, therefore, whatever vacuum is produced in the condenser, the same, or nearly so, must now be on the under side of the piston. The return stroke downwards commences on the diagram by a round falling corner marked DP; this is owing to the condensation of the steam not being instantaneous, thereby occasioning a slight back pressure for a short distance. The line marked UP, continues straight, indicating the vacuum effect, until the point marked "exhaust

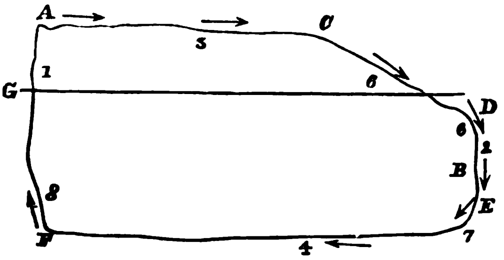
Fig. 13.


Exhaust full open
Downstroke begins
D.P.
Exhaust opens
Expansion curve
Pressure diminishing
Expansion begins
Admission closed
U.P.
Up stroke
Downstroke
Atmospheric Line
Terminal Pressure
Exhaust closed
Compression begins
Pressure increasing
Compression curve
L.P.
Admission begins
30 ins.
24 ins.
18 ins.
12 ins.
6 ins.
0
Scale 1/16 in 1lb
15 lbs
10 lbs
5 lbs
0
5 lbs
10 lbs
15 lbs
20 lbs
25 lbs
30 lbs
35 lbs
Vacuum
Steam

closed" is reached; at this point the port is closed to the condenser (corresponding to about 4 inches from the end of the stroke) and compression begins. To explain what "compression" means, it must be borne in mind that the process of condensation is not complete, else a perfect vacuum would be formed. There is always a certain amount of vapour which cannot be condensed or completely abstracted by the air-pumps. Now, suppose the exhaust to be closed, (as in the diagram), when the piston has travelled 26 inches, or 4 inches from the end of the stroke, therefore, when the piston has travelled other 2 inches, or 28 inches of the stroke, this vapour will have double the resisting effect, consequent on its being compressed into two inches of space. The effect of compression is as the space occupied, its use is that it forms a kind of elastic cushion or buffer to each side of the piston at the end of its stroke; without a certain amount of compression, there would be a straining (and often a knocking) of the cross head and crank pins; it will be evident that too much compression makes the curve large, occasioning an apparent loss of power.

The following diagram is taken from above the piston of a steam ship; and what follows will serve as an explanation of it, and assist, likewise, in elucidating what has been before stated.

Fig. 14.



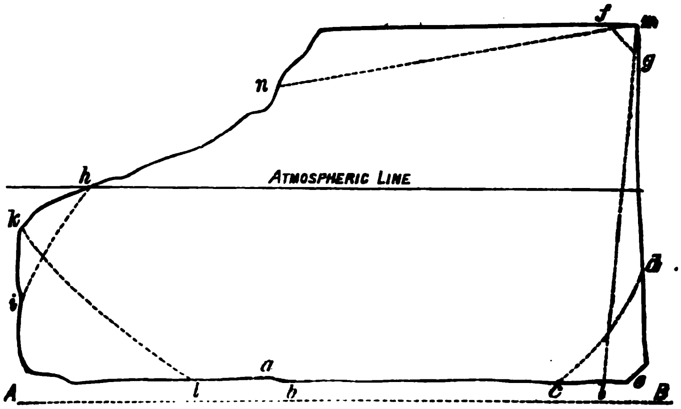
Suppose the pencil to commence at A, and trace out the curve in the direction of the arrow, we see that the steam preserves its first and highest pressure for a considerable portion of the stroke, viz., from A to C; from C to B the downward stroke continues; but the steam begins to lose its pressure, although at a variable rate, decreasing rapidly at D. At B, the motion of the piston ceases; but the steam continues to fall, till at length the pencil moves back nearly horizontally for some space, showing the pressure to continue invariable, although the piston is rising. At F, however, 8 shows the steam pressure to increase rapidly and suddenly, the piston still ascending, till, as this oblique line merges into the vertical one at G, we perceive that the piston has arrived at the upper end of its stroke, and the fresh influx of steam drives the pencil up to A. From this point the pencil will retrace the same curve.

Thus, every change and every deviation of the engine is exhibited, including those of expansion, lead, traverse of the valve, and closing of the exhaust. Expansion produces, to all appearance, an irregular or defective diagram (see figs. 13, 14, &c.), but this is not the case, because the diminished pressure of the steam at the end of the stroke, or at the termination of the piston's descent, is much below the pressure of the atmosphere, thereby reducing the steam to a low degree, which is more than compensated by the saving of fuel. A very little lead of the valve is required, if the pressure be high. The closing of the exhaust before the termination of the stroke is called cushioning the piston. This will have to be regulated according to the speed of the engine. The piston and most of the working parts of an engine having to be reversed, require to be brought to a stand before the return stroke. The steam left in the cylinder will be compressed, and check the speed of the piston at the end of the stroke. The elasticity of the compressed steam between the bottom or the top of the cylinder and the piston is ready, by its force, to commence the return stroke with ease, preventing an undue strain upon the engine. The steam side of the valve does not require to be as forward as when the exhaust is open the full length of the stroke. For very quick speed of the piston the opening on the steam side requires to be earlier, and as much steam admitted before the stroke is completed as will check the speed, and act as a cushion or buffer to the piston.

Fig. 14 is what would be termed a good diagram, so also is Fig. 15, which we will take in further elucidation of the subject.

(*a*) It appears from this diagram, however, that the indicator worked rather tightly, which caused it to stick a little in some places, as is evidenced by the steps in the expansion line, and also at *a b* in the vacuum line. If the piston of the indicator becomes much scratched, similar effects will be produced. Great care should therefore be taken in its use to see that the piston neither works too tightly nor too loosely; for on the one hand it will stick, and thereby produce an imperfect outline, and on the other hand will produce the same effect by exhibiting false vacuum and expansion lines. (See pages 314—315.)

Fig. 15.



(*b*) Should Fig. 15 instead of being as shown in full lines, have the lower right hand corner cut off as shown in dots at *c d*, the defect would have been that the exhaust valve closed too soon—at *c* instead of *e*—occasioning excessive cushioning or too much compression. With some engines, however, as has already been stated, a large amount of cushioning is necessary to prevent them thumping on the centres.

(*c*) Should the line *d m*, instead of being at right-angles to the atmospheric line, be slanted with the top inclined to the right as from *l* to *m*, it would show there is excessive lead given to the steam valve.

(*d*) Had the upper right hand corner been slanted off or rounded, as shown by the dotted line *f g*, the defect would have been that the steam-valve opened too late, caused by valves having too little lead.

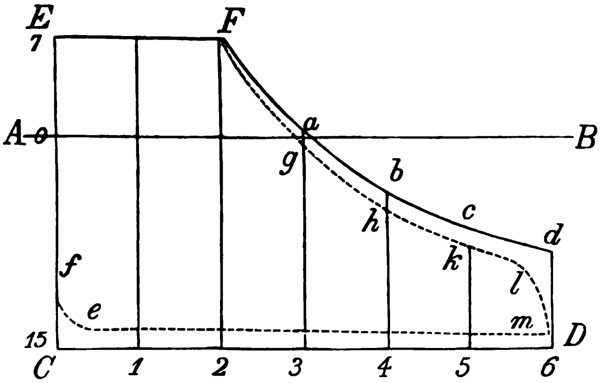
(*e*) When the exhaust corner is cut off, as shown by the dotted line *h i*, it shows that the exhaust-valve opens too soon.

(*f*) But if the exhaust corner had been in the form shown by the dotted line *k l*, the exhaust-valve would have opened too late, and after it did commence to open, would open with too slow a velocity, preventing the free escape of steam, or the exhaust passages would have been too small, which would produce a similar effect to the valve opening too slowly.

(*g*) Should the steam line, instead of being parallel to the atmospheric line, fall in as shown by the dotted line *m n*, it shows that the throttle-valve is partially closed, or the steam passages too small, preventing the full flow of steam into the cylinder. It will be seen that the pressure is not maintained uniformly for any part of the stroke, by reason of a diminished supply of steam.

297. **The manner of laying out a true Hyperbolic Curve.**—In the case of an expansive engine the theoretical curve of expansion will be an hyperbola. We will show a method of laying out the curve that will be found useful in connection with the diagrams. In fig. 16, AB is the atmospheric line, and CD is the perfect vacuum line; CE is the receiving line, EF the steam line, and at F the steam is cut off, and the pencil describes the hyperbola F *a b c d*, called the expansion line. We can lay this out merely by applying Mariotte's law. Divide the whole length of the stroke CD into any number of equal parts, and observe how many of those parts the steam follows. In the case under consideration: suppose the initial pressure of steam is 60 pounds, and the steam follows two of the spaces into which we have divided the stroke. Then, when the steam reaches the third space, its volume will be 1½ times what it was originally, and its pressure will, therefore, be ⅔ of 60 or 40 pounds. Similarly, as at the fourth division

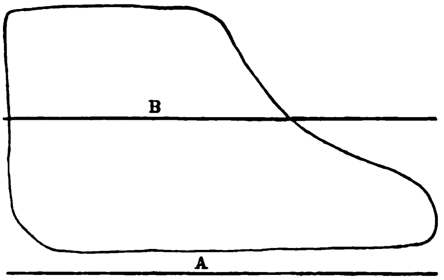
Fig. 16.



the volume is doubled, the pressure will be ½ of 60, or 30 pounds. At the fifth division the volume is 2½ times its original bulk, and the pressure is ⅖ of 60 or 24 pounds, and so on. When all the points are found, describe the curve F *a b c d*. E F *a b c d* D C is, then, the theoretical diagram taken from an engine carrying 45 pounds of steam per gauge, and following ⅕ of the stroke.

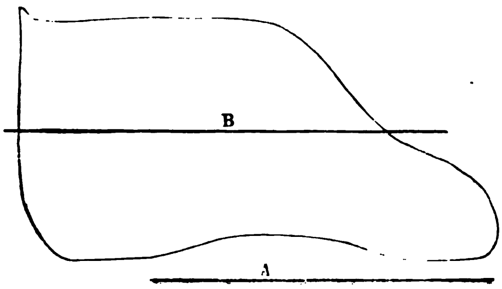
An experienced engineer can tell at a glance whether an engine is in good working order from its diagram; but, nevertheless, in most cases it would be well to draw the true curve, in order to ascertain how much the actual one differs from it, for by this means we can ascertain when under way, whether the valves or piston leaks. In drawing the theoretical curve on a diagram taken from an expansive engine, it must be remembered that all the steam in the clearance also expands. This clearance, then, must be reduced to an equivalent length of cylinder, and added to the end of the stroke. In the practical case the curve will usually fall a little below the theoretical curve. If it is very much below, it is caused by a leak, either in the exhaust valve or around the piston. If the actual curve is above the theoretical, it will be on account of a leak in the steam valve, whereby steam is admitted to the cylinder after the valve is closed. Our practical diagram is E F *g h k l m e f*.

Fig. 17.



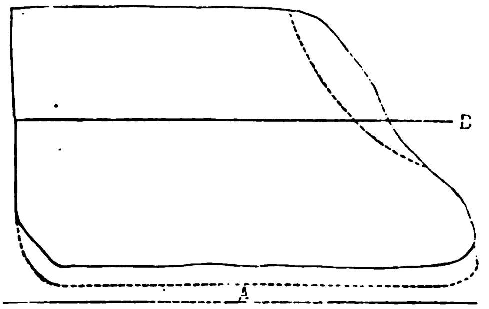
Leaky Piston, when leaking throughout the whole stroke.—This diagram shows the appearance produced by a leaky piston when it is leaking throughout the whole stroke. It will be noticed that the vacuum line is imperfect, because of a constant leakage of steam past the piston.

Fig. 18.



Leaky Piston, when leaking through a portion of the stroke.—This diagram shows the appearance produced by a leaky piston when leaking through a portion of the stroke. It will be observed that the vacuum line is wavy, and the part at A shows a vacuum vitiated by the leakage of steam past the piston.

Fig. 19.



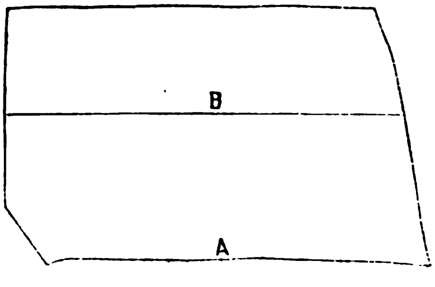
Leaky Slides.—This is an illustration of an indicator figure taken from an engine with leaky slides. The line below the diagram is the line of perfect vacuum, and the dotted line shows the figure that ought to be made if the slide-valves were tight. Should the steam valves leak while everything else remains tight, the termination of the expansion line will be too high.

298. **How to discriminate between errors due to the slide-valve and these due to the eccentric.**—In discriminating between an error due to the slide-valve and one due to the eccentric, remember that the effect of shifting the eccentric is to move all the points of opening or of closing the ports round the diagram opposite to that in which the eccentric has been shifted. The effect of an alteration in the position of the slide-valve, or of any of the edges of the slide-valve is, as a rule, to move the points affected by the alteration along the diagram in the same direction in which the valve or valve edge has been altered. When the valve does not open to admit steam until the crank has passed the centre, the point of admission is an exception to this rule. The points are all in pairs, each point of opening has a corresponding point of shutting. If one point is wrong, find whether the corresponding point on the other side of the diagram is also wrong, and whether the two points require to be shifted round upon the diagram, or whether they require to be moved along the diagram in the same direction, and state the alteration required.

299. **Effects produced on an indicator diagram by shortening and lengthening slide-rod.**

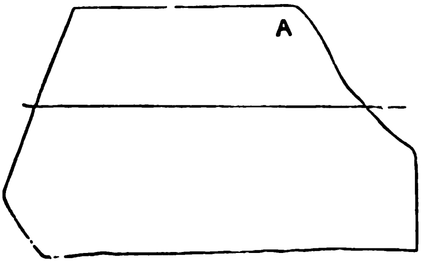
Figs. 20 and 21 show a full speed diagram with lap, and one without lap on the slides.

Fig. 20.



In the diagram, Fig. 20, it is shown that the steam is carried uniformly to the end of the stroke; but in Fig. 21 it will be seen that it is cut off when the piston has some distance to travel, which is caused by the lap on the valve. In the former case the valve only just covered its ports, therefore, it is only shut at the end of its stroke; but in the latter, by means of the lap, the port is closed sooner, and steam is admitted to the opposite side of the piston before the completion of its stroke.

Fig. 21.

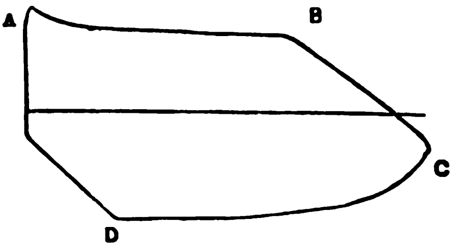


Observe that if the cushioning corner of an indicator diagram is very square, it may be attributed to too little lap being on the exhaust side of the slide-valve.

The following diagrams, Figs. 22 and 23, illustrate the effect produced by shortening the slide-rod.

Top Stroke.—*(Shortening the rod.)*

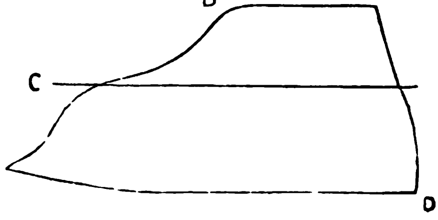
Fig. 22.



The effects on the Top Stroke (Fig. 22) are that the steam enters sooner, A; the cut-off is later, B; the exhaust is later, C; and the slide is closed sooner, D; in other words, the defects shown by the diagram are that the steam line, A B, is too long, or spread out, and the exhaust line, C D, is contracted.

Bottom Stroke.—*(Shortening the rod.)*

Fig. 23.

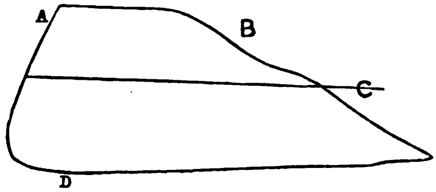


On the **Bottom Stroke** the effect produced is exactly the opposite, for we find the steam is late in entering, A. The steam is cut off quicker, B. The port is open to the exhaust sooner, C; while the closing is later.

While the effect produced by lengthening the slide-valve rod is precisely the opposite of what has just been explained.

TOP STROKE.—*(Lengthening the rod).*

Fig. 24.



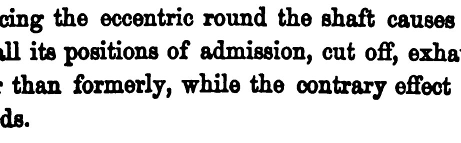
By this we find that on the Top Stroke the steam is late in entering the cylinder, A (Fig. 24), the valve not opening sufficiently early.

The slide-valve has, it will be seen, cut off the supply at an earlier stage, B.

The exhaust is also sooner, C, while at the termination of the stroke the slide is longer in closing, D. In other words, the defects of the (TOP) diagram with the slide-rod too long are, the steam line is contracted or too short, and the exhaust line is too long.

BOTTOM STROKE.—*(Lengthening the rod).*

Fig. 25.



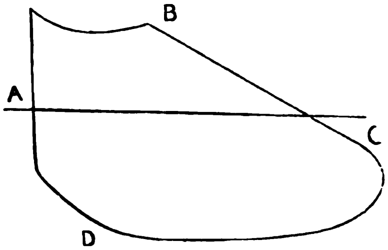
On the bottom stroke the steam enters sooner, A; the cut-off is later, B; the exhaust is later, C; but the valve at the end of the stroke is closed sooner, D. (See fig. 25.)

300. **Illustration of the effect produced by altering the position of the eccentric.**—Advancing the eccentric round the shaft causes the eccentric to put the slide into all its positions of admission, cut off, exhaust, cushioning, &c., a little earlier than formerly, while the contrary effect is produced by moving it backwards.

For example.—In a diagram taken after advancing the eccentric (Fig. 26) where the cushioning is early, D; the cut-off is early, B; and the exhaust is sooner, C. We have a diagram something similar to that given when the slide-rod is too short, because all the movements of the slides are too early, but the corners will be sharp and irregular instead of round. There is a distinction between this case and the cases where the eccentric-rod is too long or too short. In the case under consideration the *same fault* would exist in the diagrams taken from the top and bottom of the cylinder; but when the slide-rod is in fault, the opposite defects would exist at top and bottom (see Figs. 23, 24, 25, and 26).

Advancing the Eccentric.

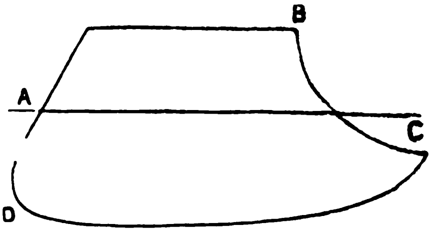
Fig. 26.



And in a diagram taken after backing the eccentric, where the *cushioning* is late, D; the steam is late, A; the cut-off is late, B; and the exhaust begins later, C. It is evident, therefore, that if the stop of the eccentric be not sufficiently forward, the diagram will be too full at every position, because all the motions of the slide will be too late.

Backing the Eccentric.

Fig. 27.



In Figs. 22 to 25 the defects are remedied by means of a liner; but in Figs. 26 and 27 they are remedied by means of the sheave. In order to determine whether it is liner or sheave that is required to remedy the defect, look to the parts of the figure which correspond to the valve openings, that is, to the part which shows when the port is opened to steam, and to the other which shows where it is opened to the exhaust. If *one* of these openings is *soon* and the other is *late*, then the defect is remedied by lining up; but if these points of opening are *both soon* or *both late*, then the defect is remedied by altering the sheave. In Fig. 22 the port is opened to steam *soon*, and the exhaust port opens *late*; also in Figs. 24 and 25 the points of opening are one *soon* and the other *late;* hence the defects in each case are remedied by means of a liner. On the other hand, it will be noticed that in Fig. 26 the steam and exhaust ports, respectively, both open *soon;* and in Fig. 27 both open *late;* and the remedy for such defects is to alter the sheave.

Top. Fig. 28. Bottom.

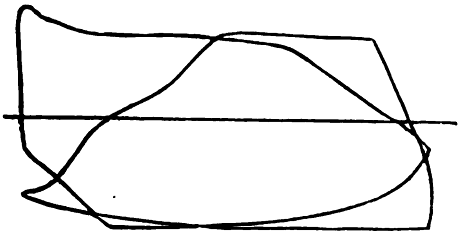


Fig. 28. *Too much lead* at top and none at bottom. *At the top* steam is *early* and the cut-off is *late*, while the exhaust is *late in opening* and *closes early*. *At the bottom* steam is *late* and the cut-off *early*, the exhaust opens early. The remedy for these defects is to *line the valve up*.

Fig. 29.

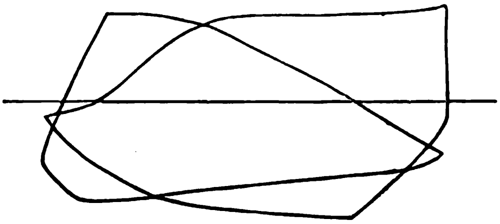


Fig. 29. This case is directly opposite to Fig. 28. No lead on the top, exhaust opens late. The bottom has too much lead, and carries the steam about ⅔ of the stroke; the exhaust is late in opening; bad vacuum line and excessive back pressure. The valve is evidently too high: take a liner out to remedy this.

Fig. 30.

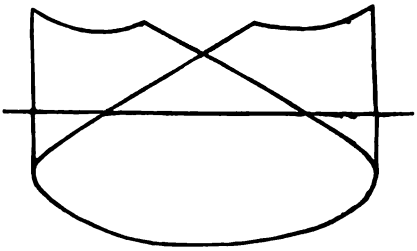


Fig. 30. *Too much lead* both top and bottom; exhaust is *late in opening* and closes *too soon*, causing excessive cushioning. Remedy:—Move the sheave back.

Fig. 31.

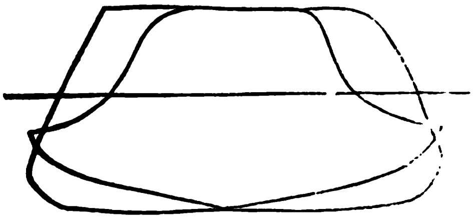


Fig. 31. There is no lead either top or bottom, steam is late, and cut-off later, exhaust late in opening. Put the sheave forward to remedy this.

Figs. 32 & 32* are two diagrams taken from a steamer; 32* was taken about ten minutes after 32. In both of these figures we have the true hyperbolic curve drawn in, with and without taking the clearance, &c., into account. The upper curve in small dots is the true curve when the clearance is taken into consideration, and the lower one in large dots is the true curve without reference to the clearance, &c. In figure 32* the steam was cut off at a very early part of the stroke; the importance of taking the clearance, &c., into consideration is very conspicuous. The dotted lines on the right of the diagrams show the amount they are lengthened by adding the clearance space in nozzles, &c., to them.

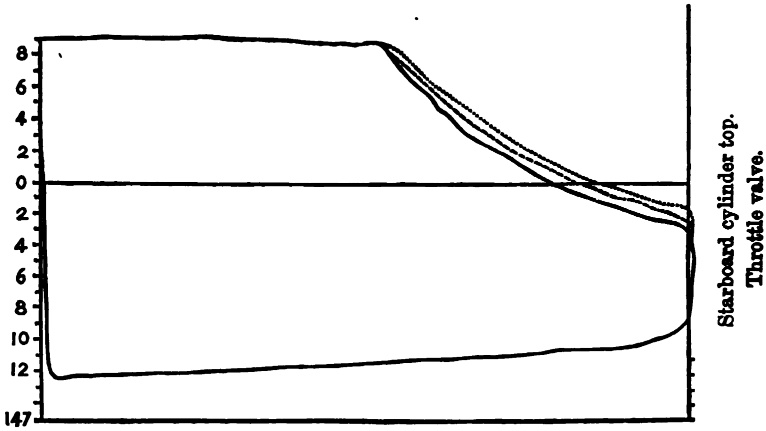


Fig. 32. Steam 10 lbs. Rev. 9. Vacuum 26. Hot well 100°.

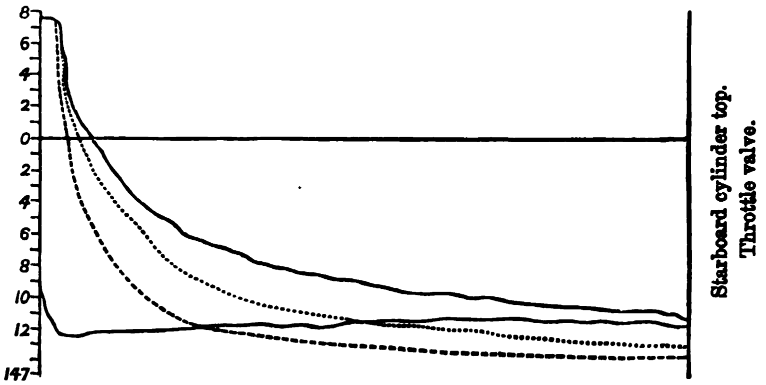


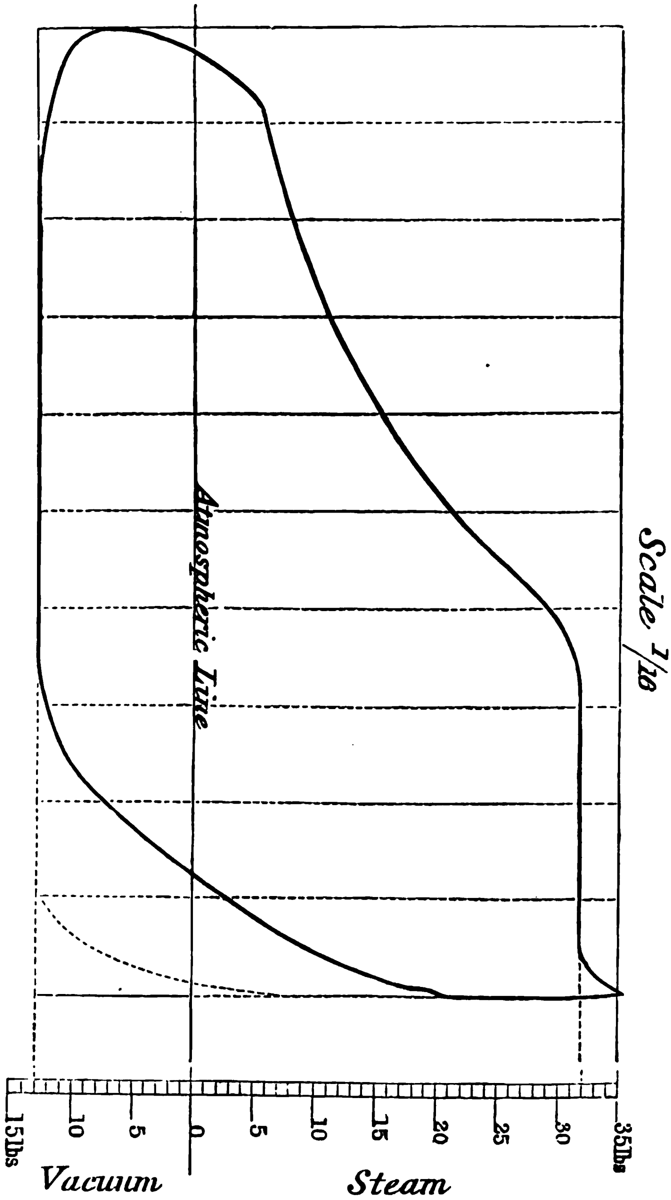
Fig. 32.* Steam 8¼ lbs. Rev. 6. Vacuum 26¼. Hot-well 82°.

From a casual inspection of these diagrams they seem to present an anomaly that at first is difficult to solve. Thus, in Fig. 32* the termination of the true expansion curve, considering clearance, &c., is about one pound *above* the actual curve, whereas in Fig. 32 it is two pounds *below* it. The first would indicate the exhaust valves or piston leaked, and the second that the steam valves leaked, while the exhaust valves and piston were tight. Now, then, since one was taken only ten minutes after the other, it is not at all probable that this sudden change was brought about in that short space of time, hence we must look for some defect in the engine that would occasion it. We account for it in this way:—In the first case the steam valve leaked, and also the steam piston, but the piston leaked to a greater extent than the valve, that is to say, more steam passed through the piston and into the condenser from the leakage of the piston than entered the cylinder from the leakage of the valve; therefore the actual curve must fall below the true curve. In the second case, the steam valve also leaked, but the pressure on the piston fell so rapidly, from expansion, that it became too low to force a passage through the piston, the elasticity of the packing being sufficient in this case, though not in the other, where it had a greater pressure to sustain to keep it tight; hence, the true curve in this case must be below the actual curve, agreeing precisely with the conditions of the figure.

There is, however, another thing which would produce diagrams similar to those before us, and which most probably caused the formation of these, viz., leakage about the cylinder heads. Thus, supposing the stuffing box, for instance, to leak. So long as the pressure in the cylinder remained above the atmosphere, steam would blow out, occasioning the curve to fall; on the other hand, when cutting off short, the pressure in the cylinder would soon fall below the atmosphere, and air would enter, causing the curve to rise exactly as shown in the figure.

Explanatory Diagram, Fig. 33.—This diagram represents the usual form when there is too much compression. This is owing to the slide-valve closing the exhaust too soon, occasioned by having lap on the exhaust edge of the valve, or in working with the link not in full gear. To remedy the former defect, a piece must be chipped off the exhaust edge of the slide-valve, but this cannot be done without affecting the point marked in diagram, Fig. 13, "exhaust opens;" for it must be borne in mind that if the exhaust is longer in being closed, it must open the sooner. Therefore, if the diagram taken shows that the exhaust does not take place soon enough (the part of the stroke can be easily determined or measured off the diagram, see Fig. 13), and a great compression exists, then a piece can with safety be taken off the exhaust edge of the valve.

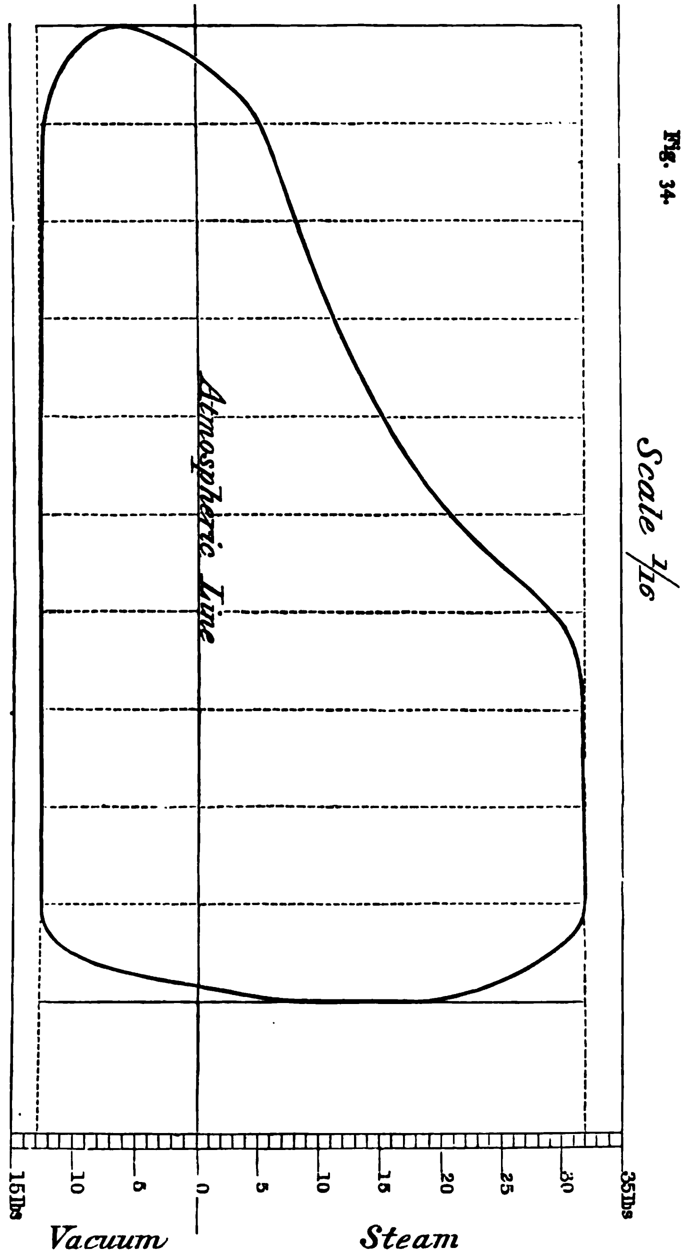
Fig. 33.



Too much compression caused by Valves closing exhaust too soon.

Explanatory Diagram, Fig. 34.—This diagram represents the usual form when there is too little "lead" or a "leaky piston." It will be seen that the piston has travelled some distance before the pressure continues uniform; this should never be, and an engineer would be much to blame for his engines showing this diagram. He can remedy the former defect on trying the "setting of the valves" (as mentioned formerly), should they have too little "lead," by taking a piece off the steam edges of the slide-valve. He must find out the latter defect by admitting steam on either side of the piston, and opening the indicator cock to the opposite side, and if the piston is leaky, he must look to the "set" of the packing ring springs.

Fig. 34.

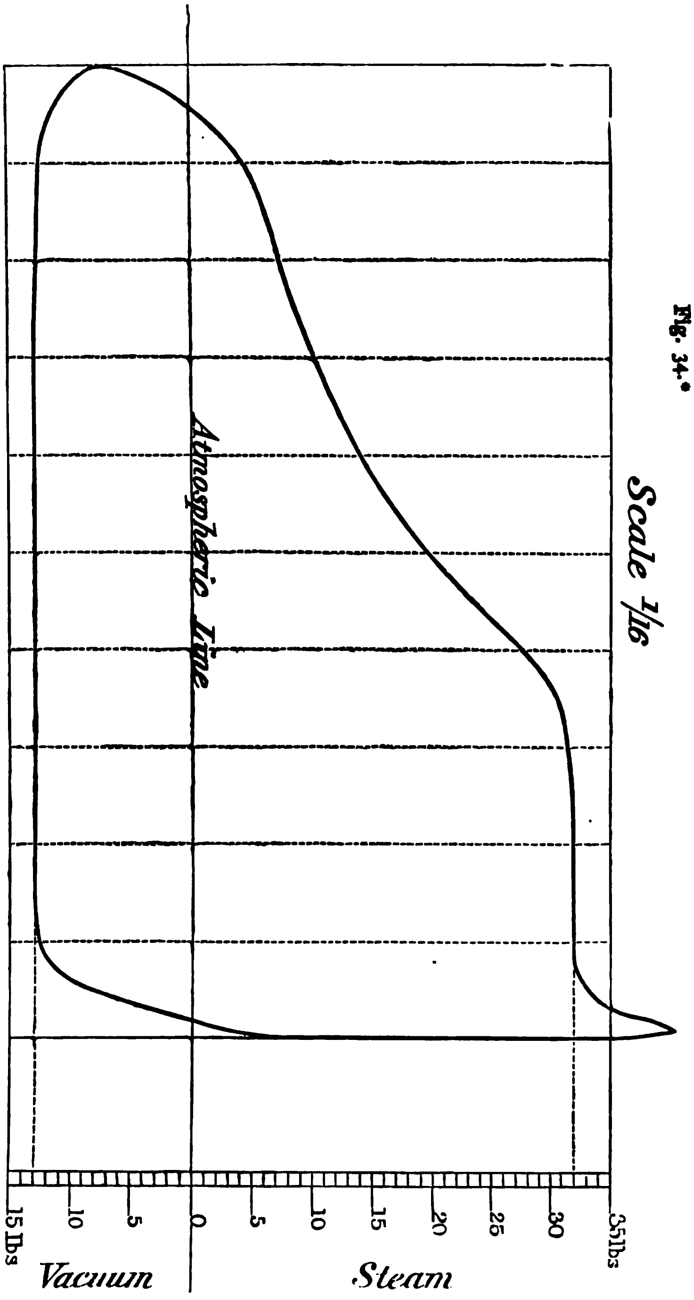


Round top corner caused by Valves having too little lead

Explanatory Diagram, Fig. 34.*—This diagram having the loop in the corner, is occasioned by the sudden inlet of steam jerking up the indicator-piston; almost all diagrams have this loop.

Fig. 34.*

Scale 1/16



Loop on top corner formed by inlet of steam jerking up Indicator piston

Explanatory Diagram, Fig. 35.—This diagram represents the rise of the expansion curve; this should never be, for whatever valve is used to "cut off," this curve should always fall; by its tendency to rise, it will be apparent that there must be an increase of pressure going on after the steam is cut off, and we cannot look for this increase anywhere but at the slide-valve which must either be "off the face" or else not tight; the dotted line shows the difference. Should the diagrams taken be of this form, the engineer should, at the first opportunity, examine his slide valves; if allowed to work long, and giving this diagram, the cylinder face will become worse.

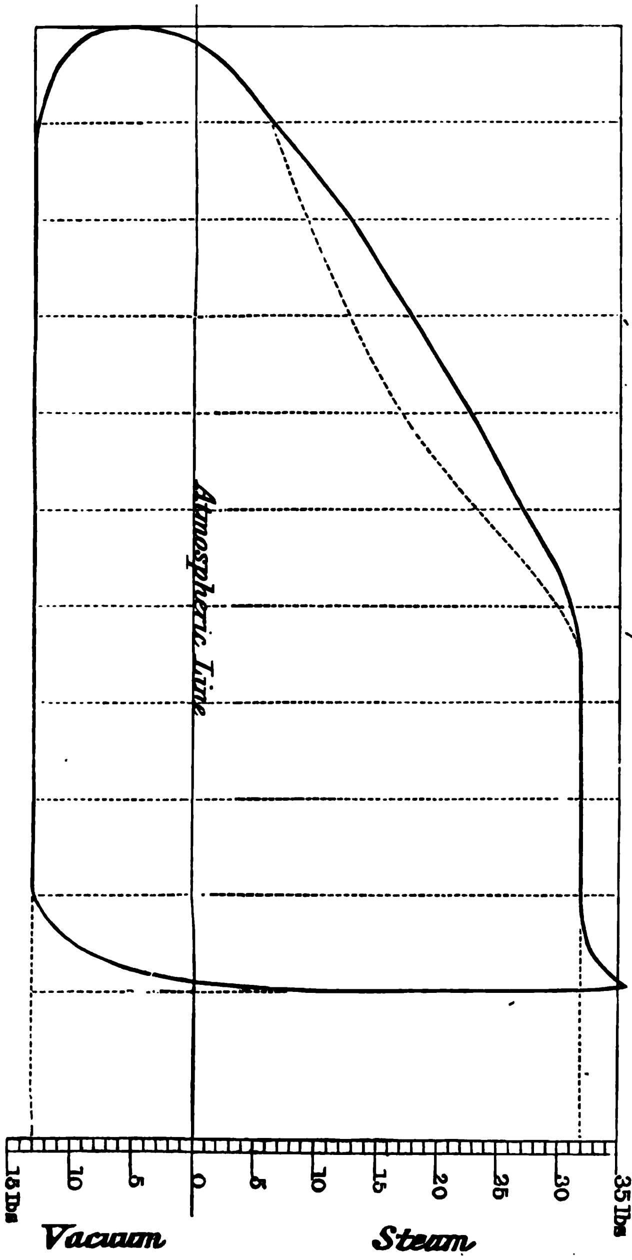


Fig. 35.

Rise of expansion curve above dotted line caused by Valves leaking

Explanatory Diagram, Fig. 36.—This diagram represents the usual form when the throttle-valve is not full open, or the ports too small. It will be seen that the pressure is not maintained uniformly for any part of the stroke, by reason of a diminished supply of steam; the dotted line, or what the diagram should be, will show the difference or loss by throttling.

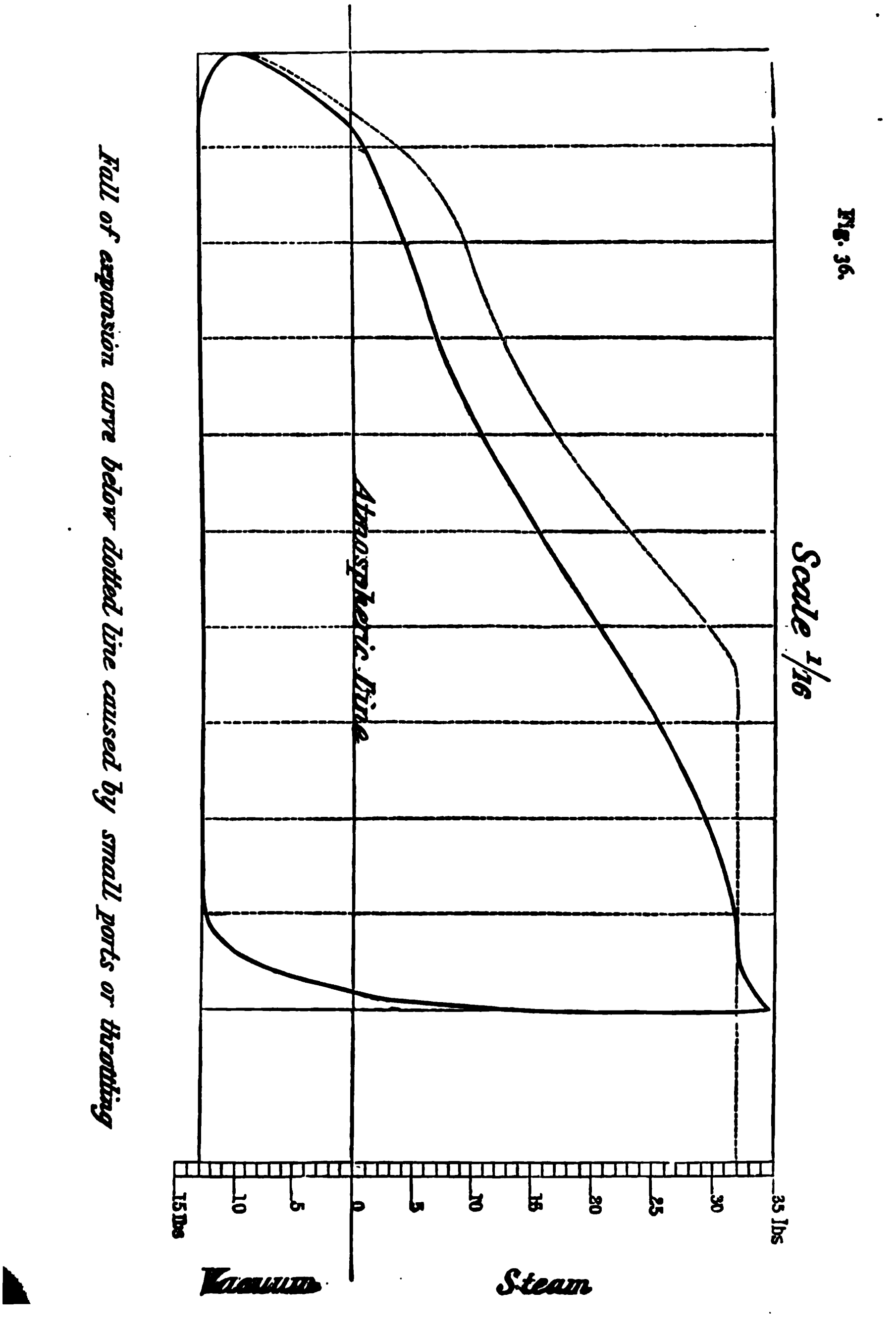
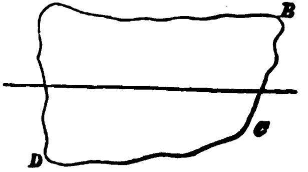


Fig. 36.

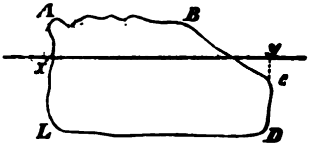
In further elucidation of the subject, a few diagrams (reduced) are added, together with some practical hints as to the important information to be obtained from them.

Fig. 37.



The "steam corner" A, or commencement of stroke, being very angular, shows that the valve is too forward at the beginning (see page 320). AB being near parallel to the atmospheric line, shows that there was no expansion, but supply of steam to the end of the stroke with pretty equal pressure. The "exhaust" or "eduction corner" C should be more nearly square and not rounded off, the action as here indicated is not very good, as the exhaust valve opens too late, therefore, power is lost, because the steam does not get into the condenser soon enough. D is the lead corner, and is too square, and shows that the valve opens too late. When the lead corner D is much slanted off, the amount of lead is great (see also Fig. 45); but the cutting away of the lead corner, to a small extent, is not to be looked upon as a defect. (See page 320).

Fig. 38.



This is a fair average diagram. The fulness of the part below the atmospheric line shows that a good vacuum was obtained; the depression at B shows that the steam was cut off there at about two-thirds the whole stroke of the piston (as measured by $x y$); hence, from B to C the steam was acting expansively. At A, however, it shows that, if not caused by the friction of the piston of the indicator itself, there was too sudden a rush of steam, which is trying to the machinery from its abruptness.

Fig. 39. This shows an inclined supply steam line, with full expansion, exhaustion, and undulated lead. It illustrates the imperfections of the gear used for the supply of the steam; if the pressure in the boiler were constant and the gear perfect, the dotted lines indicate the formation that should have occurred.

Fig. 39.

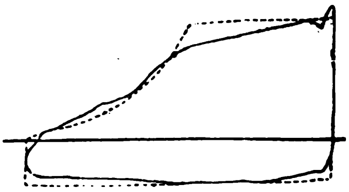


Fig. 40.

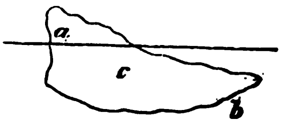


Fig. 40. This engine has small steam ways, as shown by the smallness of *a*, the steam side; the vacuum side is bad, for there is much steam uncondensed, it causes also noise in the cylinder from the passages being too small. The exhaust valve opens too late, hence the steam does not leave the cylinder at proper time (see at *b*); there wants "lead" on the exhaust side; hence waste of fuel and considerable loss of power.

Fig. 41. In this instance the steam is cut off at half stroke, at the end of which there was (as shown at *a*) some steam to condense; the fulness of *b* shows there was a good vacuum.

Fig. 41.

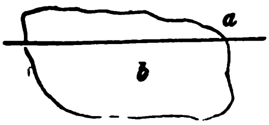


Fig. 42.

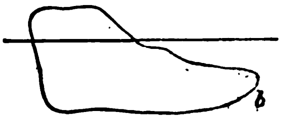


Fig. 42. Steam cut off at one-third stroke, *b* is too much rounded. Exhaust opens too soon, hence loss of power.

Fig. 43. In this case the slide-valve requires more lead and more lap. The exhaust opens too late and closes too late. The steam ways are too small altogether. The vacuum is very good, but too rounded at the exhaust corner.

Fig. 43.

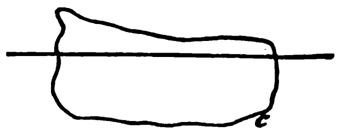


Fig. 44.

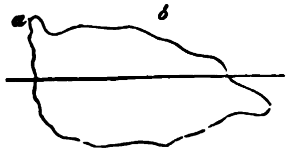


Fig. 44. In this case *a* shows that the valve has no lead, and that the pressure of steam strikes the piston with some force, to the great strain of the engine, by its suddenness; the part *b* shows that the cut-off was not good, the steam was wire-drawn, "entering after the cut-off," and, indeed, the wavy lines of the figure indicate great derangement of the valves generally.

Fig. 45. In this figure *a*, by being too much rounded and hollowed, shows that the valve has too much lead; the cut off at *b* shows expansion; the exhaust opens rather early; it is, however, a pretty good diagram.

Fig. 45.

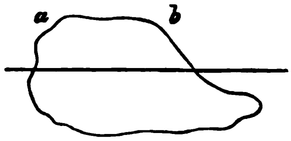


Fig. 46.

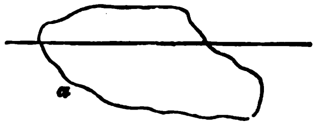


Fig. 46. In this instance the steam corner is slanted off, the steam being admitted too late to the engine (see page 320); the exhaust is too soon. Valves out of order. The rounded corner at *a* shows cushioning, caused by the too early closing of the eduction-valve before the end of the stroke, whereby steam is shut in and compressed by the piston.

Fig. 47. This is a very fair diagram, steam cut off at first grade; *a* was caused by friction of the indicator piston.

Fig. 47.

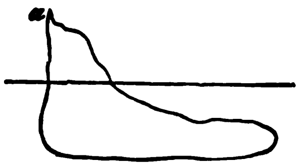


Fig. 48.

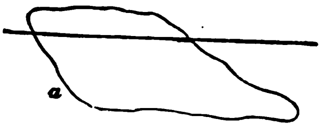
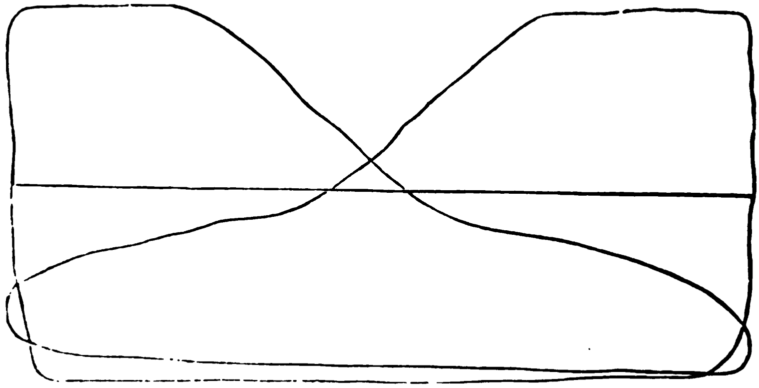


Fig. 48. In this diagram *a* shows too much compression or cushioniong, owing to the slide-valve closing the exhaust too soon, occasioned by having lap on the exhaust edge of the valve, or in working with the link not in full gear.

300. **Top and bottom Diagrams exhibited on the same card.**—It has been already stated that the indicator may be attached to a pipe connected with the top and bottom of the piston, and thus both the diagrams may be taken on the same card without shifting the instrument. As this is the plan

usually adopted at present, a specimen of the two diagrams is given, which is as follows:

Fig. 49.



It will be observed that the two figures face each other, as they evidently ought to do.

301. *Where accuracy is required, a diagram should be taken from the top and bottom of the cylinder.*

The diagram taken from the top of the cylinder shows only the pressure and vacuum on the upper side of the piston, and, therefore, cannot indicate what is going on below the piston. If our object be merely to calculate approximately the horse-power of the engine, and it be in tolerable good working condition, it is not of much consequence whether the diagram be taken from above or below; but, if the horse-power is required with any great accuracy, the mean result obtained from top and bottom diagrams must be used. It will often, and perhaps most generally, be found that two diagrams taken at different ends of the cylinder will in some respects be dissimilar. In estimating the horse-power of the engine, it is proper that this difference should be known and the average of the two taken. If the actual state of the engine is required, it is necessary to examine into what is passing both above and below the piston, because the errors in one part may have no connection with the errors in another. This will be the case if the slide is too long or too short, in which case the upper part may be properly covered, and the lower one not so; or the upper slide-face may be steam-tight and the lower one leaky; but, if the indicator be applied to top and bottom, it will detect all these inaccuracies, and prevent our attempting to improve the working of one side to the detriment of the other. It ought to be remarked here, that in direct-action engines, the diagram from below the piston is generally superior to the other. First, because, since the steam has more work to accomplish, the piston does not run away from the steam so readily, and in consequence, the steam pressure is better maintained;[*] and there is generally a little more lead to the slide, to allow a freer ingress to the steam.

There is also another reason why the one diagram is better than the other. The eccentric and crank are so connected that they revolve together, and on account of the smallness of the throw of the former compared with the length of the eccentric-rod, if the steam be cut off when the crank has described a certain angle from the top centre, it will, if the lead be the same on both ports, be again cut off when it has described the same angle from the bottom centre, but because of the shortness of the connecting-rod, the piston will not have traversed so far when the crank has *descended* through a given angle as it will when it has *ascended* through the same angle. For instance, when the crank is horizontal and descending, the piston will not be half-way down; but, when horizontal and ascending, it will be more than half-wap up; and, hence, as the crank moves from the top centre, the space traversed by the piston till the steam is cut off is less than when moving from the bottom centre. It therefore follows, that in direct-action engines the steam line is continued further in the up stroke than in the down stroke, and the reverse will be the case in beam engines; and this difference becomes more apparent as the connecting rod is shorter in comparison with the crank.

302. **To calculate the mean pressure on the piston throughout the stroke from an indicator diagram.**

For the purpose of ascertaining the *condition* of the valves, piston, &c., it will, in the hand of an experienced engineer, be quite sufficient for him to see the general outline of the diagram; but if it is to be used for ascertaining the power of the engine, its average measurement is to be ascertained. This will be be best done by

RULE CXXXII.

1°. *Divide the* diagram *in the* direction *of its* length *into* ten *equal parts by drawing* nine ordinates, or lines perpendicular *to the* atmospheric line *at* equal distances.[†]

---

[*] The difference in the speed of the piston at the opposite ends of the cylinder is, at the outer end of a direct-acting engine, from one-sixth to one-third greater than at the crank end—the difference varying according to the degree of angular vibration of the connecting-rod. In side lever or beam engines these proportions are reversed, and the speed of the piston is greater at the upper end of the cylinder.

[†] Dividing a diagram into ten equal parts is a somewhat slow process by rule and compass, and an ingenious contrivance has been introduced with Mr. RICHARDS' indicator for this purpose.
It consists of a parallel rule composed of eleven bars of thin steel, and two strips; and by bringing the edge of the first bar against a perpendicular drawn on the atmospheric line, the contrivance can be moved so that it contracts so as to be equal to the length of the diagram, whatever that may bo, and an accurate division into 10 equal parts is made with surprising facility by drawing pencil lines against each of the bars.

NOTE.—In practice 10 is the usual number of parts employed (see figure following) but any number may be made use of; and the greater the number the more correct will be the result (see Fig. 57). The advantage of taking 10 ordinates instead of 8, 9, or 11, is, that the division by 10 is accomplished by merely shifting the position of the decimal point, while 10 ordinates are enough to enable the area to be measured accurately enough for all practical purposes.

2°. *With the scale to which the indicator is constructed, measure* between *the* spaces *the* distance *from the* atmospheric line *to the* upper outline *(or the steam side) of the diagram until* this *crosses the* former, *if it does so.* Next, *repeat the process for the area* between *the* atmospheric line, *or the* expansion curve, *after it has* crossed *this line and the* lower outline *of the diagram.* (See Fig. 50 and 52 in illustration.)

3°. *Take the* sum *of the* measurements *of the* steam side, *then of the* vacuum side, *and* divide *by* 10 *in each case.*

4°. *Add* vacuum *to* steam *which gives the* total mean pressure *for each part of the stroke.*

In finding the mean effective pressure off the diagram it is not at all necessary to take the steam and vacuum effects separately. The usual and most expeditious mode is to proceed thus:—*

(*a*) *Take measurements at once* completely across *the diagram, that is, measure between the spaces* from the vacuum line to the steam line; *note these scale distances down.*

(*b*) *Add the whole* together *and* take the mean *by dividing by* 10, the result is the total mean pressure.

In the case of a double diagram, or diagrams from top and bottom of cylinder combined, proceed as directed in Nos. 2°, 3°, and 4°; or as directed in 4° (*a*) and (*b*), then,

5°. *Lastly,* add top *and* bottom *together, and* divide by 2, the result is the *average* of both, or the mean pressure *for the whole double stroke.*

NOTE.—The mode of admeasurement as given in paragraphs 2°, 3°, and 4°, of the foregoing rule is not a strictly accurate one for ascertaining the power of any condensing engine, the steam line of which expands *below* the atmospheric or datum line. The method, however, does *not* give a really wrong result when it is wished to ascertain the *gross power* of an engine by means of the indicator; if, however, it be desired to find *separately* the respective values of the *steam* and *vacuum* pressures, the method is decidedly incorrect. The *gross* pressure on the piston throughout the stroke is, by this method, obtained at the expense of double work—it not being at all necessary to measure separately the ordinates of the steam and vacuum curves. In any properly constructed condensing engine the steam line of the diagram expands *below* the atmospheric line. At the beginning of the stroke it is *above* the line, and further on it is *below* the atmospheric line. In adding up the steam pressures, only the *plus* pressures *above* the atmospheric line are taken into account, while the *minus* steam pressures *below* the atmospheric line are neglected. In adding up the vacuum pressures, instead of taking the complete vacuum up to the atmospheric line, the ordinates are measured between the steam line (that happens to be below the atmospheric line) and the vacuum and exhaust line. It will be at once perceived that the steam pressure is exaggerated by only taking the *plus* pressures into account, while the vacuum pressures are detracted from by not measuring them from the atmospheric line. The *total* amount comes *right* at last; but the *separate* values of the steam and vacuum pressures are wrong.

---

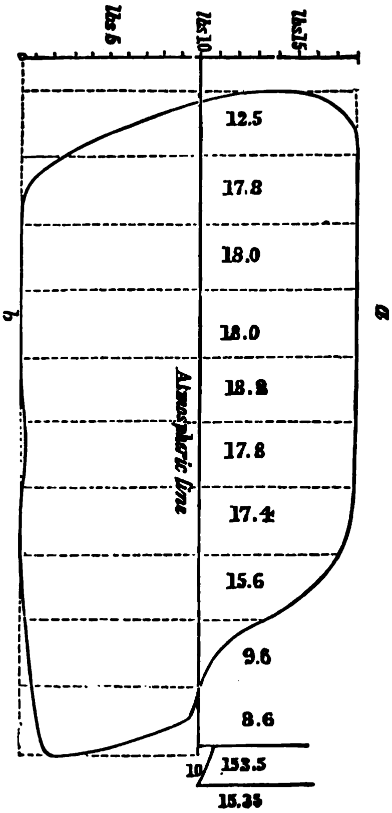
* This is the method that is always adopted by the steam department of the Admiralty, and the principal firms in the kingdom.

But it happens that there is a simple plan by which a person totally unconscious of the deep mysteries of *plus* and *minus* can take the separate values of the steam and vacuum pressures from an indicator diagram. It is one often employed in the steam department of the Admiralty. The atmospheric line is taken as equal to 15 lbs., and from this line, as a point of departure, fifteen other lines are marked by the scale, thus coming below the vacuum line of the engine up to the true vacuum line of a perfect exhaust. The *lowest line* is then taken as the *zero* or *datum* line, and the actual pressure (not the relative pressures as compared with the atmosphere) of both the steam and vacuum line can thus be obtained. The difference between the two pressures gives the gross or actual pressure on the piston.

EXAMPLES.

Ex. 1. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

Fig. 50.

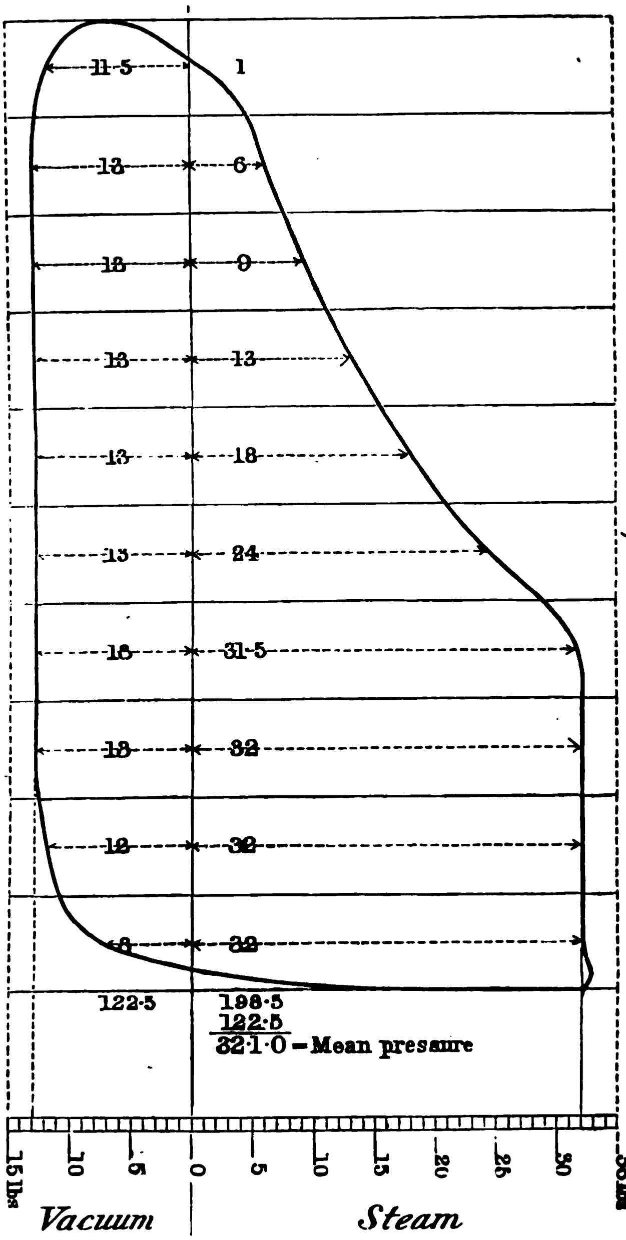


In the accompanying copy of an actual diagram, the horizontal line represents the atmospheric line; a graduated scale is on the left, which measures the pounds pressure upon every square inch of the piston surface, as previously ascertained by the maker. Nine ordinates to the atmospheric line are drawn at equal distances, thus dividing the space in length into 10 equal parts, the measurements are taken at once completely across the diagram, (see 4° *a*), and, as an example, the length *a b* indicates that in the interval, while the parts of the curves at *a* and at *b* were being traced by the indicator, there was a pressure in the cylinder of 18 pounds to the square inch; the sum of all these areas divided by 10 gives the average mean pressure of steam and vacuum in the cylinder.

Fig. 51.

Shewing method of computing Mean pressure

Scale 1/16.



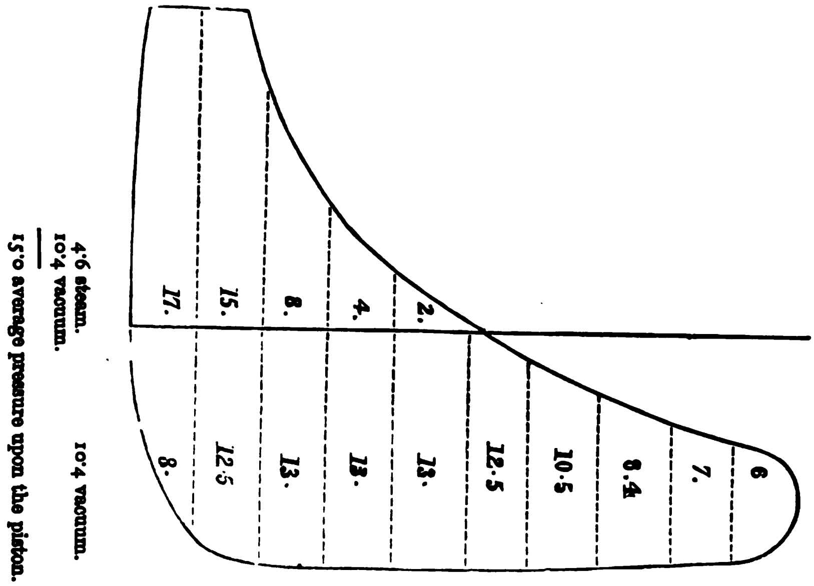
This is a diagram showing the mode of determining the effective pressure of the steam, and the mean vacuum effect, these combined are the "total effective pressure." To obtain these proceed as follows:—Divide the diagram into 10 equal spaces or points, draw lines at right-angles to the atmospheric line across the diagram at these points, and with the wooden scale corresponding to the spring used, measure between the spaces from the atmospheric line upwards to the diagram line, note the number on the scale corresponding to this distance, and write it down in this space; repeat the measuring and noting of the 10 spaces above the atmospheric line; add the whole of these measurements together, and take the mean—that is, as ten measurements have been made, the amount must be divided by 10; the quotient is the mean effective pressure of the steam upon the piston.

Next, measure between the spaces from the atmospheric line downwards to the vacuum line, write down the amounts, add the whole together, and take the mean; this quotient is the vacuum effect upon the piston.

Ex. 2. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

Fig. 52.

Scale of 10ths.



The foregoing diagram is taken from a condensing engine (not a marine one, however), which engine may be considered a good example of expanding steam in the cylinder, and also of the setting of the valves. The pressure of steam in the boiler is 30 lbs. The steam in the engine having expanded to 6 lbs. below the pressure of the atmosphere is better used up than in most cases. The steam is cut off at one-seventh the length of the stroke, the pressure of the steam decreasing as the piston advances the other six parts of the stroke.

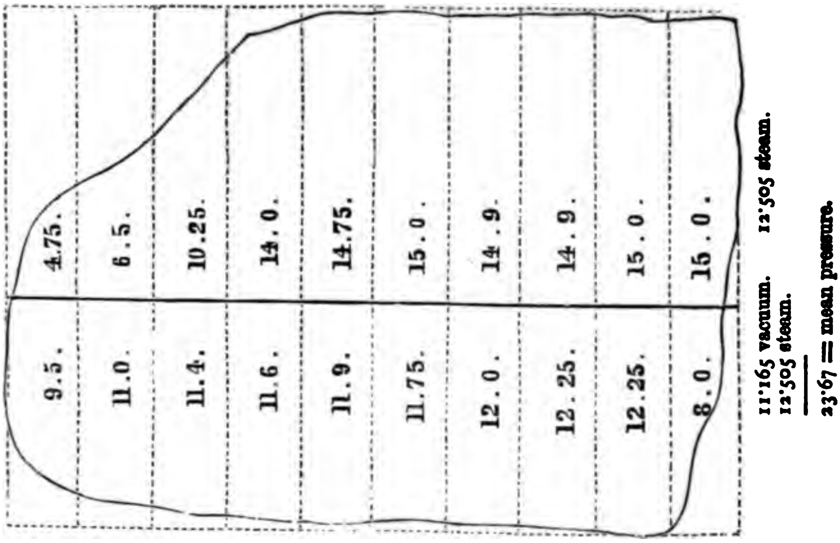
In this diagram measurements are separately made from the atmospheric line to the steam and vacuum curves respectively, but after the steam line expands *below* the atmospheric line, the ordinates are measured between the steam line and the vacuum and exhaust line. This, it will be seen, is the method alluded to in the note at page 343, No. 5°, as being a mode of admeasurement not strictly accurate for ascertaining the *power* of a condensing engine, but, as before remarked, it gives the *gross power* correctly, but in finding separately the steam and vacuum pressures it is decidedly incorrect.

Ex. 3. Work out an indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

Indicator Diagram of S.S. *Havelock*, November 9th, 1858.

Steam 20 lbs.; vacuum 13½ lbs.; revolutions 26.

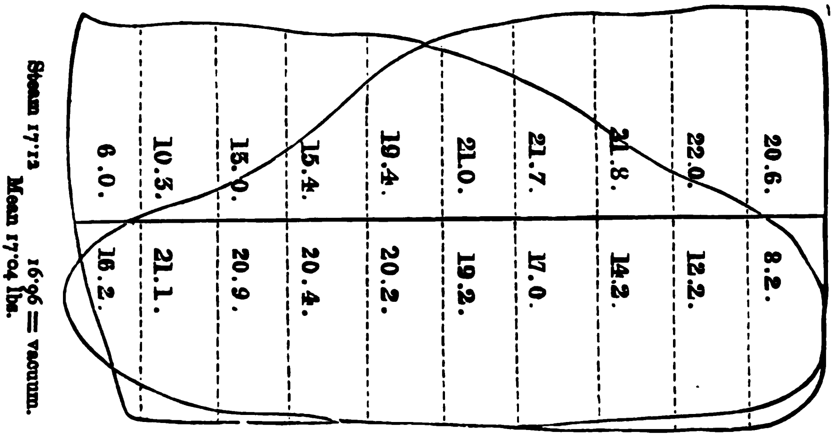
Fig. 53.
Scale of 10ths.



The diagram is divided into 10 equal parts, and the pressures are obtained by measuring each way outwards from the atmospheric line (No. 2°), in the middle between every two divisions with the scale accompanying the indicator.

Ex. 4. Work out an indicator card, and show the point where the steam is cut off, whether the slide valve is correct or requires alteration, and if the latter, what?

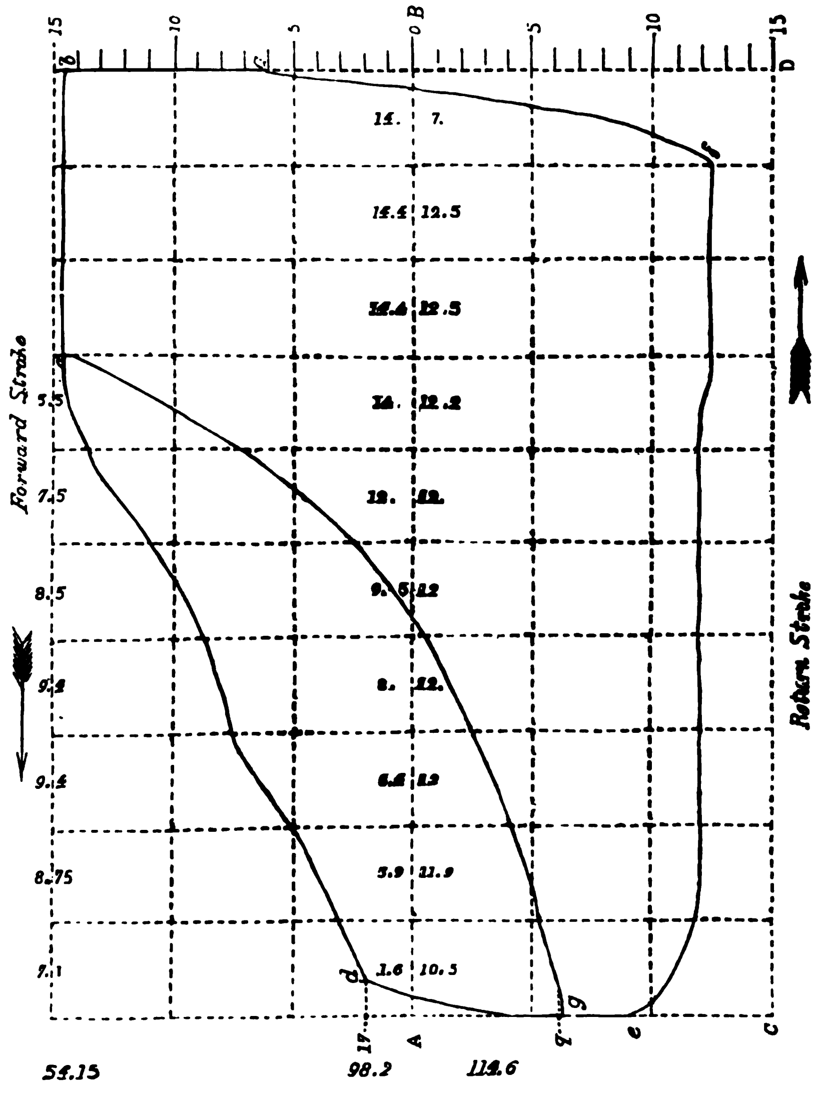
Fig. 54.
Scale of 10ths.



The double diagram is from the top and bottom of a sea-going coasting screw steamer. The cylinders, two in number, are each 36 inches diameter; the piston travelling at a speed of 280 to 301 feet per minute.

The diagrams are taken from top and bottom without shifting the indicator, by having the stop-cocks fitted into the top and bottom of the cylinders, connected by a tube, into the top of which is fitted the indicator. By this means a diagram can be taken from top and bottom by the same paper—both figures having the same atmospheric line. In oscillating engines, or those of the direct-acting kind, where it is inconvenient to erect a radius bar, it is usual to use a large and small pulley to reduce the stroke of the piston to that of the indicator. It will be convenient to make the small pulley in the form of a grooved cone, so as to be enabled, with facility, to adjust the stroke of the indicator to the proper length.

Example 5. Fig. 55.



The foregoing diagram is taken from one of the engines of a well-known steam ship. The scale of the indicator was eight pounds to the inch.

There are some points in this diagram to which we wish to direct the reader's attention.

The line *c g* is the theoretical expansion curve drawn from the point *c*. The line CD is the line of perfect vacuum. The line *b* to *c* is the steam line, which does not in fact end at *c*, but at some unknown point beyond *c*.

From an examination of this diagram we learn that the exhaust port was covered at the point *f* of the return stroke, and the vapour remaining in the cylinder was then compressed by the advance of the piston to a density at the commencement of the forward stroke, of about five pounds above the atmosphere. The port was then opened for admission, and the pressure instantly rose to fourteen and a half pounds above the atmosphere. The port being opened wider and wider, this pressure was maintained behind the advancing piston to the point *c*, at which it began to fall, at first very slowly, from the gradual closing of the port by the cut off valve. The point at which the port was covered cannot be identified. It was certainly, however, far beyond the point *c*, and strictly the steam line continues to the point of cut off; however, the pressure may fall before that point is reached. At the point *d*, the pressure had fallen by expansion to two pounds above the atmosphere. Here the valve began to open communication with the condenser, and before the piston commenced its return stroke the pressure on this side of it fell to nearly ten pounds below the atmosphere, and almost immediately after a vacuum of twelve pounds was formed; and when the return stroke was two-thirds accomplished, the counter pressure suddenly fell half a pound lower, and this vacuum was maintained until the exhaust port was closed at the port *f*.

It will be observed that the line of perfect vacuum is drawn at the bottom of the diagram, and the line of boiler pressure as shown by the gauge at the top. The line of perfect vacuum in diagrams varies in its distance from the atmospheric line; or, more correctly, the latter varies in its distance from the former, according to the pressure of the atmosphere, as shown by the barometer from 13·71 lbs. on the square inch, when the mercury stands at 28 inches, to 15·19 lbs. when it stands at 31 inches (*vide* Table I at end of work), and it should be drawn according as the fact of this can be ascertained. The engineer should always have a good aneroid in his pocket. The pressure of the atmosphere is usually reckoned at 15 lbs., which as a general rule is too high, being correct only when the barometer stands at 30·6 inches; but the error is unimportant. The principal object of knowing the exact pressure of the atmosphere is to ascertain the duty performed by the condenser and air pump. The temperature of the discharge being known, the pressure of vapour inseparable from that temperature is also known (*vide* Table at end of work), and this being deducted from the actual pressure of the atmosphere, the remainder is the total attainable vacuum *at that pressure*.

The space between the steam line *b* to *c* and the line of boiler pressure shows how much the pressure is reduced in the cylinder by throttling, or by the insufficient area of the ports, proper allowance being made for the difference of pressure necessary to give the required motion to the steam, and that between the line of counter pressure *e* to *f*, and the line of perfect vacuum, shows the amount of resistance to the motion of the piston.

When we wish to ascertain how effectually the resistance of the atmosphere is removed from the non-acting side of the piston, by those parts of the engine whose function is is, we must calculate separately the areas of the diagrams above and below the atmospheric line. In case of engines working very expansively, however, the expansion curve crosses the atmospheric line, and sometimes at an early part of the stroke, as in Fig. 52. In such cases, the whole space between the atmospheric line and the line of counter pressure should be credited to the condenser and air pump; not, of course, to be considered in estimating the power exerted, but for ascertaining the degree of economy in the consumption of steam, which depends greatly on the amount of vacuum maintained.

It will also be noted that the expansion curve *c d* does not coincide with the theoretical expansion curve *c g*.

Ex. 6. Work out an indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

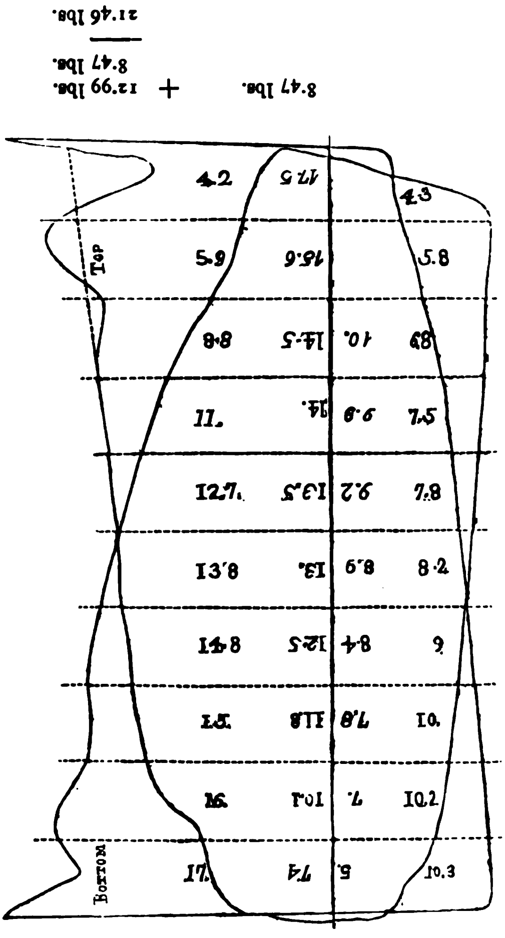


Fig. 56 is a copy of a double diagram as taken from an engine, top and bottom strokes being represented with the atmospheric line through the centre, on the right of which is shown the pressure from vacuum, and on the left that from steam. Also, that in the figure the numbers from the top stroke are next the atmospheric line, while those for the bottom are further out of the numbers.

Fig. 56.

Scale of 12ths.

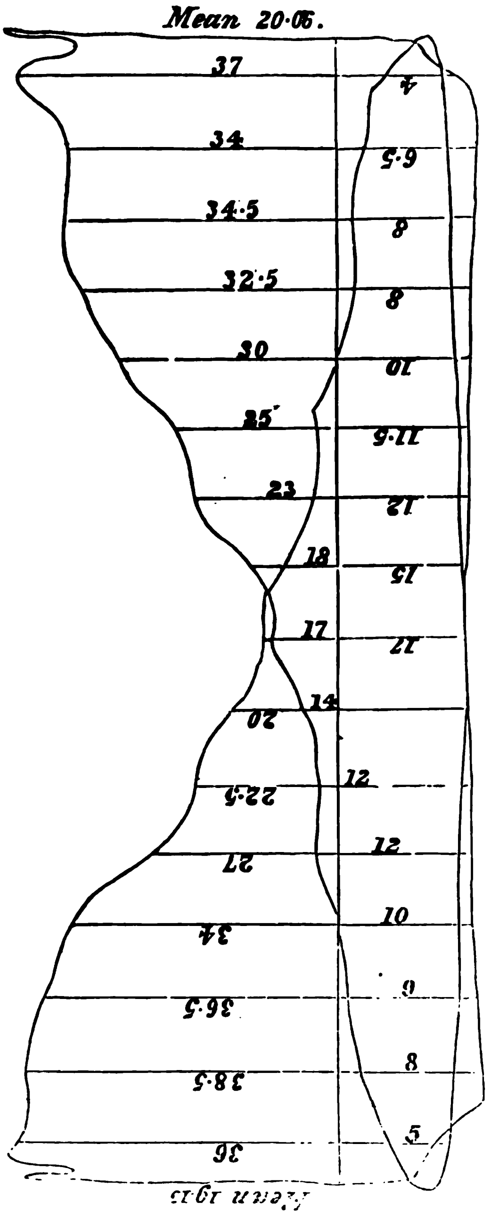
11·92 + 7·99 lbs.
11·92 lbs.

Bottom 19·91 lbs.
Top 21·46 lbs.

2)41·37

Mean 20·68 lbs.

Ex. 7. Work out an indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?



This diagram (Fig. 57) is from the forward engine of the S.S. *Hansa*, which plies between Bremen and America.
Scale of 15ths.
Fig. 57.

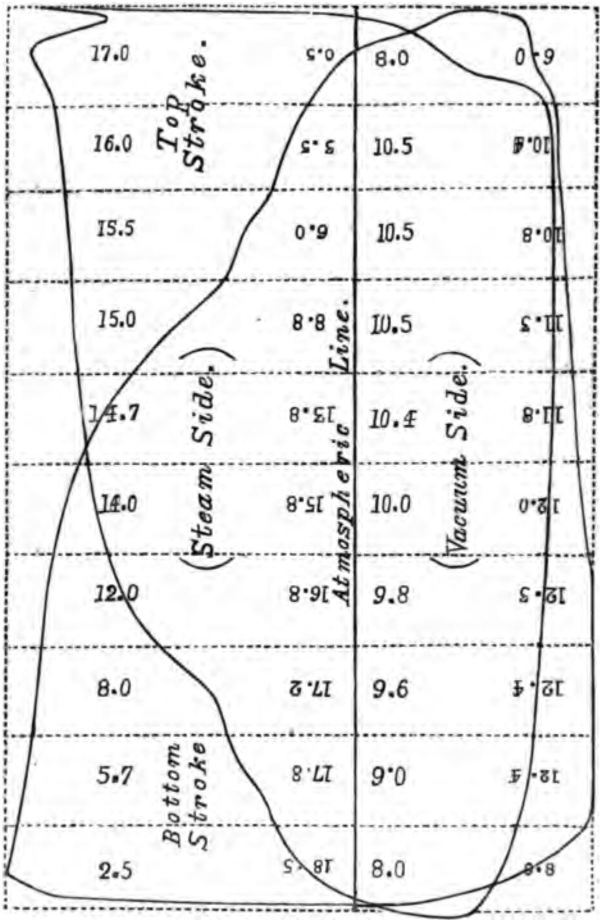
16)306·5
19·15
20·06

2)39·21
19·6 Mean.

Steam 29 lbs.; vacuum 28 inches; revolutions 50. Steam cut off at 12 inches. Mean pressure 19·6 lbs.; horse-power 1961·4; cylinder 80 inches diameter; stroke 42 inches; consumption of fuel 3801 lbs.

Ex. 7. Work out an indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

10.74 + 11.87 = 22.61 lbs. bottom.



Indicator diagram from forward engine of P. & O.S.S. *Pera*,
Revolutions 32½ per minute. Indicated H.P. 1130.

Fig. 58.

12·04 + 9·63 = 21·67 lbs. top.
22·61 lbs. bottom.

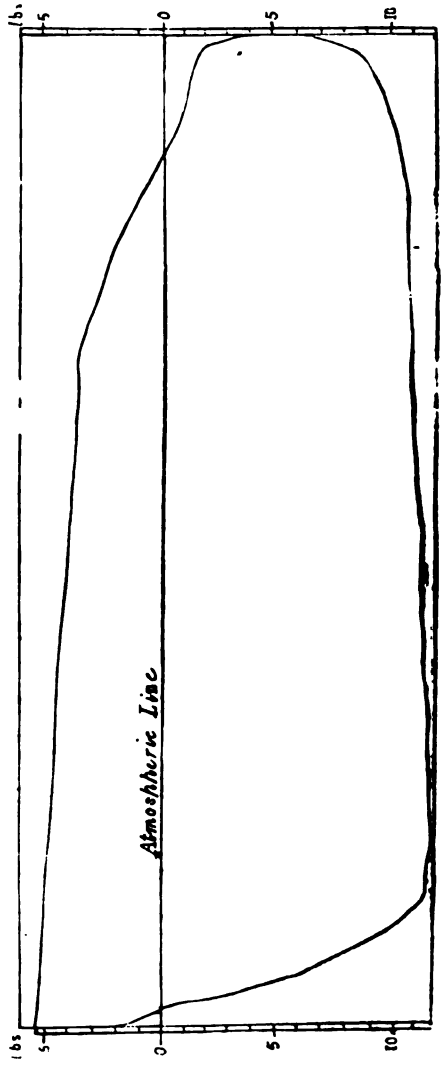
2)44·28

Mean 22·14 lbs.

In the foregoing Fig. 58, which is a double diagram taken from the forward engine of the Peninsular and Oriental S.S. *Pera*, top and bottom strokes being represented with the atmospheric line through the centre, on the right of which is shown the pressure from vacuum, and on the left that from steam. The numbers for the top stroke on the steam side are at the extreme left, and those representing the vacuum are adjoining the atmospheric line. For the bottom stroke the number representing the steam pressure is near the atmospheric line, and those representing the vacuum at the right hand.

EXAMPLES FOR PRACTICE.

Ex. 1. Work out an indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?



Indicator diagram of H.M.S.S. *Spiteful.*

Fig. 59.

Answer, Mean Pressure 13·725 lbs.

In this case the pencil starting from the atmospheric line immediately rises up to 5 lbs. pressure; but from the beginning of the stroke the pressure slowly declines, showing that the steam is wire-drawn, at about ¾ths of the stroke the pressure declines more rapidly, showing that at that point the steam is cut off, leaving the rest of the stroke to be performed by expansion. Before the completion of the stroke, the pressure in the cylinder has fallen to between 2 and 3 lbs. below the pressure of the atmosphere. The *slanting away at the eduction corner* shows that the eduction passages are of insufficient area, and the lead corner shows that there is more lead given to the valve than is usual.

Ex. 2.

Fig. 60.

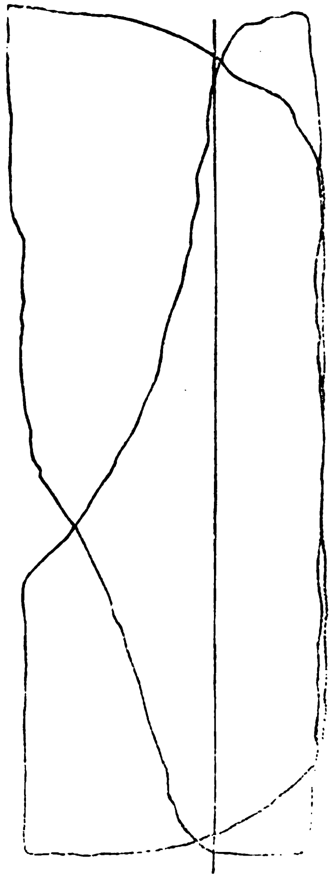


Diagram from the Paddle Engines of the *Great Eastern*.

Answer, Mean Pressure 22·625 lbs.

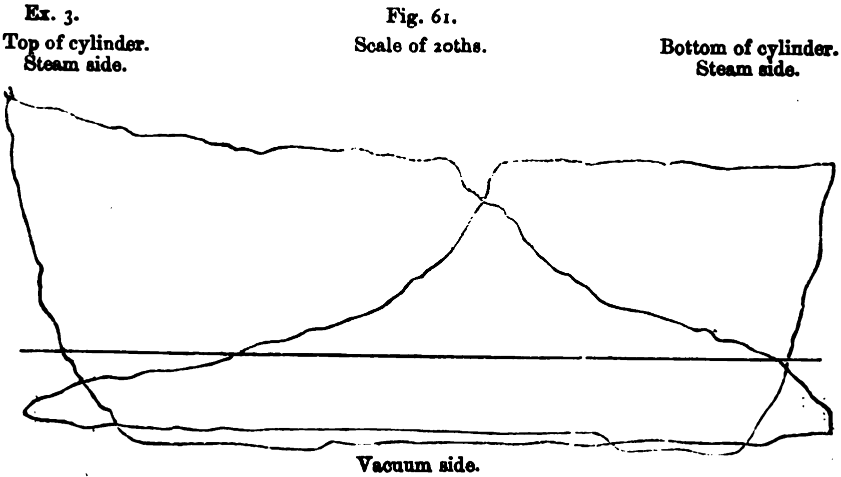
This is a double diagram taken from one of the paddle engines of the *Great Eastern* when on her trial trip in the Channel.

It will be observed that, to a great extent, the valves were unevenly set. The diagram from the top of the cylinder shows that the pressure on the piston was 20 lbs. above the atmosphere, cut off at one-third the length of the stroke, and expanded down to atmospheric pressure at the termination. The diagram from the bottom of the cylinder shows that the steam was upon the piston equal to 22 lbs. above the atmosphere, cut off at half stroke, and expanding to 4 lbs. above the atmosphere at the termination; in both cases the vacuum being 12 lbs. The number of strokes 11¼ and the speed of the piston 331 feet per minute. The exhaust closed when the piston had travelled five-sixths of the length of the cylinder, checking the progress of the piston when upwards of two feet from the end of the cylinder.

Ex. 3.

Fig. 61.

Scale of 20ths.



Top 22·5 lbs. Bottom 16·5 lbs.

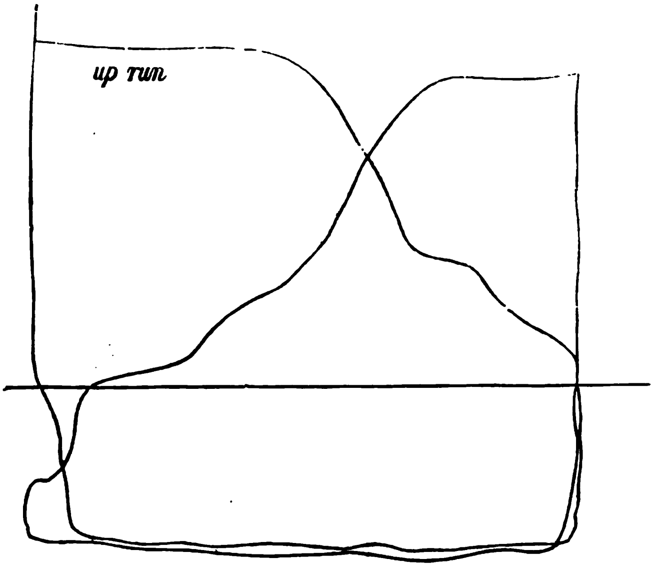
Answer, Mean Pressure 19·5 lbs.

This is also a double diagram from the paddle engines of the *Great Eastern*, taken June 26th, 1860, when she had been nine days out on her first voyage to America.

The pressure on the boiler was 26 lbs. per square inch above the atmosphere, and 21 lbs. were brought upon the piston at the bottom of the cylinder, cut off a little before half stroke, and expanding down the remainder to 4 lbs. below the atmosphere at the termination. On the top of the cylinder the steam exerts a pressure of 30 lbs. at the commencement, and cuts off a little past half stroke. The average pressure was 23 lbs. up to a little past half stroke, and expanding to 2 lbs. above the atmosphere, making only a 9 lbs. vacuum. The exhaust began to close at one end of the cylinder when the piston had to travel 2 feet of the stroke, and at the other end the exhaust began to close when the piston had to travel one foot. It will be perceived that, as the valves are now set, they are exactly the reverse to the position Mr. Scott Russel had them (see Fig. 60, page 354), when on the trial trip in the Channel, so far as regards the admission of steam and the cut off. There are objections to the setting of these valves in both instances; but on the whole, Mr. Scott Russel's diagrams are superior. The early closing of the exhaust, with engines travelling only 11¾ revolutions per minute, is a defect calculated to diminish the power considerably, from the engines having to compress such a great quantity of steam as was left in the cylinder with a 9 lbs. vacuum, acting as a retarding force against the course of the piston. The steam thus left in the cylinder had to be compressed to a greater pressure than that which was in the boiler. The diagram shows that there was a compressed force of 30 lbs., which had to be overcome by the force of the other engine when at its full power assisted by the motion of the ship. If these engines were separated, and not attached to a heavy fly-wheel or some other body in motion, the piston would stop before it arrived at the end of the cylinder, because the compressed or cushioned steam would become of much greater power or pressure than the steam on the propelling side of the piston, or that in the boiler, because the steam left in the cylinder could not escape on account of the exhaust valve closing too soon. This must be a serious drawback to the working of the engines, involving a loss of fuel, besides exposing the engines to the danger of breaking down, by having such a force as 30 lbs. of steam above the atmosphere, upon the crank and piston at plumb centre—almost as bad "as priming" in the cylinder. There need be no wonder at the comparatively slow speed those engines attained. Without taking into consideration various other defects in the arrangement of the valves, there is an almost unpardonable defect which we must notice—the amount of vacuum obtained. On the one side of the piston this amounts to 9 lbs., and on the other 10 lbs., which contrasted with the diagram taken on the trial trip shows a deficiency averaging 2½ lbs. per square inch throughout the stroke. On the one side it is 3 lbs., which proves there must be something requiring alteration. That 4 lbs. of deficiency in the vacuum proves the simple fact that an extra pressure of 4 lbs. of steam is required to give out the power that a proper condensation would have given; in other words, there is a loss of 20 per cent. in fuel, or 20 per cent. in power. The loss of vacuum, and the loss of power at each end of the stroke, accounts, to a great extent, for the slow speed of the engines and the great quantity of fuel consumed.

Ex. 4. Work out an indicator card, and show the point where the steam is cut off; whether the slide-valve is correct or requires alteration, and if the latter, what?

Scale of 10ths to an inch.



From the engines of S.S. *Sea King*, or *Shenandoah*, Confederate Cruiser, Oct. 8th, 1863. Clook & Combrae.

Fig. 62.

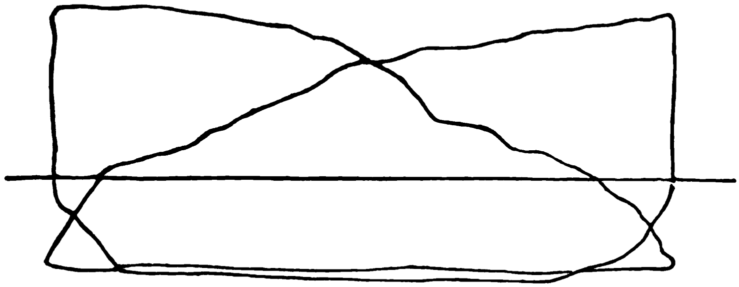
Area of cylinder 1662; I.H.P. 1814.
Steam 24 lbs., vacuum 25 in., revolutions 65 per minute.
Top 19·7 lbs. Bottom 24·9 lbs.
Mean Pressure 22·3 lbs.

Ex. 5. Work out the following indicator card, and show the point where the steam is cut off; whether the slide-valve is correct or requires alteration, and if the latter, what?

Top of cylinder Steam side.

Fig. 63. Scale of 20ths.

Bottom of cylinder Steam side.



Vacuum side.

Top 19·27 lbs. Bottom 18·55 lbs.

Mean Pressure 18·91 lbs.

The foregoing double diagram is taken from the screw engines of the *Great Eastern* on January 26th, 1860, during her first trial trip to New York.

The pressure of steam in the boiler was 20 lbs., the speed of the piston 285 feet. The [illegible]ssure of steam brought upon the piston at the bottom end was 16 lbs., and at the top end 18 lbs., with a partial cut off, and the ordinary closing of the valve leaving steam to condense at the end of the stroke, above the atmosphere, and making a vacuum of 10 to 11 lbs. per square inch. The steam being exhausted when the piston has to travel one-seventh the length of the cylinder, that portion, or nearly one-seventh of the power, is lost, the steam being condensed before it has given out its full power. The steam ought to be better used up. There are other minor points in connection with these diagrams that might be remarked upon, such as the projection on the bottom diagram; but as the reader will, by this time, have acquired a tolerably accurate knowledge of the subject, and be able of himself to point out or comprehend such defects, we do not deem it necessary to dilate upon them.

Ex. 6. Work out an indicator card, and show the point where the steam is cut off; whether the slide-valve is correct or requires alteration, and if the latter, what?

Fig. 64.

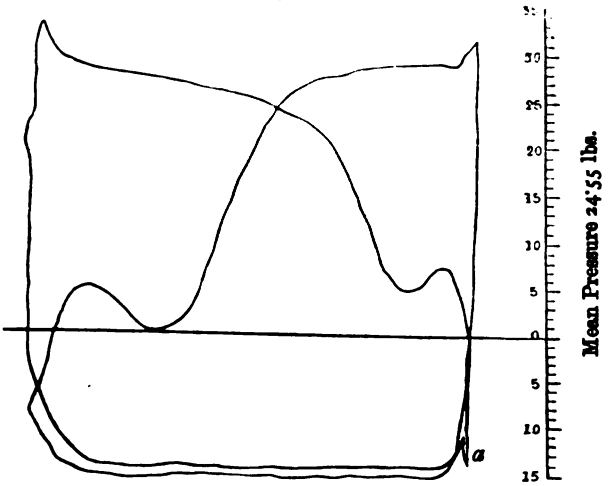
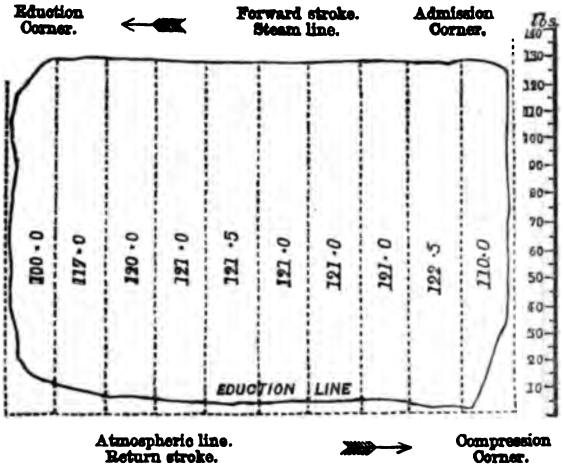


Fig. 64 represents two diagrams from the engines of the ironclad S.S. *Research*. This engine is fitted with surface condensers. The serrated deviation at *a* is caused by the momentum of the piston of the indicator.

303. **On High Pressure Diagrams.**—In the case of a diagram taken from a high pressure engine, the whole card would appear above the atmospheric line, as the steam exhausts into the open air, and the mean pressure is obtained from the measure of the depth of the diagram above the atmospheric line. The perpendicular distance between the lower line of the diagram and the atmospheric line indicates the resistance to the motion of the piston, caused by vapour in the cylinder, obstacles to a free exhaust, and the like. The back pressure is this pressure, plus the pressure of the atmosphere into which the steam exhausts.

Explanation of figure.—In the following diagram the piston moves in the forward stroke in the direction shown by the arrow, and backward on the return stroke in the direction shown by the arrow. The steam being supposed let in upon the piston of the engine, presses the piston of the indicator up to the point shown at the "admission corner," and as the piston moves forward the steam continues to press upon it with undiminished pressure, until close to the end of the stroke, at the "eduction corner," the eduction passage is opened, and as the steam consequently escapes into the atmosphere, there is no longer the same pressure on the spring of the indicator as before, and its piston consequently descends. As, however, the steam cannot instantaneously make its escape, the pressure does not descend quite so low

Fig. 65.

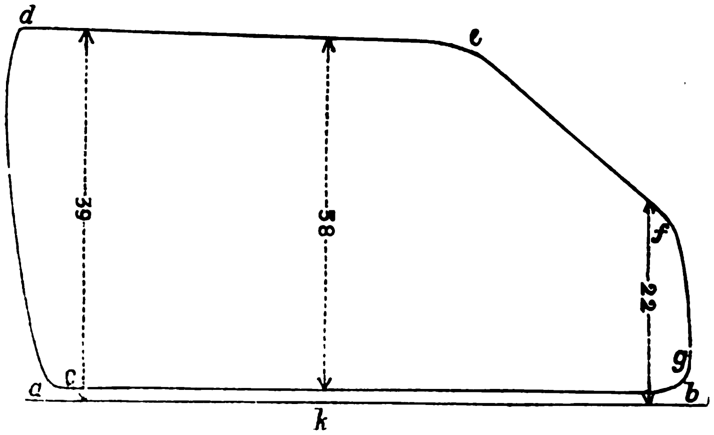


as the atmospheric line. The eduction passage begins to be opened, as will be seen from the diagram, when about nine-tenths of the return stroke has been completed; and it also begins to close when about nine-tenths of the return stroke has been completed, as appears by a reference to the "compression corner," which shows that the back pressure begins to rise before the termination of the stroke. The area comprehended between the atmospheric line and the bottom of the diagram shows the amount of back pressure resisting the piston, which, in this diagram, is of the average amount of 5·1 lbs.; and this increased back pressure at the compression corner is produced by the compression of the steam shut within the cylinder, which is accomplished by the piston as it approaches the end of the stroke.

304. **On Determining the Mean Pressure from High Pressure Diagrams.** —To ascertain the mean pressure we add together the pressure at each division or ordinate, and divide by the number of ordinates. The total amount of the pressures in Fig. 65, taken at ten places or ordinates is 1175 lbs., and the tenth of this, or 117·5 lbs. per square inch, is the mean pressure on the piston throughout the stroke.

The following, Fig. 66, is a diagram from the top side of a piston of a high pressure or non-condensing engine, the action of the steam being the same as in the condensing engine, except that the exhaust line is above the atmospheric line. The line *a b* is the atmospheric line; *d* to *e* the steam line; *e* to *f* the line of expansion; at *f* the eduction valve opens, and allows the steam to escape into the atmosphere. From *g* to *c* is the line of exhaust, and the distance from it to the atmospheric line represents the back pressure on the top side of the piston during its upward stroke.

Fig. 66.



On diagrams from non-condensing engines the line of boiler pressure should be drawn at the top, and it is well to draw the line of perfect vacuum also, that the engineer may be able to see at a glance the quantity of steam consumed, and to compare with it the amount of work done. It is not possible that the back pressure resisting the motion of the piston shall be less than the pressure of the atmosphere, but it may be a great deal more, and very commonly in non-condensing engines the line of resistance is as much as 2 or 3 lbs. above the atmospheric line, though it is quite possible to avoid this excess altogether.

COMPOUND MARINE STEAM ENGINE.

A Compound Engine, called also a **High and Low Pressure Engine**, is an engine with two cylinders, the one frequently double the diameter of the other. The use of two cylinders, as a means of uniting the full economical advantages due to complete expansion, with the practical requirements of equable motion, is attracting great attention amongst the engineers of the day; and, this class of engine having been tried with the use of high pressure steam, the beneficial results obtained have induced owners almost universally to adopt them for vessels going long voyages; indeed, so great are the advantages found to be, that most of the large steamship companies have had the single cylinder engines existing in their ships replaced by compound engines.

In this class of engine the steam from the boiler first enters the small cylinder at a high tension, where it is cut off at from one-third to one-half the stroke, and the steam having pressed the first piston to the end of the stroke in the manner of a high pressure piston, escapes or exhausts into a jacket, or into a low pressure casing, whence it then enters into another cylinder of larger dimensions, and completes its work there by expanding to the end of the stroke, being fully condensed in the usual manner. The steam is expanded by this two-fold operation to about six or eight times its original volume. The pressure urging the first or high pressure piston, is consequently the difference of pressure between the steam in the boiler and that in the second cylinder; and the pressure urging the second or low pressure cylinder piston, is the difference of pressure between the steam on the eduction side of the high pressure cylinder and that of the vapour in the condenser. There will be a small difference between the pressure in the communicating parts of the high and low pressure engines, just as there is a small difference between the vacuum in the cylinder and that in the condenser. But, in well constructed high pressure engines this difference will not sensibly detract from the power.

The arrangements of the compound engine which have been from time to time brought into use are various. In one arrangement the position of the low pressure cylinders is side by side, as for those of the ordinary kind; in some cases annular cylinders are used, viz., the high pressure cylinder within that of the low pressure. Another arrangement is that in which the high pressure or small cylinder is erected on the top of the large or low pressure cylinder, having a fly wheel to prevent sticking on the centre. A third arrangement has the smaller cylinder at the fore end of the larger. A fourth example consists of four cylinders, the two high pressure above the low, with the double cranks. The best and most convenient arrangement is the two cylinders, connected each to its crank, and having the cranks from 90° up to 135°, with starting valves.

Of course there is no theoretical gain in the use of two cylinders. The economical advantages of expansion is the same, whether developed in a single cylinder or a double cylinder engine. When, however, expansion is carried to any great extent in a single cylinder, there is a great irregularity of pressure at different parts of the stroke. The initial pressure at the beginning of every stroke produces violent concussion through the whole of the machinery, and in proportion to the degree of expansion used, the evil will increase.

When it is wished to attain a high degree of expansion in marine and other rotatory engines, with a single cylinder, the amount is limited to an extent of three or four times, from the great strain at the commencement of the stroke, compared with the mean force of the entire stroke. In these engines the initial blow is not absorbed by the inertia of a large mass to be put in motion at the beginning of the stroke, forming a reservoir of power for the steam while expanding through the remainder of the travel of the piston. Compounded engines are, in this case, peculiarly advantageous. Of course, steam engine economy almost always involves additional weight and expense in the engine. This is, however, in this case, compensated by the reduced number of boilers and furnaces required in developing a given amount of power—a fact which is not seldom lost sight of when estimating the relative advantages of compounded and single cylinder engines.

In the ordinary compound engine the larger cylinder is worked entirely by the exhaust steam from the smaller; but there is a further modification into what is called the *continuous expansion* principle, by Messrs. STEWART and NICHOLSON, in whose arrangement the steam partially acts on both pistons at the same time. (See Figs. 80 and 81).

310. **To calculate the power from a Diagram off a Compound Engine.**—We first determine by a diagram and a computation, such as in figure 66, the power exerted by a high pressure engine, and then, in like manner, we determine the power exerted by the low pressure engine (see Rule CXXXII, pages 343—344). The total power is obviously the sum of the two.

Explanatory Diagrams, Figs. 68 and 69, represent a diagram from the high pressure cylinder of a compound engine. It will be seen that the bottom of the diagram representing the exhaust line is above the atmospheric line, thus indicating that the exhaust steam has a pressure above the atmospheric pressure (the amount of this pressure can be easily measured off the scale); it represents the back or resisting pressure upon the high pressure piston, and is due to the amount of expansion the steam undergoes on being exhausted into the jacket, or receiver; therefore, the larger the area of this jacket, or receiver, the greater is the amount of expansion and consequent diminution of pressure, or back pressure upon the high pressure piston, but this back pressure is of paramount importance in a compound engine; in this way it equalizes the power developed by both cylinders, inasmuch as the greater the back pressure on the high pressure piston, the less will be the power developed by the high pressure engine, but the greater power will be developed by the low pressure engine; and should the high pressure engine exhaust at, or under the atmospheric line, the greater will be its power, consequently, the less will be the power of the low pressure engine; therefore, in compound engines, the area of the jacket, or receiver, should be such as when the high pressure steam is cut off at a certain portion of the stroke, and exhausted into this jacket, or receiver, the pressure of steam that will enter the low pressure cylinder shall be such as will develop in it equal, or nearly so, power to the high pressure cylinder, otherwise the engines are badly designed.

The other diagram represents the usual form from the low pressure cylinder of a compound engine. It will be seen that for a short distance the piston is acted upon by steam above the pressure of the atmosphere (or above the atmospheric line), this steam is the high pressure exhaust, and is nearly equivalent to the amount of the back pressure; the low pressure diagrams of compound engines have invariably a "throttled look." This is owing to the large area of piston the steam acts upon, and consequent great expansion, the initial, or entering pressure, being low.

Fig. 69.

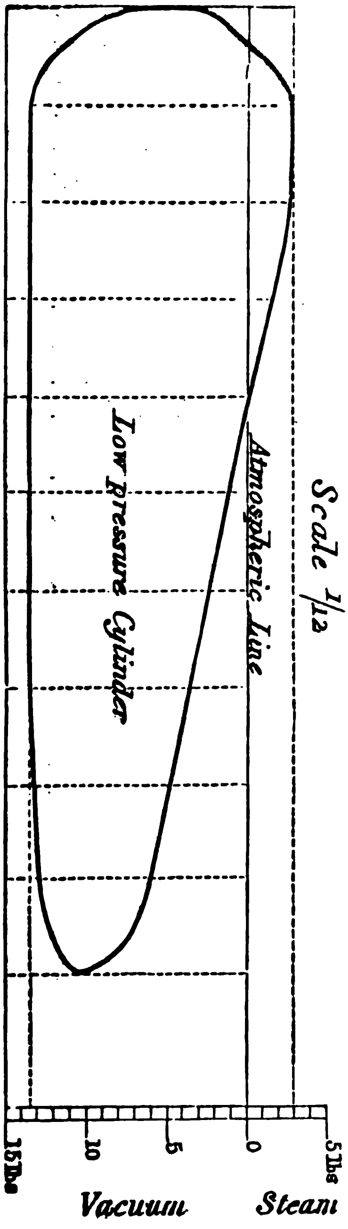
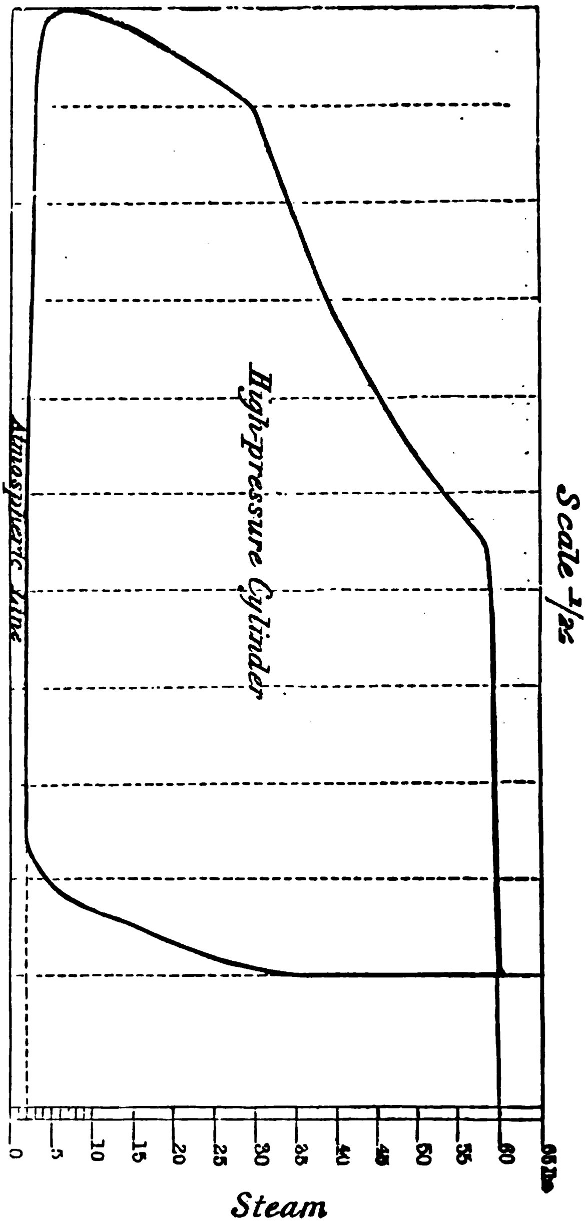
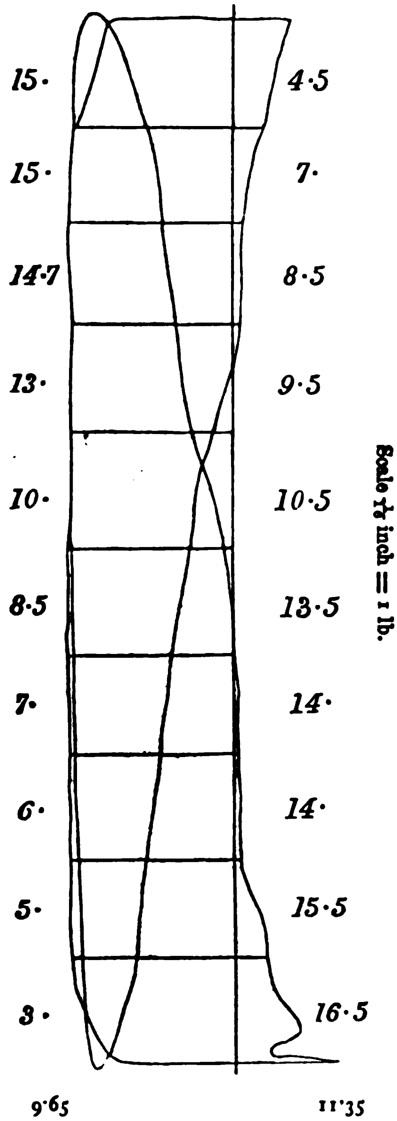

Low pressure Cylinder
Atmospheric Line
Scale 1/12
15 lbs
10
5
0
5 lbs
Vacuum
Steam

Fig. 68.


High-pressure Cylinder
Atmospheric Line
Scale 1/25
0
5
10
15
20
25
30
35
40
45
50
55
60
65 lbs
Steam

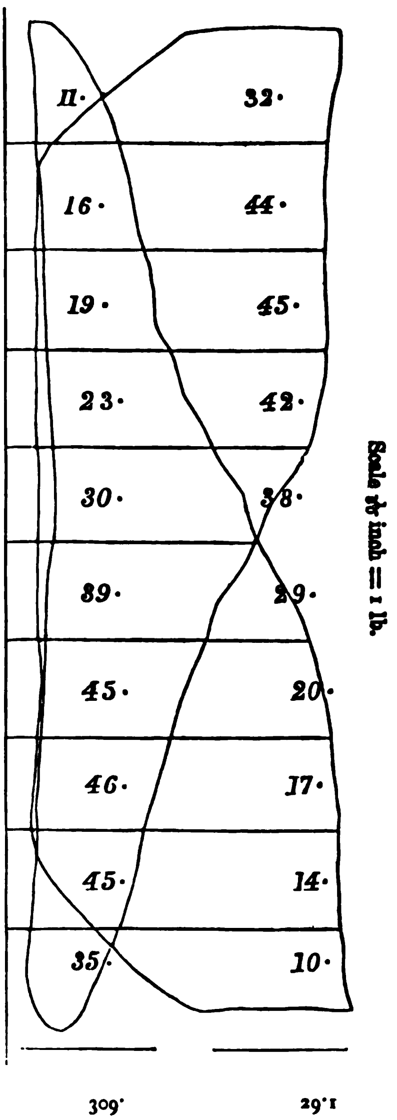
Diagrams taken off Engines (Compound) by ———, of Glasgow, November, 1870.
Steam 55 lbs.; Vacuum 27½ inches; Revolutions 48.
Diameter H.P. Cylinder 38″; Diameter L.P. Cylinder 68″; Stroke of Pistons 3′ 6″.
Pressure on compound gauge 5 lbs; Pressure at end of stroke in L.P. cylinder 6 lbs. (mean) above pure vacuum.

Fig. 70.



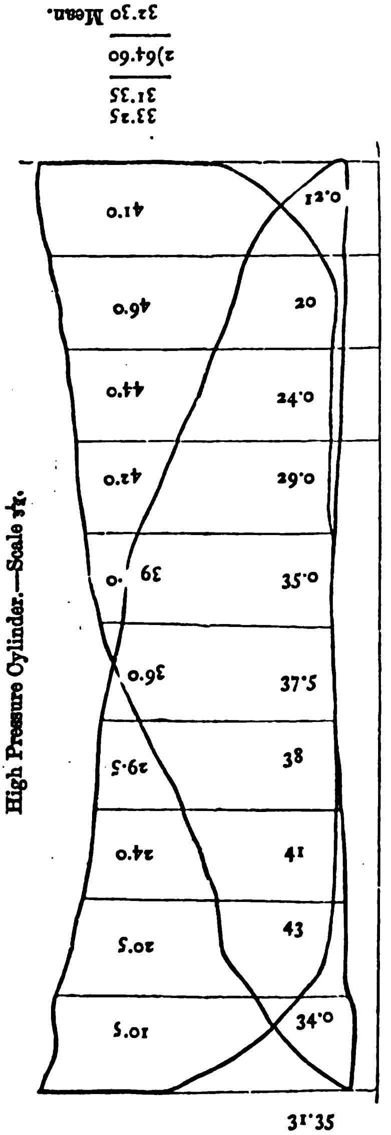
Mean L.P. 10·5.

Fig. 71.



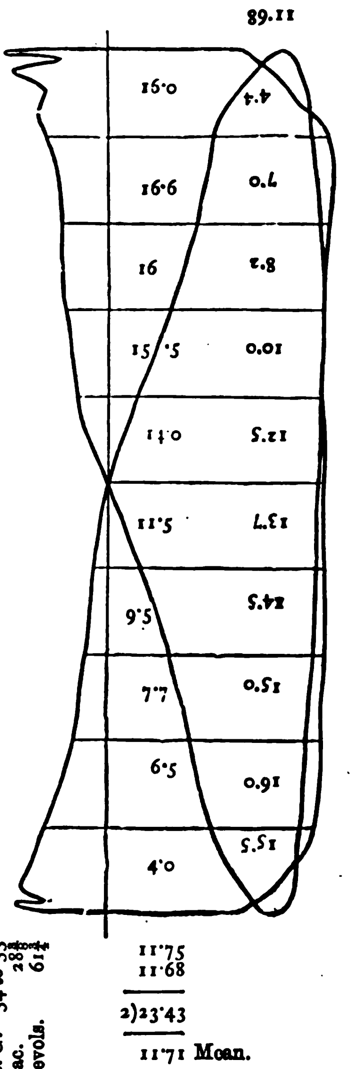
Mean Pressure 30·0 lbs.

Fig. 72.



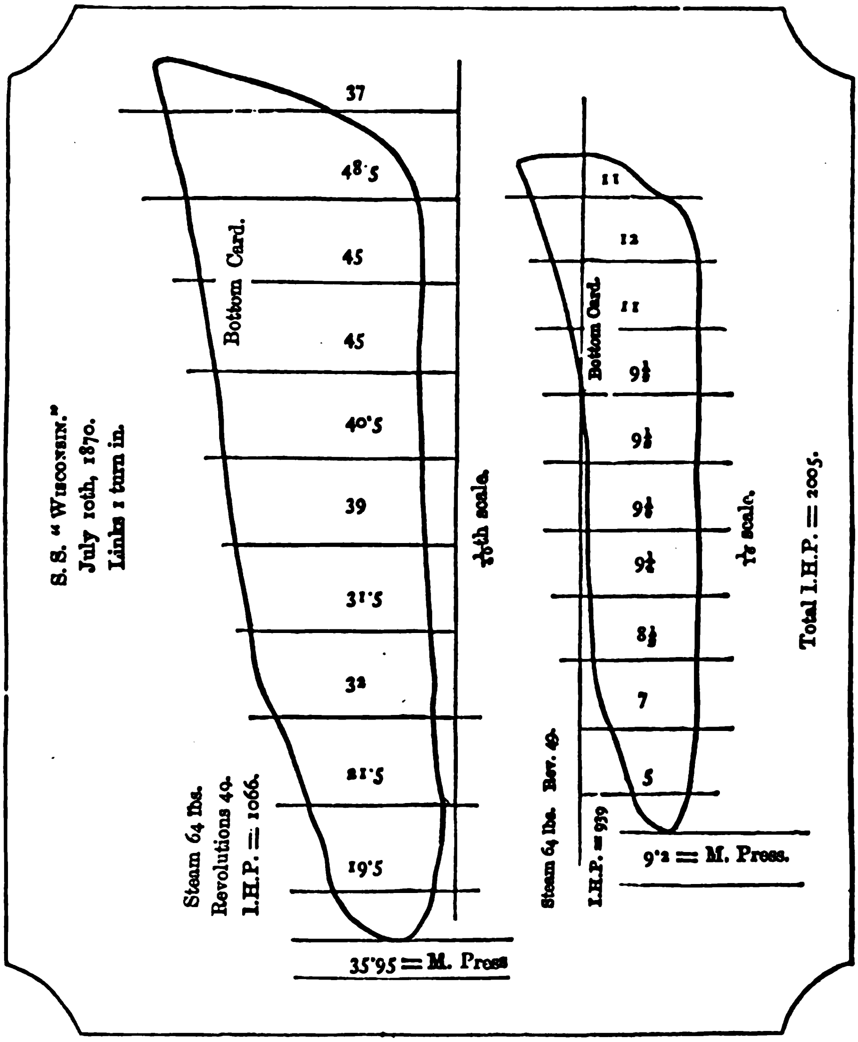
R.M.S. *Elbe.* Stokes Bay, Feb. 24th, 1870.
High Pressure Cylinder.—Scale $\frac{1}{32}$.

Fig. 73.



Low Pressure Cylinder.—Scale $\frac{1}{16}$th.

Work out the following indicator card. Figs. 74 and 75.



S.S. *Haco.* Oct. 20th, 1870.

Steam 120 lbs.
Vacuum 24½.
Revolutions 68.

Diameter of cylinder, H.P., 12".
" " L.P., 36".
Length of stroke 3 feet.

Fig. 76.

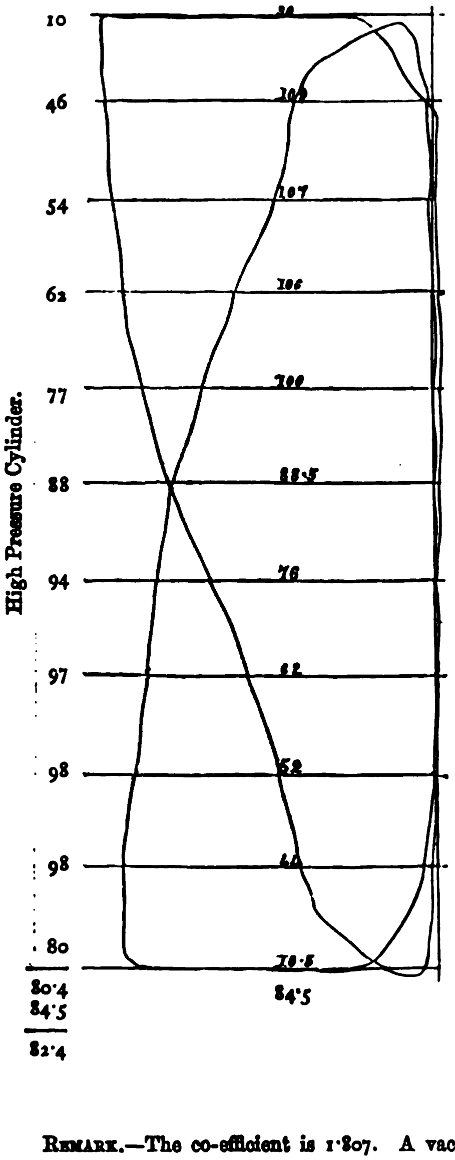
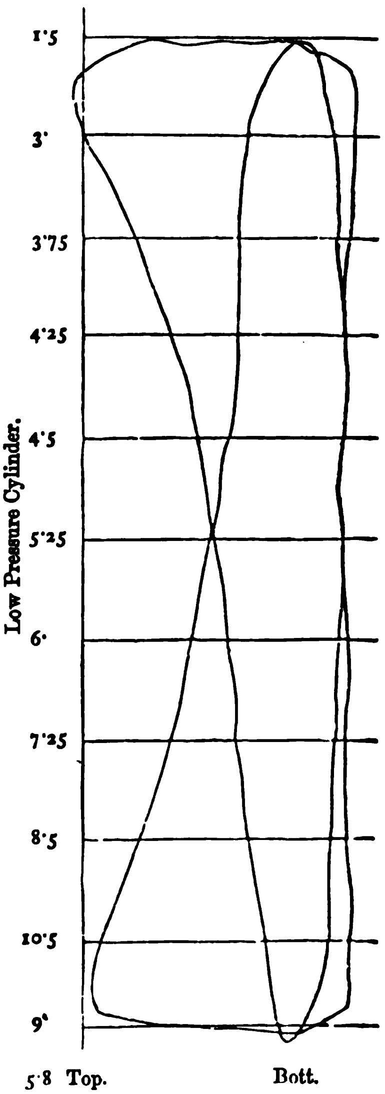


Fig. 77.



Remark.—The co-efficient is 1·807. A vacuum of 28·75 would increase the power ⅙ or co-efficient 2·087.

S.S. *Rothbury.*

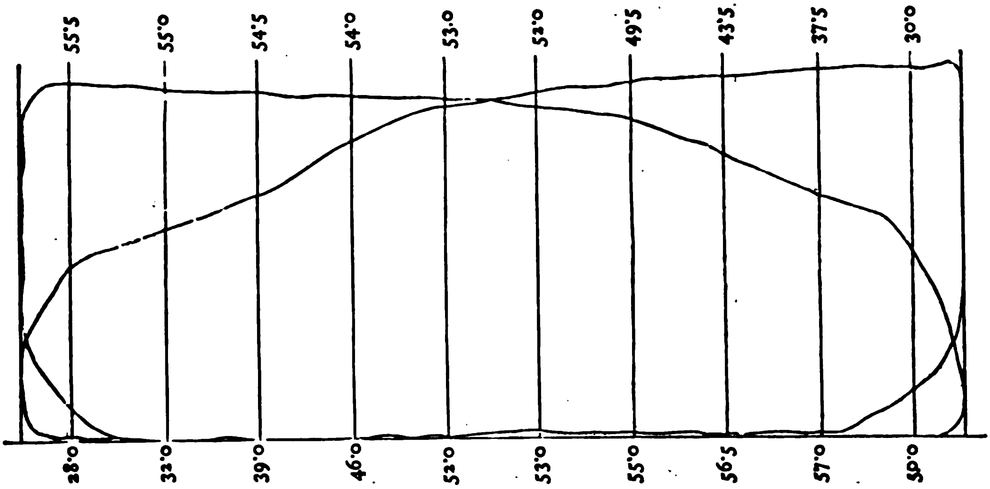
Steam 60 lbs.; Vacuum 25½; Revolutions 56.

High Pressure Cylinder.

OUTRIDGE'S PATENT SLIDE.

FULL GEAR.

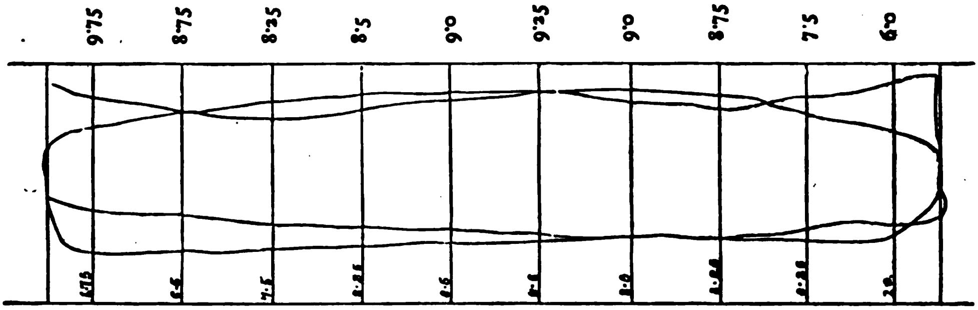
Fig. 78. Scale $\frac{1}{16}$.



Mean Pressure 47·65 lbs.

Common Slide.

Fig. 79. Scale $\frac{1}{16}$.

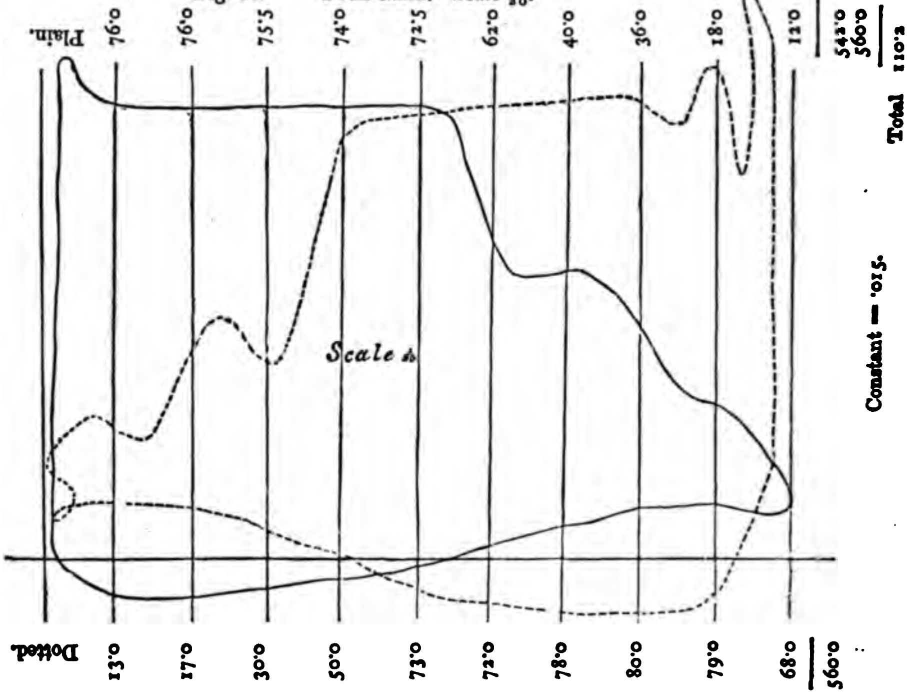


Mean Pressure 8·24 lbs.; 280 feet per minute.

I.H.P. 137·24.

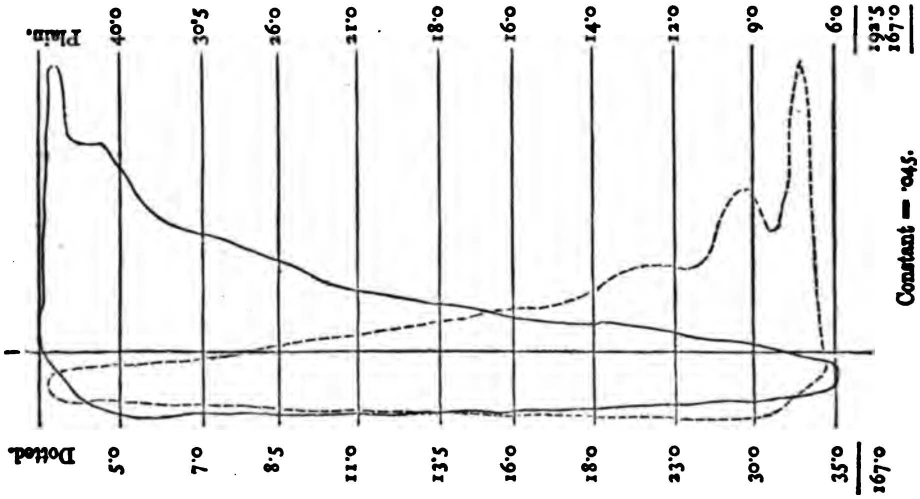
Diagram from a Compound Engine with Stewart & Nicholson's Continuous Expansion.
S.S. *Albion*, at moorings, May 7th, 1870.
Cylinder $14\frac{5}{8}''$; Stroke 16″; Vacuum $26\frac{1}{2}''$; Steam 72 lbs.; Revolutions 137.

Fig. 80. FULL GEAR.—Scale $\frac{1}{10}$.



Cylinder $26\frac{5}{8}''$; Stroke 16″; Steam 75 lbs.; Vacuum 27; Revolutions 137.

Fig. 81. FULL GEAR.



I.H.P. of large cylinder = $\frac{35\cdot95}{2}$ × ·045 × 137 = 110·80
„ of small cylinder . . . . = 113·23

Total I.H.P. . . . 224·03

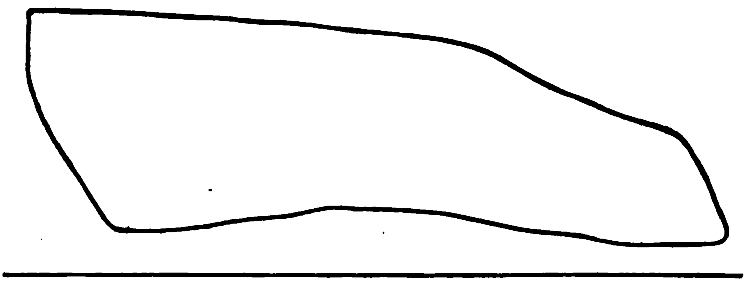
Diagrams taken from the engine of the S.S. *F——s*, trial at sea September 1st, 1870.

Steam 59½ lbs.; vacuum 27¾; revolutions 79; diameter of cylinders 52" and 30"; stroke 33"; nominal horse-power 100; speed of ship 11·1 knots per hour.

Scale of 40ths to an inch.

Fig. 82.

Full Gear.—High Pressure Cylinder.

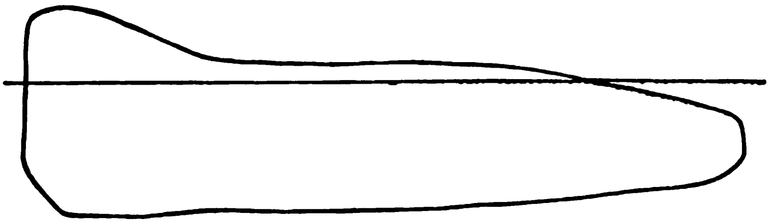


Ans. Mean Pressure 34·65 lbs.; Indicated Horse-power 322·4.

Scale of 16ths to an inch.

Fig. 83.

Low Pressure Cylinder.



Ans. Mean Pressure 11·9 lbs.; Indicated Horse-power 332·75.

The following is a "Bottom Card" from the same engine as the foregoing.

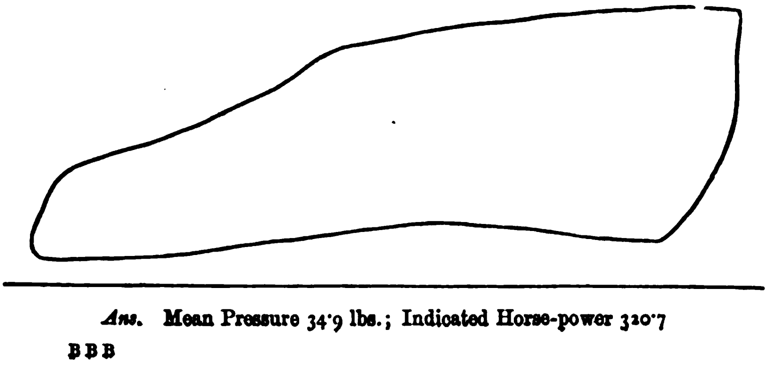
Steam 60 lbs.; vacuum 27¾; revolutions 78.

Full Gear.

High Pressure Cylinder.

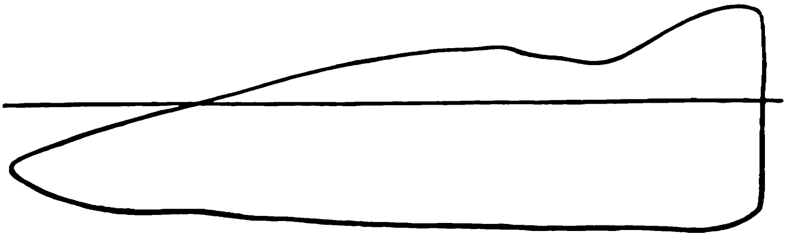
Scale 40ths to an inch.

Fig. 84.



Ans. Mean Pressure 34·9 lbs.; Indicated Horse-power 320·7

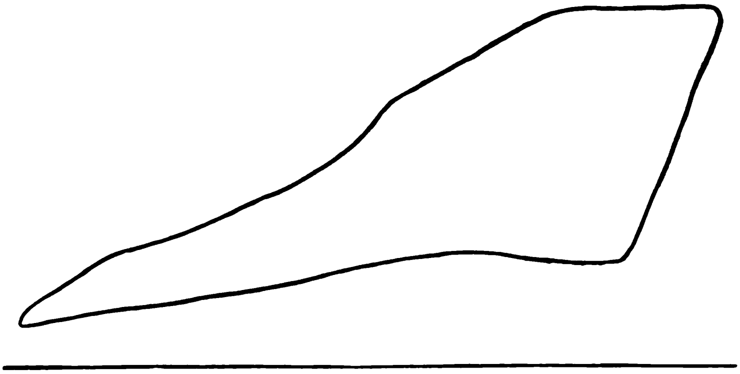
Low Pressure cylinder.
Scale 16ths to an inch.
Fig. 85.



Ans. Mean Pressure 12·25; Indicated Horse-power 338·5.

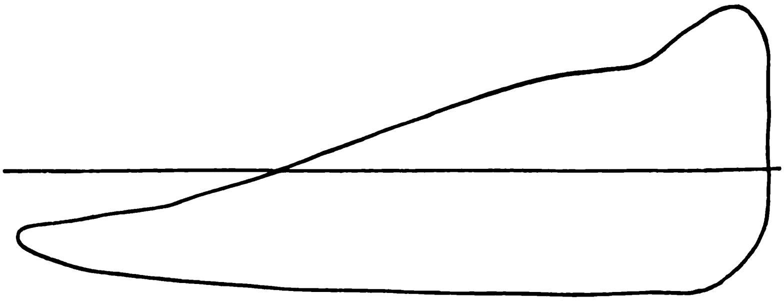
An indicator card from same engine as the foregoing. Steam 24 lbs.; vacuum 27⅞; revolutions 72.

Wheel 4 turns back.
Scale 40ths to an inch.
Fig. 86.



Ans. Mean Pressure 27·85 lbs.; Indicated Horse-power 236·23.

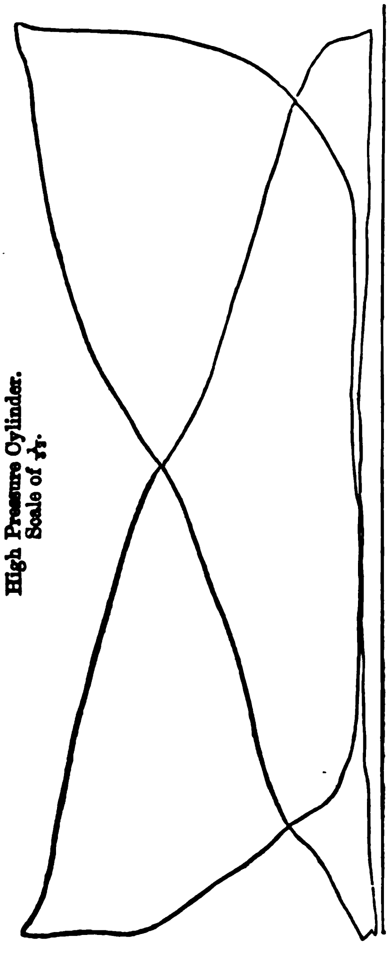
Scale 16ths to an inch.
Fig. 87.



Ans. Mean Pressure 13·125; Indicated Horse-power 334·5.

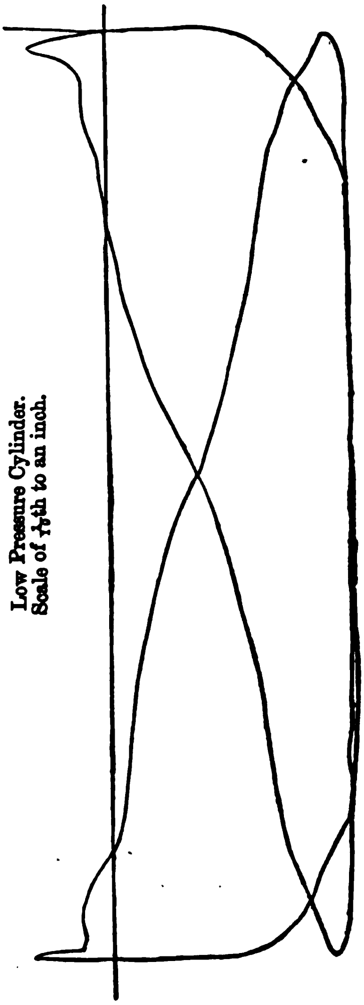
Diagrams taken from the engine of the S.S. *Burlington.*

Fig. 88.



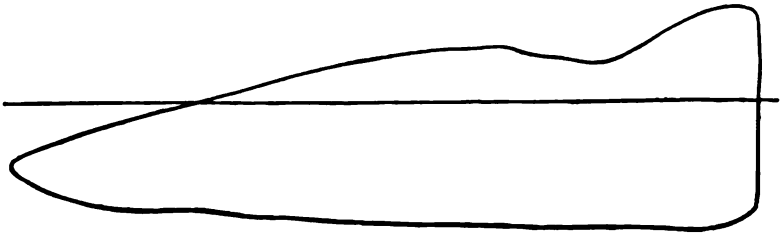
High Pressure Cylinder.
Scale of $\frac{1}{40}$.

Fig. 89.



Low Pressure Cylinder.
Scale of $\frac{1}{10}$th to an inch.

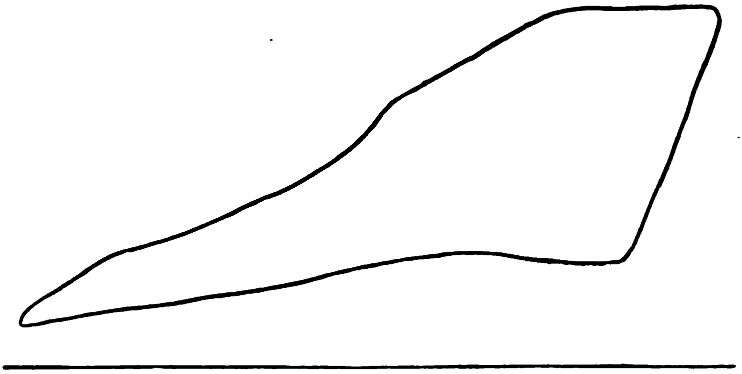
Low Pressure cylinder.
Scale 16ths to an inch.
Fig. 85.



Ans. Mean Pressure 12·25; Indicated Horse-power 338·5.

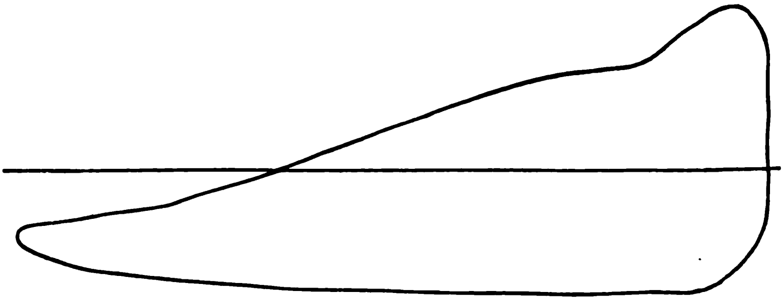
An indicator card from same engine as the foregoing. Steam 24 lbs.; vacuum 27⅜; revolutions 72.

Wheel 4 turns back.
Scale 40ths to an inch.
Fig. 86.



Ans. Mean Pressure 27·85 lbs.; Indicated Horse-power 236·23.

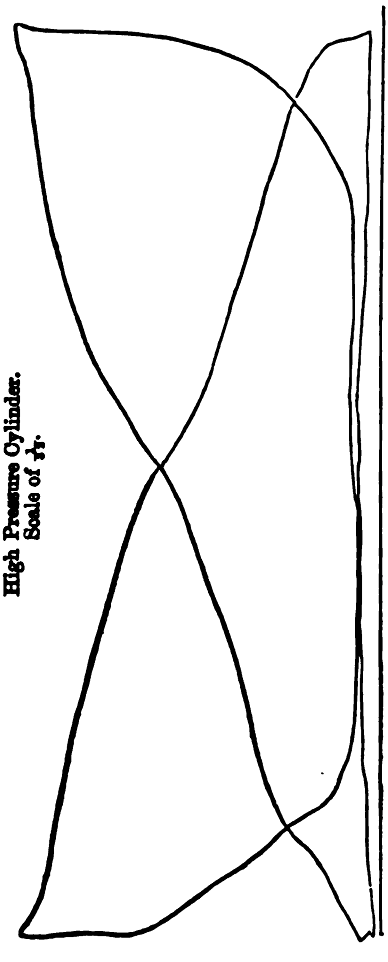
Scale 16ths to an inch.
Fig. 87.



Ans. Mean Pressure 13·125; Indicated Horse-power 334·5.

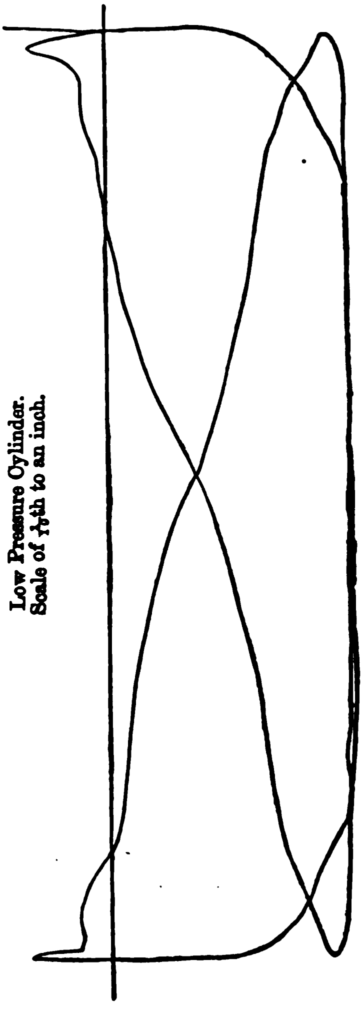
Diagrams taken from the engine of the S.S. *Burlington.*

Fig. 88.



High Pressure Cylinder.
Scale of $\frac{1}{32}$.

Fig. 89.



Low Pressure Cylinder.
Scale of $\frac{1}{10}$th to an inch.

S.S. *Fairy Dell*, at sea, 16th February, 1871.
Steam 125 lbs.; Vacuum 26½; Revolutions 80; Stroke 20″.
High Pressure Cylinder 8½″; Low Pressure Cylinder 24½″; I.H.P. 146.

Fig. 90.

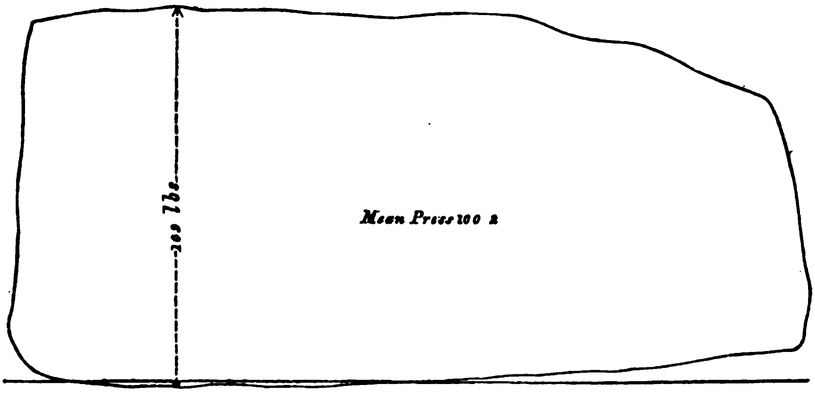


Fig. 91. Scale $\frac{1}{16}$.

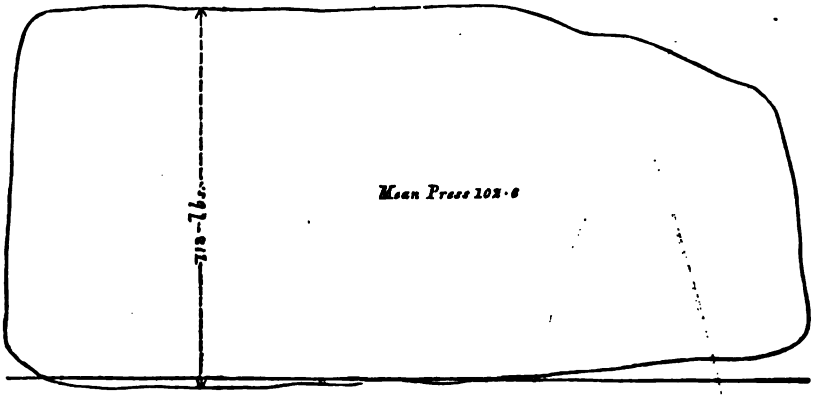


Fig. 92.

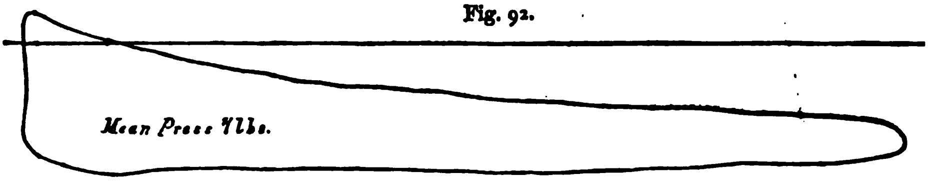
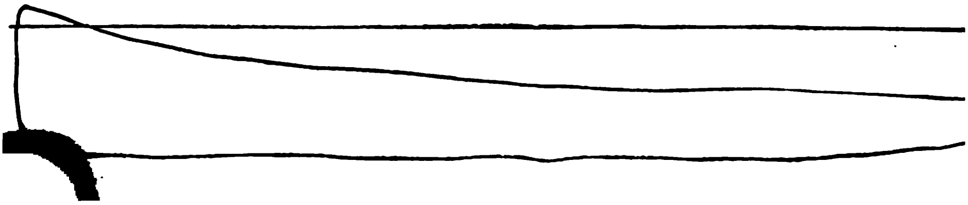
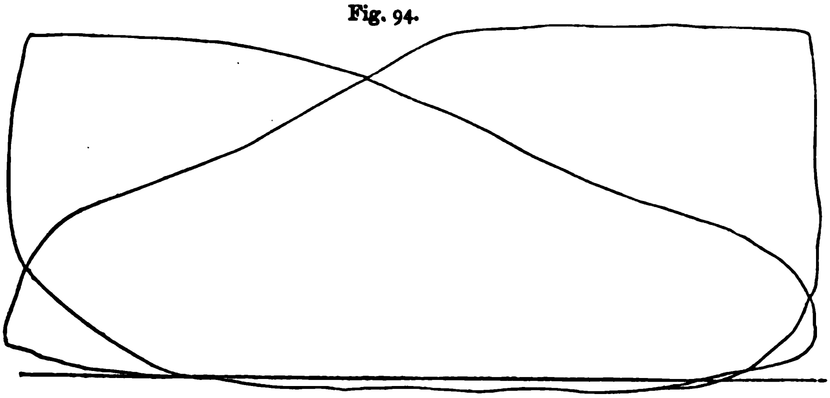


Fig. 93. Scale $\frac{1}{16}$.



S.S. *Marc Antony*, November 14th, 1870. Running Card.
Steam gauge 110; Vacuum 25; Revolutions 60.
Forward Engine. High Pressure Cylinder.

Fig. 94.



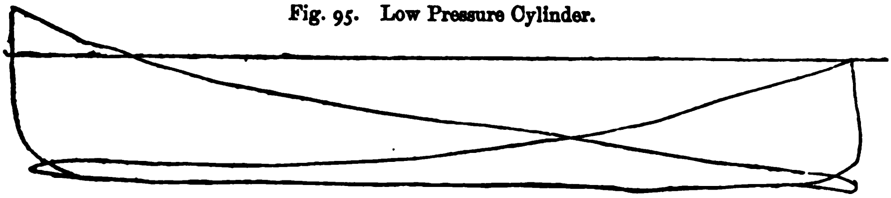


Fig. 95. Low Pressure Cylinder.

After Engine. High Pressure Cylinder.
Steam gauge 110 lbs.; Vacuum 25; Revolutions 60.

Fig. 96.

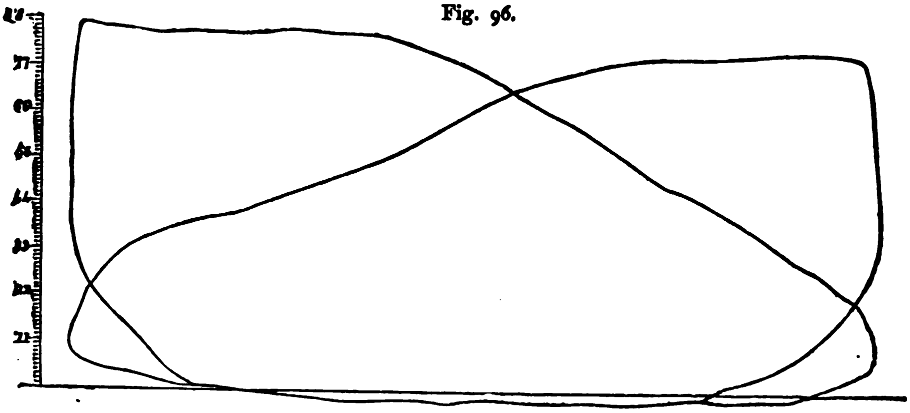
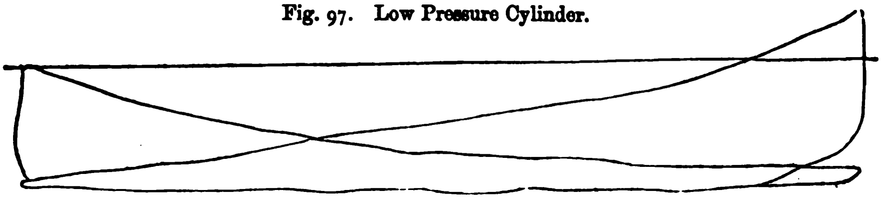


Fig. 97. Low Pressure Cylinder.

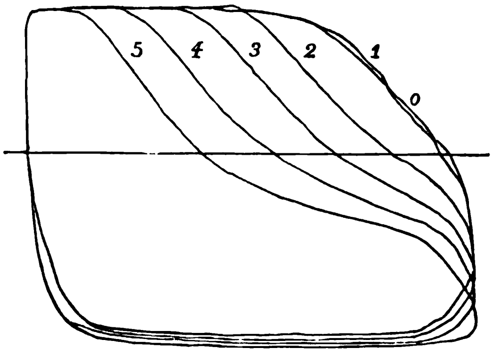


ON THE FORM OF THE FIGURE WHEN THE EXPANSION GEAR ALONE IS USED.

If the steam be cut off at any part of the stroke, the steam pressure will decrease and the pencil begin to descend. Now, if the temperature of the steam be unaltered, the pressure, as before stated (see page 326), will vary inversely as the space it occupies, and the theoretical figure will be F *a b c d* (see Fig. 16, page 327). But since the slide-valve also acts we have the modification this would produce too, for the slide-valve is placed between the expansion-valve and the cylinder; it follows, therefore, that the effective volume of the steam intercepted by the expansion-valve is the whole of the space between it and the piston, and the slide-valve interposes an additional barrier when it begins to cut off the steam.[*] The case, therefore, is somewhat similar to what it would be if there were two expansion valves, one nearer to the cylinder than the other, and the outer one acting first.[†]

This figure exhibits a series of diagrams representing various grades of expansion; here 0 gives the full power of steam without using the expansion gear; 1, that produced by the first grade of expansion; 2, that produced by the second grade, and so on. It is worthy of remark, that the diagram

Fig. 98.



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[*] Except with engines fitted with SEAWARD's slide.

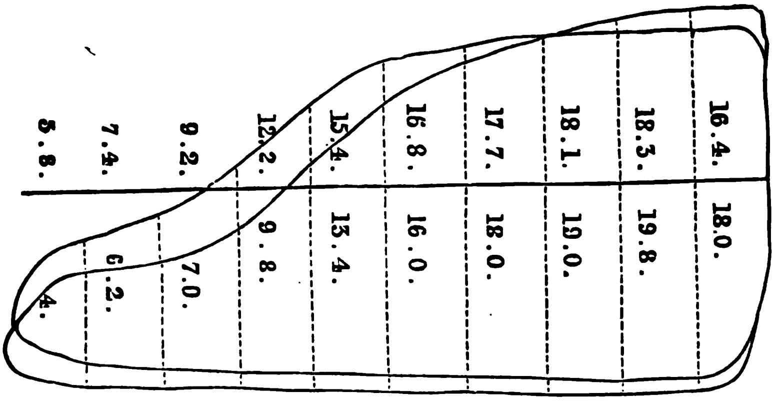
[†] The condensation of steam by radiation of heat from the cylinder, the leakage of steam through the piston or valves, or the expansion of the steam in the nozzles of the cylinder, the steam pipes, or the extra length of cylinder allowed for clearance of the piston, will vary the shape and termination of the curve. It should, therefore, always be an inquiry, upon the production of a diagram, whether the curve be theoretically correct or not. If it is not, a satisfactory reason should be found for it, in the construction of the engine or the valves, and the piston should be thoroughly examined. A ready method to determine at what point this curve should terminate, is to find the capacity of nozzles, pipe, &c., from the expansion-valve to the cylinder, and adding it to the clearance, call it so much additional length of stroke. Extend the diagram in the same proportion and then divide it into any number of equal parts that will divide it equally, and have one of the divisions cut at the point where the valve closed. Take the number of these divisions on the steam line for the numerator of a fraction, and the whole number of them for the denominator, and divide with it the whole pressure upon the piston.

marked o, which is that resulting from the slide-valve, closely assimilates to that marked 1, produced by the first grade of expansion, as it should do in well constructed engines.

Each of the following figures exhibits a series of diagrams representing different grades of expansion, it is required to determine the best point at which to cut off the steam.

Fig. 99. This is from the top side of the engines of a screw steamer, the engines being connected to the screw shaft, without intermediate gearing. The stroke is 2 feet 3 inches; the screw shaft, and of course the engines also, making from 90 to 98 revolutions per minute. This figure shows the steam cut off with the slide-valves, and also with the first grade of expansion; and it will be observed that the power is very little reduced by putting on the expansion, as, with the speed of the piston and the condenser being small, the vacuum is much improved by cutting off the steam. In this example the valves are very well adjusted.

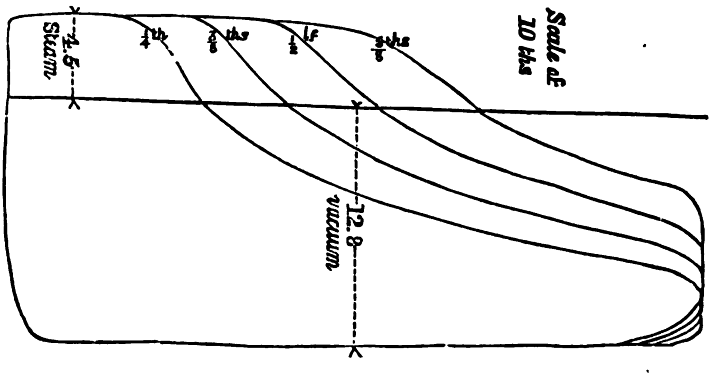
Fig. 99.



Scale of 10ths.

Indicator diagram from the engines of H.M.S.S. *Thunderbolt*, and shows the different grades of expansion.

Fig. 100.

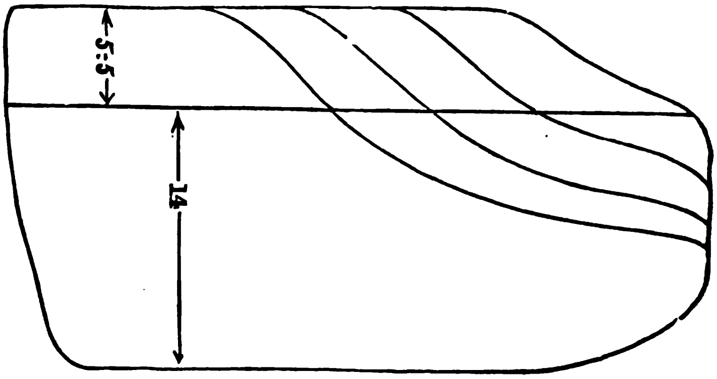


Ans. Average of $\frac{1}{4}$ 9·84 lbs.; $\frac{3}{8}$ 11·4 lbs.; $\frac{1}{2}$ 12·85 lbs.; $\frac{5}{8}$ 14·22 lbs.

From this figure it will be seen that the best point to cut off the steam with this small pressure is about half-stroke, because the steam consumed by cutting off at half-stroke is less in proportion to the power, as compared with cutting off at five-eighths; while, by cutting off at less than half-stroke, the steam line only decreases in proportion to the decrease of power. If the pressure were much greater, it might be advantageous to cut off sooner.

Indicator diagram from the S.S. *British Queen*. *Taken from the top of the cylinder.* Vacuum 30½ inches; steam in boilers 6½ lbs.; expansion tried on ⅜, ½, ⅝, as represented by the diagram.

Fig. 101.



The vacuum was the same with or without expansion, as shown by the diagram; the steam ways must have been large. The condenser was on Mr. Hall's patent principle of condensing by cold surface, without the water coming in contact with the steam.

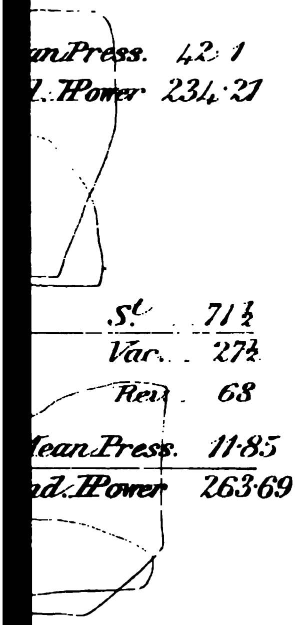
311. **Difference of Effect between throttling the Steam and cutting it off by the Expansion Valve.**—An experiment made in this manner to determine the difference of effect between expanding the steam in the cylinder by a proper system of expansion, and merely shutting it off by the throttle-valve when it is wished to work the engines to a low power, gave the following result:—An engine of the nominal power of 400 horses was selected, and was first worked on the lowest grade of expansion, the steam in the boiler having its full pressure of 8 lbs. Under these circumstances, it was found that 10·87 cubic feet of steam, at atmospheric pressure, were consumed per horse-power per minute.

The expansion-valves were then thrown out of gear, and the engines reduced by the throttle-valve to exactly the same number of revolutions. It was now found that although the gross horse-power, as shown by the indicator, was almost identical in both cases, in the latter case the power exerted was obtained at an expenditure of 14·72 cubic feet of atmospheric steam per minute, in place of 10·87 cubic feet, as formerly; 3·85 cubic feet of steam per indicated horse-power per minute being thus saved by the principle of expansion.

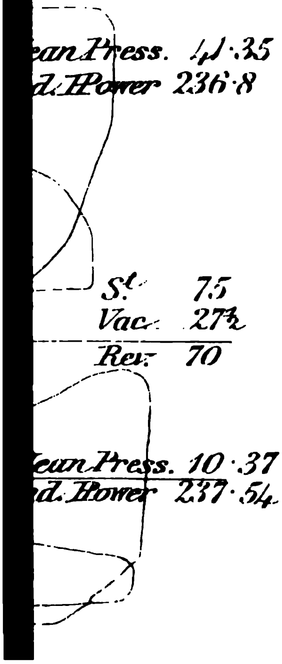
It is apparent that this calculation will not give us the *cause* of any loss of effect by the steam escaping, or being condensed in the pipes, etc., which

I,

1870.



anPress. 42·1
l.:HPower 234·27
St. 71½
Vac. 27½
Rev. 68
eanPress. 11·85
d.HPower 263·69

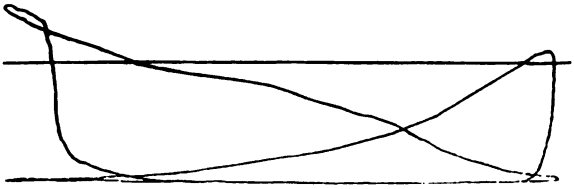


eanPress. 41·35
d.HPower 236·8
St. 75
Vac. 27½
Rev. 70
eanPress. 10·37
d.HPower 237·54

must be sought for when suspected, either in the form of the indicator diagram, or by testing the engine for leakage of steam or air.

312. **On the form of the Diagram when the Steam is throttled by the Link.**—In Fig. 102, we have diagrams taken from top and bottom of an engine, when slowed to 4½ strokes per minute, which is effected by partly closing the throttle-valve, and partly by shifting the link towards its mid-position. It will be observed that in these diagrams nearly the whole areas are below the atmospheric line. Note also that on the left-hand corner of one of the figures a loop is formed, which not unfrequently appears in engines employing the link, and the meaning of which we shall now proceed to explain. The extreme point of the diagram, in every instance, answers to the length of the stroke; and if the steam is pent up in the cylinder by the eduction passage being shut before the end of the stroke, or if it be permitted to enter before the end of the stroke, the pencil will be pushed up to its highest point before the stroke terminates, and since the paper still continues to move onward the upper part of the loop is formed. If the pressure within the cylinder, when the piston returns, were to be precisely the same as when the piston advances during this part of its course, the loop would be narrowed to a line. But as the advance of the piston, when the valve is very little opened, compresses somewhat the steam, and as its recession, when the valve is very little opened, causes it to be wire-drawn, the pressures, while the piston advances and retires through this small distance, although the cylinder is open to the boiler by means of a small orifice, will not be precisely the same; and the higher pressure will form the upper part of the loop while the lower part will be formed by the lower pressure. In following the outline of the left-hand diagram of the figure, it will be seen that the steam begins to be compressed within the cylinder when about three-fourths of the stroke has been completed, and the pencil begins to rise somewhat above its lowest point. But as the vapour within the cylinder is very rare, the rise is very little, until when the piston is about one-eighth part of its stroke, or about 8 inches from the end of the stroke, the steam valve is slightly opened, when the piston of the indicator ascends to the point answering to the pressure within the cylinder thus produced. As the opening from the boiler continues, and the piston, by advancing

Fig. 102.



against the steam, instead of receding from it, compresses rather than expands the steam admitted into the cylinder, the pressure continues to rise somewhat to the end of the stroke, when the piston of the engine having to move in the opposite direction, the steam within the cylinder will be expanded, and any that may still be entering will be wire-drawn in the contracted passage, and the pressure will fall. Under such circumstances a loop will necessarily be formed at the corner of the diagram, such as is shown to exist at the left-hand corner (see Fig. 102). The reason of there being no corresponding loop at the right-hand corner, arises simply from the valve being differently set at one end of the engine from what it is at the other; and the angles of the eccentric-rods will, in general, cause some small difference in the action of the valve at the different ends of the engine.

313. We have seen that an indicator card is run out, that is to say, the mean pressure of the card is usually ascertained by reading off with the scale the different mean pressures in each of the ten spaces, and then adding them together and dividing them by ten, and this is correct, provided each reading is a correct one; but the following is a far better and easier method. A long strip of paper, ½ inch wide (the length varies from 6 to 18 inches, according to the nature of the card), has a starting point marked on the edge near to one end. The strip of paper is then laid along the first tenth of space, and the length of the mean pressure marked off, not read off, starting from the starting point. It is then laid on the second space, and again the mean pressure is marked off, starting from the point where the first mean pressure ended, and so on to the last of the ten mean pressures. By this means the mean pressure in each of the ten spaces are laid end to end. If we now take a rule and read off how many inches there are in the whole length, and divide them by ten, we get the number of inches in the mean pressure of the whole card.

Suppose when all the mean pressures for each of the ten spaces, when added together, measured 6·95 inches, then $\frac{6·95}{10}$ is the measure in inches of the true mean pressure, and if the scale was $\frac{1}{32}$ inch, that is, 1 inch stood for 32 lbs., then ·695 inch stands for $·695 \times 32 = 22·24$.

Generally expressed, we multiply the total number of inches read off the strip by the scale, and divide by ten.

314. **Remarks on the Quality of the Vacuum.**—In designating the quality of the vacuum formed under the piston, there are two elements affecting it which ought to be noticed, viz., the weight of the atmosphere, and the temperature of the water in the condenser at the time the diagram was made. For, if the barometer stands at only 28 inches, 30 inches of mercury being equivalent to 14·7 lbs., 13·7 lbs. would be a perfect vacuum; and if the water in the condenser be at a temperature of 130°, its vapour will form a resistance of 2·17 lbs.; therefore, the lowest attainable vacuum

would be but 13·7 — 2·17 = 11·53 lbs.; whereas, if the barometer stood at 31 inches, a perfect vacuum would be 15·2; and if the water was but 100°, its vapour would give a resistance of only ·9, and, consequently, the highest attainable vacuum would be 15·2 — ·9 = 14·3 lbs., making a difference of 2·77 lbs.*

The table showing the force with which vapour or uncondensed steam resists the ascent or descent of the piston, according to the temperature of the water discharged from the air-pumps (Table II at the end of the work), and also the table of the variations of the weight of the atmosphere, as shown by the barometer (Table I at the end of the work), will make apparent the necessity of these observations.

315. Before concluding this subject, it is important to remark that in using an indicator for measuring work, that the distinction between indicated power—that is, the power developed in the cylinder—and "effective power," must never be lost sight of. The latter is the *useful effect* of the engine, while the former includes the power expended in working the pumps, overcoming the friction of the machinery, and all resistances which exist within the engine itself. In the marine engine it is very difficult to arrive at the amount of power really used in propelling a ship. That expended in driving the engine and pumps at different speeds can be approximated to by disconnecting the engines from the propeller shafting, and taking diagrams with the steam throttled down just sufficient for driving the engine at the required speed. Great care is necessary for starting the engines in taking these diagrams.

316. **Further Remarks on Indicator Diagrams.**—It is only by a proper employment of this inestimable instrument—the indicator—that tolerably true deductions as to the internal working of an engine can be arrived at with any degree of certainty—that a correct diagnosis can be formed as to the real state of the iron patient. Such being the case, it is, perhaps, surprising that more attention has not been directed to certain influences that, in some cases, undoutedly distort an indicator diagram. These disturbing actions may be greatly modified by the construction of the indicator itself; they may be increased or diminished by a proper or improper construction of the instrument; but these influences will always exist to a greater or less extent, and some allowance should be made on this account, before the diagram be accepted as a perfectly true picture of the performance of the steam in the cylinder.

(*a*) The steam engine indicator is especially liable to several objections when used for determining forces acting with great velocity and suddenness.

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* The vacuum shown by the indicator will generally vary from that shown by the vacuum gauge, when it is constructed with a glass tube, hermetically sealed at the top; for such gauges are designed to show the variation from a perfect vacuum, without reference to the weight of the atmosphere; but the vacuum shown by the indicator is affected by all its variations.

The inlet of the steam is generally throttled by the narrowed aperture of the cock, and the indicator piston in its sudden ascent has to meet the column of air above. In the ordinary instrument there is a long and tremulous spring, which, from its length and weakness, is somewhat liable to rub on one side of the casing. The working parts are also often made unnecessarily heavy. When steam of a high pressure is let into the indicator piston, the spring gives way and is suddenly thrown up; this action is aided by the momentum of the other moving parts; the pencil continues to oscillate during the remainder of the stroke; and the natural result is a jumping, serrated, zig-zag diagram (see Fig. 65, page 357). The total power indicated is, perhaps, not very far from exactitude, as the action and re-action are probably about equal; and as in interpreting the diagram *the true curve is supposed to run midway between the crests and hollows of the wavy line produced by these oscillations*—but the most important function of the indicator consists in the giving the true performance and progressive course of the steam in the cylinder, and it is evident that, under such circumstances, the lines of the diagram have not the accuracy that is demanded by the importance of the deductions the experimenter wishes to obtain.

(*b*) There can be little doubt that the larger the indicator the better. The friction of the piston and of the moving parts is less in proportion; the cock and pipe are larger, the steam is thus less liable to be throttled in its passage to the piston, &c., and the divisions on the scale and the diagram itself being of a larger size, there is less chance of error.[*] (See page 314).

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[*] The original indicators of Boulton and Watt were all of a large size, and were made with a large powerful spring, while the stroke of the piston was much shorter than in the ordinary indicator as made at present. It is stated that Watt obtained the scale of pounds by actual trial of weights pressing down the spring; but these results were taken while the indicator was cold, and therefore in a very different state, and in a different relation of the parts by measure than when the instrument was doing its work. It is evident that a more correct way would be to apply the weights on the top side of the piston while the indicator is fixed on the cylinder of an engine, the instrument being then in the same state as when performing its proper functions, the spring is of the same size and of the same elasticity as when in use, the whole apparatus on it at a similar temperature, and the piston is lubricated with the same quantity of oil.

The weights being, one after the other, applied to the top of the piston, the atmospheric line and the gradual descent of the piston are then recorded by the pencil of the instrument, and the principal divisions are thus obtained for a complete subdivision, and a true scale is the result. Perhaps the most exact way would be to use a properly graduated mercurial column. There would certainly be some difficulty with this plan, with an indicator for very high pressure. Mr. Parkes, the civil engineer, found that the pressures for the scale of the indicator were more correctly obtained by comparing them with a mercurial column than by ascertaining them by means of weights. "The instrument being then heated, was then in precisely the same state as when it was in use." He also found that a certain amount of correction was frequently necessary, as both the spring and the amount of piston friction were affected by heat.

According to Professor Mosely, the scale of ordinary indicators should not always be in equal divisions, because the wire of the spring being wound spirally into a screw of small diameter, the spiral obliquity of the thread of such a screw becomes "more oblique to the direction of the bending force as the spring is stretched, and less oblique as the spring is compressed, and hence the scale of pounds per square inch, by which the curve should be measured for summing up the results, ought to be a scale of unequal divisions."

(*c*) **The friction of the indicator,** by directly opposing the motion of its piston and pencil, tends to make the indicated forward pressure less, and the indicated back pressure greater, thus the real forward and back pressures respectively, and so to make the indicated energy less than the real energy exerted by the steam on the piston, but to what extent is very uncertain. According to some experiments by Mr. HIRN (*Bulletin de Mulhouse*, vols. XXVII and XXVIII), the diminution of the indicated energy by the friction of the indicator agrees nearly with the work performed in overcoming the friction of the steam engine, so that the indicator shows, not the whole energy exerted by the steam on the piston, but very nearly the *useful work* of the steam engine; but it is doubtful how far this principle is generally applicable; and other experiments, especially those on screw steamers, are at variance with it.

(*d*) As is well known, the part of the engine giving motion to the string of the indicator ought always to be as near the instrument as possible, as otherwise the string will slacken, and the barrel will thus be stationary for an interval, of course, the result being a wrong diagram. As it is of such great importance that the string attached to the indicator barrel should neither be too slack nor too tight and liable to stretch, it has been suggested that it might be advisable to use a kind of string worked up with fine wire, similar to the tape measures that are sold for measuring lengths of 30 or 40 feet.*

(*e*) There is also sometimes a danger to the exactitude of the diagram, from the steam getting condensed. The most convenient way of taking a double diagram, as already stated, is by fixing the indicator to a pipe leading from the top to the bottom of the cylinder. This pipe being fitted with stop cocks, the two diagrams can be taken simultaneously. The great objection, however, to this plan, consists in the natural tendency of the steam to get condensed in this small pipe.

(*f*) It appears that the steam pressure undergoes some fall during the passage from the boiler to the cylinder. The amount of such fall varies greatly in different engines, but the general result is that the highest indicated steam pressure, before expansion begins, is some two or three pounds less than the boiler pressure.

Among the points to be noticed are (1) the resistance of the steam-pipe through which the steam passes, (2) the resistance of the regulator or throttle-valve, (3) the resistance due to the ports and steam passages; and here, also, the bends or sharp angles as well as the imperfect coating of the steam-pipe must be taken into account.

RANKINE says that in the present state of our knowledge it is impossible to calculate separately the losses of pressure due to these causes, and if it were possible the resulting formulæ would be too complicated to be of much use. An observation of this kind has a wide application. It may be pointed out that steam which has been lowered in pressure by the resistance of passages, or has been *wire-drawn*, as we have termed it, is to some extent *superheated* by the friction of its molecules, the tendency of all friction being to produce heat.

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* Mechanics' Magazine, vol. 9, New Series.

(*g*) There is, in practice, a rounding of the angle at which the expansion curve begins in the theoretical diagram. This is called *wire-drawing at cut off*. It is always to be seen where a slide-valve closing gradually is employed, and is reduced to a minimum in a well formed diagram of a Corliss engine. Speaking generally, it may be said that the steam begins, as it were, to work expansively a little before the valve is completely closed; or that the energy exerted is nearly the same as if the valve had closed instantaneously at a somewhat earlier period of the stroke, which point may be termed the effective cut off. Such a point is easily obtained by carrying the expansion curve a little higher, and by prolonging the probable steam line to meet it.

(*h*) There is a rounding of the expansion curve when release begins before the end of the stroke, and it is recommended that the point of release should be so advanced that one-half of the fall of pressure takes place at the end of the forward stroke, and the other half at the beginning of the return stroke. When the release is small the expansion curve is continued to the end of the diagram, and in such a case the exhaust line slopes gradually downwards as the piston returns, instead of being nearly horizontal.

(*i*) The general effect of water in the cylinder, from whatever cause produced, but which we will suppose to be present in some degree throughout the stroke, is to lower the steam line in the first portion of the stroke, and to raise it in the latter portion. On this subject it is very easy to propound theories, but the subject-matter lies so much within the region of experiment, that any theoretical deductions, which are nearly all that we can be said to have at present, may indeed be interesting, but would perhaps admit of being classed among conclusions in which nothing is concluded.

(*j*) Another cause which affects the form of the expansion curve, and which has been the subject of much observation, is that of the partial liquifaction of the steam at the beginning of the stroke, and immediately after being cut off; and again, the re-evaporation of this condensed steam towards the termination of the stroke. This is most noticeable in engines in which the steam is greatly expanded. The effect is shown upon the diagram by the lessening of the initial pressure by the falling of the pressure immediately after cut off below the hyperbola, and again by the rise of the pencil above the hyperbolic line towards the end of the stroke. A partial explanation appears to be that the incoming steam is cooled and partially liquified by contact with the sides of the cylinder and passages, which have just been cooled by communication with the condenser, and that towards the end of the stroke the steam is re-evaporated by the heat conveyed by the sides of the cylinder, which is then at a high temperature as compared with that of the expanded steam. ZEUNER, however, a well-known investigator, has given calculations to show that the approach of the line to the hyperbola towards the end of the stroke is affected by the initial condensation of the steam as to dryness approaching it the more nearly the greater the quantity of moisture it originally contained. ZEUNER concludes, therefore, that the action of the sides of the cylinder has been over-rated,[*] and there are practical engineers who ascribe the rise of the pressure solely to leakage through the valves, contending also that it is absurd to suppose that the amount of heat transmitted to the mass of steam and water in so short a time would re-evaporate it to such an extent as to account for this phenomenon. It is to be remembered, however, that leakage through the pistons, in this case, tends to counteract the effect of leakage through the valves, and the rise of pressure is so very marked in engines of the best make, when working with great expansion, that there can be little doubt, after making allowance for other disturbing causes, that the steam is affected in the manner stated above.

In addition to the action of the sides of the cylinder, the presence of water due to the liquifaction of steam during expansion also affects the diagram, and with respect to this Prof. RANKINE states:—That liquifaction does not, when it first takes place, directly constitute a waste of heat or of energy, for it is accompanied by a corresponding performance of work. It does, however, afterwards, by an indirect process, diminish the efficiency of the engine; for the water which becomes liquid in the cylinder, probably in the form of mist and spray, acts as a distributer of heat and equalizer of temperature, abstracting heat from the hot and dense steam during its admission into the cylinder, and communicating that heat to the cool and rarefied steam which is on the point of being discharged, and thus lowering the initial pressure and increasing the final pressure of the steam, but lowering the initial pressure much more than the final pressure is increased, and so producing a loss of energy which cannot be estimated theoretically.

(*k*) Clearance will modify the form of the expansion curve of steam by removing backwards through a small space the zero line of volumes. And, as we have seen, if the steam be completely exhausted from the cylinder during the return stroke, the effect of clearance is to waste during the double stroke. But, inasmuch as it is possible to compress a portion of steam in the cylinder during the return stroke, the loss above referred to may be greatly,

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[*] ZEUNER in a paper, "Ueber die Wirkung des Drosselus und den Einfluss des schädlichen Raumes auf die bei Dampf-maschinen verbranchte Dampfmerge," Civil Ingénieur, vol. XXI, p. 1, published in 1875, states his belief that a more powerful cause must be found for some of the effects observed, and finds it in certain consequences of wire-drawing. See further on this subject "The Steam Engine considered as a Heat Engine." By JAMES H. COTTERILL, M.A., chapter X.

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or perhaps wholly, eliminated. On this subject **Rankine** recommends tha the point of compression should be adjusted in such a manner that th quantity of steam confined, or cushioned, should be just sufficient to fill th clearance with steam at the initial pressure when the piston comes to rest In such a case the work expended in compression is restored again durin expansion, and the steam spring is continually reproduced without waste.

(*l*) There are, however, disturbing influences in the diagram of an indicato that are more beyond the control of maker and user of this useful instrument and which do not seem to have been, as yet, thoroughly investigated by th writers on, and experimenters with, the steam engine indicator. We allude t the partial vacuum by the motion of a fluid and at right-angles to its course.*

This action may be easily and familiarly illustrated by placing a sheet o paper on the inside of the hand, and blowing from the back of the han through the spaces between the fingers. The paper, instead of being blow away, as might be expected, is kept up against the palm of the hand by th partial vacuum between the palm and the paper, caused by the rapid an diverging current of air. This would seem to point to the necessity o placing the indicator on the cylinder at such a place, that it may be as littl as possible influenced by the motion of the steam, out of the ports into th steam-pipes† deteriorating influence on an indicator diagram would be mos likely to occur on the vacuum side of a condensing engine, from the gre speed of the steam rushing into a vacuum, than on the steam side. It woul not improbably affect, to some extent, the diagram of a high pressure engin working at high speed. There is another enquiry, no less interesting, as t the exact effect on a diagram of the speed of the piston. There is, o course, always sufficient time for the steam to follow the piston at full pressure if the piston moves slow enough; but, at a higher speed, there i an undoubted reduction in pressure of the steam as it expands behind th piston.

(*m*) Again, where a pair of coupled engines exhaust into one blast pipe, there is probably a reciprocative intermittent action of the back pressure of the two exhausts on the exhausting sides of the piston.

The exact value of the objections just raised against the accuracy of this important instrument could only be ascertained by means of a series of carefully conducted experiments, that those objections are more or less tenable will be evident on reflection.

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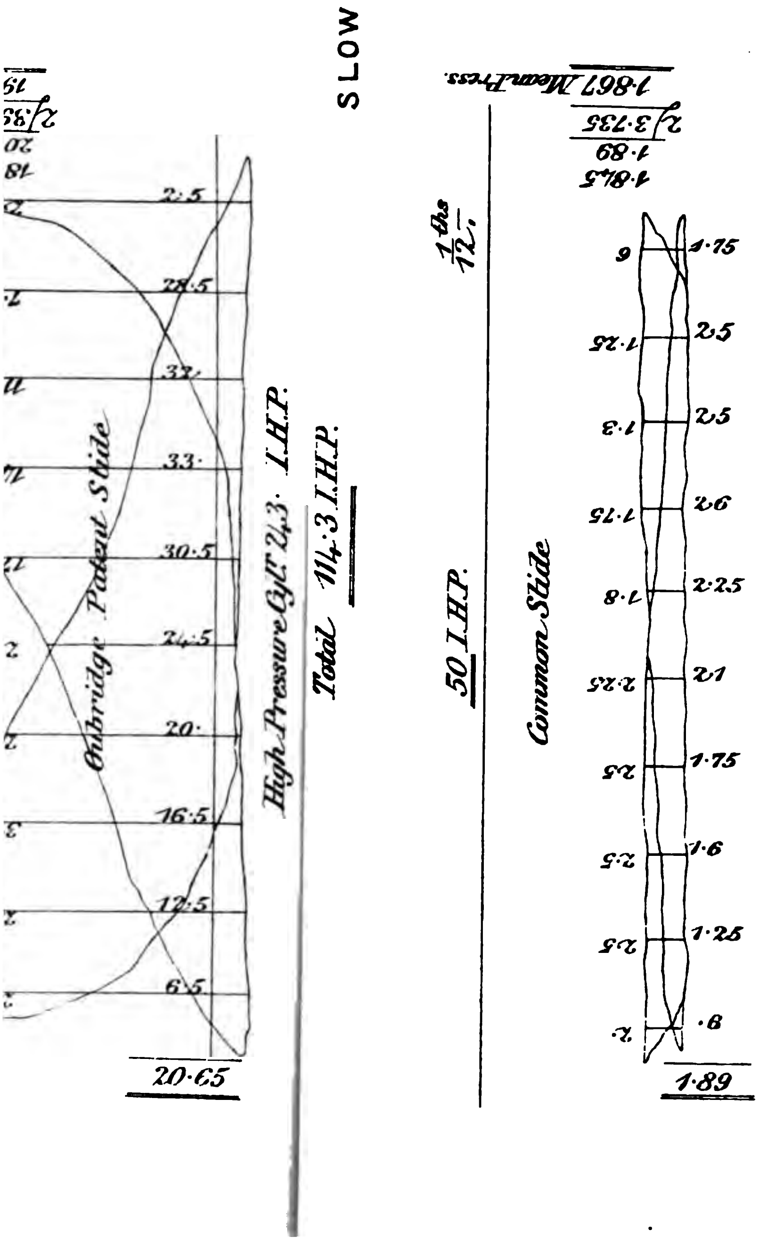
* We believe that this phenomenon was first noticed by Mr. **Richard Roberts**, the well-known engineer, while making some experiments with a blowing engine in Manchester, in the course of the year 1826.

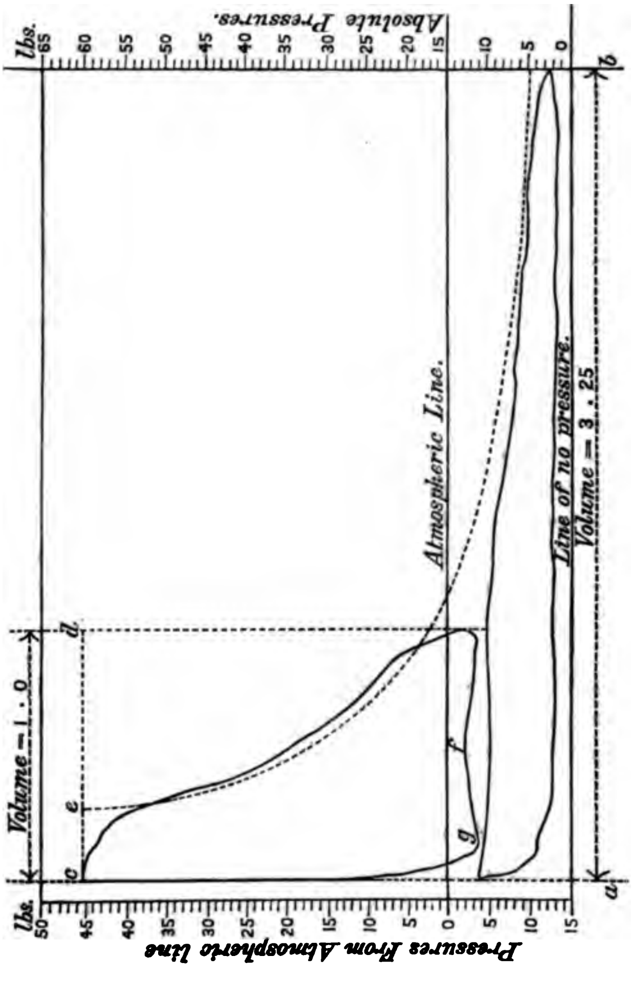
† Experiments by Messrs. **Randolph, Elder**, and Co., have proved what might have been expected from the laws of fluid motion, that when a rapid current of steam blows across the orifice of the nozzle of an indicator, the indicated pressure is less than the real pressure. Every indicator, therefore, should be fixed, if possible, in a position where it is not exposed to this cause of the error.


SLOW
Oubridge Patent Slide
High Pressure Cyl.r 24.3 I.H.P.
Total 114.3 I.H.P.
2.5
28.5
32
33
30.5
24.5
20
16.5
12.5
6.5
20.65
18
20
2)35
19
50 I.H.P.
1/12 ths
Common Slide
1.845
1.89
2)3.735
1.867 Mean Press.
9
1.25
1.3
1.75
1.8
2.25
2.5
2.5
2.5
2.
1.75
2.5
2.5
2.6
2.25
2.1
1.75
1.6
1.25
.8
1.89

317. The action of the steam in Compound Engines is best understood by combining the diagrams from the high and low pressure cylinders, and we have in Fig. 103 combined the diagrams from the *Elbe's* engines.*

Here the length *ab* represents the volume of the low pressure cylinder, and consequently the volume of the steam before being discharged into the condenser, and the length *cd* of the upper diagram represents the volume of the high pressure cylinder, while the portion *ce* of this volume represents the portion of the stroke of the high pressure cylinder during which steam is admitted. The ratio of the volumes in this case is 3·25 to 1, and the lengths

Fig. 103.



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* The diagrams are taken from a set forwarded by the firm of Messrs. John Elder & Co. as specimen diagrams to the Committee on Admiralty Designs. See also Figs. 72 and 73, page 364 of this work.

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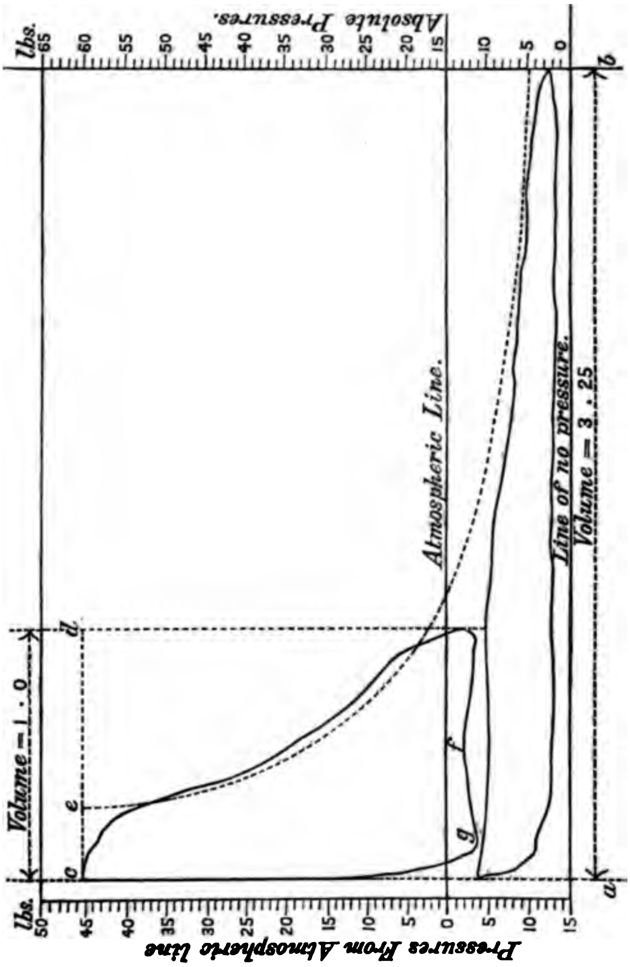

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of the diagrams, as shown, have the same proportion. The pressures being to the same scale, it will be understood that the areas of the diagrams will represent the relative amount of work performed in each cylinder. Also, as ce represents the initial volume of the steam as compared with its final volume, the initial absolute pressure also being given, the hyperbolic curve can be constructed, which is shown dotted in Fig. 103, so as to test the performance of the engine as if the steam were expanded in a single cylinder. Bearing in mind the fact that the areas of the diagrams represent the work done in the cylinders, it will be evident that if the steam were quite effective, the sum of the areas of the two diagrams would be equal to the area bounded by the hyperbolic line; it will be seen that it is the case in this diagram.

A consideration of the diagrams, Fig. 103, will show "that the total ratio of expansion, that is, the ratio of the final volume to the initial volume, or $\frac{ab}{ce}$, while dependent on the capacity of the low pressure cylinder, is independent of the capacity of the high pressure cylinder, so long as this is capable of holding the quantity of steam ce. Other considerations apart, therefore, the volume of the high pressure cylinder need not exceed ce. Any additional capacity might, in fact, be dispensed with, and this is accordingly done in many cases, the steam being carried approximately for the whole length of the stroke."* Referring to Fig. 103 it will be seen that there the steam is expanded, performing work in the high pressure cylinder until the fall of pressure at the end of stroke is the cause of no serious loss as the steam flows into the receiver. Owing to the position of the cranks, which are placed at right-angles, the steam is compressed in the receiver during the return stroke of the high pressure piston until the point f (approximately half stroke) is reached, when steam is admitted to the low pressure cylinder. The effect is shown in the diagram by the increase of the back pressure to the point f, when it decreases until the point g is reached, where the exhaust closes and compression in the high pressure cylinder begins. Although the provision of a special receiver has generally been considered necessary in this class of engines, it is not essential, and in some engines it has been dispensed with, the intermediate spaces between the two cylinders being only of sufficient capacity to admit of the free passage of the steam.

AIR-PUMP, &c., DIAGRAMS.

The indicator, though usually only applied to the cylinder, may be applied to the air-pump, the hot-well, and other parts of the engine, to show the nature of the pressures subsisting with them. When applied to the hot-well there is usually found to be at sometimes pressure and at sometimes exhaustion. The principles of the indicator diagram taken off pumps are precisely the same as those from the steam cylinder; but the mechanical construction is entirely different. In this case the motion for the pencil when showing the pressure of the water is *rising* during the motion of the paper barrel, whereas with steam it is stationary. To make this matter clear, "imagine a pump with the piston at one end, and the barrel *filled* with water; the action of the piston on moving is to compress the water until the discharge valves open, when the water will be driven out with a velocity due to that of the piston."

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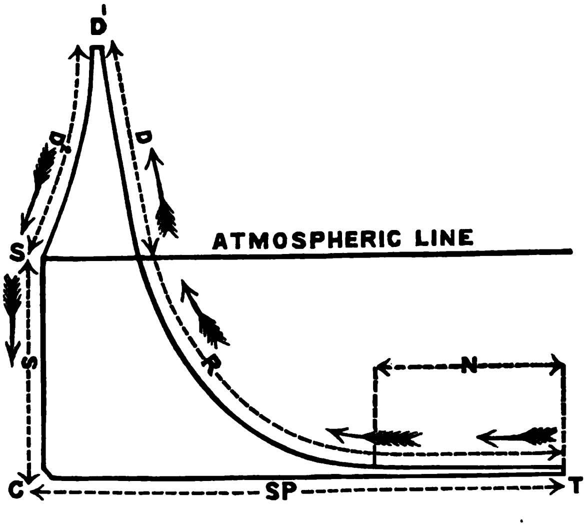
* *Naval Science*, vol. II, page 470.

"Again, assume that the same piston is at the end of its stroke, but the barrel only *partially* filled with water, the result will be that on the piston moving for a certain distance there will be *no* pressure, because the water is being merely pushed, and maintaining its level due to its gravity during that time; but immediately the water fills the space in front of the piston then the pressure occurs, because the water is lifted above its natural level."

As a practical introduction we introduce the diagram Fig. 104.*

Fig. 104.



S is the starting point; *s* the vacuum limit; SP the stroke of piston; N down pressure; R denotes return stroke; D denotes compression and commencement of discharge; D^1 denotes full discharge; C is commencement of stroke; T is termination of stroke.

This diagram was described by the pencil under the following circumstances: —First, the paper vibrated and the atmospheric line was described. The communication from the air-pump was then formed by turning the stop-plug, the pencil then instantly fell to *s*, in consequence of the piston of the pump forming a *suction* directly it moved, and thus causing the atmosphere to force the indicator piston down to the point of resistance. The pencil at this point becomes stationary, and was kept in that position by the atmospheric pressure during the stroke of the pump piston, marked SP in diagram, or the length from C to T. This piston then returned and forced the water directly in front of it in the barrel for the length of N, which shows that during that portion of the stroke there was but little, if any, pressure in the pump. The pencil having traced the (nearly) parallel line for the length of N, it rose and formed the remainder of the line R, while the piston of the pump moved until it reached the atmospheric line.

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* The Indicator Diagram practically considered, by N. P. Burgh. 5th Edition. London: C. & F. N. Spon.

It may perhaps be necessary, in further explanation, to state that while the pencil formed the line R, the water in the pump and in front of the piston constantly maintained its *natural* level; but as there was some *air* amongst the water, that vapour rose as the liquid was disturbed, and thus caused the pressure to be *below* the atmospheric line.

The compression and discharge of the water commenced, and thus the *lead* on the piston caused a further pressure under the indicator piston which forced the pencil to the height of D. Here it stopped, and the pressure maintained its level for a moment, which allowed the pencil to form D¹. The main load on the piston was here overcome, because the bulk of the water left was a less quantity than what had been discharged; then, as the pressure decreased, the pencil descended and formed the line D², and it then resumed its original position at S. The arrows marked in the diagram indicate the motion of the pencil.

Fig. 105.

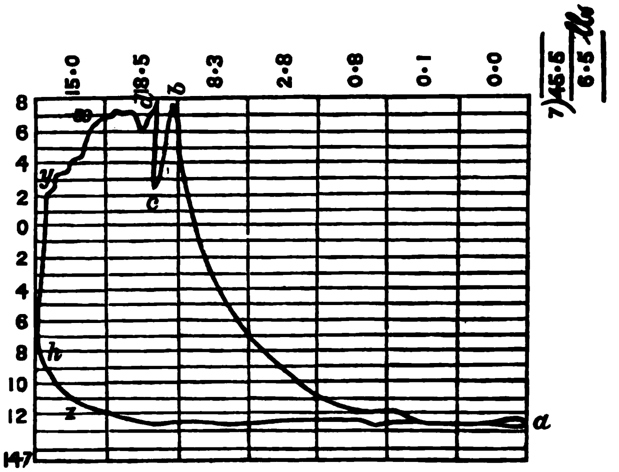


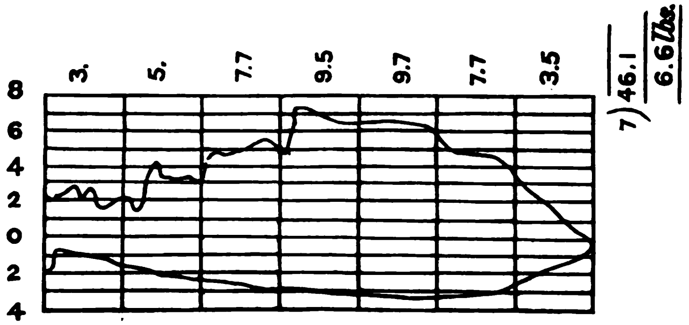
Fig. 105 is a diagram taken from a steamer's starboard air-pump, which is of the lifting kind, and the piston fitted with a large brass conical valve. The following is an explanation of the diagram:—At *a*, the piston being at the bottom of the stroke, starts to rise, compressing the air and vapour above it, until it arrives at *b*, at which place a sudden discharge of air and vapour seems to have taken place, and the pressure fell to *c*, from which point the pressure again gradually rose until it arrived at *d*, where the water began to be delivered, and continued to the end of the stroke.

Attached to the top of the air-pump is a pipe, running down into the bilge, for the purpose of pumping off the bilge water. Where this pipe is attached to the pump is fitted a valve, operating like an ordinary check-valve, a handle being made to screw down on the top of it to keep it firmly in its seat when there is no water in the bilge.

Steam 15 lbs.; revolutions 10; hot-well 106°; vacuum gauge out of order.

Starboard air-pump.

Fig. 106.



Resistance of vapour and water in air-pump = (6·6 + 312 × ·0969 =) ·6697 lb. per square inch of steam piston.

Calculated for ⅓ full of water.

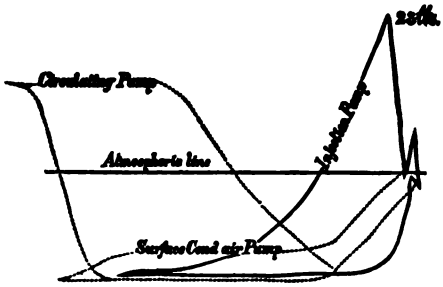
There being no water in the bilge at the time Fig. 105 was taken, Fig. 106 was taken five minutes after, for the purpose of ascertaining what effect the opening of this valve and admitting air would produce. It shows that no extra power, from the admission of this air, was required to work the pump, the average pressure being about the same as in Fig. 105, and that the vacuum in the pumps at no time was more than 4½ lbs. There was no alteration in the vacuum, as shown by the gauge, however, attached to the condenser, and the engines continued to work in the same manner as before the bilge-valve attached to the air-pump was opened.

In these diagrams, the pressure at the termination of the up stroke, it will be seen, is about 2½ lbs. per square inch above the atmosphere, which is due to the height of the level of the water surrounding the ship above the top of the air-pump. The pressure increased to between 7 and 8 lbs. per square inch, as shown in other parts of the diagram, is occasioned by the friction of water and vapour through the delivery-pipes and valves. The slanting off in the diagram, Fig. 105, from *x* to *y*, we think partly owing to two causes: First, the decreased velocity of the piston as it approaches the end of the stroke does not expel the water with such force, and hence there is not so much friction; but this would not occasion the slanting off from *h* to *s* on the return stroke; and secondly, therefore, we are inclined to think that the string slipped or stretched a little from *s* to *y*, and recoiled again to its original place from *h* to *s*.

Fig. 107 is an illustration of diagrams from vertical pumps, and shows the difference in the discharge pressure. It will be seen that the injection-pump diagram shows a rapid discharge up to a pressure of 23 lbs. from the vacuum line.

The diagram from the circulating-pump indicates a continuous discharge, and that there was little clearance for the water in the barrel before pressure occurred, as the pressure or angular line commences at the beginning of the diagram, thus showing that the water began to leave the barrel at about one-third of the stroke of the piston instead of two-thirds, as in the injection-pump. The reason of the discharge limit line being horizontal for such a length is, that the piston had to force the water through the tubes of the condenser, and thus the resistance was continuous during that process.

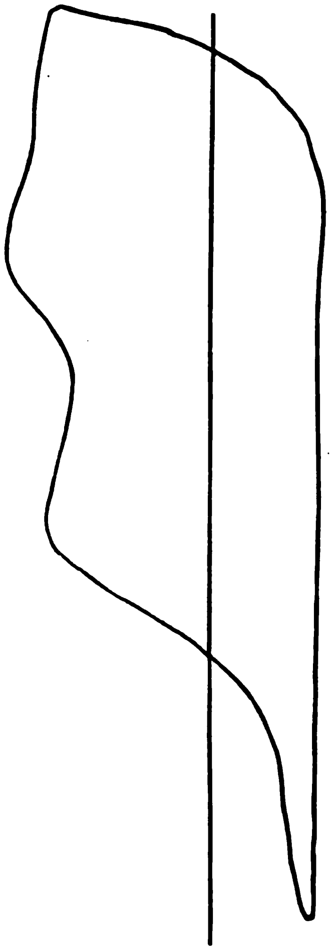
Fig. 107.



Double-acting pumps. Vertical motion. Screw engines.

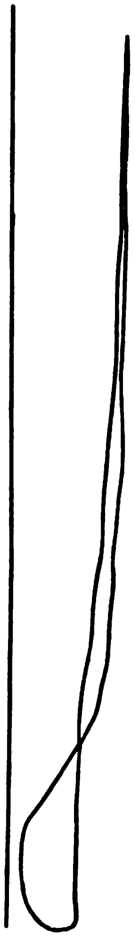
The surface-condenser air-pump diagram indicates below the atmospheric line, and shows that the discharge was never above the atmosphere, and that at the return stroke there was a little pressure in the barrel, as indicated by the angular line.

Fig. 108.



December 25th, 1874. S.S. *Sibylla.* Full gear; Throttle-valve full open; Kingston 1¼ open.
Barometer 29·80. Scale $\frac{1}{16}$.
Circulating Pump. Top.
Steam 68 lbs.; Vacuum 26; Rev. 60.

Fig. 109.



December 25th, 1874. S.S. *Sibylla.* Full gear; Throttle-valve full open; Kingston 1¼ open.
Barometer 29·80. Scale $\frac{1}{16}$.
Circulating Pump. Bottom.
Steam 68; Vacuum 26; Rev. 60.

We now proceed to ascertain the

Power required to work the Air-pump.

318. Ascertain the capacities of the steam cylinder and air-pump, by multiplying the areas of their cross-sections by the lengths of their stroke, and divide the latter by the former, which will give the ratio of the cylinder capacity to that of the air-pump. But the air-pump makes but *one* delivery stroke to every *two* strokes of the steam piston, consequently, divide this ratio by two, which will give the co-efficient for our present calculation, and this co-efficient, multiplied by the mean pressure per square inch of air-pump

Ex. 2. The required saturation of the water is $\frac{3}{32}$; then, since the saturation of the water in the boiler is trebled, it must have received $\frac{2}{32}$ from the evaporated water; so that in this case we blow off $\frac{1}{3}$ of the water that enters the boiler, and the quantity blown off equals $\frac{1}{2}$ of the quantity evaporated; or,

Water blown off : water evaporated :: 1 : 2 $= \frac{1}{2}$.

Similarly, we may show that—

If required saturation $= \frac{4}{32}$, water blown off $= \frac{1}{4}$ of water entering the boiler, and

Water blown off : water evaporated :: 1 : 3 $= \frac{1}{3}$.

Ex. 3. If required saturation $= \frac{5}{32}$, water blown off $= \frac{1}{5}$ of water entering the boiler, and

Water blown off : water evaporated :: 1 : 4 $= \frac{1}{4}$.

Ex. 4. If required saturation $= \frac{1\frac{3}{4}}{32}$, water evaporated $= \frac{3}{4}$ of water blown off, and

Water blown off : water evaporated :: 1 : $\frac{3}{4}$.

Ex. 5. If required saturation $= \frac{1\frac{1}{2}}{32}$, water evaporated $= \frac{1}{2}$ of water blown off, and

Water blown off : water evaporated :: 1 : $\frac{1}{2}$.

Ex. 6. The water in a boiler connected with a jet condenser shows $\frac{1\frac{7}{8}}{32}$: what proportion of the feed is used as steam?

If 1 be the quantity blown off, then $1\frac{7}{8}$ will be the full feed and $\frac{7}{8}$.

Then the answer to the question will be the ratio $\frac{7}{8}$ to $1\frac{7}{8}$; clearing the terms of the ratio of fractions by multiplying by 8, we have $\frac{7}{8} \times 8 = 7$, and $1\frac{7}{8} \times 8 = 15$,

$\therefore$ the ratio is 7 to 15, *i.e.*, 7 gallons out of 15 gallons of feed.

The water in the boiler is $\frac{2\frac{7}{8}}{32}$: what proportion of the feed is used as steam?

Answer. 7 gallons out of 15 gallons.

The water in the boiler is $\frac{2\frac{1}{2}}{32}$: what proportion of the feed is used as steam?

Answer. 3 gallons out of 5 gallons.

Ex. 7. The boiler contains 50 tons of fresh water on starting on a voyage; the water in the hot-well, through some leak, is $\frac{1}{8}$ of the saltness of the sea: how many tons of water must have been evaporated during the voyage, if at the end of the voyage the boiler has $2\frac{1}{2}$ times the saltness of the sea, there having been no blow-off?

The number of times the boiler was filled $= 2\frac{1}{2} \div \frac{1}{8} = 20$.

The weight of water $= 50$ tons $\times 20 = 1000$ tons. *Answer.*

QUESTIONS RELATING TO TEMPERATURE.

EXAMPLES.

Ex. 1. Fahrenheit's thermometer is marked 32° at freezing point, and 212° at boiling point, while Centigrade is marked 0° at freezing and 100° at boiling: what mark on Fahrenheit should agree with 89° on Centigrade. (See article on "Thermometers, &c.")

There are 180° between freezing and boiling point on Fahrenheit.
„ 100° „ „ Centigrade.

Now this temperature, 89° C., is 89° above the temperature of the melting point of ice. Find how many of the F. degrees will correspond to this by the proportion

$$100° : 180° :: 89° : 160°{\cdot}2 \text{ F., the answer.}$$

$$100° : 180° :: 89°$$

$$x = \frac{\overset{9}{\cancel{180}} \times 89}{\underset{5}{\cancel{100}}} = \frac{801}{5} = 160°{\cdot}2 \text{ F. above freezing point.}$$

$$\begin{array}{r} +\ 32 \\ \hline 192{\cdot}2 \text{ F.} = \text{required reading.} \end{array}$$

To this 160°·2 F. degrees above the freezing point add 32°, and we get 192°·2 F. as the required reading.

Ex. 2. What mark on the Centigrade scale will indicate the temperature of 47° Fah.

Here 47° F. means 15° of Fahrenheit's degrees (47° — 32° = 15°) above the melting point of ice. Find the number of degrees on the Centigrade scale which corresponds to 15 on the Fahrenheit scale by this proportion 180° : 100° :: 15° : 8°·3 the answer.

$$180° : 100° :: 15° : x$$

$$x = \frac{\overset{5}{\cancel{100}} \times 15}{\underset{9}{\cancel{180}}} = \frac{75}{9} = 8°{\cdot}3$$

That is, 47° F. is the same as 8°·3 of Centigrade degrees above the freezing point, or 8°·3 C.

For further exercises on the changing of the readings of the different thermometer scales see article "Thermometers."

Ex. 3. The sea water is at 54°, and the water in the hot-well is at 115°; if the sea water gets to 74°, and if you cannot put on more injection, what will now be the temperature of the hot-well?

74° — 54° = 20°. Then 115° + 20° = 135°. *Answer.*

Ex. 4. What weight of water at 60° F. must be mixed with 20 lbs. of steam at atmospheric pressure to produce water at 120° F.?

The formula for working the above is obtained as follows: —

Let x lbs. of steam at a temperature t be injected into y lbs. of water at a temperature t', and let the resulting temperature of the condensed steam and water be T. Also let H be the number of units of heat in steam at a temperature t, reckoning from zero of Fahrenheit's scale.

The steam loses x (H — T) units of heat, and the water gains y (T — t') units of heat.

$$\therefore x(H - T) = y(T - t).$$

$$\text{Here } x = 20,\ t = 212,\ T = 120,\ H = 1178{\cdot}6,\ t = 60.$$

$$\therefore y(120 - 60) = 20(1058{\cdot}6) \ \therefore y = 352{\cdot}87 \text{ lbs.}$$

Ex. 5. **If 37 lbs. of water at 32° Fah. be added to 43 lbs. at 212° Fah.: what will be the resulting temperature?**

Thermal units in the 1st quantity = 37 lbs. × 32° = 1184
" " 2nd " = 43 × 212° = 9116

Total = 10300

Weight of the mixture = 37 lbs. + 43 lbs. = 80 lbs.
∴ Temperature of mixture = 10300 ÷ 80 = 128·75 lbs.

Ex. 6. If 1 lb. of steam raises the temperature of 1000 lbs. of water 1°, **the sea water is** 50°, and the discharge water is 110°: how many pounds of circulating water **passes the condenser** for each pound of steam?

Temperature of discharge water 110°
" sea water 50
Difference 60

6,0)100,0 lbs.
16·66 lbs. *Ans.*

Ex. 7. **What is the pressure per square inch when the temperature is 75° Fahrenheit, and the barometer shows 29·4 inches of mercury?**

Formula $\frac{4907 \times h}{9057 + t}$ $\frac{4907 \times 29\cdot4}{9057 + 75} = \frac{144265\cdot8}{9132} = 15\cdot798$ lbs. *Ans.*

Ex. 8. **The temperature in the funnel is 593°, with a consumption of 25 tons per day; after a few days it increases to 29 tons per day to raise the same amount of steam: what is now the temperature of the funnel, nothing else having altered; and what is the cause of this increased consumption?**

Note.—If the cost of evaporation be increased by (*n*) per cent., the alteration in the temperature of the escaping gases will be increased by 22 (*n*) degrees.

29 tons.
25 "
Increase = 4 "

Here the increase is 4 tons on 25 tons; **what is this per cent.?**

25 : 100 :: 4 : *x*
4
25)400(16 per cent. = *n*.
25
150
150

∴ Increase of temperature = 16 × 22 = 352°
Original temperature 593
Present temperature 945

Ex. 9. Suppose the temperature of the injection water to be 60°, **steam as it enters the** condenser 212°, and water in the condenser at 110°: required the proportion of **injection water to the water evaporated in the boiler.**

1178°·6 = **total heat of steam at the sensible temperature of 212°.**
110°·0 = **temperature of the water after condensation.**

1068°·6 = **heat to be destroyed.**

110° = **temperature of the water after condensation.**
60° = **temperature of the injection water.**

50° = **difference.**

Now, then, we have 1068°·6 of heat to be destroyed, and only 50° to do it with; therefore, we must make up this difference in quantity; hence 1068°·6 ÷ 50° = 21°·372 times the evaporated water to be admitted into the condenser to condense the steam and keep the condenser at the temperature of 110°.

Ex. 10. If the sea water be at 60° Fah., and the water in the hot-well at 100° Fah.: how many lbs. of sea water will be required to condense 1 lb. of steam?

Rule. $\frac{1149^{\circ}\cdot 6 - t'}{t' - t}$, where t = the temperature of sea water,
and t' = ,, hot-well.

$$\frac{1149^{\circ}\cdot 6 - t'}{t' - t} = \frac{1149^{\circ}\cdot 6 - 100}{100 - 60} = \frac{1049\cdot 6}{40} = 26\cdot 24 \text{ lbs.}$$ *Ans.*

Ex. 11. The total heat of steam may be found from the following formula, $1115^{\circ} + \cdot 3\ T^{\circ}$, where T is the temperature or sensible heat of the steam. (See chapter on "Properties of Steam)."

Example.—What is the total units of heat in steam of 212°?

$$1115^{\circ} + \cdot 3 \times 212^{\circ} = 1115^{\circ} + 63^{\circ}\cdot 6 = 1178\cdot 6,$$ the total heat.

Ex. 12. And what will be the latent heat in this case?

Total heat = 1178·6
Sensible heat = 212

Latent heat = 966·6

Ex. 13. Find the total and latent heat of steam that is 60 lbs. by the gauge.

60 lbs. + 15 lbs. = 75 lbs. gross.

And the temperature of steam at 75 lbs. pressure is 309°,

$$\therefore\ 1115^{\circ} + \cdot 3 \times 309 = 1115^{\circ} + 92\cdot 7 = 1207^{\circ}\cdot 7$$ total heat.

$$1207^{\circ}\cdot 7 - 309^{\circ} = 898^{\circ}\cdot 7$$ latent heat.

Then if we know the temperature of the feed water, and subtract this temperature from the total heat of the steam, the remainder will be the units of heat to each pound of water turned into steam.

Or, $1115^{\circ} + \cdot 3\ T^{\circ} - t$ = units of heat required to turn 1 lb. of water into steam.

Ex. 14. If the steam in the boiler be at 309°, and the feed water be at 110°: how many units of heat will it be necessary to add to this water to convert a pound of it into steam?

$$1115^{\circ} + \cdot 3 \times 309 - 110 = 1097^{\circ}\cdot 7.$$ *Ans.*

Ex. 15. The temperature of the hot-well was 112°, and the evaporation was at the rate of 8 lbs. of water per 1 lb. of coal: what would be the evaporation if the temperature of the hot-well was increased to 150°?

Increase $\frac{F}{1100}$

$$\therefore\ \frac{150 - 112}{1100} \times 8 \text{ lbs.} = \cdot 276 \text{ increase,}$$

$$\therefore\ 8 + \cdot 276 = 8\cdot 276$$ the evaporation. *Answer.*

Ex. 16. The I.H.P. of an engine is 682, and the weight of steam per hour per I.H.P. is 21 lbs.; if 1 lb. of steam condensed gives out sufficient heat to raise 1000 lbs. of water 1° Fah., the injection water having a temperature of 61°, and is discharged at 104°: how many tons of injection water are used per day of 10 hours?

Temperature of discharge 104°
,, injection 61
43

Then $\frac{1000}{43} = 23\cdot 256$ lbs. of water to condense 1 lb. of steam.

$682 \times 21 = 14322$ lbs. of steam condensed per hour.

$14322 \times 23\cdot 256 = 333072\cdot 432$ lbs. of cold water per hour.

$$\therefore\ \frac{333072\cdot 432 \times 10}{2240} = 1486\cdot 93$$ tons of injection water used per day. *Ans.*

Ex. 17. If 1 lb. of coal evaporates 9 lbs. of water from and at 212° Fah.: how much will 1 lb. evaporate if the steam was at 320° Fah., and the feed water at 120° Fah.?

First, $1115^{\circ} + \cdot 3 \times 212^{\circ} - 212^{\circ} = 966\cdot 6$ units of heat to 1 lb. of steam,
Second, $1115^{\circ} + \cdot 3 \times 320^{\circ} - 120^{\circ} = 1091$,, ,,

$$\therefore\ 1091^{\circ} : 966^{\circ}\cdot 6 :: 9 \text{ lbs.} : x = 7\cdot 97 \text{ lbs.}$$ *Ans.*

Ex. 18. The temperature of the water in the hot-well is 120°, and the vacuum is $11\frac{1}{2}$ lbs., but by some accident the temperature of the water in the hot-well is increased to 150°: what should now be the vacuum?

$$\text{Decrease} = \frac{(T - t)\ (T - 50)\ (t - 50)}{100000}$$

$$\therefore \frac{150 - 120)\ (150 - 50)\ (120 - 50)}{100000} = \frac{30 \times 100 \times 70}{100000} = 2{\cdot}1 \text{ lbs.}$$

$$\therefore \text{new vacuum} = 11{\cdot}5 \text{ lbs.} - 2{\cdot}1 \text{ lbs.} = 9{\cdot}4 \text{ lbs.} \quad \textit{Answer.}$$

Ex. 19. On starting on a voyage the consumption was $8\frac{1}{2}$ tons per day, and the temperature of the funnel was 560°, but at the end of the voyage the temperature was found to be 690°: what was the increased consumption?

$\frac{F}{2200}$ = increase, where F = diff. of temperature × by the original consumption.

$$\frac{690 - 560}{2200} \times 8\tfrac{1}{2} \text{ tons} = {\cdot}502 \text{ increase of consumption.}$$

$$\therefore \text{present consumption} = 8{\cdot}5 + {\cdot}502 = 9{\cdot}002 \text{ tons.} \quad \textit{Answer.}$$

Ex. 20. If the loss by blowing off is ·11 of the fuel used, the temperature of the steam being 250°, and that of the feed being 110°: what should the boiler water be at above the feed density?

$$n = \frac{T - t}{l\ (1115^\circ + {\cdot}3\,T - t} + 1$$

$$= \frac{250 - 110}{{\cdot}11\ (1115^\circ + {\cdot}3 \times 250 - 110)} + 1 = \frac{140}{118{\cdot}8} + 1 = 2{\cdot}17. \quad \textit{Answer.}$$

320. In condensing engines, the steam is condensed (when it exhausts), either by being brought into contact with a jet of cold water, as in the case of a common jet condenser, or by passing through or around a series of tubes made cool by water being passed around or through them, which is the action when a surface condenser is employed. When a jet condenser is used, salt water is pumped into the boilers, but a surface condenser, if tight, saves the fresh water derived by condensation, and it can be returned to the boilers again and again. By this means nearly all the feed water is fresh, and but little blowing off is required to keep the water at a certain point of saturation. A calculation of the loss by blowing off will show us the gain derived from fresh water condensers, supposing them to be tight, and to condense all the steam used by the engines. We will show the method of calculating the loss by blowing off.

The number of degrees of heat that must be imparted to the water converted into steam will be the number in the total heat of the steam minus the number of degrees in the feed water. The heat lost by blowing off will be the difference between the temperature of the feed water and the sensible heat of the steam. A comparison of this loss with the entire number of degrees of heat required, being the sum of the heat imparted to the steam, and that to the water blown off, will give the per cent. of loss.

EXAMPLES.

Ex. 1. What is the per cent. of loss by blowing off, supposing the water entering the boiler to be $\frac{1}{32}$, and that the water is carried at a saturation of $\frac{2}{32}$; supposing also the temperature of the water entering the boiler to be 100° Fah., and the temperature of the water in the boiler to be 248°. (The total heat in steam, having 248° for the sensible heat, is 1189°·58).

1189°·58 = total heat.
100°·00 = temperature of water entering the boiler.

1089°·58

But since as much water is made into steam as is blown off

1089°·58 × 1 = 1089°·58 = heat required for the water to be evaporated.
248° = temperature of the water in the boiler.
100 = " " entering "

148° = heat lost by blowing off.

Therefore, since one part (requiring 1089°·58) is converted into steam, and the other part (requiring 148°) is blown off, the total heat imparted to the water is (1089°·58 + 148° =) 1237°·58, and as 148° of this is blown off, we have

1237°·58 : 148° :: 100 : 11·95 &c., per cent. of heat lost by blowing off.

Ex. 2. What is the per cent. of heat lost carrying the water at a density of $\frac{1\frac{3}{4}}{32}$, the other data being the same as in the preceding example?

Here only $\frac{3}{4}$ as much water is converted into steam as is blown off, so that

1189°·58 — 100° = (1089°·58) × $\frac{3}{4}$ = 817°·185 = heat required from the fuel for the water to be evaporated.

248° — 100° = 148° = heat lost by blowing off.

817°·185 + 148° = 965°·185 = total heat imparted to the water.

Therefore, 956°·185 : 148° :: 100° : 15·33, &c., per cent. of heat lost by blowing off.

Ex. 3. What is the per cent. of loss by blowing off, where the water which enters the boiler at $\frac{1}{32}$, having a temperature of 110°, and is carried at a density of $\frac{3}{32}$, and having a temperature of 260°. (The total heat of steam having 260° for the sensible heat is 1193°·45.)

1193°·45 — 110° = 1083°·45 = heat imparted to the water converted into steam.

But twice as much water is made into steam as is blown off.

1083°·45 × 2166°·9 = heat producing effect for every 260° — 110° = 150° heat lost by blowing off.

2166°·9 + 150° = 2316°·9 = total quantity of heat imparted to the water.

2316°·9 : 150° :: 100 : 6·47 per cent. of heat lost by blowing off.

These per centages are the losses in fuel, combustible, minus that lost by radiation and heated gases passing up the chimney.

The above calculations apply only to cases when the water enters the boiler at a density of $\frac{1}{32}$; should it enter at a lower density, the loss will be less, or a greater density more, because to retain the water at the density assumed in the above examples, there would either have to be a less or greater quantity blown off than we have considered to be the case.

321. *Gain by pumping water into the boiler at an increased temperature.*

EXAMPLES.

Ex. 1. Steam 248°, feed water 100°, and density $\frac{2}{32}$: now suppose the feed water, instead of entering the boiler at 100°, is made to enter at 150°: what will be the saving of fuel by so doing?

1189°·58 (= total heat of steam) — 100° (= temperature of feed water) = 1089°·58
= heat required from the fuel to evaporate one part of water.

248° (= temperature of water blown off) — 100° (= temperature of feed water)
= 148° (heat lost by blowing off),

and 1089°·58 + 148° = 1237°·58 = total heat required from the fuel when the water enters the boiler at 100°. Let us now see what the total heat will be when the water is pumped in at 150°, and the difference of these results will be, of course, the saving.

1189°·58 — 150° = 1039°·58 = heat required from the fuel to evaporate one part of water;
248° — 150° = 98° = heat lost by blowing off,

and 1039°·58 + 98° = 1137°·58 = total heat required from the fuel (when the water enters the boiler at 150°). Therefore

$$\begin{array}{l} 1237^{\circ}\cdot58 \\ 1137^{\circ}\cdot58 \\ \hline 100^{\circ} = \text{saving in degrees;} \end{array}$$

whence 1237°·58 : 100° :: 100 : 8·08 per cent.; that is to say, if with the feed water at 100° the boilers consumed 100 tons of coal per day; with the increased temperature of 150° they would produce the same quantity of steam with (100 — 8·08 =) 91·92 tons.

Ex. 2. Suppose that the density of the water in the above example was $\frac{1\frac{1}{3}}{32}$ and all the other conditions remain the same: what would be the saving in that case?

1189°·58 — 100 = 1089°·58 × $\frac{3}{4}$ = 817°·185 = heat required from the fuel for the water that is evaporated.

248° — 100 = 148° = heat lost by blowing off.

817°·185 + 148° = 965°·185 = total heat required from the fuel when the water is pumped into the boiler at 100°.

1189°·58 — 150° = 1039°·58 = heat required from the fuel to evaporate one part of water.

1039°·58 × $\frac{3}{4}$ = 779°·685 = heat required from the fuel for the water that is evaporated.
248° — 150° = 98° = heat lost by blowing off.

779°·685 + 98° = 877°·685 = total heat required from the fuel when the water is pumped into the boiler at 150°.

Therefore, 965°·185 : 87°·5 :: 100 : 9·06 per cent.

Ex. 3. Water in the boilers, carried at a density of 1$\frac{1}{3}$ per saline hydrometer; temperature of the condenser and water entering the boilers 105° Fah.; vacuum in condenser 27·82 inches. Compare the economic performance of the engine under these circumstances with the same engine carrying the water in the boilers at the same density, but the water in the condenser at 120° Fah.; the mean pressure of steam in both cases 20 lbs. per sq. inch.

Neglecting the difference of power in the two cases required to work the air-pump, taking the boiler pressure at 20 lbs., and 2 inches of mercury to be equal to 1 lb. pressure, we proceed thus—

1184 — 105 × $\frac{3}{4}$ + 228·5 — 105 : 228°·5 — 105° :: 100 : 13·23 per cent. loss by blowing off in the first case.

1184 — 120 × $\frac{3}{4}$ + 228·5 — 120 : 228°·5 — 120° :: 100 : 11·96 per cent. loss by blowing off in the second case.

20 × 2 : 2·18 (back pressure) :: 100 : 5·45 per cent. of the effect of the engine lost by back pressure in the first case.

20 × 2 : 3·33 (back pressure) :: 100 : 8·325 per cent. of the effect of the engine lost by back pressure in the second case.

Now, then, letting the fuel represent the power, we observe, in the first case, that only (100 — 13·23 =) 86·77 per cent. reaches the engine, of which 5·45 per cent. of 86·77 per cent. = 4·73 per cent. of the total effect lost by back pressure, leaving (86·77 — 4·73 =) 82·04 per cent. to be applied to working the engine.

In the second case (100 — 11·96 =) 88·04 per cent. of the power reaches the engine, of which 8·325 per cent. is lost in back pressure, and 8·325 per cent. of 88·04 per cent. = 7·33 per cent. of the total effect lost by back pressure; leaving (88·04 — 7·33 =) 80·71 per cent. to be applied to working the engine.

Therefore, under the conditions of the question, the engine in the first case performs the same amount of work with (82·04 — 80·71 =) 1·33 per cent. less fuel.

This calculation can be made accurate by taking diagrams from the cylinder and air-pump, under the conditions of the example, and estimating the power in each case; then, the power to work the air-pump is considered.

EXAMPLES FOR PRACTICE.

1. How much water should you allow for the condensation of each cubic foot of steam at 212° during the working of an engine?

2. What quantity of water is required to obtain one cubic foot of steam at 212°?

3. A pound of steam at 212° Fah. is passed into 20 lbs. of water at 70° Fah.: what is the temperature of the water at the close of the operation? *Answer* 122°·79 Fah.

4. If a pound of water at 212° be mixed with x pounds of water at 60°: what is the value of x when the resulting temperature is 100°? Again, if a pound of steam at 212° be mixed with y pounds of water at 60°: find y when the resulting temperature is 100°. Account for the difference between x and y. *Ans.*, 2·8 and 26·965.

5. Zinc boils at 1204° Fah., mercury at 608° Fah.; change the readings into Centigrade scale. *Answer.* 650° C. and 320° C.

6. Change the following readings from Fah. scale into C. scale:—Polished steel is of a deep blue colour at 580° Fah. Polished steel is of a pale straw colour at 460° Fah. Sea water freezes at 28° Fah. *Answer.* 304°·5 C., 237°·75 C., and — 2°·2 C.

7. The following table of *remarkable temperatures* is given according to the C. scale: what are the corresponding readings on the Fah. scale?

Greatest artificial cold	140°
Mercury freezes	39·4
Ice melts	0
Greatest density of water	4
Blood heat	36·6
Water boils	100
Red heat	526
Cast iron melts	1530

Answers. 220° Fah.; 28°·90; 32°; 39°·2; 97°·88; 212°; 968°·8; 2786°.

8. A unit of heat is the heat given to 1lb. of water to raise its temperature 1° C.; how many units of heat are involved in raising 3 lbs. of water through 30 of the Fah. degrees? *Answer.* 50.

9. The latent heats of 1 lb. of water and 1 lb. of steam are respectively 79 and 573 units, according to the C. scale: what are they according to the Fah. scale? *Answer.* 142 and 966·6 units.

10. A mixture is made of 4 lbs. of water at 7° C. with 6 lbs. of water at 12° C.: find the temperature. *Answer.* 10° C.

QUESTIONS RELATING TO STRAINS, &c.

322. Given the strain by which a bar one inch square is torn asunder; it is required to find the force that will break another bar a given number of inches square.

It has been found by experiment that a strain of 4500 lbs. can tear asunder an iron bar of 1 inch square, so that we have

RULE CXXXIV.

As the square *of* 1 *inch is to the* square *of the number of inches in the bar, so is* 4500 *pounds to the force required to break the bar.*

EXAMPLES.

Ex. 1. If a bar of iron one inch square is torn asunder by a strain of 4500 lbs.: what force will be required to break a bar 3 inches square?

$$\begin{array}{ccccccc} 1^2 & : & 3^2 & :: & 4500 & : & x \\ 1 & : & 9 & :: & 4500 & & \\ & & & & \underline{\quad 9} & & \end{array}$$

$$40500 \text{ lbs., the force required.}$$

Ex. 2. If a bar of iron one inch square is torn asunder by a strain of 4500 lbs.: what force will be required to break a bar 4½ inches square?

$$1^2 : 4{\cdot}5^2 :: 4500 : x$$

$$x = 4500 \times 4{\cdot}5^2 = 4500 \times 20{\cdot}25 = 91125 \text{ lbs.}$$

$$\begin{array}{cccr} 1^2 & : & 4{\cdot}5^2 & :: \; 4500 \\ & & \underline{4{\cdot}5} & \underline{20{\cdot}25} \\ & & 225 & 225 \\ & & 180 & 90 \\ & & \overline{} & 900 \\ & & 20{\cdot}25 & \overline{} \\ & & & 91125{\cdot}00 \text{ lbs.} \end{array}$$

EXAMPLES FOR PRACTICE.

In each of the following examples it is required to find the force that will break a bar of a given number of inches square.

1.	4·4 square inches.	4500 lbs. breaking bar 1 inch square.	4.	3·5 square inches.
2.	5 ”		5.	4·3 ”
3.	5·2 ”		6.	5·7 ”

Ex. 3. A bar of iron 2½ inches by ¾ inch: what is the strain at 5000 lbs. per sq. inch?

$$\begin{array}{rr} \text{Bar of iron} & 2{\cdot}5 \\ & \underline{\cdot 75} \\ & 125 \\ & \underline{175} \\ & 1{\cdot}875 \\ & \underline{5000} \\ \text{Strain} & 9375{\cdot}000 \text{ in lbs.} \end{array}$$

EXAMPLES FOR PRACTICE.

No.	Inches.	Inches.	Strain per sq. inch.	*Ans.*: Strain.
1.	$2\frac{1}{2}$	$\frac{3}{4}$	5000	9375 lbs.
2.	$4\frac{1}{2}$	$1\frac{1}{2}$	5000	33750 „
3.	6	$2\frac{1}{4}$	5000	67500 „
4.	$2\frac{3}{4}$	$1\frac{1}{4}$	5000	17187·5 „

Ex. 4. If the shearing strength of rivets per square inch of section is taken as equal to the tensile strength of the plates per square inch of section: what is the strength of a double seam of the following dimensions:

Diameter of rivets $\frac{1}{2}''$, pitch $1\frac{3}{4}''$, breadth of plate $10\frac{1}{2}''$, thickness $\frac{1}{4}''$, and tensile strength of plate = 20 tons.

$$\text{Section of plate} = 10{\cdot}5'' \times {\cdot}25 = 2{\cdot}625 \text{ square inches.}$$

$$\text{Breaking strength of plate} = 2{\cdot}625 \times 20 \text{ tons} = 52{\cdot}500 \text{ tons.}$$

$$\text{Relative strength of seam} = \frac{\frac{1}{2}'' \times \frac{1}{2}'' \times {\cdot}7854 \times 2}{1\frac{3}{4}'' \times \frac{1}{4}''} = \frac{{\cdot}3927}{{\cdot}4375} = {\cdot}8976.$$

$$\therefore \text{ Strength of seam} = 52{\cdot}5 \times {\cdot}8976 = 47{\cdot}124 \text{ tons.} \quad \textit{Answer.}$$

Ex. 5. A pull of 1 ton per square inch on a bar of wrought iron draws it ·000075 of its length.

When a wrought stay of the following length is loaded to a strain of 3 tons per square inch of section: how much will it lengthen, the original length of stay being 15 ft. 2 in.?

$${\cdot}000075 \times 3 \times 182 \text{ inches} = 0{\cdot}4095 \text{ inches.} \quad \textit{Answer.}$$

Ex. 6. How many bolts $1\frac{1}{2}''$ diameter are required for the cover of a cylinder 60″ diameter; the boiler pressure is 50 lbs. by gauge, and the strain per square inch on the bolts measuring over the threads) is not to exceed 2000 lbs.?

$$\frac{60^2 \times {\cdot}7854 \times 50}{1\frac{1}{2}^2 \times {\cdot}7854 \times 2000} = \frac{141372}{3534{\cdot}3} = 40 \text{ bolts.}$$

323. In the modern marine engine the diameters of some of the principal parts are found to be according to the undermentioned proportions; the pressure on the safety-valve is supposed to be 70 lbs. on the square inch.

The proportions suppose the length of stroke to bear the ordinary ratio to the diameter of the cylinder.

Where D = diameter of the low pressure cylinder,
and L = length of stroke.

Diameter of body of piston-rod $= \frac{D}{10}$

Piston-rod at bottom of thread $= \frac{D}{14}$

Connecting-rod bolts $\frac{D}{20}$ (supposing there are only 2).

	STRONG.		LIGHT.
Crank-shaft journals	$\frac{D+L}{9}$	and	$\frac{D+L}{10}$
Tunnel shaft	$\frac{D+L}{10}$	and	$\frac{D+L}{11}$

Ex. 7. The high pressure cylinder is 36″ diameter, the low pressure cylinder 80″ diameter, the length of the stroke is 42″: what are the diameters of the parts specified according to the foregoing scale of proportions, to the nearest eighth of an inch, more or less?

$$\text{Body of piston-rod} = \frac{D}{10} = \frac{80}{10} = 8 \text{ inches.}$$

$$\text{Piston-rod at bottom of threads} = \frac{D}{14} = \frac{80}{14} = 5\tfrac{3}{4} \text{ inches (nearly).}$$

$$\text{Connecting-rod bolts} = \frac{D}{10} = \frac{40}{10} = 4 \text{ inches.}$$

$$\text{Crank shaft journals (strong)} = \frac{D+L}{9} = \frac{80+42}{9} = 13\tfrac{1}{2} \text{ inches.}$$

$$\text{do. do. (light)} = \frac{D+L}{10} = \frac{80+42}{10} = 12\tfrac{1}{4} \text{ inches (nearly).}$$

$$\text{Tunnel shaft (strong)} = \frac{D+L}{10} = \frac{80+42}{10} = 12\tfrac{1}{4} \text{ inches.}$$

$$\text{do. (light)} = \frac{D+L}{11} = \frac{80+}{11} = 11\tfrac{1}{8} \text{ inches.}$$

Ex. 8. If the piston-rod of a compound engine of ordinary construction be made to $\frac{1}{10}$ of the diameter of the larger cylinder, what will be the compression per square inch on the high pressure rod when the pressure on the top of the piston is 60 lbs., back pressure 17 lbs., and diameters of cylinders 90″ and 50″?

60 lbs. — 17 lbs. back pressure = 43 lbs. effective pressure on piston-rod.
60 lbs. — 15 lbs. atmospheric pressure = 45 lbs. " "

$$\text{Diameter of rod} = 90'' \div 10 = 9''.$$

$$\text{Area of rod} = 9^2 \times \cdot 7854 = 63 \cdot 6174.$$

$$\text{Pressure on top of rod} = 9^2 \times \cdot 7854 \times 45 \text{ lbs.} = 2862 \cdot 7830$$

$$\text{,, piston} = (50^2 - 9^2) \times \cdot 7854 \times 43 \text{ lbs.} = 81694 \cdot 9518$$

$$\text{Total} = 84557 \cdot 7348$$

$$\text{Compression per square inch} = \frac{84557 \cdot 7348}{63 \cdot 6174} = 1329 \cdot 1 \text{ lbs.} \quad \textit{Answer.}$$

Ex. 9. Supposing everything the same as in the last example, saving the back pressure to be 15 lbs.

$$\text{Pressure on top of rod} = 9^2 \times \cdot 7854 \times 45 \text{ lbs.} = 2862 \cdot 7830$$

$$\text{,, piston} = 9^2 \times \cdot 7854 \times 43 \text{ lbs.} = 85494 \cdot 7170$$

$$\text{Total} = 88357 \cdot 5000$$

$$\therefore \text{Compression per square inch} = \frac{88357 \cdot 5}{63 \cdot 6174} = 1388 \cdot 8 \text{ lbs.} \quad \textit{Answer.}$$

Ex. 10. The diameter of the piston-rod at the smallest part is $\frac{1}{14}$ part of the diameter of the piston; if the effective pressure on the piston is equal to 24 lbs. per square inch, what is the tensile strain per square inch section of the rod when the piston is loose or slack on the rod, that is, when the friction of the rod in the eye of the piston is not to be taken into account?

NOTE.—The area of the rod need not be deducted, although in strictness it would be correct to do so.

Let diameter of piston = 14″, then diameter of rod = 1.

$$\therefore \text{Strain} = \frac{14^2 \times \cancel{\cdot 7854} \times 24}{1^2 \times \cancel{\cdot 7854}} = 14^2 \times 24 = 4704 \text{ lbs.} \quad \textit{Answer.}$$

Ex. 11. The diameter of a piston-rod at the smallest part is $\frac{1}{15}$ of diameter of piston: what is the tensile strain per square inch on the piston-rod when the pressure on the piston is 40 lbs., back pressure 3·4 lbs., and the diameter of the body of the rod is $\frac{1}{11}$ the diameter of the piston?

In this example the size of the body of the rod is given, and is to be allowed for.

Let diameter of piston = 15″; then the least diameter of piston-rod is $\frac{1}{15}$ of 15″ = 1″, and the greater diameter = $\frac{1}{11}$ of 15″ = 1″·3636.

Effective pressure = steam pressure — back pressure = 40 lbs. — 3·4 lbs. = 36·6 lbs.

Vacuum 15 lbs. — back pressure 3·4 lbs. = 11·6 lbs.

Area of piston — area of rod = $(15^2 - 1{\cdot}363^2) \times {\cdot}7854 = 175{\cdot}2559$.

Pressure on this = 175·2559 × 36·6 lbs. = 6414·36594
Pressure on piston-rod = $(1''{\cdot}363^2 \times {\cdot}7854) \times 11{\cdot}6$ lbs. = 16·92544

Total = 6431·29138

∴ Tensile strain = $\frac{6431{\cdot}29138}{1^2 \times {\cdot}7854}$ = 8188·56 lbs. *Answer.*

The short way as in last example is

$15^2 \times 36{\cdot}6 = 15 \times 15\ (= 225) \times 36{\cdot}6 = 82350$ lbs. *Answer.*

Ex. 12. The piston is 51″ diameter, and at the beginning of the stroke the pressure is 38 lbs. per square inch; if the crank-pin is 9″ diameter, what length should it be in the bearing if the pressure on the pin is not to exceed 700 lbs. per square inch?

Area of piston = $51^2 \times {\cdot}7854 = 2042{\cdot}8254$.

Pressure on piston = 2042·8254 × 38 lbs. = 77627·3682 lbs.

Area of longitudinal section of pin = 77627·3682 ÷ 700 = 110·89626,

∴ Length of bearing = 110·89626 ÷ 9 = 12·32 &c., inches. *Answer.*

Or thus: $\frac{51 \times 51^2 \times {\cdot}7854 \times 38}{700 \times 9}$ = 12·32 inches, &c. *Answer.*

Ex. 13. The diameter of the crank-pin journal is, according to the larger of the two proportions, given in page 403, and the length of journal is 1·5 times its diameter.

The diameter of the L.P. cylinder is 81″, length of stroke 48″, and the areas of the cylinder are to each other as 1 to 3. The effective pressure per square inch at the beginning of the stroke is 20 lbs. on the L.P. piston, and 46 lbs. on the H.P. one.

Diameter of crank-pin = $\frac{D + L}{9} = \frac{81 + 48}{9} = \frac{129}{9} = 14\frac{1}{3}$ inches.

Length of journal = 1·5 times its diameter = $14\frac{1}{3} \times 1\frac{1}{2} = 21\frac{1}{2}$ inches.

And the area = $308\frac{1}{6}$ square inches.

Area of low pressure piston = $81^2 \times {\cdot}7854$ = 5153·0094.

„ high „ = $\frac{1}{3}$ of 5153·0094 = 1717·6698.

Pressure on low pressure piston = 5153·0094 × 20 = 103060·1880.

„ high „ × 1717·6698 × 46 = 79012·8108.

Pressure per square inch on L.P. piston = 103060·1880 ÷ $308\frac{1}{6}$ = 334·43 lbs.

„ „ H.P. „ = 79012·8108 ÷ $308\frac{1}{6}$ = 256·39 lbs.

RECTANGULAR BEAMS.

Examples.

Ex. 1. A rectangular beam is 16″ in section, and 64″ long; it is supported at the end by a weight of 2240 lbs.: what is the depth of the beam, the strain being 12000 lbs.?

(1) $\frac{S \times B \times D^2}{6} = W \times L$. Where S = strain, B = breadth, D² = depth, W = weight, and L = leverage.

(2) Or, $\frac{S \times B \times D \times D}{6} = W \times L$, Where B × D = sectional area.

$$\therefore D = \frac{W \times L \times 6}{S \times (B \times D)}$$

$\frac{2240 \times 64 \times 6}{12000 \times 16} = \frac{860160}{192000}$ = 4·48 inches. *Answer.*

Otherwise by (2) $\frac{12000 \times 16 \times D}{6} = 2240 \times 64$.

32000 × D = 143360,

∴ D = 143360 ÷ 32000 = 4·48 inches. *Answer.*

Ex. 2. If B in the last example is 3″, D 5″, length 50″, and weight 5000 lbs.: what would the strain have been per square inch of section?

$$\text{Since } \frac{S \times B \times D^2}{6} = W \times l \qquad \therefore S = \frac{W \times l \times 6}{B \times D^2}$$

$$S = \frac{5000 \times 50 \times 6}{3 \times 5^2} = \frac{1500000}{75} = 20000 \text{ lbs. } \textit{Answer.}$$

Or, by cancelling, thus:— $$S = \frac{5000 \times \overset{2}{\cancel{50}} \times \overset{2}{\cancel{6}}}{\cancel{3} \times \cancel{25}} = 20000 \text{ lbs. } \textit{Answer.}$$

Ex. 3. A red pine beam is 20 feet long, 18 inches thick, by 15 inches broad: what weight can be laid in the centre of the beam if the working strain is $\frac{1}{10}$ of the breaking strain?

$$\textit{Rule.} \quad \frac{4 \times B \times D^2}{\text{length in feet}} = \text{breaking strain in cwts.}$$

$$\text{Breaking strain} = \frac{4 \times 15 \times 18^2}{20} = \frac{19440}{20} = 972 \text{ cwt.}$$

$$\therefore \text{Working strain} = \tfrac{1}{10} \text{ of } 972 = 97{\cdot}2 \text{ tons. } \textit{Answer.}$$

Ex. 4. A beam, 14 feet long, and weighing 11 cwts., is supported at its extremities by two props A and B; a weight of 4 tons is placed 7 feet from B, and another of 7 tons at 6 feet from A: required the pressure on each prop?

Let l = length of beam, W = weight suspended at a feet from A.

NOTE.—a^1 feet from A = l — distance from B.

W = weight suspended at a^1 feet from A.

$$\text{Then pressure on B} = \frac{W a + W^1 a^1}{l} \times \tfrac{1}{2} a.$$

$$\text{Pressure on A} = \frac{W(l - a) + W^1 (l - a^1)}{l} + \tfrac{1}{2} a.$$

$$\text{Pressure on B} = \frac{7 \text{ tons} \times 6 + 4 \text{ tons} \times 7}{14} + 5\tfrac{1}{2} \text{ cwt.} = \frac{42 \text{ tons} + 28 \text{ tons}}{14} + 5\tfrac{1}{2} \text{ cwt.} =$$

$$\frac{70 \text{ tons}}{14} + 5\tfrac{1}{2} \text{ cwt.} = 5 \text{ tons } 5\tfrac{1}{2} \text{ cwt.}$$

$$\text{Pressure on A} = \frac{7 \text{ tons} \times 8 + 4 \text{ tons} \times 7}{14} + 5\tfrac{1}{2} = \frac{56 + 28}{14} + 5\tfrac{1}{2} = \frac{84}{14} = 6 \text{ tons } 5\tfrac{1}{2} \text{ cwt. } \textit{Ans.}$$

324. The strength of a rectangular bar, or its moment of resistance to transverse breaking, is equal to the breadth multiplied by the square of the depth multiplied by the maximum strain per square inch divided by 6. The moment of load is the weight multiplied by the leverage with which it acts. Take all dimensions in inches.

Let W = weight.
L = length of lever.
D = depth.
B = breadth.
S = maximum strain, allowing a factor of safety of 6, then

$$W \times L = \frac{B \times D^2 \times S}{6}$$

$$\therefore D^2 = \frac{W \times L \times 6}{B \times S}$$

which expressed in words is as follows:—

RULE CXXXV.

1°. *Multiply the weight* (reduced to pounds) *by length of lever and by* 6.

2°. *Multiply the breadth (or thickness) by the strain in pounds.*

3°. Divide *the result obtained in* 1° *by the product found by* 2°, *and take the square root of the quotient;* the result is the depth of fulcrum of lever.

EXAMPLES.

Ex. 1. If the maximum strain to be allowed on iron be taken as 50000 lbs. per square inch, what should be the depth at the fulcrum of an iron lever whose thickness is 2 inches, to support 12 tons at the extremity, the lever arm being 26 inches long? (Neglecting the shearing strain.)

$$\text{Depth at fulcrum} = \sqrt[2]{\frac{12 \times 2240 \times 26 \times 6}{2 \times 50000}} = 6''{\cdot}47$$

$$\begin{array}{r}
50000 \\
2 \\ \hline
100000
\end{array}
\qquad
\begin{array}{rl}
2240 & \text{lbs.} = 1 \text{ ton.} \\
12 & \text{tons.} \\ \hline
26880 & \text{lbs.} \\
26 & \text{length of lever.} \\ \hline
161280 & \\
53760 & \\ \hline
698880 & \\
6 & = \text{factor of safety.} \\ \hline
41{\cdot}93280 &
\end{array}
\qquad
\begin{array}{r|l}
6 & 41{\cdot}9328(6{\cdot}47 \\
6 & 36 \\ \hline
124 & 593 \\
4 & 496 \\ \hline
1287 & 9728 \\
 & 9009 \\ \hline
\end{array}$$

Answer. Depth = 6·47 inches.

Otherwise:—Since all the terms but one in the general formula above are known, that one may be found by multiplying the terms on one side, and also those on the other side (the unknown one excepted), and then dividing. In this instance all the terms are known excepting the depth.

$$\text{Then } (50000 \times 2 \div 6) \times d^2 = (2240 \times 12) \times 26.$$

$$d^2 \times 16666\tfrac{2}{3} = 698880, \qquad \therefore d^2 = \frac{698880}{16666\frac{2}{3}} = 41{\cdot}9328.$$

$$\sqrt[2]{41{\cdot}9328} = 6{\cdot}47 \text{ inches. } \textit{Answer.}$$

Ex. 2. If the maximum strain to be allowed on iron be taken as 51520 lbs. (23 tons) per square inch, what should be the depth at the fulcrum of an iron lever whose thickness is 2½ inches to support 17 tons at the extremity, the lever arm being 32 inches long? (Neglecting the shearing strain.)

$$\text{Depth at fulcrum} = \sqrt[2]{\frac{17 \times 2240 \times 32 \times 6}{2\frac{1}{2} \times 51520}} = 7{\cdot}53 \text{ inches nearly.}$$

$$2240 \times 17\ (= 38080) \times 2\ (= 1218560) \times 6 = 7311360 \div 128800\ (= 2\tfrac{1}{2} \times 51520)$$

$$= 56{\cdot}7652. \quad \sqrt{56{\cdot}7652} = 7{\cdot}53 \text{ inches.}$$

EXAMPLES FOR PRACTICE.

No.	Strain allowed in lbs.	Thickness of Lever. Inches.	Weight. Tons.	Length of Lever Arm in inches.	*Ans.*: Depth at the Fulcrum.
1.	4500	1¾	14	28	25·86 in. nrly.
2.	7000	3	20	32·25	20·32 „
3.	8000	3½	28	36	21·99 „
4.	9000	4	30	27	17·39 „
5.	7500	2½	18	25·5	18·14 „
6.	5500	1½	6	24	15·316 „

326. To find the transverse strength of a beam.

Ex. 1. What weight will be required to break a beam of English oak supported at one end and loaded at the other; length 9 feet, depth 11 inches, breadth 3 inches.

Rule. (1) $\dfrac{\text{Constant} \times \text{breadth} \times \text{depth}^2}{\text{Length}}$ or (2) $\dfrac{\text{Constant} \times \text{depth} \times \text{area of cross section}}{\text{Length}}$

NOTE.—The constant for English oak is 1672, and is the transverse strength of English oak 1 inch square, or 1 inch in section, and 12 inches long.

$$\text{Weight required} = \frac{1672 \times 3 \times 11^2}{108} = 5619{\cdot}7$$

$$1672 \times 3\ (= 5016) \times 121\ (= 606936),$$

$$\therefore\ 606936 \div 108 = 5619{\cdot}7.$$

$$\text{Or, (2)}\quad \frac{1672 \times 11 \times 33}{108} = 5619{\cdot}7.$$

$$1672 \times 11\ (= 18392) \times 3 \times 11\ (\text{area of cross section}) = 606936$$

$$\therefore\ 606936 \div 108 = 5619{\cdot}7.$$

If the weight, instead of being suspended from the extremity of the beam, was distributed uniformly along its length, it would sustain twice the weight that it does when loaded at the extremity. If, in the above case, the load were distributed along its length, it would be 5619·7 × 2 = 11239·4 lbs. required to break the beam.

But the weight of the beam itself constitutes a part of the weight it has to resist. The weight of the beam in this case is about 125 lbs., because 11 × 3 × 108 = 3564 cubic inches in the beam, and $\frac{3564}{1728}$ = 2·0625 cubic feet in the beam.

Then according to the specific gravity of bodies, the weight of a cubic foot of English oak is 60·625 lbs., and 60·625 × 2·0625 = 125·039 lbs. Subtract half the weight of the beam itself. Then, the weight that the beam would bear in addition to half its own weight in the case of being loaded at its extremity, would be 5619·7 — 62·5 = 5557·2 lbs. Uniformly distributed, 11239·4 — 62·5 = 11176·9 lbs. If a beam is fixed at both ends, and loaded in the middle, it will sustain four times the weight as when fixed at one end and loaded at the other. The weight that may be safely placed upon a beam is one-third its breaking weight for a steady load, and one-sixth for a moving or suddenly applied load, and in the case of timber bearers that have to bear a permanent load, should not be more than one-tenth, in order to allow for the effect of decay.

325. **On Torsion Strains.**—Given the maximum strain per square inch to be allowed on iron, it is required to find what pressure may be applied at the end of a crank to be transmitted by a shaft, the length of crank and diameter of shaft being given.

The strength of a cylindrical bar to resist torsion, or its moment of resistance to torsion is equal to the cube of its diameter multiplied by the maximum strain per square inch divided by 5·1.

Let P = pressure.
L = leverage of crank.
D = diameter of shaft.
S = maximum strain; then,

$$P \times L = \frac{D^3 \times S}{5{\cdot}1} \qquad \therefore\ P = \frac{D^3 \times S}{L \times 5{\cdot}1}$$

$$\text{and } D = \sqrt[3]{\frac{P \times L \times 5{\cdot}1}{S}}$$

RULE CXXXVI.

1°. *Multiply the cube of the diameter of shaft by the maximum strain per square inch (in pounds).*

2°. *Multiply the length or leverage of crank by* 5·1.

3°. Divide *the product found by* No. 1 *by the result obtained by* No. 2; the quotient is the pressure that may be applied at the end of the crank.

EXAMPLES.

Ex. 1. If the maximum strain to be allowed on iron be taken as 50000 pounds per square inch: what pressure may be applied at the end of a crank 16 inches long to be transmitted by a shaft 10 inches diameter? (The torsional strain only to be taken into account).

$$\text{Pressure} = \frac{10^3 \times 50000}{5{\cdot}1 \times 16} = \frac{10 \times 10 \times 10 \times 50000}{5{\cdot}1 \times 16} = \frac{1000 \times 50000}{81{\cdot}6} = \frac{50000000}{81{\cdot}6} = 612745{\cdot}11 \text{ lbs.}$$

$$612745{\cdot}11 \text{ lbs.} \div 2240 = 273{\cdot}55 \text{ tons.}$$

5·1
16
———
306
51
———
81·6

10
10
———
100
10
———
1000
50000
———

81·6)50000000(612745·11 lbs.
4896
———
1040
816
———
2240
1632
———
6080
5712
———
3680
3264
———
4160
4080
———
800
816
———
840

2240)612745·11(273·55 tons.
4480
———
16474
15680
———
7945
6720
———
12251
11200
———
10511

Or thus:—

$$\text{Since by general formula above, } P \times \text{leverage} = \frac{d^3 \times \text{strain}}{5{\cdot}1}$$

$$P \times 16' = \frac{10 \times 10 \times 10 \times 50000}{5{\cdot}1} = 9803921{\cdot}5, \text{ \&c.}$$

$$\therefore \text{pressure} = 9803921{\cdot}5, \text{ \&c.}, \div 16 = 612745{\cdot}11 \text{ lbs.} = 273{\cdot}5 \text{ tons.}$$

Ex. 2. If the maximum strain to be allowed on iron be taken as 6000 lbs. per square inch: what pressure may be applied at the end of a crank 21 inches long to be transmitted by a shaft 13 inches diameter? (The torsional strain only to be taken into account).

$$\text{Pressure} = \frac{13^3 \times 6000}{5{\cdot}1 \times 21} = \frac{13 \times 13 \times 13 \times 6000}{5{\cdot}1 \times 21} = \frac{2197 \times 6000}{5{\cdot}1 \times 21} = \frac{13182000}{107{\cdot}1} = 123081{\cdot}232 \text{ lbs.}$$

$123081{\cdot}232$ lbs. ÷ 2240 = $54{\cdot}94$ tons. *Answer.*

EXAMPLES FOR PRACTICE.

In each of the following examples given the strain per square inch, length of crank, and diameter of shaft, to find the pressure to be applied to the end of the crank.

No.	Strain per sq. in.	Length of crank.	Dia. of shaft.	*Ans.:* Pressure to be applied to end of crank.
1.	4500	15 inches.	8 inches.	13·4454 tons.
2.	7000	21 "	14¾ "	93·63465 "
3.	9000	15 "	9 "	38·28781 "
4.	6500	20 "	14½ "	86·72996 "
5.	5400	16 "	16½ "	132·71115 "
6.	7400	17 "	9 "	27·77743 "

EXAMPLES.

Ex. 1. If the maximum strain allowed on iron be taken as 51520 lbs. per square inch: what should be the length of a lever arm to support 1½ tons at the extremity, the thickness being 1¼ inches, and depth at the fulcrum 2½ inches?

$$\text{The general formula as before is } W \times L = \frac{B \times D^2 \times S}{6}$$

In this instance all the terms are known but the leverage, hence,

$$\therefore L = \frac{B \times D^2 \times S}{W \times 6}$$

$$\text{Leverage} = \frac{1\tfrac{1}{4} \times 2\tfrac{1}{2}^2 \times 51520}{(1\tfrac{1}{2} \times 2240) \times 6} = \frac{402500}{20160} = 19{\cdot}96, \text{ \&c., inches, or 20 inches, nearly. } \textit{Ans.}$$

Otherwise:—

$$\tfrac{1}{6}\,(51520 = 2{\cdot}5 \times 2{\cdot}5 \times 1{\cdot}25) = (1{\cdot}5 \text{ tons} \times 2240) \times \text{leverage}$$
$$= 67083{\cdot}33 = 3360 \times \text{leverage},$$
$$\therefore \text{leverage} = \frac{67083{\cdot}33}{3360} = 19{\cdot}96, \text{ \&c. } \textit{Answer.}$$

Ex. 2. A lever 20 inches long, 1¼ inches thick, and 2½ inches deep: what weight must be placed at the end of the lever to be equal to a strain of 51520 lbs.?

$$\text{Here } \tfrac{1}{6}\,(51520 \times 1{\cdot}25 \times 2{\cdot}5^2) = \text{weight} \times 20$$
$$= 67083{\cdot}33 \div 20 = 3354{\cdot}16 \text{ lbs., the weight,}$$
$$\text{and } 3354{\cdot}16 \text{ lbs.} = 1{\cdot}5 \text{ tons (nearly). } \textit{Answer.}$$

327. Given the diameter of piston, steam pressure, length of crank, and diameter of shaft, to find the strain per square inch.

Formula $S = \frac{5{\cdot}1\,W\,l}{d^3}$ Where S is the strain in lbs. per square inch section of shaft; W is the weight on crank-pin; l is the leverage; and d is the diameter of the shaft.

Question.—The piston is 47 inches diameter, steam pressure 21 lbs. per square inch, length of crank 18 inches, diameter of shaft 9 inches: what is the strain per square inch, and is the shaft strong enough?

$$\text{Strain} = \frac{5{\cdot}1 \times 47^2 \times {\cdot}7854 \times 21 \text{ lbs.} \times 18}{9 \times 9 \times 9} = 3032{\cdot}419, \text{ \&c., lbs.}$$

And as this is less than 5000 lbs. it shows that the shaft is strong enough.

328. The strength of cylindrical bar against bending is one-half of its strength against torsion.

Let W = weight of propeller.
L = leverage.
D = diameter of shaft.
S = maximum strain.

$$\text{Then } W \times L = \frac{D^3 \times S}{10{\cdot}2} \qquad \therefore S = \frac{W \times L \times 10{\cdot}2}{D^3}$$

$$\text{And } D = \sqrt[3]{\frac{W \times L \times 10{\cdot}2}{S}}$$

EXAMPLES.

Ex. 1. What is the maximum strain per square inch on a screw-shaft due to the overhang of a propeller weighing 3·75 tons, the centre of weight being 15 inches from the point of support, and the shaft being 9 inches diameter? (Neglecting the shearing strain).

$$\text{Maximum strain} = \frac{3{\cdot}75 \times 2240 \times 15 \times 10{\cdot}2}{9^3 = (9 \times 9 \times 9)}$$

Weight of propeller 3·75 tons.
2240 lbs. = 1 ton.

1500
750
750

8400·00 lbs.
15

420
84

126000
10·2

252
1260

1285200·0

Dia. 9 in.
9

81
9

$D^3 = 729$ inches.

729)12852000(1762·9
729

5562
5103

4590
4374

2160
1458

7020
6561

Or thus:—

$$729\left\{\begin{array}{l}9)1285200\\ 9)142800\\ 9)15866·6\end{array}\right.$$

1762·9 lbs.

Ans.: Maximum strain 1762·9 lbs.

Otherwise, thus:—

$$\frac{D^3 + S}{10·2} = W \times L = \frac{9^3 \times S}{10·2} = (2240 \times 3·75) \times 15 = 126000.$$

$\therefore$ S = 126000 × 10·2 ÷ 9³ = 1285200 ÷ 729 = 1762·9 lbs., maximum strain. *Ans.*

Ex. 2. What is the maximum strain per square inch on a screw-shaft due to the overhang of a propeller weighing 3·6 tons, the centre of weight being 11 inches from the point of support, and the shaft being 9¼ inches diameter? (Neglecting the shearing strain).

$$\text{Maximum strain} = \frac{3·6 \times 2240 \times 11 \times 10·2}{9\tfrac{1}{4}^3}$$

$$2240 \times 3·6 = 8064 \times 11 = 88704 \times 10·2 = 904780·8.$$

$$9\tfrac{1}{4}^3 = 9·25 \times 9·25\ (= 85·5625) \times 9·25 = 791·453125.$$

$$904780·8 \div 791·453 = 1143·19 \text{ lbs.}$$

Or thus:—

$$\frac{9\tfrac{1}{4}^3 \times S}{10·2} = (3·6 \times 2240) \times 11 = 88704.$$

$\therefore$ S = 88704 × 10·2 = 904780·8 ÷ (9·25³ =) 791·453125 = 904780·8 ÷ 791·453 = 1143·19 lbs. *Answer.*

EXAMPLES FOR PRACTICE.

No.	Weight of Propeller.	Centre of Weight.	Dia. of Shaft.	*Ans.:* Maximum strain per sq. in. on Screw Shaft.
1.	4·3 tons.	12 inches.	10 inches.	1178·9568 lbs.
2.	2·94 „	11½ „	9⅝ „	1016·704 „
3.	3 „	24 „	9 „	2256·6 „
4.	5 „	18 „	9½ „	2398·4 „
5.	3½ „	23 „	8¼ „	3275·5 „
	Tons cwt. qrs. lbs.			
6.	3 6 2 17	27 „	11 „	1544·6 „

Ex. 1. A sling chain when lifting a 4 ton weight forms a triangle with two sides, each 5 feet; the base is 6 feet, and the height of triangle is 4 feet: what strain is there upon each half of the sling chain?

$$\text{Strain} = \frac{\text{Weight} \times \text{length of side}}{\text{twice height}} = \frac{4 \times 5}{4 \times 2} = \frac{20}{8} = 2\tfrac{1}{2} \text{ tons.}$$

Ex. 2. A sling chain when lifting a 6 ton weight forms a triangle with two sides, each 9 feet; the base is 6 feet, and the height of the triangle 7½ feet: what strain is there upon each half of the sling chain?

$$\text{Strain} = \frac{\text{Weight} \times \text{length of side}}{\text{twice height}} = \frac{6 \times 9}{7\frac{3}{4} \times 2} = \frac{54}{15\frac{1}{2}} = 3{\cdot}5 \text{ tons, nearly.}$$

$$\begin{array}{r} 7\tfrac{3}{4} = 7{\cdot}75 \\ 2 \\ \hline 15{\cdot}50 \end{array} \qquad \begin{array}{l} \phantom{15{\cdot}5)}6 \\ \phantom{15{\cdot}5)}\underline{9} \\ 15{\cdot}5)540(3{\cdot}5 \text{ nearly.} \\ \phantom{15{\cdot}5)}\underline{465} \\ \phantom{15{\cdot}5)}750 \end{array}$$

Ans.: 3½ tons, nearly.

EXAMPLES FOR PRACTICE.

No.	Weight.	Side of triangle	Base of triangle.	Height of triangle.	*Ans.:* Strain upon each half of sling chain.
1.	3½ tons.	7 feet.	5¼ feet.	6½ feet.	1·73 tons, nearly.
2.	2¾ „	5½ „	7 „	4 „	1·89 „
3.	4¼ „	10 „	6½ „	9½ „	2·24 „
4.	2¾ „	5 „	5¼ „	4 „	1·72 „

To find the weight necessary to break a stud.

Ex. 1. If it take 480 lbs. at a leverage of 12″ to break a stud 1″ diameter: what weight would be necessary to break a stud ½″ diameter at a length of 15″?

$$15'' : 12'' :: 480 \text{ lbs.} : x$$

$$\begin{array}{r} 480 \\ \underline{12} \\ 15 \left\{ \begin{array}{r} 5)5760 \\ \hline 3)1152 \\ \hline \end{array} \right. \\ 384 \text{ lbs. to break } 1'' \text{ stud.} \end{array}$$

Since the strains are in proportion to the cubes of the diameters, therefore,

$$1^{3''} : \tfrac{1}{2}^{3''} (= {\cdot}5^3) :: 384$$

$$\begin{array}{r} 1 : {\cdot}125 :: 384 \\ \underline{{\cdot}125} \\ 1920 \\ \underline{4608} \\ 48{\cdot}000 \text{ lbs.} \end{array}$$

Answer.

FRICTION.

329. Friction is the resistance experienced when one body is rubbed upon another body. "When motion in opposite directions is given to smooth surfaces, the asperities of one surface must mount upon those of the other, and both will be abraded and worn away, in which act power will be expended. The friction of smooth rubbing substances is less when the composition of those substances is different than when it is the same, the particles being supposed to interlock less when the opposite prominences or asperities are not coincident."

(1) Friction does not increase with the extent of the rubbing surfaces, and so long as there is no violent heating or abrasion, the friction is simply in the proportion of the pressure keeping the substances together, or nearly so. It is, therefore, an obvious advantage to have the bearing surfaces of steam engines as large as possible, as there is no increase of friction by extending the surface, while there is a great increase in their durability.

(2) "Friction does not increase with the velocity of the rubbing body, if the friction over a given amount of surface be considered, but it increases as the velocity, if the comparison be made with the time during which the friction acts. Thus the friction of each stroke of a piston is the same, whether it makes 20 strokes in the minute or 40; in the latter case, however, there are twice the number of strokes made, so that, though the friction per stroke is the same, the friction per minute is doubled. The force expended in overcoming friction varies with the nature of the rubbing bodies."

330. **The Co-efficient of Friction** is a certain fraction which, when multiplied by the normal pressure, gives the friction.

EXAMPLES.

Ex. 1. If the co-efficient of friction be ·07 of the pressure, what horse-power is expended on the friction of a thrust bearing with 5 collars, the mean diameter of the collars being 15 inches, the shaft making 49 revolutions per minute, and the average pressure being 10 tons?

Here, although there are 5 collars, we take the friction as being concentrated on 1 collar only.

Total pressure on collars = 10 tons × 2240 = 22400 lbs. The drag of friction = 22400 lbs. × ·07 = 1568 lbs. Rubbing surfaces in feet = 15 × 3·1416 ÷ 12 = 3·927. Foot pounds = 3·927 × 1568 = 6157·53.

$$\text{Horse-power expended} = \frac{6157{\cdot}53 \times 49}{33000} = 9{\cdot}143.$$

1 ton =	2240 lbs.	3·1416	Rubbing surfaces in feet	3·927
Pressure	10 tons.	15	Drag of friction	1568
Total press. on collars	22400 lbs.	157080		31416
Co-efficient	·07	31416		23562
				19635
Drag of friction =	1568·00 lbs.	12)47·1240		3927
	Rubbing surfaces in feet	3·927	Foot pounds	6157·536

To find horse-power expended.

Foot pounds 6157·536
49 revolutions.

554178244
24630144

33,000 { 3,000)30719·264
11)100·573

9·143 horse-power.

Ex. 2. If the drag of friction be ·25 of the pressure, what horse-power is expended in the friction of a thrust bearing with 5 collars, the mean diameter of the collars being 10 inches, the shaft making 60 revolutions, and the average pressure being $7\frac{1}{2}$ tons?

$$7\cdot5 \text{ tons} = 7\cdot5 \times 2240 = 16800 \text{ lbs.}$$

$$\text{The drag of friction} = \frac{16800}{4} = 4200 \text{ lbs.}$$

NOTE.—To divide by 4 is equivalent to multiplying by ·25.

$$\text{Then the H.P. expended would be } \frac{4200 \times 10 \times 3\cdot1416 \times 60}{12 \times 33000} = \frac{7916832\cdot0}{396000} = 19\cdot98 \text{ H.P.}$$

EXAMPLES FOR PRACTICE.

No.	Drag of Friction.	No. of Collars.	Dia. of Collars.	Revolutions.	Pressure Tons.	*Ans.:* H.P. expended on Friction.
1.	·1	6	17	66	12	23·92
2.	·11	8	17	44	6	8·773
3.	$\frac{1}{8}$	6	12	36	9	8·637
4.	$\frac{1}{16}$	8	14	69	11	11·802
5.	·09	7	18	62	7	12·494
6.	$\frac{1}{32}$	4	18	75	5	3·748
7.	·06	3	19	63	8	10·21

Ex. 1. If the drag of friction be ·07 of the whole pressure, and if the pressure of the connecting-rod on the crank-pin be uniform, what per centage of that is lost by friction on the crank-pin, the stroke being 36 inches, and the diameter of the crank-pin 10 inches?

$$\text{Distance travelled by piston in one revolution} = 72 \text{ inches.}$$

$$\text{Circumference of crank-pin} = 3\cdot1416 \times 10 = 31\cdot416 \text{ inches.}$$

$$\text{Ratio of distances} = \frac{31\cdot416}{72} = \cdot43633.$$

$$\text{Ratio of power lost on crank-pin} = \cdot43633 \times \cdot07 = \cdot0305431.$$

$$\text{Per centage} = \text{Ratio} \times 100 = \cdot030543 \times 100 = 3\cdot0543. \quad \textit{Ans.}$$

Or, thus:—

Work done by the pressure on the piston during one revolution may be expressed by the formula.

$$\text{Work done} = P \times \text{twice the stroke} = P \times 72 \text{ inches.}$$

$$\text{And work done by the friction in one revolution} = P \times \cdot07 \times (3\cdot1416 \times 10) = 2\cdot19912.$$

$$\begin{array}{r} 3\cdot1416 \\ 10 \\ \hline 31\cdot4160 \\ \cdot07 \\ \hline 2\cdot199120 \end{array}$$

$$\text{Then, as } P \times 72 \; : \; P \times 2\cdot19912 \; :: \; 100 \; : \; \text{per centage.}$$

$$\begin{array}{r} 2\cdot19912 \\ 100 \\ \hline \end{array}$$

$$72\left\{\begin{array}{l} 8)219\cdot91200 \\ \hline 9)27\cdot48900 \\ \hline \end{array}\right.$$

$$3\cdot0543 \text{ per centage.}$$

Ex. 2. If the drag of friction be ·08 of the pressure, and if the pressure of the connecting-rod on the crank-pin be uniform: what per centage of that is lost by friction on the crank-pin, the stroke being 38 inches, and the diameter of the crank-pin 12 inches?

(1). Distance travelled by the piston in one revolution (38 × 2) = 76 inches.

(2). Circumference of pin = 3·1416 × 12 = 37·6992.

(3). Ratio of distances $= \frac{37·6992}{76}$ = ·4960.

(4). Ratio of work lost = ·4960 × ·08 = ·03968.

(5). Per centage = ·03968 × 100 = 3·968.

(1) Stroke 38
2

Distance travelled by piston 76 inches.

(2) 3·1416
Diam. 12

Dis. travelled by cir. of pin 37·6992 inches.

(3) 76)37·6992(·4960
304

729
684

459
456

30

(4) ·4960
·08

Ratio of lost work ·039680

(5) ·039680
100

Per centage 3·968000

Or thus: work done by the pressure on the piston during one revolution
= P × twice the stroke = P × (38 × 2) = P × 76.

Work done by the friction in one revolution
= P × ·08 × (3·1416 × 12) = 3·015936.

Then, as P × 76 : P × 3·015936 :: 100 : 3·968 per cent.

Whence, per centage lost $= \frac{3·1416 \times 12 \times ·08 \times 100}{38 \times 2}$ = 3·968.

3·1416
12

37·6992
·08

3·015936

3·015936
100

76)301·593600(3·968 per cent.
228

735
684

519
456

633
608

Examples for Practice.

No.	Drag of Friction.	Stroke.	Diameter of Crank-pin.	*Ans.*: Per centage of Power lost by friction.
1.	·1	40 inches.	17 inches.	6·675
2.	·068	36 "	16 "	4·7473
3.	·09	34 "	9 "	3·7422
4.	$\frac{1}{32}$	44 "	18 "	2·008
5.	·03	42 "	14 "	1·5708
6.	·09	33 "	9 "	3·8555
7.	·11	48 "	20 "	7·1999
8.	·06	42 "	15 "	3·3666

ON THE MECHANICAL POWERS.

331. The mechanical powers are the elements by the combinations of which are formed all the varieties of machines for enabling a smaller force to keep at rest, or put in motion, a larger weight, or to overcome a greater resistance.

332. Def.—The force which is applied to any machine to sustain or raise any weight, or to produce any other desired effect, is called the *power*.

333. The mechanical powers are, in fact, but three in number—the *lever*, the *inclined plane*, and *cords;* but it has always been customary to add to these the *wheel* and *axle*, the *wedge*, and the *screw*.

Cords, as a mechanical power, are usually denominated pulleys, from their being passed round pulleys to change their direction.

334. Def.—Forces are said to balance, or to be balanced about a point or axis, when they keep a rigid body at rest about the point or axis.

335. Def.—A lever is a rigid rod, movable in one plane about a fixed point, and acted upon by forces which tend to turn it round the point.

336. Def.—The fixed point about which the lever is movable is called the *fulcrum*, or *centre of motion*.

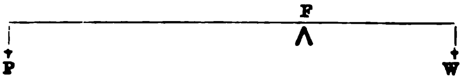
337. Def.—The portions of the lever between the fulcrum and the points upon which the forces are impressed are called the arms of the lever.

The lever will be supposed to be without weight, and the forces impressed upon it to act in the plane in which it is movable, unless the contrary be expressed.

338. Def.—The product of the numerical representatives of the magnitude of a force and the perpendicular let fall upon its direction from any fixed point, or centre of motion, is called the *moment*[*] *of the force* about the centre.

THE LEVER.

Let P W be a rod or lever turning upon the fulcrum or centre F. P and W are weights which balance each other, or maintain the lever in equilibrium; then, when this is the case, P × PF = W × WF; that is, the units of weight in P multiplied by the units in its distance from F, will be equal to the units of weight in W multiplied by the units in its distance from F. Here P is called the power, and W the weight or resistance, and a mechanical advantage is gained, inasmuch as a small weight or pressure is used to balance a greater one. The product of any weight, by its distance from the centre of motion, or fulcrum, is called the moment of that weight (Def. No. 338), and, therefore, when the sum of the moments tending to turn the lever in one direction, is equal to the sum of the moments tending to turn the lever in the opposite direction, then the lever will be in equilibrium.



Or, the power and weight are reciprocally as the distance at which they act, and therefore

$$P : W :: WF : PF$$

For instance, if WF is one quarter of PF, then P must be one quarter of F.

Note.—The weight is often called the *resistance*.

---

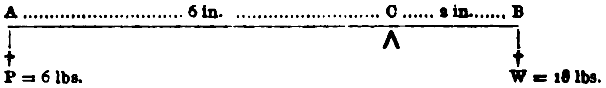
[*] The moment of a force is so named because it measured the value of the force in balancing, or assisting to balance, other forces about the centre of motion.

339. **Levers are sometimes divided into three classes, according to the position of the points of application of the power and the weight with respect to the fulcrum. In the first class the power and the weight act on opposite sides of the fulcrum. In the second class the power and the weight act on the same side of the fulcrum, the weight being the nearer to the fulcrum. In the third class the power and the weight act on the same side of the fulcrum, the power being the nearer to the fulcrum. Hence we may say briefly that the three classes of levers have respectively the fulcrum, the weight, and the power in the middle position.**

I.—LEVERS WITHOUT WEIGHT IN THE BEAM.

340. The First Kind.

Fig. 125.



$$P : W :: BC : AC$$
$$\text{or } 6 : 9 :: 2 : 6$$

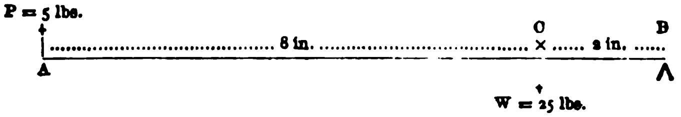
From which we have,

$$P = \frac{W \times BC}{AC} = \frac{18 \times 2}{6} = 6$$
$$W = \frac{P \times AC}{BC} = \frac{6 \times 6}{2} = 18$$
$$AC = \frac{W \times BO}{P} = \frac{18 \times 2}{6} = 6$$
$$\text{and } BO = \frac{AO \times P}{W} = \frac{6 \times 6}{18} = 2$$

A crow bar, a claw hammer, the handle of a common pump, a pair of scissors, a fire poker, &c., belong to the first kind of lever.

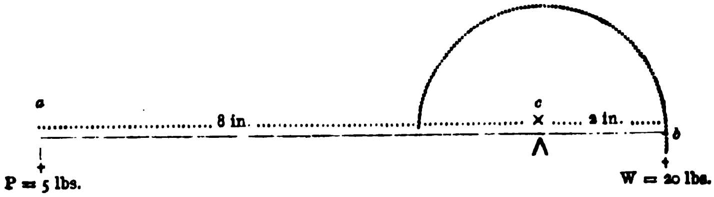
341. **The Second Kind.**—It is best to resolve the second and third kinds of levers into the first kind, for whether, in the following figures, a force

Fig. 126.



Resolved into the first kind, thus:—

Fig. 127.



draws downwards with a force of 25 lbs. at C, or with a force of 25 lbs. at a distance equal to BC beyond the fulcrum of the lever produced, as at *b* (in

the *resolved*), will not signify, so that we are careful to reverse the direction of the power also. In like manner, if we reverse a sign in an equation, we must reverse all. By doing so in this case we resolve the second kind into first, as under. Using, therefore, the formula for the first (340) we say

$$P : W :: bc : ac$$

$$\text{or } 5 : 25 :: 2 : 10$$

$$\therefore P \times ac = W \times bc$$

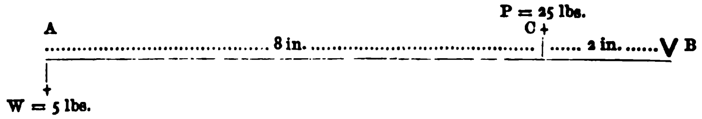
$$\text{and } P = \frac{W \times bc}{ac} = \frac{25 \times 2}{10} = 5,$$

&c., &c.

Of this kind is the oar, the blade being the momentary fulcrum, the crutch representing the place of the weight, and the hand the power, the rod of the safety-valve, &c.

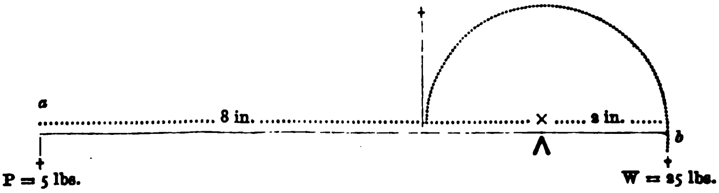
342. **The Third Kind.**—In this, when resolving, we have only to interchange the letter P and W, calling *the power weight and the weight power* for the occasion. That is to say, in the first figure, that which stands at A as W = 5 lbs. will be altered in the second figure, at *a*, to P = 5 lbs.; and that which in the first stands at C as P = 25 lbs. will, when resolved in the second figure, be W = 25 lbs.

Fig. 128.



Resolved into the first kind, thus:—

Fig. 129.



It will be evident that the positions of the actual forces themselves will not be altered, *but only the letters change places in order to bring all levers under the one formula given at* (340) for the first kind; for it is quite immaterial whether a weight of say 25 lbs. draw upwards at *c*, or downwards at an equal distance beyond the fulcrum at *b*, provided we reverse the fulcrum.

The definition refers to the designation of the three kinds, and not to them when *resolved*.

Taking again the formula for the first kind, and retaining the positions of P and W as in the first and second kinds, although really reversed in signification, we have as before (340):—

$$P : W :: bc : ac$$

$$\text{or } 5 : 25 :: 2 : 10$$

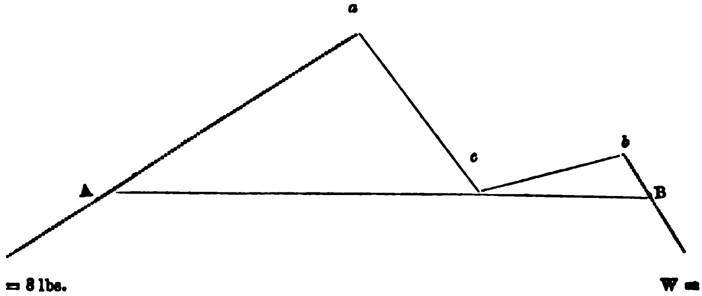
$$\therefore P = \frac{W \times bc}{ac} = \frac{25 \times 2}{10} = 5,$$

&c., &c.

343. Sometimes bent levers are spoken of as a fourth kind, but they differ in principle in no respect from the straight lever, into which they are easily resolved (see Fig. 125), the line *a*A being the direction of the force.

We have hitherto considered levers as acted upon at right-angles to the bar; but when the forces are applied to the end of the lever obliquely, we proceed thus:—

Fig. 130.



Suppose, in the above figure, the power be applied in the direction AP, and the weight in the direction BW, produce the line P*a*, and let fall a perpendicular, *ca*, from the fulcrum upon it; do the same with W*b*, and thus get the points *a* and *b*.

In the figure we may suppose *acb* to be a bent lever.

Then, in the bent lever, the two forces are inversely as the arms *ca* and *cb*; or, as again, in the first kind (125):—

$$P : W :: bc : ac$$

$$\text{or } 8 : 10 :: 2{\cdot}4 : 3$$

$$\therefore P \times ac = W \times bc$$

$$\text{and } P = \frac{W \times bc}{ac} = \frac{10 \times 2{\cdot}4}{3} = 8 \text{ lbs.}$$

By this means we bring all levers which are without weight under a simple and single equation.

It is worthy of notice that—

In the first kind there is a gain of power when the power is less than the resistance of weight, but a loss of power when the power is *necessarily* greater than the weight.

In the second kind there is always a gain of power, because the power must always be less than the weight.

In the third kind there is always a loss of power, since the power must always be greater than the weight.

Examples.

Ex. 1. What power applied at 36 inches from the fulcrum will balance a weight of 300 lbs. applied at 8 inches from the fulcrum?

$$P : W :: 8 : 36$$

$$P = \frac{W \times \overset{2}{\cancel{8}}}{\underset{9}{\cancel{36}}} = \frac{300 \times 2}{9} = \frac{600}{9} = 66\tfrac{2}{3} \text{ lbs.} \quad \textit{Answer.}$$

Or thus:

$$W \times 8 = P \times 36,$$

$$\therefore P = \frac{300 \times 8}{36} = \frac{2400}{36} = 66\tfrac{2}{3} \text{ lbs. as before.}$$

Ex. 2. How far from the fulcrum must the power of 75 lbs. be applied to balance a weight of 300 lbs. applied at 9 inches from the fulcrum?

$$300 \text{ lbs.} \times 9 \text{ inches} = 75 \times \text{distance},$$

$$\therefore \text{distance} = \frac{300 \times 9}{75} = 36 \text{ inches.} \quad \textit{Answer.}$$

Ex. 3. What weight 9 inches from the fulcrum can be balanced by a power of 75 lbs. at 36 inches from the fulcrum?

$$W \times 9 = 75 \times 36,$$

$$\therefore W = \frac{75 \times 36}{9} = 75 \times 4 = 300 \text{ lbs.} \quad \textit{Answer.}$$

Ex. 4. A bar 12 feet long, resting on two supports A and B; a weight of 18 cwt. is hanging from the bar at a distance from B of 8 feet 6 inches: what weight must be placed on A to prevent it springing?

A being the shorter arm will, by the principle of the lever, have the greater weight to support.

Hence, $12 \text{ feet} : 8{\cdot}5 \text{ feet} :: 18 \text{ cwt.} : x$

$$\begin{array}{r} 18 \\ \hline 680 \\ 85 \\ \hline 12)153{\cdot}0 \\ \hline 12{\cdot}75 \text{ cwt.} \end{array}$$

And since this weight is acting at a distance of 3 feet 6 inches from A, we have

$$12{\cdot}75 \text{ cwt.} \times 3{\cdot}5 = 44{\cdot}625 \text{ cwt.}$$

Ex. 5. An air-pump lever has a weight of 5 tons at a distance of 1′ 4″ from its turning point: what would be the weight on the centre bearing when the other arm is 3′ 1″?

$$\begin{array}{llcl} & \underline{3} \text{ ft. } 1 \text{ in.} & : & \underline{1} \text{ ft. } 4 \text{ in.} \quad :: \quad 5 \text{ tons} \quad : \quad x \\ \text{Or,} & 37 \text{ in.} & : & 16 \text{ in.} \end{array}$$

$$x = \frac{5 \times 16}{37} = 2{\cdot}16 \text{ tons.}$$

Hence, total weight $= 5 + 2{\cdot}16 = 7{\cdot}16$ tons. *Answer.*

Ex. 6. In a lever of the first kind the long arm is 2 feet, the short arm 3 inches: what must be the power to balance a weight of 64 lbs.? *Answer*, 8 lbs.

Ex. 7. What will be the weight in Ex. above when the power is 14 lbs.? *Answer*, 1 cwt.

Ex. 8. How far from the fulcrum must a power of 7 lbs. be applied so as to balance a weight of 126 lbs. placed at the distance of 2 inches from the fulcrum? *Answer*, 1 foot.

Ex. 9. In a lever of the second kind W = 20 lbs., OB = 5 inches, and AB = 25 inches; required P (see Fig. 126).

$$\text{Here } P \times 25 = W \times 20, \therefore P = 4 \text{ lbs.}$$

Ex. 10. In a lever of the second kind P = 6 lbs., W = 72 lbs., and OB = 3 inches: required AB.

$$6 \times AB = 72 \times 3 = 36 \text{ inches.}$$

Ex. 11. A man exerts a pressure of 80 lbs. upon a crow-bar at a distance of 4 feet from the fulcrum: what weight will he balance at the distance of 4 inches from the fulcrum?

$$W \times 4 = 80 \times 48 = 960 \text{ lbs.}$$

Or thus:—In this example the leverage of the power is 12 times the leverage of the weight, for 4 feet are 12 times 4 inches, therefore W will be 12 times P; that is, W = 12 times 80 lbs. = 960 lbs.

Ex. 12. Required W, as in example above, when the pressure of 60 lbs. is exerted at the distance of 6 feet from the fulcrum, and the weight W is at 3 inches. *Answer*, 1440 lbs.

Ex. 13. What is the mechanical advantage of the lever in the last example? *Answer*, 24, because the weight is 24 times the power.

344. If several weights hang upon a lever, some on one side of the fulcrum, some on the other, then there will be an equilibrium, when the sum of the product of the weights into their respective distances on one side is equal to the several products of weights and distances on the other side.

For, the product of the weight into its distance from the fulcrum F, is its *moment* about the point F, and, therefore, the above proposition is a direct consequence of the principle of the equality of the moments of any forces in equilibrium about any fixed point.

345. When two, three, or more levers act one upon the other in succession, then the entire mechanical advantage which they afford is found by taking, not the *sum*, but the *product* of their separate advantages. Thus, if the arms of three levers, acting thus in connection, are as 3 to 1, 4 to 1, and 5 to 1, then the joint advantage is that of 3 × 4 × 5 to 1, or 60 to 1, so that 1 lb. would, through their intervention, balance 60 lbs.

Ex. 14. In a lever of the first kind 5 and 8 lbs. are placed on one side of the fulcrum at the distance of 4 and 2 inches respectively from the fulcrum: required the power P acting at the distance of 12 inches from the fulcrum to maintain equilibrium.

The efficacy of the 5 lbs. to turn the lever will be its weight multiplied by its leverage, or 5×4; and the efficacy of the 8 lbs. to turn the lever will be its weight multiplied by its leverage, or 8×2. Now the sum of these two moments, or tendencies to turn the lever, will be equal to the moment of P, or $P \times 12$, tending to turn the lever in the opposite direction. $\therefore P \times 12 = 5 \times 4 + 8 \times 2$; $\therefore P = 3$ lbs.

Ex. 15. In a combination of three levers of the first kind, with long arms 9, 10, and 12 inches respectively, and the short arms 3, 2, and 4 inches: what weight will be balanced by a power of 5 lbs.?

$$1\text{st}, \quad 5 \text{ lbs.} \times 9 = W \times 3; \; W = 15.$$

$$2\text{nd}, \quad 15 \text{ ,,} \times 10 = W \times 2; \; W = 75.$$

$$3\text{rd}, \quad 75 \text{ ,,} \times 12 = W \times 4; \; W = 225.$$

$$\text{or } 5 \text{ lbs.} \times 9 \times 10 \times 12 = W \times 3 \times 2 \times 4.$$

$$\therefore W = \frac{5 \times \overset{3}{\cancel{9}} \times \overset{5}{\cancel{10}} \times \overset{3}{\cancel{12}}}{\cancel{3} \times \cancel{2} \times \cancel{4}} = 225 \text{ lbs. } \textit{Answer.}$$

Ex. 16. In a combination of levers of the first kind, the long arms are 2, 3, and 5 *feet* respectively, and the short arms 4, 6, and 12 inches respectively: if a pressure of 8 lbs. be applied, what weight will be balanced? *Answer*, 3·8 tons.

Ex. 17. In a lever of the third kind W = 28, AB = 20 inches, and CB = 7 inches: required P.

$$P \times 7 = 28 \times 20 = 560$$

$$\therefore P = 80 \text{ lbs.}$$

346. THE PRINCIPLE OF WORK AS APPLIED TO THE LEVER.

Ex. 1. Let PF = 5 feet, WF = 1 foot, and P = 2 lbs.: required W. See Fig. 125, page 417.

Now, in order to raise W one foot, P must descend 5 feet, because PF is 5 times WF, hence we have

$$W \text{ of } P = 2 \times 5; \; \text{work of } W = W \times 1.$$

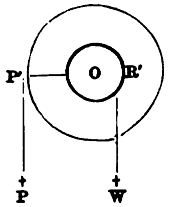
$$\therefore W \times 1 = 2 \times 5, \text{ and } W = 10 \text{ lbs.}$$

This result is evidently the same as that which would be obtained by the preceding method. Assuming the equality of moments, we may readily establish the principle of the equality of work, and conversely.

347. It will be evident that a very small power might be made to balance a very great weight by using a suitable lever, that is, by making the distance of the power from the fulcrum very large, and the distance of the weight from the fulcrum very small. But machines are, in general, used rather to *produce* motion than to *prevent* it; and this leads to a very important remark. Suppose in Fig. 125, page 417, that BC is one-third of AC, then when there is equilibrium the power is one-third of the weight. It is true then that a force just exceeding one-third of the weight will be sufficient to move the weight, but on the other hand it will be found that if the weight is to be raised *one* inch, the power end of the lever must descend through *three* inches. Thus, although by the aid of a lever we can move any weight by a force much less than that weight, yet the force must be exerted through a distance which is greater in a corresponding degree. This important principle is found to apply to the other mechanical powers, and to combinations of them, and it is usually briefly stated thus:—*What is gained in power is lost in speed.*

THE WHEEL AND AXLE.

348. The wheel and axle is one of the simplest and most useful modifications of the lever; it consists, as its name expresses, of a large wheel, or simply spokes fastened perpendicularly on an axle, which is supported by two rings or sockets in which it revolves, which act as fulcra, and must sustain the pressure caused by the resultant of the power and resistance. The power is applied to the circumference of the wheel, and the resistance to the circumference of the axle, in directions at right-angles to their radii. The annexed figure represents a cross section of the wheel and axle, in which OP' is the radius of the wheel, OR' the radius of the axle, and P and W the power and resistance, both acting vertically downwards. It is evident from inspection of the figure, $P'OR'$ may be considered as a lever of the first order, having its fulcrum at O, and the weights P and W applied at its extremities; and in order that there may be equilibrium, P must be to W in the same proportion as OR' to OP'.



If the wheel and axle revolve uniformly: in a single revolution the power P passes through a space which is the circumference of the wheel, and the weight or resistance W passes through a space which is the circumference of the axle. Hence

Power × circumference of wheel = weight × circumference of axle. (1).

Or, since the circumference of circles are proportional to their radii,

Power × radius of wheel = weight × radius of axle. (2).

and, power × diameter of wheel = weight × diameter of axle. (3).

349. We now proceed to consider the equilibrium on the principle of work. While the wheel makes one revolution, the axle also makes one.

In one turn P' descends a space $= 2 \times OP' \times 3{\cdot}1416$, and W ascends a space $= 2 \times OR' \times 3{\cdot}1416$.

$$\therefore \text{Work of P in 1 revolution} \times 2 \times OP' \times 3{\cdot}1416 \times P.$$

$$\text{Work of W in 1 revolution} = 2 \times OR' \times 3{\cdot}1416 \times W.$$

Then, as the work done is equal to the work applied (abstracted from friction), we have

$$2 \times OP' \times 3{\cdot}1416 \times P = 2 \times OR' \times 3{\cdot}1416 \times W.$$

$$\therefore P \times OP' = W \times OR'$$

which is the relation already established.

350. The circumference of the wheel may be removed, and the power may be applied at the extremity of a radius. This is the case in the capstan, windlass, and other machines; indeed, to this power belong all turning or wheel machines of different radii. The windlass, however, has more than one fixed handle for the convenience of working it, or there may be a movable

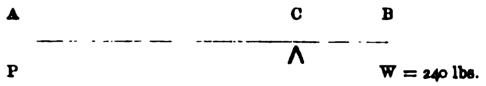
handle which can be shifted from one place to another. In the capstan the fixed axis round which the machine turns is vertical; the hand which supplies the power describes a circle in a horizontal plane, and the weight is some heavy body which is attached to the axle by a rope passing in a horizontal direction. In all these instances the power being to the weight always as the radii of lever, at which the weight acts, to that at which the power acts, so that they are always in the reciprocal ratio of the velocities.

351. But all this, however, is on the supposition that the ropes, or cords, sustaining the weights are of no sensible thickness. For, if the thickness be considerable, or if there be several folds of them over one another on the roller or barrel, then we must measure to the middle of the outermost rope for the radius of the roller, or to the radius of the roller and half the thickness of the cord when there is but one fold.

EXAMPLES.

Ex. 1. Taking the common winch as an example.—Let diameter of circle described by the handle when working = 18 inches, and let 6 inches = diameter of axle, and weight W = 240 lbs.

Fig. 131.



If on a straight line we lay off AC, as above, = 18 inches, and CB, the diameter of axle, = 6 inches, and W = 240 lbs., then AB represents a lever of the first kind of which C is the fulcrum, and W = 240 lbs.

$$\text{Then, } P : W :: BC : AC.$$

$$\text{or } P : W :: \text{diameter of axle} : \text{diameter of wheel.}$$

$$P : 240 :: 6 : 18,$$

$$\therefore P = \frac{240 \times 6}{18} = 80 \text{ lbs. } \textit{Answer.}$$

Or, since W × diameter of axle = P × diameter of wheel (see No. 348 (3).

$$240 \times 6 = P \times 18,$$

$$\therefore P = \frac{240 \times 6}{18} = 80 \text{ lbs. } \textit{Answer.}$$

Or again: work of W in 1 revolution = 240 × 6 × 3·1416 (No. 349).

$$\text{,, } P \text{ in 1 revolution} = P \times 18 \times 3{\cdot}1416,$$

$$\therefore P \times 18 \times 3{\cdot}1416 = 240 \times 6 \times 3{\cdot}1416, \text{ and } P = 80 \text{ lbs.}$$

Ex. 2. The handle of a windlass is 2 feet, the radius of the axle 2½ inches, and the power applied 80 lbs.: required the weight which would be raised were there no friction.

By the principle of the lever:

$$P \times \text{radius of wheel} = W \times \text{radius.}$$

$$80 \times 2 \times 12 = W \times 2\tfrac{1}{2},$$

$$\therefore W = 1920 \div 2\tfrac{1}{2} = 768 \text{ lbs. } \textit{Answer.}$$

By the principle of work:

$$\text{Work of } P \text{ in 1 revolution} = 80 \times 4 \times 3{\cdot}1416.$$

$$\text{,, } W \text{ ,, } = W \times \tfrac{5}{12} \times 3{\cdot}1416,$$

$$\therefore W \times \tfrac{5}{12} \times 3{\cdot}1416 = 80 \times 4 \times 3{\cdot}1416, \text{ and } W = 768 \text{ lbs. } \textit{Answer.}$$

Ex. 3. Sixteen sailors, exerting each a force of 29 lbs., push a capstan with a lever equal to 8 feet, the radius of the capstan being 1 foot 2 inches. Calculate the resistance which this force is capable of sustaining.

If each sailor exerts a force of 29 lbs., then 16 men will exert a force $= 29 \times 16 =$ 464 lbs.

$$P \times 8 \text{ ft.} = W \times 1\tfrac{1}{6} \text{ ft.},$$

$$\text{Hence, } 464 \text{ lbs.} \times 8 \text{ ft.} = W \times 1\tfrac{1}{6} \text{ ft.},$$

$$\therefore W = 3712 \text{ lbs.} \div 1\tfrac{1}{6} = 3181\tfrac{5}{7} \text{ lbs.} = 1 \text{ ton } 8 \text{ cwt. } 1 \text{ qr. } 17\tfrac{5}{7} \text{ lbs.} \quad \textit{Answer.}$$

Ex. 4. In this last example, suppose 10 per cent. of the effort be lost through friction and the rigidity of the cord: what would have been the weight lifted?

Since 10 per cent., that is, $\frac{1}{10}$ of effort, is lost, and $\frac{9}{10}$ is left,

then, $\frac{9}{10}$ of 1 ton 8 cwt. 1 qr. 17 lbs. = 1 ton 5 cwt. 2 qrs. 7 lbs. *Answer.*

Otherwise, $\frac{9}{10}$ of 464 lbs. $\times 8 = W \times 1\frac{1}{6}$ ft.

$$\therefore W = 3340{\cdot}8 \text{ lbs.} \div \tfrac{7}{6} = 2863{\cdot}5 \text{ lbs.} = 1 \text{ ton } 5 \text{ cwt. } 2 \text{ qrs. } 7 \text{ lbs.} \quad \textit{Answer.}$$

Ex. 5. Supposing (see Ex. 3) them to have wound the rope round the capstan so that it doubles back on itself, the radius of the axis is then increased by the thickness of the rope. If this be 2 inches, how much will the power of the instrument be diminished?

Radius of capstan + thickness of rope = 1 ft. 2 in. + 2 in. = 1 ft. 4 in. = $1\frac{1}{3}$ ft.

$$P \times 8 \text{ ft.} = W \times 1\tfrac{1}{3} \text{ ft.}$$

$$464 \times 8 \text{ ft.} = W \times 1\tfrac{1}{3} \text{ ft.},$$

$$\therefore W = 3712 \div \tfrac{4}{3} = 2784 \text{ lbs.}$$

Power of capstan for radius, 1 ft. 2 in., or $1\frac{1}{6}$ ft. (Ex. 3) = 3181 lbs.
" " 1 ft. 4 in., or $1\frac{1}{3}$ ft. = 2784 "

Difference 397

Loss o power.
3181 : 397 :: 100 : P.

$$\text{Per cent. of loss of power} = \frac{397 \times 100}{3181} = 12{\cdot}5. \quad \textit{Answer.}$$

Or, $\frac{397}{3181} = \frac{1}{8}$, loss of power.

352. In the state of rest the wheel and axle is the same machine with the lever. In the state of motion the lever is the most simple machine for overcoming a large resistance through a small distance; while the wheel and axle is applicable to overcoming resistance or raising weights through any desired distance, a new lever coming into existence every instant, so that the power always acts upon these levers to the greatest advantage.

353. **The Toothed Wheel.**—It is frequently necessary to communicate rotation from one axis to a parallel axis, and there are various methods by which this can be accomplished. One of these methods will be here considered. Let two wheels of wood or metal have their circumferences cut into equal teeth at equal distances. Let the wheels be movable about their centres and in their own planes, and let them be placed in the same plane so that their edges touch, one tooth of one circumference lying between two teeth of the other circumference. If one of the wheels of the pair be turned round its centre by any means, the other wheel will also be made to turn round its centre. Or a force which tends to turn one wheel

round may be balanced by a suitable force which tends to turn the other wheel round in a contrary direction. The two forces may be supposed to act by means of strings on axles belonging to the toothed wheels.

354. The conditions of equilibrium is somewhat complex; the reader may take it as verified by experiment. *When there is equilibrium on a pair of toothed wheels the moment of the power round the centre of the axle must be to the moment of the weight round the centre of the axle in the same proportion as the radius of the power wheel is to the radius of the weight wheel.* The principle of Art. 347 may be shown to hold with respect to this machine.

In the case of toothed wheels, when one of these wheels revolves, it necessarily causes the other wheel to revolve also. At the place where the two wheels are in contact it is clear that the teeth on each wheel must be moving with the same velocity. Suppose, for example, that 10 teeth on the small wheel pass the line joining the centres of the two wheels in one minute, then 10 teeth on the large wheel will pass the same line in one minute. Since all the teeth of the two wheels are of the same size, it appears that the number of teeth on each of the wheels will be proportional to its circumference, and since the circumferences are proportional to the radii, the number of teeth on each wheel will be proportional to its radius.

If the large wheel contains 44 teeth and the small wheel 20 teeth, so that when 20 teeth have passed the line of centres, then the small wheel must have made one revolution, and when 44 teeth have passed the line of centres, then the large wheel must have made one revolution. It therefore appears, that when the large wheel has made one complete revolution, the small wheel must have made 44 ÷ 20 revolutions, and, in general, have the following result:—

When two toothed wheels are working together the number of revolutions made by each of the wheels in a given time is inversely proportional to the number of its teeth.

355. Sometimes the teeth are placed on a straight bar instead of a wheel. Such a toothed bar is termed a *rack*, and the wheel in contact with it is called a *pinion*.

356. The *jack* used for lifting great weights through a small height is a rack and pinion. The handle by which the rack is raised is termed a winch. The instrument is an example of a double wheel and axle; the winch takes the place of the first wheel, the lower pinion being its axle. The upper pinion is the second axle and supports the rack. The large toothed wheel of the upper axle works in gear with the lower pinion. The two pinions are usually of the same size.

357. By increasing the number of wheels too, the power may be multiplied to any extent, making always the less wheels to turn greater ones as far as we please, and this is commonly called *tooth and pinion*, the teeth of one circumference working in the rounds or pinions of another to turn the wheel. And then in case of equilibrium, *the power is to the weight as the continual product of the radii of all the axles to that of all the wheels.* So if the power P turns the wheel Q, and this turn the small wheel or axle R, and this turn the wheel S, and this turn the axle T, and this turn the wheel V, and this turn the axle X, which raises the weight W; then

$$P : W :: CB \times DE \times FG : AC \times BD \times EF.$$

And in the same proportion is the velocity of W slower than that of P. Thus, if each wheel be to its axle as 10 to 1, then P : W :: $1^3 : 10^3$, or as 1 to 1000, so that a power of 1 lb. will balance a weight of 1000 lbs.; but then, when put in motion, the power will move 1000 times faster than the weight.

358. If ropes are used for the action of the power, we must consider the forces applied to the axis of the rope. Hence, if R r denote the radii of the rope of the wheel and axle, and T t half the thickness of the ropes, we have

$$P : W :: r+t : P+t.$$

EXAMPLES.

Ex. 6. A weight of 3 cwt. 2 qrs. 14 lbs. is suspended from a chain wound round a drum 8¼″ diameter: what we put will balance this if suspended from a chain wound round a sheave on the same centre as the drum, if the distance from chain to chain is 1′ 9″.

$$3 \text{ cwt. } 2 \text{ qrs. } 14 \text{ lbs.} = 406 \text{ lbs.}$$

$$\tfrac{1}{2} \text{ of diameter } 8''{\cdot}25 = 4''{\cdot}125.$$

$$1'\ 9'' \text{ or } 21'' - 4''{\cdot}125 = 16''{\cdot}875.$$

$$W \times 4{\cdot}125 = P \times 16{\cdot}875,$$

$$\therefore P = \frac{406 \times 4{\cdot}125}{16{\cdot}875} = \frac{1674{\cdot}75}{16{\cdot}875} = 99{\cdot}24 \text{ lbs. } \textit{Ans.}$$

Ex. 7. In a wheel and axle the barrel is 9″ diameter, the rope is 1¼″ diameter, the crank handle is 14″ radius, and the weight to be lifted 300 lbs.: what is the force applied to the handle, adding 10 per cent. for friction?

The radius of crank being 14″, the diameter will be 14″ × 2 = 28″.

$$\text{Diameter of barrel} + \text{diameter of rope} = 9'' + 1\tfrac{1}{4}'' = 10\tfrac{1}{4}''.$$

$$W \times \text{diameter of axle} = P \times \text{diameter of crank handle.}$$

$$W \times 10\tfrac{1}{4}'' = P \times 28.$$

$$300 \times 10{\cdot}25 = P \times 28.$$

$$\therefore P = \frac{300 \times 10{\cdot}25}{28} = \frac{3075}{28} = 109{\cdot}8214, \text{ \&c., lbs.}$$

$$P \text{ with friction} = 109{\cdot}8214 + (\tfrac{1}{10} \text{ of } 109{\cdot}8214) = 109{\cdot}8214 + 10{\cdot}9821 = 120{\cdot}803 \text{ lbs. } \textit{Ans.}$$

THE DIFFERENTIAL AXLE OR CHINESE WHEEL.

359. An ingenious contrivance for enabling the exertion of a comparatively small power to overcome a large resistance is presented in the machine called the Chinese windlass. This machine may be considered as a combination of the wheel and axle with a single movable pulley. Two cylinders of different radii have a common axis with which they are firmly connected; the axis is supported in a horizontal position, so that the two cylinders can turn as one body round the axis. A string has one end fastened to the larger cylinder, is coiled several times round the cylinder, then leaves it, passes under a movable pulley, and is coiled round the smaller cylinder to which the other end is fastened. The string is coiled in opposite ways round the two cylinders, so that as it winds off one it winds on the other.

A weight W is hung from the movable pulley, and the equilibrium is maintained by a power P applied at the end of the handle attached to the axis. It is found by theory and by trial that there is equilibrium on this machine *when the power is to the weight in the same proportion* as half the difference of the radii of the two cylinders is to the length of the arm at which the power acts. Thus, by making the difference of the radii of the two cylinders sufficiently small, we can secure any amount of mechanical advantage.

We could ascertain the power necessary to raise a given load directly, but for the purpose of illustration we shall make use of the principle of work.

Let us suppose that the circumference of the circle described by the handle is 5 feet, and that the effective circumference of the cylinders are respectively 2 feet and 1 foot 8 inches. When the handle has completed one revolution, the power has been applied through a distance of 5 feet; consequently, 2 feet of the rope has been wound up upon the larger cylinder, while 1 foot 8 inches has been unwound from the smaller cylinder. The part of the rope which is not in contact with the cylinders has, therefore, been shortened by the difference between 2 feet and 1 foot 8 inches, that is, by 4 inches; and as this must be equally distributed between the two portions of the rope going to the pulley, it follows that each of them will be shortened by 2 inches, and that, therefore, the movable pulley, and consequently, the load suspended from it will be raised through 2 inches. It thus appears that when the power has been applied through a distance of 5 feet, the load will have been raised through 2 inches. If, therefore, friction be overlooked, it follows, from the principle of work, that a given power applied at the handle will be able to raise a load at the pulley which is 30 times as great as the power.

In this case, the work done by the power in a single revolution is the power multiplied by the circumference described by the winch handle; and since, in a single revolution, the rope is diminished by a length equal to the difference of the circumferences of the axles, the weight will be lifted through *half* this difference, and, therefore, by the principle of constancy of work done by a machine, we have—

$$P \times C = W \times \frac{c - c'}{2}$$

where C is the circumference of the circle described by the winch handle, and c and c' are the circumferences of the axles. Hence we find—

The power is to the weight as the difference of the circumferences, or radii of the axles is to twice the circumferences, or radius of the circle described by the winch handle.

$$\text{Or, } 2\,R \times P = \frac{D - d}{2} \times W,$$

$$\therefore W = \frac{4\,R \times P}{D - d}$$

360.—It will thus be seen that this contrivance enables a small force to be turned into a very much larger one. The efficiency of the instrument will be the more increased according as the diameters of the two cylinders become more nearly equal. On the other hand, the machine will become more slow in its action in proportion as it becomes more powerful; in fact, it is evident that if the two cylinders were actually equal the load would never be raised at all.

EXAMPLES.

Ex. 1. The diameters of the axles are $12''$ and $9''$ respectively; the length of the handle 2 feet, and the weight 400 lbs.: required the power P.

$$2\,\mathrm{R} \times \mathrm{P} = \frac{\mathrm{D} - d}{2} \times \mathrm{W},$$

$$\therefore \mathrm{P} = \frac{\mathrm{D} - d}{2} \times \mathrm{W} \div 2\,\mathrm{R}.$$

Expressing the diameter of axles in feet and decimals of feet, we have—

$$\mathrm{D} = 1 \text{ ft., and } d = \tfrac{9}{12} \text{ ft., or } {\cdot}75 \text{ ft.,}$$

$$\therefore \mathrm{P} = \frac{1 \text{ ft.} - {\cdot}75}{2} \times 400 \div 4 = {\cdot}125 \times 100 = 125 \text{ lbs. } \textit{Answer.}$$

Ex. 2. The diameter of axles, ·7 ft. and ·5 ft. respectively, the length of handle 2 feet, the effort put forth is 10 lbs.: required the weight.

$$\mathrm{W} = \frac{4\,\mathrm{R} \times \mathrm{P}}{\mathrm{D} - d}$$

$$4\,\mathrm{R} = 2 \text{ ft.} \times 4 = 8 \text{ ft.; and } \mathrm{D} - d = {\cdot}7 \text{ ft.} - {\cdot}5 \text{ ft.} = {\cdot}2 \text{ ft.}$$

$$\mathrm{W} = \frac{8 \times 10}{{\cdot}2} = \frac{80}{{\cdot}2} = 400 \text{ lbs. } \textit{Answer.}$$

Or, by the principle of work,

$$\text{Work done upon W in 1 revolution} = \frac{{\cdot}7 \times 3{\cdot}1416 - {\cdot}5 \times 3{\cdot}1416}{2} \times \mathrm{W}.$$

$$\therefore \text{Work done upon P in 1 revolution} = 4 \times 3{\cdot}1416 \times 10,$$

$$\therefore \frac{{\cdot}7 \times 3{\cdot}1416 - {\cdot}5 \times 3{\cdot}1416}{2} \times \mathrm{W} = 4 \times 3{\cdot}1416 \times 10.$$

Dividing each side of equality by 3·1416,

$$\frac{{\cdot}7 - {\cdot}5}{2} \times \mathrm{W} = 4 \times 10, \therefore \mathrm{W} = 40 \div {\cdot}1 = 400 \text{ lbs. } \textit{Answer.}$$

Ex. 3. A toothed wheel of 8 teeth turns on the same axle as a wheel A of 2 feet diameter, and gears into another toothed wheel of 56 teeth, which turns on the same axle as an axle B of 3 inches diameter: if a power of 30 lbs. be applied to the wheel A, what weight will be lifted by the axle B?

If the toothed wheels had been of the same diameter,

$$\mathrm{P} \times 2 \text{ ft.} = \mathrm{W} \times \tfrac{1}{4} \text{ ft.}$$

$$30 \times 2 \text{ ft.} = \mathrm{W} \times \tfrac{1}{4} \text{ ft.}$$

$$\therefore \mathrm{W} = 60 \div \tfrac{1}{4} = 240 \text{ lbs.}$$

But as the number of teeth are in the proportion of 8 to 56, the diameters will be in the same proportion; that is, one diameter is 7 times the other,

$$\therefore \text{Weight lifted will be } 240 \times 7 = 1680 \text{ lbs. } \textit{Answer.}$$

Ex. 4. A geared engine has 60 teeth in the driving wheel and 20 on the shaft, the engines are making 42 revolutions per minute: what are the revolutions of the propeller per hour?

$$\text{Revolutions of engine per hour} = 42 \times 60 = 2520,$$

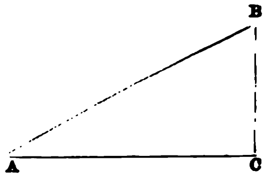
$$\therefore 20 : 60 :: 2520 : \text{revolutions of propeller.}$$

$$\text{Rev. of propeller per hour} = \frac{2520 \times 60}{20} = 7560.$$

THE INCLINED PLANE.

361. **An Inclined Plane** in Mechanics is a smooth plane supposed to be made of wood or metal, or some other rigid material, and fixed in a position inclined to the horizon. It is supposed to be capable of resisting, in a direction perpendicular to its surface, to any required amount. When an

inclined plane is used as a mechanical power, the straight lines indicating the directions in which the power and weight act, are supposed to be both in one vertical plane, namely, in the plane perpendicular to the straight line in which the inclined plane meets the horizon. Thus, the inclined plane is represented by a right-angled triangle, such as A B C; the horizontal side A C is called the base, the vertical side B C is called the height, the angle B A C is the inclination of the inclined plane to the horizon.



362. Suppose a heavy body placed on an inclined plane. The weight of the body tends vertically downwards, but owing to the resistance of the plane the body cannot move in that direction, it will, however, slide down the plane unless prevented by a suitable force, and the amount of the force which we must use depends upon the direction in which it acts. We will suppose that the force acts *along* the plane, or *parallel* to it; the proposition which applies in this case is the following:—*When a weight is put on an inclined plane and kept in equilibrium by a power acting parallel to the plane, the power is to the weight in the same proportion as the height of the plane is to its length.*

The preceding statement may be taken as an experimental truth, or it may be established by reasoning, for which we have not space here.

363. If we suppose the power to be a little greater than is necessary for equilibrium, the weight will be moved along the plane. Suppose the weight to be drawn along the plane from A to B, so that the power has passed over the *length* of the plane, then the weight has passed over as much space as the power, but the *vertical* height through which the weight has passed is B C. Thus we have here a fresh illustration of the important principle of No. 347, and at the same time an indication of the way in which the principle is to be understood; the motion of the weight, *estimated in the direction of the weight*, bears the same proportion to the motion of the power, *estimated in the direction of the power*, as the power bears to the weight in equilibrium.

364. There is another case with regard to the *inclined plane* which it is usual to notice, namely, that in which the power acts *horizontally;* the proposition which applies to this case is the following:—*When a weight is put on an inclined plane acting horizontally, the power is to the weight in the same proportion as the height of the plane is to its base.* The preceding statement may be taken as an experiment.

If we suppose the power to be a little greater than is necessary for equilibrium, the weight will be moved along the plane. Suppose the weight to be drawn along the plane from A to B so that the power has passed *horizontally* over the space A C; then the weight has passed *vertically* over the space BC. Hence, the space passed over by the weight, *estimated in the* *direction of the weight,* is to the space passed over by the power, ***estimated in the direction of the power,*** as the power is to the weight in the state of equilibrium. Thus we have here a fresh illustration of the important principle in No. 347, and an indication, as in No. 363, of the way in which it may be understood.

The only use of the inclined plane which is of any practical importance is that in which the power is applied parallel to the plane. In this case, as has just been shown, No. 363, while the power moves through the length of the plane, the weight is lifted through the height of the plane, and, therefore,

Power × length of plane = weight × height of plane.

Or, ***the power is to the weight as the height of the plane is to its length.***

EXAMPLES.

Ex. 1. Length of plane 20 feet, height of plane 9 feet, and weight 500 lbs.: required the power P.

P : W :: height of plane : length of plane.
P : 500 :: 9 : 20

$$P = \frac{500 \times 9}{20} = 225 \text{ lbs.} \quad \textit{Answer.}$$

Or thus:—

Work in raising $W = 500 \times 9$.

Work due to the descent of $P = P \times 20$,

$\therefore P \times 20 = 500 \times 9$; and $P = 225$ lbs. *Answer.*

Ex. 2. Suppose length of inclined plane 100 feet, height 15 feet, and weight 20 cwt. required the power.

P : W :: height of plane : length of plane.
P : 20 cwt. :: 15 : 100

$$P = \frac{20 \times 15}{100} = 3 \text{ cwt.}$$

THE SCREW.

365. A screw may be considered an inclined plane wound round a cylinder.

The projecting coils are termed the threads of the screw.

The distance between the upper edge of one thread and the corresponding edge of the next, measured on a line parallel to the axis, is termed the ***distance between the threads.*** The screw is usually connected with a concave cylinder, termed a nut, on the interior surface of which a spiral cavity is cut, corresponding exactly to the thread of the screw which moves in it.

Suppose the axis of the screw to be vertical, and let a weight W be placed on the screw. Then the weight, by its tendency to descend, would cause the screw to turn round in the block, unless this motion were prevented by some power. We will suppose the power P to act at the *end* of a horizontal arm perpendicular to the lever, and horizontally, the arm being firmly attached to the cylinder. The distance between the axis of the cylinder and the point of application of the power we shall call the power-arm. It is found when there is equilibrium, P is to W in the same proportion as the distance between two consecutive threads of the screw is to the circumference of the circle, having the power-arm for the radius. The reasoning on which this depends is not simple enough to find a place here, so that this may be taken as an experimental fact. It is interesting, however, to observe its agreement with No. 347, as illustrated in Nos. 363 and 364; for, supposing the power to be a little greater than necessary for equilibrium, then the weight will be moved; by turning the screw once round, the weight will be raised through a space equal to the distance between two consecutive threads. Also, the whole space passed over by the end of the power-arm, estimated in the direction of the power, consists of a multitude of small spaces, which are together equivalent to the circumference of the circle, having the power-arm for radius.

Power × circumference of circle through which it moves =
weight × interval between the threads.

Or, *the power is to the weight as the interval between the threads is to the circumference of the circle described by the power.*

EXAMPLES.

Ex. 1. The lever of a simple screw is 2 feet, the distance between the threads is ·2 foot: if a pressure of 100 lbs. be applied to the lever, what pressure will be produced on the press-board?

Circumference of circle through which power moves $= 4 \times 3{\cdot}1416 = 12{\cdot}5664$.

$$P : W :: {\cdot}2 : 12{\cdot}5664.$$

$$W = \frac{100 \times 12{\cdot}5664}{{\cdot}2} = \frac{1256{\cdot}64}{{\cdot}2} = 6283{\cdot}2 \text{ lbs.} \quad \textit{Answer.}$$

Or thus:—In one turn of the lever the press-board is moved over the distance between the threads of the screw.

$\therefore$ Space moved over by P in 1 revolution $= 4 \times 3{\cdot}1416$.
" " W " $= {\cdot}2$.
Work of P in 1 revolution $= 4 \times 3{\cdot}1416 \times 100$.
" W " $= {\cdot}2$.
$\therefore W \times {\cdot}2 = 4 \times 3{\cdot}1416 \times 100$, and $W = 6283{\cdot}2$ lbs.

Obs.—The equation here obtained shows that we may increase the mechanical force of the screw either by diminishing the distance between the threads, or by lengthening the arm of the lever, or by both.

Ex. 2. Required the same as in the last example, when the distance between the threads is ½ in., and the pressure applied to the lever is 60 lbs. *Ans.*, 8 tons 1 cwt. 2 qrs. 7 lbs.

Ex. 3. Required the advantage of the power in the last example. *Ans.*, 301.

Ex. 4. The lever of the screw is 2½ ft., and the distance between the threads ¾ in.: what pressure must be exerted on the lever so as to move a 5-ton weight?

$$5 \text{ tons} = 5 \times 2240 = 11200 \text{ lbs.}; \ 2\tfrac{1}{2} \times 2 = 5 \text{ ft.} = 60''.$$

$$P \times 60'' \times 3{\cdot}1416 = 11200 \times \tfrac{3}{4}'',$$

$$\therefore P = \frac{11200 \times \frac{3}{4}}{60 \times 3{\cdot}1416} = \frac{8400}{188{\cdot}496} = 44{\cdot}56 \text{ lbs.} \quad \textit{Answer.}$$

THE COMPOUND SCREW.

366. This mechanical power consists of two screws, one of which screws with the other, so that whilst the larger one is descending the small one is relatively rising within the large one. In consequence of this compound motion, in one revolution of the lever, the press board only descends a distance equal to the difference of the thickness of the threads of the screw. Hence the advantage depends upon the smallness of this difference and not upon the absolute size of the threads. In this respect the machine is analagous to the compound wheel and axle.

367. It is found, by theory and by trial, that there is equilibrium in this machine when the power is to the weight in the same proportion as the difference of the distances between two consecutive threads in the two screws is to the circumference of the circle having the power-arm for radius.

EXAMPLES.

Ex. 1. In a compound screw the length of the lever is 1½ ft., the distance between the threads of the large or hollow screw is ¾ in., and that of the small one ½ in.: if 30 lbs. pressure be applied to the lever, what will be the pressure on the press-board?

P : W :: difference of distance between two threads of screws : circumference of circle described by lever.

$$P : W :: \left(\tfrac{3}{4} - \tfrac{1}{2} = \tfrac{1}{4}\right) = \tfrac{1}{48} \text{ ft.} : 3 \times 3{\cdot}1416.$$

$$W = \frac{30 \times 3 \times 3{\cdot}1416}{\frac{1}{48}} = \frac{282{\cdot}744 \times 48}{1}$$

$= 13571{\cdot}712$ lbs. $=$ 6 tons 1 cwt. 0 qr. 19·7 lbs. *Answer.*

In one revolution the large screw descends ¾ inch, but at the same time the small screw (by turning within the larger one) ascends ½ inch, therefore, the press-board must descend a space $= \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ inch $= \frac{1}{48}$ foot.

Work done in 1 revolution $= W \times \frac{1}{48}$.

Work applied in 1 revolution $= 30 \times 2 \times 1{\cdot}5 \times 3{\cdot}1416 = 282{\cdot}744$,

$\therefore W \times \frac{1}{48} = 282{\cdot}74$; and $W = 13571{\cdot}7$ lbs.

Ex. 2. Required the same as in the last example, when the threads are ⅞ and ¾ respectively, and the pressure applied to the lever is 1 lb. *Answer*, 904.

Ex. 3. What pressure must be applied to the lever in Ex. 1, so that a pressure of 4 tons may be given to the press-board? *Answer*, 19·8 lbs.

Ex. 4. If the length of the lever be 2 feet, the thickness of the thread of the large screw 1 in.: what must be the thickness of the small screw, so that 1 lb. applied to the lever may produce a pressure of ¼ ton?

Here, we have, by the principle of equality of work,

$$1 \times 4 \times 3{\cdot}1416 = 560 \left(\tfrac{1}{12} - \text{thickness}\right),$$

$$\therefore \text{Thickness} = {\cdot}06 \text{ ft.} = {\cdot}72 \text{ inch.}$$

Ex. 5. The connecting-rod end is adjusted by a screwed cutter, the screw has 14 threads to the inch, the taper of the cutter is ⅝ in. to the foot, the nut is 6 canted: required the number of turns and cants of the nut to bring the brasses ·01 in. closer.

$\frac{5}{8} = {\cdot}625$

12)·625 of an inch.

$$14\left\{\begin{array}{l} 2)\,{\cdot}052083 \text{ of an inch to an inch.} \\ \hline 7)\,{\cdot}026041 \end{array}\right.$$

·003720 of an inch for 1 turn of the nut.

$$\begin{array}{rrl} \cdot 00372\ )\ \cdot 010000\ (& 2\cdot 688 & \text{complete turns.} \\ 744 & 6 & \\ \hline 2560 & 4\cdot 128 & \text{cants.} \\ 2232 & & \\ \hline 3280 & & \\ 2976 & & \\ \hline 3040 & & \end{array}$$

Answer, 2 turns and 4 cants.

Ex. 6. The outer or hollow screw of a screw-jack has a pitch of 1 inch, the smaller or inner one a pitch of $\frac{7}{8}$ inch, the lever is 1 foot in length: what weight will be eased up by a man exerting a force equal to 150 lbs. at the end of the lever? *Ans.*, 40·39 tons.

THE PULLEY.

368. A force may be transmitted from one point to another by means of a cord, and if there were no friction, this force might be transmitted undiminished in any direction. In order to diminish friction in thus transmitting a force from one point to another by means of a cord, we make the cord pass over a wheel with a groove cut in its circumference, which turns with the cord, and so diminishes the friction. The wheel is called the *sheave*, and the wooden frame in which it works, and to which its axle or pivot is fastened, is called the *block* of the pulley. The principle on which the equilibrium of a system of pulleys is determined is the following:—

The tension of each cord employed is constant, and equal to the power applied at its extremity.

369. Let us consider the case of a pulley, the block of which is fixed, and suppose a weight attached to the end of a string passing round the pulley. If the string be pulled at the other end by a power equal to the weight, there will be equilibrium; if the string be pulled by a power somewhat greater, the weight will be raised. Thus, a *fixed* pulley is a machine by the aid of which we can change the *direction* of a force without changing its *magnitude*. For example, we might have a power which could most conveniently act in a direction *inclined* at a certain angle to the horizon, and it might be used in supporting a weight, that is, in balancing a *vertical* force; then by transmitting the power by means of a string, and passing the string round a fixed pulley, we can support a weight equal to the power. A fixed pulley is often used when weights are to be raised, as for example, in setting the sails of ships. Thus, a *fixed* pulley, though it may be very convenient, does not afford us any *mechanical advantage*.

370. It might at first sight appear that nothing is gained by making the circular plate of the pulley capable of turning round its axis; but practically this is very important. When the circular plate can turn round, it is found by trial that in the state of equilibrium the tension of the string is almost the same on both sides of the pulley, so that a weight can be moved by a

power which is very slightly greater. But when the circular plate cannot turn round, it is found that there may be a considerable difference between the tension of the string on the two sides of the pulley, and so a weight could not be moved unless the power was considerably greater. This is owing to friction.

371. Now consider the case of the single movable pulley. Let a string pass round the pulley having one end fixed, and to be pulled vertically upwards by a power P at the other end. Let a weight be attached to the block of the pulley. Thus it is found on trial that there is equilibrium if the power is equal to half the weight. In fact we may consider the block to be acted on by three parallel forces; namely, the weight downwards and the two forces upwards arising from the tension of the two parts of the string. Thus the weight must be equal to the sum of the two upward forces. But the tension of the string is throughout equal to the power: therefore twice power (P) is equal to the weight (W). It will be apparent that there is nothing strange in this result. The weight has to be supported in some manner, and on examining we find that the result is the same as if half of the weight were supported by a fixed beam (to which one end of the string is fastened) and half by the power.

372. If the power be only a little greater than half the weight, the weight will be raised. According to the principle of No. 347, if the weight is raised through any space, the end of the string at which the power acts must be raised through *twice* that space. This may be easily shown.

For, suppose the weight to be raised through any space, say one inch, the part of the string between the fixed end and round the pulley must be shortened by one inch, and to keep the string stretched the end at which P acts must be raised through two inches. Thus the power end of the string moves through twice the space through which the weight moves.

373. Sometimes the two parts of the string are not parallel. But in order that there may be equilibrium, the two parts of the string must be inclined to the vertical at the *same angle;* for, if they were not, the pulley would be drawn towards the side where the string was most inclined to the vertical. Then, when there is equilibrium, the weight is equal to twice that part of the power which acts in a vertical direction; suppose the power resolved into two components, one vertical and the other horizontal. We now pass on to consider various combinations of pulleys.

374. **First System of Pulleys.**—In a system of pulleys in which each pulley hangs by a separate string, and all the strings are parallel, it is usually called the *First System of Pulleys.* In this system the string which passes round any pulley, except the highest, has one end attached to a fixed point, and the other end supported by the power. We saw that for equilibrium in the single movable pulley, the weight must be twice the power; in

the *First System of Pulleys* it is found that for equilibrium, if there are two movable pulleys, the weight must be four times the power; if there are three movable pulleys, the weight must be eight times the power; if there are four movable pulleys, the weight must be sixteen times the power, and so on. Thus, for every additional movable pulley the weight that can be supported by a given power is doubled.

375. There is no difficulty in the reasoning by which this result is established. Suppose there are four movable pulleys. By the principle of the single movable pulley, the tension of string which passes under the lowest pulley is $\frac{W}{2}$; thus the next pulley is drawn downwards by a force equal to $\frac{W}{2}$, and, consequently, the tension of the string which passes round it is $\frac{W}{4}$; in like manner, the next pulley is drawn downwards by a force equal to $\frac{W}{4}$; and, consequently, the tension of the string that passes round it is $\frac{W}{8}$; in like manner, the next pulley is drawn downwards by a force equal to $\frac{W}{8}$, and, consequently, the tension of the string which passes round it is $\frac{W}{16}$. This last tension must be equal to the power which acts at the end of the string, so that P is equal to $\frac{W}{16}$.

376. As in former cases, if the power be a little greater than is necessary for equilibrium, the weight will be moved, and the principle No. 347 will be found to hold.

377. **Second System of Pulleys.**—In this system of pulleys the same string passes round all the pulleys, and the parts of it between the pulleys are parallel. Let us suppose that there are *four* strings at the lower block, and when there is equilibrium the weight is four times the power. In like manner, if there are *six* strings at the lower block, the weight is six times the power. Suppose, in this case, the string is fastened to the upper block, and the number of strings at the lower block is an *even* number. But the end of the strings might be fastened to the lower block, and then the number of strings at the lower block would be an *odd* number.

The reasoning by which the condition of equilibrium in the *Second System of Pulleys* is established is as follows:—The tension of the string is the same throughout, and is equal to the power, so that if there are *four* strings at the lower block, we may regard that block as drawn up by four parallel forces each equal to the power, and drawn downwards by the weight; therefore the weight must be equal to four times the power. It will be seen that there are five strings at the upper block, so that the fixed point must support altogether five times the power, that is, the sum of W and P. A remark similar to No. 376 may be repeated here.

378. **Third System of Pulleys.**—In this system of pulleys each string is attached to the weight, and all the strings are parallel. The string which

passes round any pulley except the lowest, has one end attached to the block of the next lower pulley; the string which passes round the lowest pulley has one end attached to the weight, and the other end supported by the power. The highest pulley is fixed, and the others are movable. The condition of equilibrium for the *Third System of Pulleys* can be expressed most easily by stating what proportions the *sum* of the weight and the power must bear to the power. If there is one pulley the sum is twice the power, if there are two pulleys the sum is four times the power, if there are three pulleys the sum is eight times the power, if there are four pulleys the sum is sixteen times the power, and so on. Thus for every additional power the sum is doubled. If the sum is twice the power, the weight is equal to the power; if the sum is four times the power, the weight is three times the power; if the sum is eight times the power, the weight is seven times the power; if the sum is sixteen times the power, the weight is fifteen times the power, and so on.

There is no difficulty in the reasoning by which the condition of equilibrium for the *Third System of Pulleys* is established. Suppose that there are four pulleys. Let W denote the weight to which all the strings are fastened, and P the power which acts vertically downward at the end of the string which passes over the lowest pulley. The tension which passes over the lowest pulley is P, hence this pulley is drawn downwards by a force equal to 2 P, and, consequently, this must be the tension of the string which is fastened to it and draws it upwards, passing over the second pulley. Hence, the second pulley is drawn downwards by a force equal to 4 P, and, consequently, this must be the tension of the string which is fastened to it and draws it upwards, passing over the third pulley. Similarly, with respect to the tension of the string which passes over the fourth pulley. Now, all the strings are fastened to the weight, and so help to support it; then W must be equal to the turn of P, 2 P, 4 P, and 8 P, that is, W must be equal to 15 P.

Or we might shorten the process a little, thus:—The tension of the string that goes over the highest pulley is 8 P, so that this pulley is drawn downwards by a force equal to 16 P, but the whole weight supported at the point to which the highest pulley is fixed must be equal to the sum of W and P; therefore, the sum of W and P is equal to 16 P, and, consequently, W is equal to 15 P. A remark similar to that of No. 376 may be repeated here.

In what has preceded we have supposed that the weights of the pulleys themselves are neglected, but in practice it may be necessary to take these weights into account; it will be sufficient to take one case as an example. Consider the *Third System of Pulleys*, and suppose, as in No. 376, that there are four pulleys. The weight of the lowest pulley here assists the power, and acts just like the power, except that it has a system of *three* pulleys above it instead of four; thus it will support seven times its own weight. Similarly, the weight of the next pulley will support three times its own weight, and the weight of the next to that will support just its own weight. The weight of the highest pulley will not give any aid. Thus, finally, we have the following result:—The weight W is equal to the sum of 15 P, together with seven times the weight of the lowest pulley, three times the weight of the next, and the weight of the next to that.

EXAMPLES.

Ex. 1. A weight of 3 tons is lifted by a pair of blocks of two sheaves each, the rope is fastened to the upper block: what strain is on each single of the rope?

$$3 \text{ tons or } 60 \text{ cwt.} \div 4 = 15 \text{ cwt.} \quad \textit{Answer.}$$

$$\text{Mechanical efficiency} = \frac{60 \text{ cwt.}}{15 \text{ cwt.}} = 4.$$

Ex. 2. A boiler weighing $7\frac{1}{2}$ tons has to be lifted by a pair of treble blocks: what is the strain on each single of the rope?

$$7{\cdot}5 \times 2240 = 16800 \text{ lbs.}$$

$$16800 \text{ lbs.} \div 6 = 2800 \text{ lbs.} = 1 \text{ ton } 5 \text{ cwt.} \quad \textit{Answer.}$$

Ex. 3. A pair of double blocks are used to lift 2 tons weight: what would be the pull if 20 per cent. be lost by friction?

$$2 \text{ tons} \times 2240 = 4480 \text{ lbs.}$$

$$\text{Pull without friction} = 4480 \text{ lbs.} \div 4 = 1120 \text{ lbs.}$$

With friction: Since 20 per cent. is lost by friction, if we suppose the pull to be 100 lbs., then the weight lifted would only be 80 lbs.; therefore, the pull without friction, viz., 1120 lbs., must be increased in the ratio of 80 : 100, or as 8 : 10,

$$\therefore \tfrac{10}{8} \text{ of } 1120 = 1400 \text{ lbs.} \quad \textit{Answer.}$$

$$\text{Otherwise,} \quad 80 : 100 :: 1120 : x.$$

$$\frac{1120 \times 100}{80} = 1400 \text{ lbs.} \quad \textit{Answer.}$$

Ex. 4. A weight of 4 tons is lifted by a pair of blocks of two sheaves each, the rope is fastened to the upper block: what must be the pull at the free end of the rope, supposing 10 per cent. to be lost at each sheave?

If 10 per cent. is lost at each sheave, the first lifting rope will exert a lifting force equal to ·9 of the pull of the free end of the rope.

The others will exert a force equal to ·9 of the preceding rope.

The sum of these calculated for all the ropes is the divisor to be used instead of the number of sheaves.

100 That is, the rope, after passing the 1st sheave, will have only $\frac{90}{100}$, or $\frac{9}{10}$
10 per cent. of the full pull. ·9 is the first power of ·9. The rope, after passing the
— 2nd sheave, will have ·9 of ·9 = ·81 of the full pull, which is the second
90 power of ·9, and so on to as many powers as there are sheaves, and the
work may be written thus:—

·9 = 1st power.
·9
—
·81 = 2nd power.
·9
—
·729 = 3rd power.
·9
—
·6561 = 4th power.

·9
·81
·729
·6561
—
3·0951

Weight 4 tons = 4 × 2240 = 8960 lbs.

3·0951)8960·0000(2894·9 lbs.
61902
―――
276980
247608
―――
293720
278559
―――
151610
123804
―――
278060

Ans.: 2894·9 lbs., the pull required.

Ex. 5. A weight of 2 tons is lifted by a pair of blocks, there are 3 sheaves in each block, the rope is fastened to the upper block: what must be the pull at the free end, supposing 12 per cent. be lost at each sheave?

Calculating as many powers as there are sheaves, thus:—

100
12
―――
88

$\frac{88}{100} = \cdot 88$

·88 = 1st power.
·88
―――
704
704
―――
·7744 = 2nd power.
·88
―――
61952
61952
―――
6814,72 = 3rd power.
or, 6815
·88
―――
54520
54520
―――
·5997,20 = 4th power.
·88
―――
47976
47976
―――
·5277,36 = 5th power.
·88
―――
42216
42216
―――
·4643,76 = 6th power.

·88
·7744
·6815
·5997
·5277
·4644
―――
3·9277

Note.—It is quite unnecessary to retain more than four places of decimals in the work.

Weight 2 tons = 2 × 2240 = 4480 lbs.

3·9277)4480·0000(1140·6 lbs.
39277
―――
55230
39277
―――
159530
157108
―――
242200
235662
―――

EXAMPLES FOR PRACTICE.

1. What weight on the end of a lever 8 feet in length will exert a force equal to 100 lbs. on the other end, 6 inches being the distance of the fulcrum from the 100 lbs.? *Ans.*, 6·667 lbs.

2. What weight on the end of a lever 12 feet in length will equipoise 2 tons on the other end, the fulcrum being distant 9 inches from the weight to be equipoised, and the weight of the lever not taken into account? *Ans.*, 194·7 lbs.

3. Four weights, 1, 3, 5, and 7, are at equal distances of 8 inches from each other on a straight lever: how far from each is the fulcrum on which they will balance? *Ans.*, 17 inches, 9 inches, 1 inch, and 7 inches.

4. The arms of a lever are respectively 15 and 16 inches: find what weight at the end of the short arm will balance 30 lbs. at the end of the long arm, and what weight at the end of the long arm will balance 30 lbs. at the end of the short arm. *Ans.*, 32 lbs.; 28 lbs. 2 ozs.

5. What force will be required to work the handle of a windlass, the resistance to be overcome being 1156 lbs., the radius of the axle being 6 inches, and of the handle 2 feet 8 inches. *Ans.*, 216·75 lbs.

6. A weight of 17 lbs. just balances a weight of 79 lbs. on a wheel and axle: what will be the radius of the axle if that of the wheel be 17 inches? *Ans.*, 3·65 inches.

7. In a wheel and axle, the radius of the wheel is 20 in., and that of the axle 4 in.; if the power applied is 70 lbs.: what must be the weight? *Ans.*, 350 lbs.

8. If the radius of the axle be 3 in.: what will then be the weight? *Ans.*, 466⅔ lbs.

9. What power must be applied, in Ex. 7, in order to support a weight of 400 lbs.? *Ans.*, 80 lbs.

10. With a wheel and axle a power of 8 lbs. sustains a weight of 12 lbs.: what is the radius of the axle, that of the wheel being 24 in.? *Ans.*, 16.

11. The circumference of wheel and axle are respectively 1 yd. and 15 in.: what power will sustain a weight of 1½ tons. *Ans.*, 12·5 cwt.

$$P \times 36 = W \times r = 30 \times 15, \quad \therefore P = 12{\cdot}5 \text{ cwt.}$$

12. Twelve sailors, exerting each a force of 80 lbs., push a capstan with a length of lever equal to 6 ft.; the diameter of capstan is 2 ft. Calculate the resistance which this force is capable of sustaining. *Ans.*, 2 tons 11 cwt. 1 qr. 20 lbs.

13. In the last example, suppose 10 per cent. of the effort is lost through friction and the rigidity of the rope: what would have been the weight lifted? *Ans.*, 2 tons 6 cwt. 1 qr. 4 lbs.

14. The diameter of a steering wheel is 5 ft., and the diameter of barrel is 15 in.; if a man applies a power equal to 200 lbs.: what resistance would be overcome? *Ans.*, 800 lbs.

15. If the diameter of the barrel of the wheel be 8″, and the diameter of the rope 1″; weight 200 lbs.: what force must be applied to the handle whose radius is 18″? *Ans.*, 15 lbs.

16. In a wheel and axle the diameter of barrel was 6″, and that of the rope ⅞″, the crank handle was 14″, and the weight 174 lbs.: what was the force resisted, allowing 16 per cent. for friction? *Ans.*, 49·5589 lbs.

17. In a wheel and axle the barrel is 9″ diameter, the rope is 1¼″ diameter, the crank handle is 14″ radius, and the weight to be lifted 300 lbs.: what is the force applied to the handle, adding 10 per cent. for friction? *Ans.*, 120·8035 lbs.

18. A machine is formed by the combination of three wheels and axles; the radius of each wheel is four times that of the corresponding axle: if the power is 4 lbs., find the weight. *Ans.*, 256 lbs.

19. In the chinese wheel the length of the power-arm is 30 in., the radius of the larger cylinder is 3½ in., and the radius of the smaller 3 in.; if the power is 4 lbs., find the weight. *Ans.*, 480 lbs.

20. A power P acting by means of a rope going over a wheel whose diameter is 6 feet, supports a weight of 320 lbs., the diameter of the axle is 8 inches, and the rope is 2 inches thick: what is the value of P, supposing the thickness of the rope to be taken into account?

21. The length of an inclined plane is 80 feet, height 2 feet, the weight of the body 2 tons: what pressure will be required to sustain the body on tho plane, friction being neglected? *Answer*, 1 cwt.

22. If the height of the plane be 3 feet: what will be the pressure? *Answer*, 1½ cwt.

23. If the friction in Ex. 21 be $\frac{1}{30}$ of the load: what must be the pressure to move the body up the plane? *Answer*, 261⅓ lbs.

24. If the friction be $\frac{1}{20}$ of the load: what will then be the pressure? *Answer*, 3 cwt.

25. The diameter of a screw is 7 inches, and the distance between the threads ¼ inch: what power applied at the circumference of the screw will support a weight of 110 lbs. *Ans.*, 1·25 lbs.

26. When the circumference of the screw is 12 inches, and there are three threads to the inch: find the weight that will be supported by a power of 10 lbs. *Ans.*, 360 lbs.

27. If the circumference described by the end of the lever be 10 feet, the power 10 lbs., and there be three threads in 2 inches: find the resistance supported. *Ans.*, 1800 lbs.

28. If the interval between the threads of a screw be $\frac{1}{11}$ of an inch, and the circumference of the circle described by the powers be 2 feet: what resistance will a power of 13 lbs. sustain. *Ans.*, 1 ton 10 cwt. 2 qrs. 16 lbs.

29. If the interval be ⅜ of an inch, and the radius of the circle described by the power be 2 feet: find resistance which a power of 32 cwt. will sustain. *Ans.*, 643 tons 8 cwt.

30. In Hunter's screw the circumference of the circle which has the power-arm for radius is 6 feet; the threads of one screw are half-an-inch apart, and those of the other one-third of an inch: show that the advantage of the machine is 432.

31. A weight of 30 lbs. balances a weight of 20 lbs. at the extremities of a straight lever 15 feet long: find the length of the arms.

$$(30\text{ lbs.} + 20\text{ lbs.}) : 30\text{ lbs.} :: 15\text{ ft.} : x$$

$$\therefore x = 9\text{ ft., and } 15\text{ ft.} - 9\text{ ft.} = 6\text{ ft.}\quad \textit{Ans.}$$

32. If one end of a bar rests on a beam, and a weight of 50 lbs. be suspended from it at one-fifth of its length from the beam, what power at the other end will support the weight, and what will be the pressure on the beam?

Let $l =$ the length of beam. Then,

$$\tfrac{1}{5} l \times W = l \times P, \therefore P = \tfrac{1}{5} W = 10\text{ lbs.}$$

$$\text{Pressure on beam} = W - P = 50 - 10 = 40\text{ lbs.}$$

33. A weight of 5 tons is lifted by a pair of blocks, there are 4 sheaves to each block, the rope is fastened to the upper block: what must be the pull at the free end, supposing 7 per cent. be lost on each sheave?

Ans.—$·93 + ·8649 + ·8044 + ·7481 + ·6957 + ·6470 + ·6017 + ·5596 = 5·8514$ the divisor; and 5 tons $= 5 \times 2240 = 11200$ lbs.; then $11200 \div 5·8514 = 1914·0$ lbs., the pull required.

34. A weight of 7 tons is lifted by a pair of blocks, there are 2 sheaves to each block, the rope is fastened to the upper block: what must be the pull on the free end, supposing 12 per cent. be lost at each sheave?

Ans.—$·88 + ·7744 + ·6815 + ·5997 = 2·9356$; and 7 tons $= 7 \times 2240 = 15680$ lbs.; then $15680 \div 2·9356 = 53413$ lbs., the pull required.

MISCELLANEOUS EXAMPLES.

Ex. 1. The square of the diameter of the funnel in feet is equal to the whole width of the fire grate: the fire bars are 5 feet 6 inches long, how many square feet of fire grate surface are there to the square foot of funnel?

Let d represent diameter of funnel.

Then by the question $d^2 =$ whole width of fire grate,

and $d^2 \times 5{\cdot}5 =$ square feet of surface of fire grate,

also $d^2 \times {\cdot}7854 =$ square feet of section of funnel.

$$\frac{d^2 \times 5{\cdot}5}{d^2 \times {\cdot}7854} = \frac{5{\cdot}5}{{\cdot}7854} = 7{\cdot}031 \text{ nearly.}$$

Ex. 2. 4 boilers with 2 furnaces each, the diameter of each fire grate is 2 feet 10 inches: how many tons of coal will be consumed per day, and how many H.P. will be indicated?

In this question it is assumed that 1 ton of coal is burnt for every foot width of fire bars (see Answers to Elementary Questions, No. 181), and that $2\frac{1}{2}$ lbs. of coal per hour is the consumption for 1 H.P.

Hence $2'\ 10'' \times 4 \times 2 = 22'\ 8'' = 22\frac{2}{3}$ tons per day.

And $22\frac{2}{3}$ tons $\times\ 2240 \div 24 = 2115{\cdot}55$ lbs. per hour,

$\therefore\ 2115{\cdot}55 \div 2{\cdot}5 = 846{\cdot}2$ I.H.P.

Answer. $22\frac{2}{3}$ tons per day, and $846{\cdot}2$ I.H.P.

Ex. 3. The steam pipe is 11 inches diameter and goes inside the boiler; the part inside has slots $5\frac{3}{4}$ inches long and $\frac{1}{4}$ inch wide: how many slots must there be to make an area equal to double the sectional area of the pipe, and what are those slots for?

Double the area $= 11^2 \times {\cdot}7854 \times 2 = 190{\cdot}0668$.

Area of 1 slot $= 5\frac{3}{4} \times \frac{1}{4} = 5{\cdot}75 \times {\cdot}25 = 1{\cdot}4375$,

$\therefore$ No. slots $= 190{\cdot}0668 \div 1{\cdot}4375 = 132{\cdot}2$.

Or, $\dfrac{11^2 \times {\cdot}7854 \times 2}{5\frac{3}{4} \times \frac{1}{4}} = \dfrac{11 \times 11 \times {\cdot}7854 \times 2}{5{\cdot}75 \times {\cdot}25} = \dfrac{190{\cdot}0668}{1{\cdot}4375} = 132{\cdot}2$, No. of slots.

The slots are placed there to prevent the water coming into the cylinder when priming.

Ex. 4. The formula for rectangular cross bars for combustion chamber crowns is—

$B = \dfrac{72000\, d^2 t}{D L^2}$ Where B = boiler pressure.

D = dist. apart of bars.

L = length of bars.

d = depth of bars.

t = thickness or width of bars.

The cross bars are $5'' \times \frac{5}{8}''$ and $24''$ apart, length of bars $34''$: what pressure will be allowed in the boiler if there are 3 stays?

$$B = \frac{72000 \times 5^2 \times \frac{5}{8}}{24 \times 34^2} = \frac{72000 \times 25 \times \frac{5}{8}}{24 \times 1156} = \frac{1125000}{27744} = 40{\cdot}549 \text{ lbs. } \textit{Answer.}$$

Ex. 5. What would be the pressure if there were four stays?

Increase $= \dfrac{B}{N\,(N+2)}$ Where N = No. of stays.

$$\frac{40{\cdot}549}{4\,(4+2)} = \frac{40{\cdot}549}{24} = 1{\cdot}689 \text{ increase of pressure.}$$

Former pressure $40{\cdot}549 + 1{\cdot}689 = 42{\cdot}238$ lbs. pressure. *Answer.*

Ex. 6. A stay is fastened by a double eye to a T angle iron, the bolts are 1″ diameter, the plates ½″ thick: what space must be left between the hole and the edge of the plate?

The area of the hole should be equal to the area of section of plate between the hole and the edge.

$$\text{Here length} \times \tfrac{1}{2}'' = 1 \times 1 \times {\cdot}7854,$$

$$\therefore \text{Length} = 1 \times 1 \times {\cdot}7854 \div {\cdot}5 = 1{\cdot}5708 \text{ inches. } \textit{Answer.}$$

Ex. 7. If 8000 lbs. is the strain allowed on iron plates, what would be the pressure on a boiler 12 feet in diameter, double rivetted, ⅞″ rivets, and 2½″ pitch, thickness of plate ¾″, crossings not to be taken into account?

$$P = \frac{S \times 2t''}{d''} = \frac{8000 \times \frac{3}{4} \times 2}{144} = 83{\cdot}33 \text{ lbs.}$$

$$\text{Strength of seams} = \frac{\frac{7}{8}^2 \times {\cdot}7854 \times 2}{\frac{3}{4} \times 2\frac{1}{2}} = \frac{{\cdot}875 \times {\cdot}875 \times {\cdot}7854 \times 2}{{\cdot}75 \times 2{\cdot}5} = \frac{1{\cdot}2026375}{1{\cdot}875} = {\cdot}64141.$$

$$\text{Then } 83{\cdot}33 \times {\cdot}64141 = 53{\cdot}445 \text{ lbs. pressure.}$$

Ex. 8. The stays of a boiler are 12″ apart and one of them breaks, thus throwing more strain on the four stays that surround the broken one; they are 1⅝″ diameter, and the pressure is 70 lbs. per square inch: what extra strain per square inch will be on the stays if we allow ⅓ more than each had at first?

$$1\tfrac{5}{8} = 1{\cdot}625 \qquad \frac{12 \times 12 \times 70 + \frac{1}{3}}{1{\cdot}625^2 \times {\cdot}7854} = 6480{\cdot}5 \text{ lbs., strain per square inch.}$$

Ex. 9. How far apart should the stays be pitched when the pressure is 48 lbs. per square inch and the plates $\frac{7}{16}$ thick?

Formula. $S = \frac{60\,(T + 1)^2}{B} + 6$

S = surface supported by 1 stay.
B = boiler pressure.
T = thickness of plate in 16ths.

Caution.—Remember that T must be sixteenths, then if the thickness of plate were ⅞, then ⅞ = $\frac{14}{16}$.

$$\frac{60\,(7 + 1)^2}{48} + 6 = \frac{60 \times 64}{48} + 6 = \frac{3840}{48} + 6 = 86 \text{ square inches. } \textit{Answer}$$

$$\therefore \sqrt[2]{86} = 9{\cdot}273 \text{ inches, the pitch. } \textit{Answer.}$$

Ex. 10. What is the least diameter of one of these stays?

$$\text{Total pressure} = S \times B = 86 \times 48 = 4128.$$

$$\text{Area of stay} = 4128 \div 5000 = {\cdot}8256.$$

$$\therefore \sqrt[2]{\frac{{\cdot}8256}{{\cdot}7854}} = \sqrt[2]{1{\cdot}0512} = 1{\cdot}02 \text{ inch. } \textit{Answer.}$$

Ex. 11. The stays of a boiler are to be fixed by collars, the depth of the collars being 3 times the thickness, the diameter of stays 1⅞″: what size must the end be swelled out to to have uniform strength of bar?

Rule. $D = 1 + \left(\frac{{\cdot}08}{n} + \frac{{\cdot}4}{\sqrt{n}}\right) d.$ Where $n = 4$, $d = 1\frac{7}{8}''$.

$$D = 1 + \left(\frac{{\cdot}08}{4} + \frac{{\cdot}4}{\sqrt{4}}\right) 1{\cdot}875 = 1 + ({\cdot}02 + {\cdot}2 =)\ 0{\cdot}22 = 1{\cdot}22.$$

$$\text{Then } 1{\cdot}22 \times 1{\cdot}875 = 2{\cdot}28750. \textit{ Answer.}$$

$$4)\underline{{\cdot}08}$$

$${\cdot}02 \quad \therefore \sqrt{4} = 2 \text{ is } \frac{{\cdot}4}{2} = {\cdot}2, \text{ and } \begin{array}{r} 1{\cdot}875 \\ 1{\cdot}22 \\ \hline 3750 \\ 22500 \\ \hline 2{\cdot}28750 \end{array}$$

According to the Board of Trade instructions, circular furnaces with the longitudina joint welded or made with a butt strap,

$$\frac{90000 \times \text{the square of the thickness of plates in inches}}{(\text{length in feet} + 1) \times \text{diameter in inches}} = \text{the working pressure per sq. in.}$$

Without the Board's special approval of the plates the pressure in no case to exceed

$$\frac{8000 \times \text{thickness in inches}}{\text{diameter in inches}}$$

Ex. 12. Find the working pressure when the diameter is 37", length 6' 9", and thickness of plates $\frac{3}{8}$".

$$\text{First,} \qquad \frac{90000 \times \frac{3}{8} \times \frac{3}{8}}{(6{\cdot}75 + 1) \times 37} = 44{\cdot}1 \text{ lbs.}$$

$$\text{Second,} \qquad \frac{8000 \times \frac{3}{8}}{37} = 81 \text{ lbs.}$$

Therefore the working pressure must be 44·1 lbs.

Ex. 13. Upon every square foot of fire grate is burnt 11 lbs. of coal per hour, each lb. of coal evaporates 7·1 lbs. of water, the superheater has 3·1 cubic feet of capacity for every square foot of fire grate, and the volume is 512 times that of water: what is the average time each portion of steam remains in the superheater?

$$\text{Evaporation, in cubic feet of water} = \frac{7{\cdot}1 \times 11}{62{\cdot}5} = 1{\cdot}2496.$$

$$\text{Evaporation, in cubic feet of steam} = 1{\cdot}2496 \times 512 = 639{\cdot}7952 \text{ volume,}$$

$$\therefore\ 639{\cdot}7952 \ :\ 3{\cdot}1 \ ::\ 3600^{s} \ :\ 17{\cdot}44 \text{ seconds.} \quad \textit{Answer.}$$

Ex. 14. If 1 lb. of coal evaporates 7·5 lbs. of water when the gauge is at 55 lbs.: how many cubic feet of steam will this be for 1 lb. of coal?

$\frac{410 + P \div 4}{P + 1}$ (where P = gross pressure) may be taken as what 1 lb. of water will expand to.

$$\text{Here } \frac{410 + 70 \div 4}{70 + 1} = \frac{410 \times 17{\cdot}5}{71} = 6{\cdot}021 \text{ cubic feet of steam for 1 lb. of coal.}$$

$$\therefore\ 6{\cdot}021 \times 7{\cdot}5 = 45{\cdot}175 \text{ cubic feet of steam.} \quad \textit{Answer.}$$

Ex. 15. The boiler contains 60 tons of fresh water on starting on a voyage. The water in the hot-well, through some leak, is ·1 of the saltness of the sea: how many tons of water must have been evaporated during the voyage, the water in the boiler at the end of the voyage being 2·2 times the saltness of the sea?

$$\frac{2{\cdot}2}{{\cdot}1} = 22 \text{ times as much salt as at starting,}$$

$$\text{then } 22 \times 60 = 1320 \text{ tons evaporated.} \quad \textit{Answer.}$$

Ex. 16. The horse-power of an engine is 140, the boiler 15 feet long, diameter 16 feet, the water shows in the glass 7 inches. The engine uses 20 lbs. of steam per H.P. per hour: how long will the water remain in the glass after the feed is shut off?

$$7 \text{ inches} = \tfrac{7}{12} \text{ of 1 foot.}$$

$$\text{Time water remains in boiler} = \frac{15 \times 16 \times \frac{7}{12} \times 64}{140 \times 20} = 3^{m}{\cdot}1998.$$

Ex. 17. A boiler with 140 cubic feet at water level and 60 lbs. pressure; a $\frac{7}{8}$" rivet is out of the boiler bottom: how long will it take to blow 8" down?

Rule. $2\frac{1}{2}\, d^2 \sqrt{P}$ = cubic feet blown out per minute.

$$\text{Cubic feet to be blown out} = 140 \times \tfrac{2}{3}' = 93{\cdot}333, \text{ \&c.,}$$

$$\text{Cubic feet blown out per minute} = \tfrac{7}{8}^2 \times 2\tfrac{1}{2}\sqrt{60} = {\cdot}875 \times {\cdot}875 \times 2{\cdot}5 \times 7{\cdot}746$$
$$= 14{\cdot}826328125,$$

$$\therefore \text{ Time to blow out contents of boiler} = 93{\cdot}333, \text{ \&c.,} \div 14{\cdot}826328125 = 6^{m}\ 18^{s} \text{ nearly.}$$

Ex. 18. A rectangular boiler is 16 ft. 2 in. long, 11 ft. 1 in. wide, and 9 ft. 3 in. high: what would be the cost of cementing the two sides and the top at 8s. 4d. per square yard?

$$\begin{array}{rl} \text{Area of top} = 16'\,2'' \times 11'\,1'' = 194'' \times 133'' = & 25802 \text{ sq. inches.} \\ 2 \text{ sides} = 2\,(16'\,2'' \times 9'\,3'') = 2 \times 194 \times 111 = & \underline{43068} \quad ,, \\ & 68870 \quad ,, \end{array}$$

$\therefore$ 1 sq. yard = 9 sq. feet = 144 sq. inches × 9 = 1296 sq. inches.

$\therefore$ 1296 : 68870 :: 8s. 4d. : x.

$$\frac{68879 \times 100}{1296} = 5314\tfrac{7}{162}\text{d.} = £22\ 2\text{s. } 10\tfrac{7}{162}\text{d.} \quad \textit{Answer.}$$

Ex. 19. A boiler 14 ft. 6 in. by 13 ft. 9 in. is to be covered with lead, and this must lap over 7½ inches: what will be the cost at 5d. per lb. if 1 sq. foot of this lead weighs 8 lbs.?

The sheet of lead must be twice 7½″ broader and longer than boiler, that is, 14′ 6″ + 1′ 3″ = 15′ 9″ long, and 13′ 9″ + 1′ 3″ = 15′ broad.

$\therefore$ 15·75 × 15 = 236·25 square feet.

But from this we have to deduct 4 corners 7½″ square = 7·5 × 7·5 × 4 ÷ 144 = 1·5625 square feet.

$$\begin{array}{rl} & 236{\cdot}25 \text{ square feet.} \\ \text{Deduct for 4 corners} & \underline{1{\cdot}5625} \\ & 234{\cdot}6875 \\ & \underline{\phantom{234{\cdot}687}8} \\ & 1877{\cdot}5000 \text{ lbs.} \\ \text{Cost per lb.} & \underline{5} \\ & 12)9387\ \ 5\text{d.} \\ & 2,0)76,5\ \ 7 \\ & £38\ \ 5\text{s. } 7\tfrac{1}{2}\text{d.} \quad \textit{Answer.} \end{array}$$

Ex. 20. A boiler 13 ft. 9 in. by 12 ft. 8 in. has to be covered with lead, and this lead must overlap 8½ inches: what will be the cost at 3¾d. per lb. if 1 square foot of lead weighs 8 lbs.?

It is evident that the sheet of lead must be twice 8½ inches broader and longer than the boiler, that is, 13 ft. 9 in. + 1 ft. 5 in. = 15 ft. 2 in. long; and 12 ft. 8 in. + 1 ft. 5 in. = 14 ft. 1 in. broad = 213·5972.

But from this deduct 4 corners 8½ inches square, that is, 8·5 × 8·5 × 4 ÷ 144 = 2·0069 square feet.

$$\begin{array}{rl} & 213{\cdot}5972 \\ - & \underline{2{\cdot}0069} \\ & 211{\cdot}5903 \text{ square feet.} \\ & \underline{\phantom{211{\cdot}590}8 \text{ lbs.}} \\ & 1692{\cdot}7224 \text{ lbs. of lead.} \end{array}$$

$$\begin{array}{rr} \text{Then} & 1692{\cdot}7224 \text{ lbs.} \\ & \underline{3{\cdot}75\text{d.}} \\ & 84636120 \\ & 118490568 \\ & \underline{50781672} \\ & 12)6347{\cdot}709000 \\ & 2,0)52,8\ \ 11 \\ & £26\ \ 8\text{s. } 11\tfrac{3}{4}\text{d.} \end{array}$$

Ex. 21. The top of a rectangular boiler is to be covered with lead, which has to be turned down over the sides a certain distance: what will be the cost of the lead? Length of lead 13 ft. 9 in.; breadth 12 ft. 8 in.; over-lap 8½ in.; weight of lead per sq. ft. 8 lbs.; cost 3¾d. per lb.

In this example the lead is 13 ft. 9 in. by 12 ft. 8 in. $=$ 174·1666 sq. ft.
From this deduct the 4 corners $= 8·5 \times 8·5 \times 4 \div 144 = -$ 2·0764 „

172·0902 „

Lbs. of lead $=$ 172·0902 sq. ft. $\times$ 8 lbs. $=$ 1376·7216,
And 1376·7216 $\times$ $3\frac{3}{4}$ $=$ 5162·7060d.
$=$ £21 10s. 2·7d., or £21 10s. $2\frac{3}{4}$d., nearly.

Ex. 22. A compound engine has a steam receiver round the H.P. cylinder equal in diameter to the L.P. cylinder; steam chest for slides is between the cylinders; distance between the centres of cylinders 61 in.; diameter of L.P. cylinder 47 in., and depth 50 in.: how many square feet of felting will be required to cover it?

Since there is a half circumference at each end, therefore,

Circumference $= 3·1416 \times 47 = 147·6552$.

$\therefore$ $147·6552 \times 50 = 7382·7600$ inches for the two half cylinders.

$61 \times 50 \times 2 = 6100$ square inches for sides of chest.

Then $7382·76 + 6100 = 13482·76$ square inches.

And 13482·76 square inches $\div$ 144 $=$ 93·630, &c., square feet.

Answer, 93·63 square feet of felt.

Ex. 23. Formula $\frac{6·55\,d^3}{l} = D^2$. Where d = diameter of crank-shaft,
l = length of stroke in feet,
D = diameter of cylinder.

Question.—A crank-shaft is 15″ diameter, the length of the stroke is 3′ 6″, the pressure about 30 lbs.: what diameter of cylinder will that size crank require according to the above formula?

$$\frac{6·55\,d^3}{l} = \frac{6·55 \times 15 \times 15 \times 15}{3·5} = 6316·07 \text{ square of diameter.}$$

$$\therefore \sqrt[2]{6316·07} = 79·47 \text{ inches.} \quad \textit{Answer.}$$

CYLINDERS, SLIDE-VALVES, &c.

Ex. 24. An inverted cylinder, the length of the stroke is 48 inches, the length of the connecting-rod is 8 feet, the piston is 18 inches from the bottom of the cylinder: what is the distance from the centre of the cross-head to the centre of the crank-shaft?

If the piston were at half stroke, that is, 24 inches from the bottom, then the distance between the required points would be exactly equal to the length of the connecting-rod, viz., 8 feet; but as the piston is only 18 inches from the bottom, the cross-head must be 6 inches down, therefore the distance is 7 feet 6 inches.

½ stroke	24 in.	Length of connecting-rod	8 ft. 0 in.
	18	Down (subtract)	6
Down	6		7 ft. 6 in. *Answer.*

Example.—Stroke 42 inches, connecting-rod 7½ feet, piston 16 inches from bottom. 7 feet 1 inch. *Answer.*

Example.—Stroke 36 inches, connecting-rod 7 feet, piston 14 inches from bottom. 6 feet 8 inches. *Answer.*

Ex. 25. The length of a connecting-rod is 6 feet, length of piston-rod, from the collar to [illegible] centre of the cross-head, 66 inches, thickness of piston is 4 inches, and the cover enters [illegible] cylinder 2½ inches, the length of stroke is 40 inches, and the clearance is ½ inch: what [illegible] distance from the top of cylinder to centre of crank-shaft?

Rule.—Add together *half the length of stroke,*
length of connecting-rod,
length of piston-rod,
depth, or thickness of piston,
what cover enters cylinder,
and clearance.

The sum of these is the distance from the top of cylinder to centre of shaft.

½ length of stroke (= ½ of 40")	= 1 ft. 8 in.
Length of connecting-rod	= 6 „ 0 „
Length of piston-rod	= 5 „ 6 „
Thickness of piston	= 0 „ 4 „
Cover enters cylinder	= 0 „ 2½ „
Clearance	= 0 „ 0½ „
Dist. from top of cylinder to centre of shaft	= 13 „ 9 „

Example.—The length of the connecting-rod is 7 feet 9 inches, the piston is 72 inches from the collar to the centre of the cross-head, the piston 4¾ inches thick, length of stroke 48 inches, cover is 1¾ inch into cylinder, and the clearance is ¾ inch: what is the length from the top of the cylinder to the centre of the shaft?

Length = 7′ 9″ + 6′ 0″ + 4¾″ + 2′ 0″.

Length = 2′ 0″ (= ½ stroke) + 7′ 9″ + 6′ 0″ + 4¾″ + 1¾″ + ¾″ = 16′ 4¼″. *Answer.*

Ex. 26. A slide-valve has 9″ travel, 2¼″ lap on the steam side and $\frac{3}{16}$″ on the exhaust side, and ⅛″ lead: to what extent is the exhaust port open when the piston is at the extreme end of the stroke.

Distance from mid-position to where open for lead = 2¼ + ⅛ = 2⅜.

Distance exhaust port is open = 2⅜ − $\frac{3}{16}$ = 2$\frac{3}{16}$.

Ex. 27. A cylinder port is 20″ long × 2½″ broad, the valve has 1½″ lap, and 6½″ travel: find the greatest area the valve is open for steam.

½ full travel	= 3¾″	Depth of port	= 2½″
½ actual travel	= 3¼	Unopened	0½
Unopened	½	Open	2

Area = 20″ × 2″ = 40 square inches. *Answer.*

379. The pressure forcing the slide upon the cylinder face depends a good deal upon the state of the surface; but in a common slide it is never far from equal to the difference between the pressure per square inch in the casing and the back pressure in the exhaust passage, calculated for an area equal to the area of one steam port.

Ex. 28. Port in slide 22 in. by 9 in., cylinder face 22 in. by 1½ in., pressure by steam gauge 45 lbs., back pressure 4 lbs.

Area of valve = 9″ × 22″ = 198 sq. inches.
„ steam port = 1½″ × 22″ = 33 „
Sum of areas = 231 „

Gauge pressure	45 lbs.
Atmospheric „	15 „
Gross „	60 „
Back „	− 4 „
Effective „	56 „

∴ Pressure on valve = 231 × 56 = 12936 lbs. *Answer.*

Ex. 29. A common slide port is 22″ by 18″, and the steam port 3′ 5″ wide; steam gauge shows 60 lbs., pressure in the exhaust 19 lbs.: what is the effective pressure pressing the valve to the face?

In this example the lead is 13 ft. 9 in. by 12 ft. 8 in. = 174·1666 sq. ft.
From this deduct the 4 corners = 8·5 × 8·5 × 4 ÷ 144 = − 2·0764 „

172·0902 „

Lbs. of lead = 172·0902 sq. ft. × 8 lbs. = 1376·7216,
And 1376·7216 × 3¾ = 5162·7060d.
= £21 10s. 2·7d., or £21 10s. 2¾d., nearly.

Ex. 22. A compound engine has a steam receiver round the H.P. cylinder equal in diameter to the L.P. cylinder; steam chest for slides is between the cylinders; distance between the centres of cylinders 61 in.; diameter of L.P. cylinder 47 in., and depth 50 in.: how many square feet of felting will be required to cover it?

Since there is a half circumference at each end, therefore,

Circumference = 3·1416 × 47 = 147·6552.

∴ 147·6552 × 50 = 7382·7600 inches for the two half cylinders.

61 × 50 × 2 = 6100 square inches for sides of chest.

Then 7382·76 + 6100 = 13482·76 square inches.

And 13482·76 square inches ÷ 144 = 93·630, &c., square feet.

Answer, 93·63 square feet of felt.

Ex. 23. Formula $\frac{6{\cdot}55\,d^3}{l} = D^2$. Where d = diameter of crank-shaft,
l = length of stroke in feet,
D = diameter of cylinder.

Question.—A crank-shaft is 15″ diameter, the length of the stroke is 3′ 6″, the pressure about 30 lbs.: what diameter of cylinder will that size crank require according to the above formula?

$$\frac{6{\cdot}55\,d^3}{l} = \frac{6{\cdot}55 \times 15 \times 15 \times 15}{3{\cdot}5} = 6316{\cdot}07 \text{ square of diameter.}$$

$$\therefore \sqrt[2]{6316{\cdot}07} = 79{\cdot}47 \text{ inches. } \textit{Answer}.$$

CYLINDERS, SLIDE-VALVES, &c.

Ex. 24. An inverted cylinder, the length of the stroke is 48 inches, the length of the connecting-rod is 8 feet, the piston is 18 inches from the bottom of the cylinder: what is the distance from the centre of the cross-head to the centre of the crank-shaft?

If the piston were at half stroke, that is, 24 inches from the bottom, then the distance between the required points would be exactly equal to the length of the connecting-rod, viz., 8 feet; but as the piston is only 18 inches from the bottom, the cross-head must be 6 inches down, therefore the distance is 7 feet 6 inches.

½ stroke 24 in.
18

Down 6

Length of connecting-rod 8 ft. 0 in.
Down (subtract) 6

7 ft. 6 in. *Answer.*

Example.—Stroke 42 inches, connecting-rod 7½ feet, piston 16 inches from bottom. 7 feet 1 inch. *Answer.*

Example.—Stroke 36 inches, connecting-rod 7 feet, piston 14 inches from bottom. 6 feet 8 inches. *Answer.*

Ex. 25. The length of a connecting-rod is 6 feet, length of piston-rod, from the collar to the centre of the cross-head, 66 inches, thickness of piston is 4 inches, and the cover enters the cylinder 2½ inches, the length of stroke is 40 inches, and the clearance is ½ inch: what is the distance from the top of cylinder to centre of crank-shaft?

Rule.—Add together *half the length of stroke,*
length of connecting-rod,
length of piston-rod,
depth, or thickness of piston,
what cover enters cylinder,
and clearance.

The sum of these is the distance from the top of cylinder to centre of shaft.

½ length of stroke (= ½ of 40″) = 1 ft. 8 in.
Length of connecting-rod = 6 „ 0 „
Length of piston-rod = 5 „ 6 „
Thickness of piston = 0 „ 4 „
Cover enters cylinder = 0 „ 2½ „
Clearance = 0 „ 0½ „
Dist. from top of cylinder to centre of shaft = 13 „ 9 „

Example.—The length of the connecting-rod is 7 feet 9 inches, the piston is 72 inches from the collar to the centre of the cross-head, the piston 4¾ inches thick, length of stroke 48 inches, cover is 1¼ inch into cylinder, and the clearance is ¼ inch: what is the length from the top of the cylinder to the centre of the shaft?

Length = 7′ 9″ + 6′ 0″ + 4¾″ + 2′ 0″.

Length = 2′ 0″ (= ½ stroke) + 7′ 9″ + 6′ 0″ + 4¾″ + 1¼″ + ¼″ = 16′ 4½″. *Answer.*

Ex. 26. A slide-valve has 9″ travel, 2¼″ lap on the steam side and $\frac{3}{16}$″ on the exhaust side, and ⅛″ lead: to what extent is the exhaust port open when the piston is at the extreme end of the stroke.

Distance from mid-position to where open for lead = 2¼ + ⅛ = 2⅜.
Distance exhaust port is open = 2⅜ − $\frac{3}{16}$ = 2$\frac{3}{16}$.

Ex. 27. A cylinder port is 20″ long × 2½″ broad, the valve has 1¼″ lap, and 6½″ travel: find the greatest area the valve is open for steam.

½ full travel = 3¼″
½ actual travel = 3¾
Unopened ½

Depth of port = 2½″
Unopened 0½
Open 2

Area = 20″ × 2″ = 40 square inches. *Answer.*

379. The pressure forcing the slide upon the cylinder face depends a good deal upon the state of the surface; but in a common slide it is never far from equal to the difference between the pressure per square inch in the casing and the back pressure in the exhaust passage, calculated for an area equal to the area of one steam port.

Ex. 28. Port in slide 22 in. by 9 in., cylinder face 22 in. by 1½ in., pressure by steam gauge 45 lbs., back pressure 4 lbs.

Area of valve = 9″ × 22″ = 198 sq. inches.
„ steam port = 1½″ × 22″ = 33 „
Sum of areas = 231 „

Gauge pressure 45 lbs.
Atmospheric „ 15 „
Gross „ 60 „
Back „ − 4 „
Effective „ 56 „

∴ Pressure on valve = 231 × 56 = 12936 lbs. *Answer.*

Ex. 29. A common slide port is 22″ by 18″, and the steam port 3′ 5″ wide; steam gauge shows 60 lbs., pressure in the exhaust 19 lbs.: what is the effective pressure pressing the valve to the face?

Rule.—Add the area of the valve port to the area of the steam port, and multiply the sum by the effective pressure of the steam.

$22'' \times 18'' = 396;\ 22'' \times 3''\cdot5 \times 77\cdot0;\ 60 + 15\ (= 75) - 19 = 56$ lbs.

Then $396 + 77 = 473$, and $473 \times 56 = 26488$ lbs.

Ex. 30. The pressure in the high pressure slide-valve casing is 70 lbs., the slide face is 28 in. by 13 in., the steam port is 28 in. by 3 in.: what is the weight on the slide-valve face?

Slide face $= 28 \times 13 = 364$ sq. inches. Steam port $= 28 \times 3 = 84$ sq. inches.

$\therefore 364 - 84 = 280.$

Weight on slide-valve face $= 280 \times 70 = 19600$ lbs. *Answer.*

BOLTS ON CONDENSER DOORS, CYLINDER COVERS, &c.

Ex. 31. If 14 bolts are pitched at $4\frac{1}{2}''$, how far is it from outside centre to outside centre?

As there are 14 holes there must be 13 pitch spaces between them; therefore, $4''\cdot5 \times 13 = 58''\cdot5 = 4'\ 10\frac{1}{2}''$. *Answer.*

Ex. 32. From outside centre to outside centre is $3'\ 9''$: what must be the pitch when there are 9 bolts? *Answer*, $5\frac{5}{8}''$.

Ex. 33. How many bolts must there be when the distance from outside centre to outside centre is $5'\ 1\frac{1}{8}''$, and the pitch is $5\frac{5}{8}''$.

No. of pitch spaces $= \frac{61\cdot125}{5\cdot625} = 11$, $\therefore$ 12 holes. *Answer*, 12 holes.

Ex. 34. A condenser door is $5'\ 6''$ by $4'\ 3''$; there are 20 bolts one way and 16 the other: find the number of bolts and the pitch, if the centres of the outside bolts be $1\frac{1}{4}''$ from the edge of the plate.

Length between centres of outside bolts $= 66'' - 2\frac{1}{2}'' = 63\frac{1}{2}''$.

Pitch (one way) $63\cdot5 \div 19 = 3\cdot3421$.

Breadth between centre of outside bolts $= 51'' - 2\frac{1}{2}'' = 48\frac{1}{2}''$.

Pitch (the other way) $48\cdot5 \div 15 = 3\cdot23$.

Number of bolts $20 + 20 + 14 + 14 = 68$. *Answer.*

$$\text{No. of bolts} = \frac{(\text{diameter of cover} - 2 \text{ distance of bolts from edge}) \times 3\cdot1416}{\text{Space between bolts.}}$$

Ex. 35. The bolts of a cylinder cover are to be spaced not wider than $2\frac{3}{16}$ inches, centre to centre, measuring on the circle; the cover is 27 inches diameter, and the centres of the bolts are 2 inches from the edge of the cover: how many bolts are there?

23 in. $\times 3\cdot1416 = 72\cdot2568 =$ circumference of bolt circle.

27	inches	= diameter of cover.
4	„	= twice space from edge.
23	„	= diameter of bolt circle.

$$\begin{array}{r} 3\cdot1416 \\ 23 \\ \hline 94248 \\ 62832 \\ \hline 72\cdot2568 \end{array}$$

$\frac{3}{16}$ inch $= \cdot1875$

$$16\left\{\begin{array}{l} 4)3\cdot00 \\ \hline 4)\ \ \cdot75 \\ \hline \ \ \ \ \cdot1875 \end{array}\right.$$

Then prefixing the 2 we have,

$2\frac{3}{16}$ inches $= 2\cdot1875$.

Then taking $2\frac{3}{16}$ inches $(= 2\cdot1875)$ as the space between the centres of the bolts we get

$\frac{72\cdot2568}{2\cdot1875} = 33\cdot03 =$ the number of bolts.

$$\begin{array}{l} 2\cdot1875)72\cdot2568(33\cdot03 \text{ No. of bolts.} \\ \qquad\quad\ \ 65625 \\ \qquad\quad\ \ \overline{\ \ 66318} \\ \qquad\quad\ \ \ \ 65625 \\ \qquad\quad\ \ \ \ \overline{\ \ 69300} \\ \qquad\quad\ \ \ \ \ \ 65625 \\ \qquad\quad\ \ \ \ \ \ \overline{} \end{array}$$

EXAMPLES FOR PRACTICE.

	Dia. of Cover.	Space apart.	Dist. of Bolts from edge.		Dia. of Cover.	Space apart.	Dist. of Bolts from edge.
1.	70 inches.	4½ inches.	2¼ inches.	3.	44 inches.	5½ inches.	1¾ inches.
2.	38 „	5 „	1½ „	4.	74 „	6¾ „	2⅝ „

Answers.

1. 70 − 4½ = 65½ × 3·1416 ÷ 4½ = 250·77480 ÷ 4·5 = 45·727 or 46 bolts.
2. 38 − 3 = 35 × 3·1416 ÷ 5 = 109·9560 ÷ 5 = 21·9912 or 22 bolts.
3. 44 − 3½ = 40½; 40·5 × 3·1416 ÷ 5·5 = 127·23480 ÷ 5·5 = 23·13 bolts.
4. 74 − 5¼ = 68¾; 68¾ × 3·1416 ÷ 6¾ = 215·985 ÷ 6·75 = 31·99 or 32 bolts.

WEIGHT OF MATERIAL.

Ex. 36. The seat of a rectangular brass is 14″, depth 15″, length 16″, the shaft is 12″ diameter, and the flanges are 17″ square and 1½″ thick: what is the weight of two half brasses at ·3 of a lb. per cubic inch?

FLANGES.	SEAT.	SHAFT.
17	14	·7854
17	15	12^2 = 144
119	70	31416
17	14	31416
		7854
289	210	
− 113·0976	− 113·0976	113·0976
175·9024	96·9024	
Length 3	Length 16	
5277072	5814144	
+ 15504382	969024	
2078·1454	1550·4384	
·3		

Ans. 623·44362 lbs.

Ex. 37. A T headed piston-rod's dimensions are 11′ 9″ over all, and 7″ diameter, the T head being 28″ long, 10″ wide, and 4″ thick: what is the weight if 3·6 cubic inches weigh 1 lb.?

Cubic inches in the rod part = 7^2 × ·7854 × (141″ − 4″) = 5272·3902
„ T head = 28 × 10 × 4 = 1120
Total = 6392·3902

Then 6392·3902 ÷ 3·6 = 177·566, &c., lbs. = 1 cwt. 2 qrs. 9·566 lbs. *Answer.*

Ex. 38. If a piston is 43″ diameter, and assuming it to be 4¼″ thick throughout: what should it weigh if 3·8 cubic inches of cast iron weigh 1 lb.?

$$\text{Weight} = \frac{43^2 \times \cdot 7854 \times 4\cdot 25}{3\cdot 8} = 1624\cdot 1762 \text{ lbs.}$$

43 × 43 (= 1849) × ·7854 (= 1452·2046) × 4·25 = 61718·6955.

61718·6955 ÷ 3·8 = 1624·1762 lbs.

1624·1762 lbs. = 14 cwt. 2 qrs. *Answer.*

Ex. 39. A packing ring for a cylinder 31 inches diameter, before being cut was 32 inches diameter: how much should be cut out of the circumference to make it fit the cylinder?

Difference of diameters × 3·1416 = inches to be cut out.

Diameter 32 inches.	3·1416
„ 31	1
Diff. of dia. 1	3·1416 inches.

Ans.: 3·1416 inches to be cut out.

Ex. 40. A packing ring for a cylinder is 89 inches diameter, before being cut was 91¼ inches: how much should be cut out of it to make it fit the cylinder?

Difference of diameters = 91¼ — 89 = 2¼ inches.

2¼ × 3·1416 = 7·0686 inches to be cut.

$$\begin{array}{l} 91\tfrac{1}{4} \\ 89 \\ \hline 2\tfrac{1}{4} \end{array} \qquad\qquad \begin{array}{r} 3{\cdot}1416 \\ 2\tfrac{1}{4} \\ \hline 62832 \\ 7854 \\ \hline 7{\cdot}0686 \text{ inches.} \end{array}$$

Ex. 41. Find the H.P. exerted by a pump in pumping out a tank 36 feet long, 24 feet wide, and 4 feet deep, the top of which is 20 feet below the level of the sea. A cubic foot of sea water weighs 64 lbs., and 31000 lbs. of water raised 1 foot high in one hour are equal to one H.P.; it took 6 hours to pump the tank out.

Weight of water to be lifted = 36 × 24 × 4 × 64 = 221184 lbs.

Total height the water is lifted 20 + 4 = 24 feet.

Foot-pounds expended = 221184 × 24 = 5308416.

" " in 1 hour = 5308416 ÷ 6 = 884736.

∴ H.P. = 884736 ÷ 31000 = 28·54. *Answer.*

Ex. 42. A tank 38 feet by 22 feet 9 inches, and 4 feet 6 inches high: how long will it take to pump it out by a double-acting donkey, the diameter of whose piston is 8 inches, stroke 12 inches, revolutions per minute 85, if 12 per cent. is non-efficient?

12 per cent. non-efficient = 88 per cent. efficient.

Work to be done = 38′ × 22′·75 × 4′·5 = 3890·25 cubic feet.

Work per minute done by pump = $\cdot7854 \times 8^2 \times 12 \times 170 \times \frac{88}{100} \div 1728$ = 52·22 cu. ft.

∴ Time required to pump out = 3890·25 ÷ 52·22 = 74·49, &c., minutes. *Ans.*

Ex. 43. Diameter of high pressure cylinder is 32″, stroke 33″, the steam is carried for ⅓ stroke: if there are 1200 cubic feet of steam capacity in the boiler, how many strokes will be required to draw that amount off?

Cubic feet each stroke = $\cdot7854 \times 32^2 \times \frac{33}{3} \div 1728$ = 5·1196, &c.

∴ No. of strokes = 1200 ÷ 5·1196 = 234·3, &c. *Answer.*

Ex. 44. The effective pressure per square inch on the surface of a piston is 40 lbs., the total pressure 30 tons: what is the diameter of the piston?

$$\text{Diameter of piston} = \sqrt[2]{\frac{2240 \times 30}{40 \times \cdot7854}}$$

Total pressure in lbs. = 2240 × 30 = 67200 lbs.

Area of piston = 40 × ·7854 = 31·416.

∴ $\sqrt[2]{67200 \div 31{\cdot}416} = \sqrt{2139{\cdot}0374}$ = 46·24 or 46¼ inches. *Answer.*

Ex. 45. The effective pressure per square inch of a piston is 30 lbs., the total pressure is 20 tons; what is the diameter of the piston? *Answer*, 43·6 inches.

Ex. 46. Total pressure 11 tons 7 cwt. 0 qr. 13 lbs., effective pressure per square inch 16 lbs. *Answer*, 44·99 inches.

Ex. 47. A lead wire ·049 inch diameter is used for testing the adjustment of the crank-shaft brasses; it is flattened to be ¼ inch broad: how many thicknesses of it will make one inch?

area = d^2 × ·7854.

$\text{Thickness} = \frac{\text{area}}{\text{breadth}}$ $\qquad$ $\frac{\text{one inch}}{\text{thickness}}$ = number of thicknesses to an inch.

NOTE.—In this question it is necessary to take great care in placing the decimal point. For Multiplication of Decimals see p. 102, and for Division of Decimals see pp. 103-105.

Thickness $= \frac{·049 \times ·049 \times ·7854}{·25}$ $\frac{1''}{·007543} = 132·5$, number of thicknesses to an inch.

```
    ·049            ·7854                    ·007543)1"·000000(132·5 Ans.
    ·049            ·002401                          7543
    ----            -------                          -----
     441             7854                            24570
    196            314160                            22629
   -----          15708                              -----
  ·002401         ---------                           19410
             ·25)·0018857454(·007543 thickness of lead. 15086
Or thus:—         175                                 -----
     ( ·5)·0018857454  ---                             43240
 ·25 { -------------   135                             37715
     ( ·5)·0003771490  125                             -----
       -------------   ---                              5525
  ·00754298 = thick-   107
       ness of lead.   100
                       ---
                        74
                        75
                        --
```

Ex. 48. A lead wire ·125 inch diameter is used for testing the adjustment of the crank-shaft brasses; it is flattened to be ¾ inch broad: how many thicknesses of it will make one inch?

·125 × ·125 = ·015625 × ·7854 = ·012271875 ÷ ·75 = ·016362 **thickness of lead.**

1 inch ÷ ·016362 = 61·1 number of thicknesses to the inch.

Ex. 49. A piston has an area of 3064·28 sq. inches: find the breadth of a shoe 18 inches long, allowing 10¼ sq. inches of piston to 1 sq. inch of shoe.

Area of shoe = 3064·28 ÷ 10·25 = 298·9541 sq. inches.

∴ Breadth = 298·9541 ÷ 18 = 16·6085 inches. *Ans.*

380. **In altering the pitch of a screw propeller it is found that when the mean effective pressure of the indicator diagram is not altered, the product of the pitch by the square of the knots per hour is not altered.**

Ex. 50. If the pitch be altered as under, what will be the speed of the vessel, the diagram remaining the same?

Pitch 20 feet, speed 9 knots, new pitch 18 feet.

By the above rule $S^2 \times 18 = 9^2 \times 20.$

$$S^2 = \frac{9^2 \times 20}{18} = \frac{81 \times 20}{18} = 90.$$

Then the square root of 90 or $\sqrt[2]{90} = 9·48 = 9\frac{1}{2}$ **knots, nearly.**

381. **The weight of steam that will escape into a space in which the pressure does not exceed 58 per cent. of the gross pressure of the steam is, in 70 seconds, just equal to the gross pressure of the steam upon an area equal to the area of the orifice.**

Ex. 51. There is a crack in the metal between the low pressure cylinder exhaust port and the steam jacket, the sectional area of which is equal to ½ a square inch, and the steam in the jacket is 55 lbs. by the gauge: if 1 lb. of coal evaporates 10 lbs. of water, what is the loss in steam and in coal in 24 hours?

Gauge 55 lbs. + atmosphere 15 lbs. = 70 lbs. = gross pressure.

Steam escape in 70 seconds = 70 lbs. × ½ square inch = 35 lbs.

24 hours = 24 × 60 × 60 = 86400 seconds.

∴ Steam escape in 24 hours $= \frac{86400 \times 35 \text{ lbs.}}{70 \text{ seconds}} = 43200$ lbs. = 19·285 tons.

And coal lost in 24 hours $= \frac{19·285}{10} = 1·9285$ tons.

Ex. 52. If 1 lb. of steam condensed raises 1000 lbs. of water 1° Fah., and if the temperature of sea water is 58°, and that of the discharge 107°: what weight of water is required to condense the steam in the above crack?

$$\begin{array}{lr} \text{Temperature of discharge} & 107^\circ \\ \text{,,} \quad \text{sea water} & 58^\circ \\ \hline & 49^\circ \end{array} \qquad 49\left\{\begin{array}{l} 7)1000 \\ \hline 7)142{\cdot}875 \\ \hline \end{array}\right.$$

20·408 lbs. of cold water necessary to condense 1 lb. of steam.

Hence 20·408 × 19·285 tons of steam = 393·56828 tons of cold water.

Ex. 53. If, when the engines are making 41 revolutions per minute, the mean effective pressure on the piston is 27 lbs.; supposing from some cause the vacuum showed 1·5 lbs. less: how many revolutions would you expect the engine to make now?

$$\begin{array}{lr} \text{Old pressure} & 27{\cdot}0 \text{ lbs.} \\ \text{Vacuum} & 1{\cdot}5 \text{ ,,} \\ \hline \text{New pressure} & 25{\cdot}5 \text{ ,,} \end{array} \qquad \begin{array}{l} 27{\cdot}0 \text{ lbs.} : 25{\cdot}5 \text{ lbs.} :: 41^2 : x^2 \\ x^2 = \frac{25{\cdot}5 \times 41^2}{27} = 1587{\cdot}61. \end{array}$$

∴ No. of revolutions now made $= \sqrt{1587{\cdot}6} = 39{\cdot}8$. *Answer.*

Ex. 54. An engine makes 62 revolutions per minute when the steam pressure is 30 lbs.: what should be the pressure if the engine is eased down to 50 revolutions?

In this example you are allowed to use the square of the revolutions instead of the cubes.

$$62^2 \text{ rev.} : 50^2 \text{ rev.} :: 30 \text{ lbs.} : x$$

$$x = \frac{30 \times 50 \times 50}{62 \times 62} = \frac{75000}{3844} = 19{\cdot}4 \text{ lbs.} \quad \textit{Answer.}$$

Ex. 55. A ship has gone 1200 miles at 8 knots per hour with a consumption of 90 tons of coal; it is found that she has only 75 tons left, and has yet to go 1500 miles: at what rate must she steam to do it?

$$90 \text{ tons} : 75 \text{ tons} :: 8^2 \times 1200 : x^2 \times 1500$$

$$x^2 = \frac{75 \times 8^2 \times 1200}{90 \times 1500} = \frac{\overset{5}{\cancel{75}} \times 64 \times \overset{2}{\cancel{1200}}}{\underset{6}{\cancel{90}} \times \underset{3}{\cancel{1500}}} = \frac{128}{3} = 42{\cdot}66.$$

Extracting square root of 42·66 gives 6·5 knots. *Answer.*

Ex. 56. If 50 lbs. pressure give 60 revolutions, what number of revolutions should 58 lbs. give?

$$50 \text{ lbs.} : 58 \text{ lbs.} :: 60^2 : x^2$$

$$x^2 = \frac{58 \times 60 \times 60}{50} = \frac{20880}{5} = 4176$$

$\sqrt{4176} = 64{\cdot}6$ or 65 revolutions, nearly.

382. To find the exact atmospheric pressure at the time of taking an indicator diagram, and thence to determine the line of perfect vacuum.

Rule.—Divide the height of the weather barometer in inches by 2·04, the quotient is the atmospheric pressure in lbs., &c., per square inch.

Ex. 57. What is the atmospheric pressure per square inch in the following diagram (the weather barometer showing 29 inches) if 2·04 inches of mercury were equal to 1 lb. per square inch?

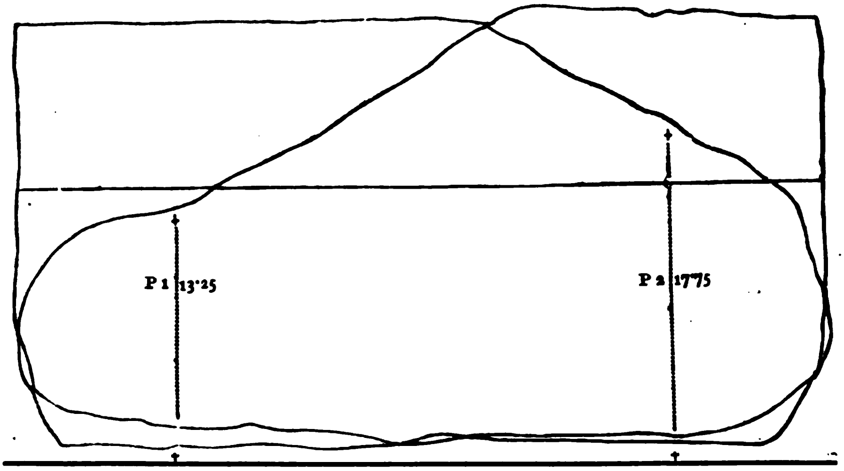
$$\begin{array}{l} 2{\cdot}04)29{\cdot}000(14{\cdot}21 \text{ lbs. per sq. inch.} \\ \quad\;\; 204 \\ \quad\;\; \overline{\;\;860} \\ \quad\;\;\;\; 816 \\ \quad\;\;\;\; \overline{\;\;440} \\ \quad\;\;\;\;\;\; 408 \\ \quad\;\;\;\;\;\; \overline{\;\;\;32} \end{array}$$

383. To determine the line of perfect vacuum diagram for a given indicator diagram.

Rule.—Take a pair of dividers and measure on the same scale as given for the indicator card, and lay this downward from the atmospheric line at both ends, then join the points thus obtained which will give the perfect vacuum line.

Ex. 58. Taking the following diagram, Fig. 132, measuring 14·2 lbs., say 14¼ lbs., on the same scale as given for the card, lay this downward from the atmospheric line at both ends, and join the points thus obtained which gives the perfect vacuum line.

Fig. 132.



Vacuum side.

NOTE.—According to the density of steam at 18 lbs. gross pressure, that is, about 3 lbs. above the atmosphere by the steam gauge, it would take just 35 lbs. weight of steam per hour to produce one I.H.P., if worked without expansion, and also without back pressure or other loss. At higher pressures (less than 35 lbs.), steam would produce the same result; but there are several considerations which lead to the adoption of 35 lbs. as a convenient standard in estimating approximately the cost of evaporation and of H.P. from indicator diagrams.

Given an indicator diagram: to find the consumption in tons for 24 hours' steaming.

Rule.—Divide the length of the card into 10 equal parts, then, counting $\frac{1}{10}$ from the beginning of the stroke at both ends, measure from perfect vacuum the pressure shown on the card at that point, and call the measurement of the down stroke P^1, *and that of the up stroke call* P^2, *then,*

Add these together, multiply the sum by the square of the diameter in inches, and the length of stroke in feet, and by the revolutions; divide the product by 140000, the quotient is the consumption of steam in tons, as shown by the card for 24 hours' steaming.

$$\text{Or, } \frac{(P^1 + P^2)\, d^2\, l r}{140000} = \text{tons of steam for 24 hours.}$$

The above is exclusive of steam condensed in the cylinder, and exclusive of the effect of clearance; the rule is stated for one cylinder.

Ex. 59. Divide the line of perfect vacuum on the diagram and measure off the pressure denoted by P^1 and P^2 in the above rule, viz., the gross pressure at $\frac{1}{10}$ of the stroke from the beginning for both strokes, P^1 being the down stroke, and P^2 the up stroke.

In this instance, $P^1 = 13\cdot 25$, and $P^2 = 17\cdot 75$,

$$\therefore (13\cdot 25 + 17\cdot 75) \times 66^2 \times 3\cdot 5 \times 60 = 28357560,$$

$$\text{And } 28357560 \div 140000 = 202\cdot 554 \text{ tons of steam for 24 hours.}$$

Dia. of cylinder

$$\begin{array}{r} 66 \\ \underline{66} \\ 396 \\ \underline{396} \\ d^2 = 4356 \end{array} \qquad \begin{array}{r} P^1 = 13\cdot 25 \\ P^2 = \underline{17\cdot 75} \\ 31\cdot 0 \end{array} \qquad \begin{array}{r} 4356 \\ \underline{31} \\ 4356 \\ \underline{13068} \\ 135036 \\ \underline{3\cdot 5} \\ 675180 \\ \underline{405108} \\ 472626\cdot 0 \\ \underline{60} \\ 14{,}0000 \left\{ \begin{array}{r} 2)2835{,}75600 \\ \hline 7)1417\cdot 87800 \\ \hline \end{array} \right. \\ 202\cdot 554 \text{ tons.} \end{array}$$

Ex. 60. What is the I.H.P. of a pair of engines, the diameter of cylinders being 66″, length of stroke 3′ 6″, revolutions per minute 60, mean pressure (from Diagram, Fig. 132) 18·36 lbs.?

$$\text{I.H.P.} = \frac{66^2 \times \cdot 7854 \times 7 \times 60 \times 18\cdot 36 \times 2}{33000} = \frac{52763151\cdot 89376}{33000} = 1598\cdot 88, \text{ \&c.}$$

$$66 \times 66\ (= 4356) \times \cdot 7854\ (= 3421\cdot 2024) \times 7 \times 60\ (= 1436905\cdot 008) \times 18\cdot 36$$
$$(= 26381575\cdot 94688) \times 2 = 52763151\cdot 89376 \div 33000 = 1598\cdot 883, \text{ \&c.}$$

Ex. 61. And what is the least consumption per I.H.P. (see preceding example for I.H.P.) that would give 7·5 lbs. evaporation for 1 lb. of coal?

The tons of steam consumed in 24 hours divided by the evaporation for 1 lb. of coal gives tons of coal used in 24 hours,

$$\therefore 202\cdot 554 \div 7\cdot 5 = 27\cdot 007 \text{ tons of coal in 24 hours.}$$
$$27\cdot 007 \text{ tons} \times 2240 = 60495\cdot 680 \text{ lbs. of coal in 24 hours.}$$
$$60495\cdot 680 \div 24 = 2520\cdot 653 \text{ lbs. per hour.}$$

Next, lbs. of coal per hour divided by I.H.P. gives coals per hour per I.H.P.

$$\therefore 2520\cdot 653 \div 1598\cdot 88 = 1\cdot 58 \text{ lbs. per I.H.P.}$$

$$\begin{array}{l} 7\cdot 5)202\cdot 554(27\cdot 007 \text{ tons.} \\ \quad\ \underline{150} \\ \quad\ \ 525 \\ \quad\ \ \underline{525} \\ \qquad\ \ 540 \\ \qquad\ \ \underline{525} \end{array} \qquad \begin{array}{r} 27\cdot 007 \text{ tons.} \\ \underline{2240\phantom{\text{ tons.}}} \\ 108028\phantom{\text{ tons.}} \\ 54014\phantom{0\text{ tons.}} \\ \underline{54014\phantom{00\text{ tons.}}} \\ 24 \left\{ \begin{array}{r} 4)60495\cdot 680 \\ \hline 6)15123\cdot 920 \\ \hline \end{array} \right. \\ 2520\cdot 653 \text{ lbs. per hour.} \end{array}$$

$$\begin{array}{l} 1598\cdot 88)2520\cdot 653(1\cdot 58 \text{ lbs. nearly.} \\ \qquad\qquad\ \underline{159888} \\ \qquad\qquad\ \ 921773 \\ \qquad\qquad\ \ \underline{799440} \\ \qquad\qquad\ \ 1223330 \end{array}$$

Ex. 18. A rectangular boiler is 16 ft. 2 in. long, [illegible] what would be the cost of cementing the two sides [illegible] per square yard?

Area of top = 16′ 2″ × [illegible] = [illegible]
2 sides = 2 (16′ 2″ × 3′ 5″) = [illegible]

[illegible]

∴ 1 sq. yard = 9 sq. feet = [illegible]
∴ 1296 [illegible]
[illegible] = [illegible] Answer.

Ex. 19. A boiler 14 ft. 6 in. by [illegible] over 7½ inches: what will be the cost at [illegible] per lb. [illegible]

The sheet of lead must be twice [illegible] = 15′ 9″ long, and 13′ 9″ [illegible] = [illegible] broad.

∴ [illegible] square feet.

But from this we have to deduct 4 corners [illegible] square feet.

[illegible] square feet.
Deduct for 4 corners [illegible]
[illegible]
Cost per lb. [illegible]
[illegible]
[illegible] Answer.

Ex. 20. A boiler 13 ft. 9 in. by 12 ft. 8 in. has to be covered with lead, and the lead must overlap 8½ inches: what will be the cost at 3¾d. per lb. if 1 square foot of lead weighs 8 lbs.?

It is evident that the sheet of lead must be twice 8½ inches broader and longer than the boiler, that is, 13 ft. 9 in. + 1 ft. 5 in. = 15 ft. 2 in. long; and 12 ft. 8 in. + 1 ft. 5 in. = 14 ft. 1 in. broad = 213·5972.

But from this deduct 4 corners 8½ inches square, that is, 8·5 × 8·5 × 4 ÷ 144 = 2·0069 square feet.

213·5972
− 2·0069
211·5903 square feet.
8 lbs.
1692·7224 lbs. of lead.

Then 1692·7224 lbs.
3·75d.
84636120
118490568
50781672
12[illegible]

Ex. 21. The top of a rectangular boiler is to be [illegible] turned down over the sides a certain distance: what [illegible] of lead 13 ft. 9 in.; breadth 12 [illegible] over-lap 8[illegible] cost 3¾d. per lb.

DATA FOR EXAMINATION FOR ENGINEERS.

SECOND CLASS.

1	**Ten tons, fifteen hundredweights, one quarter, and twenty pounds.**							
2	1500	90	600	45				
3	$2\frac{1}{4}$	$2\frac{1}{3}$	$3\frac{3}{4}$	90				
4	8	12	3	$\frac{5}{8}$				
5	17	15						
6	180	185	190	176	160	165	50	
7								
8								
9								
10								
11								
12								

SPECIMEN OF PART OF A SECOND-CLASS EXAMINATION PAPER.

WITH DIRECTIONS HOW TO FILL IT UP.

Port.	*Class for which examined.*
Date.	*Candidate's name.*

1. **Express in figures:**

Applicant's answer.

2. **A ship has to make a voyage of miles with a stock of tons of coal; after steaming miles, and consuming tons, find how much coal she will have left at the end of the voyage, steaming at the same rate.**

Applicant's answer. tons.

3. **A tank feet deep, and feet by feet, has gallons of oil in it: find how far down from the top is the oil?**

Applicant's answer. inches.

4. **A wrought iron plate feet inches by feet inches, and thick: what is its weight?**

Applicant's answer. cwt. qrs. lbs.

5. **A voyage of days with a supply of gallons of oil: how many gills was this per day?**

Applicant's answer. gills.

6. **The tally of baskets of coal for 24 hours was , , , , , , , each basket weighed lbs.: what was the consumption in tons per day?**

Applicant's answer. tons.

DIRECTIONS HOW TO FILL IN THE ABOVE.

Look at the Data Paper, and the numbers in the left hand or first column are the same with the numbers in the left-hand column above.

Then No. 1 must be filled up by writing what is opposite No. 1 in the Data Paper.

No. 2. Look at No. 2 on the Data Paper; the 1500 is to be put in the first blank space in No. 2 above; the 90 in the second blank space; the 600 in the next blank; and the 45 in the last blank space. (See Ex. 2 of Paper IV following).

No. 3. Look at No. 3 on the Data Paper; the $2\frac{1}{4}$ is to go into the first blank space; the $2\frac{1}{2}$ into the next; the $3\frac{1}{4}$ into the next, and 90 into the last. (See Ex. 3, Paper IV, following).

All the rest fill up in precisely the same manner.

EXAMINATION PAPERS FOR SECOND AND FIRST-CLASS ENGINEERS.

EXAMINATION PAPER—No. I.

FOR SECOND-CLASS ENGINEERS.

1. Express in figures:—Six millions, six thousand, six hundred and six. See pages 11 and 12.

2. Add together 62 tons 15 cwt. 1 qr. 20 lbs., 16 tons 4 cwt. 0 qrs. 13 lbs., and 7 tons 19 cwt. 3 qrs. 13 lbs., and multiply the sum by 8. Rule XIII, page 44, and Rule XV, page 47.

3. What is the product of $3·08 \times 8·8 \times 1·006$? Rule XLVI, page 102.

4. Divide the product (3) by 4·923. Rule XLVII, page 103.

5. The quantity of coal on board on leaving port is 148 tons; the consumption in 12 days, 128 tons 14 cwt. 2 qrs. 17 lbs.: what quantity remains on board? See page 211.

6. A coal bunker is 9 ft. 8 in. long, 7 ft. 4 in. high, and 3 ft. 6 in. wide: required the quantity of coal it will contain at 36 cubic feet per ton (by decimals). See page 202.

7. If the consumption of coal is 48 tons per day to produce a speed 10¼ knots: what is the consumption in a voyage of 3704 knots? Rule XCVIII, page 211.

8. The pressure of steam on the piston is 12 lbs. per square inch above the atmosphere, and the barometer on the condenser shows 26 inches of mercury: required the effective pressure per square inch on the piston. Rule CXVI, page 249.

9. What pressure of steam in the boiler is necessary to blow off the brine through the blow-off cocks, the bottom of the boiler being 12½ feet below the water-line of the ship allowing a column of water 2·305 feet high equal 1 lb.? Rule C, page 221.

10. How many revolutions must an engine make to drive a paddle-wheel 13¾ knots per hour, the diameter of the paddle-wheel 30 feet, and the knot being 6080 feet? See page 272.

11. The pitch of a screw is 14 feet: how many revolutions must it make to advance 12 knots per hour, a knot being 6080 feet? See page 272.

12. A safety-valve is loaded to 30 lbs. per square inch by a direct weight of 2015¾ lbs.: how much would be required to be taken off to reduce the pressure to 17½ lbs.? See page 253.

13. To what height must a safety-valve of 7½ inches diameter be lifted to allow a free escape of steam equal to the area of the valve? Rule CXXII, page 254.

14. How many cubic feet of water will be extracted in an hour by a brine pump 5⅝ inches diameter, and 17½ inches stroke, making 11 strokes per minute, the pump being ¾ full at each stroke? Rule CVI, page 234.

15. What is the horse-power of an engine, the diameter of the cylinder being 94 inches, length of stroke 4 feet 6 inches, revolutions per minute 62·4, and the effective pressure per square inch on the piston 21·5 lbs.? Rule CXII, page 243.

16. If a bar of iron 1 inch square is torn asunder by a strain of 4500 lbs.: what force will be required to break a bar 5 inches square? See page 402.

ADDITIONAL FOR FIRST-CLASS ENGINEERS.

1. Find the side of a square whose area is 900·180009. Rule LX, page 166.

2. Some repairs to the engines have occupied 8 men 24 weeks, and cost £315 10s. for wages; larger repairs are necessary, which it is expected will occupy 28 men 16 weeks: how much will be required to pay them at the same rate of wages? See page 155.

3. Required the weight to be placed at the end of a safety-valve lever to be equal to 28 lbs. per square inch on the valve, the diameter of valve being 5½ inches, the distance from the fulcrum to valve 3 inches, and from the valve to the weight 32 inches. The effective weight of the lever is 87 lbs. 10 oz., and the weight of the valve 13 lbs. 6 oz. See page 251.

4. Required the weight to be placed at the end of a double-beat valve to be equal to 26 lbs. per square inch on the valve, the diameter of valve being 14½ and 12½ inches. See page 252.

5. What is the total pressure on the flat bottom of a boiler 23 feet 4 inches by 12 feet 8 inches, the pressure of steam in the boiler being 30 lbs. per square inch, and the height of water in the boiler 5 feet 6 inches? Also the number of stays 2 inches in diameter required, so that each square inch of section of the stays shall bear 6500 lbs. Rule CII, page 223.

6. Required the heating surface in tubes and tube plates of a boiler, the diameter of tubes being $2\frac{1}{4}$ inches, length 6 feet 6 inches, and 134 in number, and the plate 6 feet 9 inches by 4 feet wide. Rule CI, page 221.

7. The pitch of a propeller is 11 feet 6 inches, and makes 108 revolutions per minute: what will be the progression of the propeller in knots per hour, and the rate of the ship, allowing 17·5 per cent. slip? Rule CXXVI, page 273.

8. Sea water contains $\frac{1}{33}$ of solid matter, and it is required that the water in the boiler may not contain more than $\frac{2}{33}$: what proportion must the quantity blown off bear to the quantity evaporated? Rule CXXXIII, page 393.

9. Steam is admitted into the cylinder at 25 lbs. per square inch above the atmosphere, and is cut off at $\frac{3}{8}$ of the stroke: required the mean pressure on the piston throughout the stroke, supposing the vacuum to be perfect. Rule CXXX, page 285.

10. Work out an indicator card, and show the point where the steam is cut off; whether the slide-valve is correct or requires alteration, and if the latter, what?

Fig. 133.

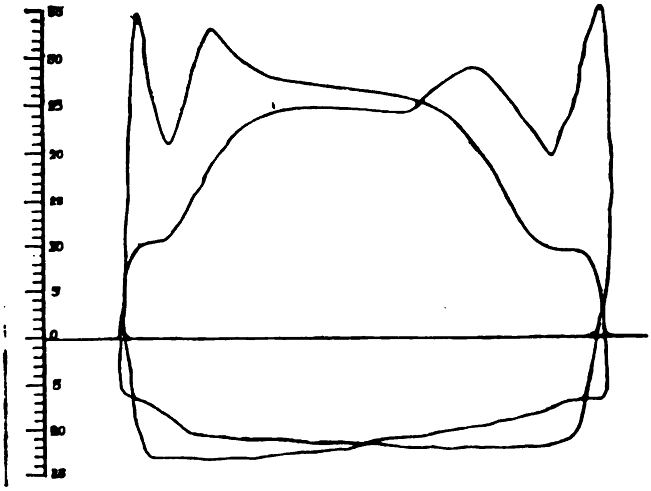


Diagram taken from H.M. Troop Steamer *Orontes*. Steam in boiler $21\frac{1}{2}$ lbs., vacuum $11\frac{1}{2}$ lbs., revolutions 60.

In these diagrams the waving lines formed by the pencil, owing to the momentum of the piston of the indicator, is very plainly shown, and although such irregularities will not materially impair the accuracy of the result, if a sufficient number of ordinates be taken correctly to measure the irregularity, yet it is greatly preferable to employ an indicator which will be as free as possible from the disturbing influence of the momentum of its own moving parts. It will be noticed that in this engine, as in most of Boulton and Watt's engines, there is a great similarity in the diagrams taken from each end of the cylinder—a result in the main produced by giving a suitable length to the eccentric-rods, by moving them up and down the links vertically by a screw, instead of by means of a lever moving in the arc of a circle, and placing the projecting side of the eccentric suitably with the curvature of the link, because if placed in one position it will aggravate the distortion produced by the angle of the eccentric rods, and if placed in the opposite position it will correct this distortion.

EXAMINATION PAPER—No. II.

FOR SECOND-CLASS ENGINEERS.

1. Express in figures:—Five hundred and six million, fifty-five thousand and eighty-four. See pages 11 and 12.

2. Add together 436 tons 15 cwt. 2 qrs. 15 lbs., 323 tons 9 cwt. 2 qrs. 5 lbs., and 521 tons 15 cwt. 3 qrs. 27 lbs., and multiply the sum by 16. See pages 44 and 48.

3. What is the product of 3487·507 by 952·0156? See page 102.

4. Divide the product of (3) by 7048624.

5. The quantity of coal on board on leaving port is 276 tons 19 cwt. 3 qrs. 27 lbs., the consumption per day is 17 tons 3 cwt. 3 qrs. 26 lbs. for 9 days: what quantity remains on board? See page 211.

6. A coal bunker is 28 feet 9 inches long, 10 feet 6 inches high, and 8 feet 3 inches wide: required the quantity of coal it will contain at 42 cubic feet per ton. (By decimals). See page 202.

7. If the consumption of coal is 20 tons per day to produce a speed of 9½ knots per hour: what is the consumption in a voyage of 1507 knots? See page 211.

8. If the pressure of steam on the piston is 26 lbs. per square inch above the atmosphere, and the barometer on the condenser gauge shows 29½ inches of mercury: required the effective pressure per square inch on the piston. See page 249.

9. What pressure of steam in the boiler will be necessary to blow off the brine through the blow-off cocks, the bottom of the boiler being 13 feet 3 inches below the water-line of the ship, allowing a column of water 2·305 feet high to equal 1 lb.? See page 221.

10. How many revolutions must an engine make to drive a paddle-wheel 9¾ knots per hour, the diameter of the wheel being 28 feet 6 inches, and the knot being 6080 feet? See page 272.

11. The pitch of a screw is 25 feet: how many revolutions must it make to advance 13·3 knots per hour, a knot being 6080 feet? See page 272.

12. A safety-valve is loaded to 24 lbs. per square inch by a direct weight of 240 lbs.: how much would be required to be taken off to reduce the pressure to 10½ lbs. per square inch? See page 253.

13. To what height must a safety-valve 9 inches diameter be lifted to allow a free escape of steam equal to the area of the valve? See page 254.

14. How many cubic feet of water will be extracted in an hour by a brine pump 7·3 inches diameter, and 15 inches stroke, making 13 strokes per minute, and the pump being ⅔ full at each stroke? See page 234.

15. What is the horse-power of an engine, the diameter of cylinder being 64 inches, the length of stroke 3 feet, revolutions per minute 66, and the pressure per square inch on the piston 22·3 lbs.? See page 243.

16. If a bar of iron 1 inch square is torn asunder by a strain of 4500 lbs.: what force will be required to break a bar 4·6 inches square? See page 402.

ADDITIONAL FOR FIRST-CLASS ENGINEERS.

1. Find the side of a square whose area is 970·299. See page 166.

2. Some repairs to the engine have occupied 21 men 7 weeks, and have cost £244 12s. 6d. for wages; some larger repairs are necessary, which it is expected will occupy 18 men 6 weeks: how much will be required to pay them at the same rate of wages? See page 155.

3. Required the weight to be placed at the end of a safety-valve lever to be equal to 28¼ lbs. per square inch on the valve, the diameter of the valve being 3¾ inches, the distance from the fulcrum to the valve 4 inches, and from the valve to the weight 32 inches. The effective weight of the lever is 25 lbs. 4 oz., and the weight of the valve 6 lbs. 6 oz. See page 251.

4. Required the weight to be placed on a double-beat valve to be equal to 20 lbs. per square inch, the diameters of the valve being 10½ and 12 inches. See page 252.

5. What is the total pressure on the flat bottom of a boiler 18 feet 4 inches long by 11 feet 8 inches broad, the pressure of steam in the boiler being 26 lbs. per square inch, and the height of water in the boiler 5 feet? also the number of stays 1½ inches in diameter is required, so that each square inch of section of the stays shall bear 5300 lbs. See page 223.

6. Required the heating surface in the tubes and tube plates of a boiler, the tubes being 2½ inches in diameter, 7 feet 3 inches in length, and 156 in number, and the plates 7 feet long by 3 feet 10 inches wide. See page 221.

7. The pitch of a propeller is 11 feet 9 inches, making 106 revolutions per minute: what will be the progression of the propeller in knots per hour, and the rate of the ship, allowing 17·5 per cent. for slip? See page 273.

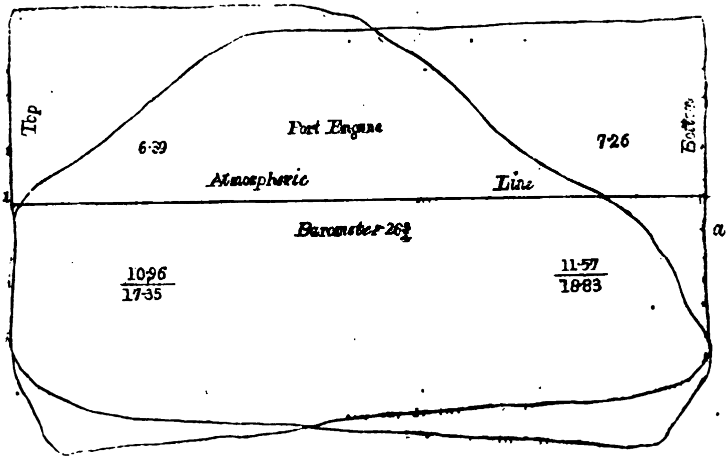
8. Sea water contains $\frac{1}{33}$ of solid matter, and the water in the boiler is not to contain more than $\frac{4\cdot 2}{33}$: what proportion must the quantity blown off bear to the quantity evaporated? See pp. 393-4.

9. Steam is admitted into the cylinder at 30 lbs. per square inch above the atmosphere, and is cut off at ⅓ of the stroke: required the mean pressure on the piston throughout the stroke, supposing the vacuum to be perfect. See page 285.

10. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

From the S.S. *Penelope.*

Fig. 134. Scale of 10ths.



EXAMINATION PAPER—No. III.

1. 46712389467; express this in words. See page 13.

2. The bolts of a cylinder cover are placed 6 inches apart, centre to centre, diameter of cover 64 inches, and the bolts 2¼ inches from the cover: how many bolts? See Ex. 45, page 448.

3. A single rivetted boiler has 7 courses of 6 plates in each course, and each plate has 36 by 14 holes: how many rivets? See Rule CIV, page 232.

4. An engine counter stood at 6001 at the commencement of the voyage, and after 10^{d} 10^{h} stood at 801001: how many revolutions per minute? See page 270.

5. A tank is 18 feet 6 inches by 5 feet 9 inches, and contains 4800 gallons: what is its height? See Ex. 1, page 205.

6. What are the wages of 27 men, 34 days, 9s. 6d. per day?
46 „ 39 „ 5s. 4d. „
93 „ 28 „ 2s. 8d. „

7. The consumption per day is 33 tons of coal, the average H.P. indicated is 1480: what is the consumption per hour per I.H.P.? See page 214.

8. How many cubic feet of steam will be used per hour by a pair of engines 61 revolutions per minute, diameter of cylinder 38 inches, stroke 33 inches, and cut-off at ⅝ of the stroke? See Rule CIX, page 238.

9. The indicated H.P. of a pair of engines is 1166, cylinder 63¾ inches diameter, stroke 36 inches, revolutions per minute 58: what is the pressure on the piston? See page 248.

10. How many tons must be pumped into a boiler whose area of water level is 10 feet by 8 feet, to raise 5 inches in the glass? See Ex. 1, page 239.

11. If the total consumption of coal is 21 tons per day: how many baskets, each 66 lbs., will be required in a watch of 4 hours? See page 215.

12. A screw shaft is 19 feet 3 inches over all, 8½ inches diameter, coupling flanges 18 inches diameter, and 3 inches thick: find the weight. See pages 216-217.

EXAMINATION PAPER—No. IV.

FIRST AND SECOND-CLASS ENGINEERS.

1. Express in words: Six tons, fifteen hundredweights, one quarter, and twenty-two pounds.

2. A ship has to make a voyage of 4500 miles with a stock of coal of 250 tons; after steaming 2450 miles and consuming 120 tons, find how much coal she will have left at the end of the voyage, steaming at the same rate. See page 219.

3. A tank 2½ feet high, and 2 ft. 6 in. by 4 ft. 3 in., has 94 gallons of oil in it: how far from the top is the oil? See Ex. 1, page 260.

4. A plate 6 ft. 4½ in. by 2 ft. 8 in., $\frac{5}{16}$ thick: what is its weight? See Ex. 1, page 260.

5. A voyage of 21 days with a supply of 9 gallons of oil: how many gills per day?

6. The tally of coal for 24 hours was 39, 41, 39, 31, and 41 baskets for each watch, each basket weighed 76 lbs.: what is the consumption in tons per day? See Exs., page 216.

7. A safety-valve 6½ in. diameter was loaded with dead weights, 5 of 63 lbs., and 2 of 49 lbs., the valve itself weighed 13½ lbs., and the spindle 19 lbs.: what is the pressure of steam per square inch on the valve? See Ex. 5, page 258.

8. A stay 15 in. apart, centre to centre, and is eaten away to 1½ in. diameter, the pressure is 25 lbs. per square inch: what is the strain per square inch section of the stay? See page 226.

9. A barrel of tallow weighed 2 cwt. 0 qr. 2 lbs., and cost £8 15s. 0d., the barrel itself weighed 26 lbs.: how much did the tallow cost per lb.?

10. What is the total cost of 2 cwt. 3 qrs. of tallow at 7½d. per lb.; 2 cwt. 1 qr. of waste at 5½d. per lb.; and 10 gallons of oil at 5s. 3d. per gallon?

11. Up to the present time, which is Friday at 8h A.M., the ship has steamed 149, 150, 147, and 160 knots each day: when will she reach her port, 1500 miles from starting, at 8 knots per hour? See Ex., page 277.

12. A water ballast donkey 10″ diameter, 14″ stroke, double-acting, 95 strokes per minute, and $\frac{1}{16}$ empty each stroke: how long will it take to pump out 350 tons? See Ex. 1, p. 235.

EXAMINATION PAPER—No. V.

FOR SECOND-CLASS ENGINEERS.

1. Express 15 feet and 7¾ inches as in a drawing.

2. $24 \div 4 + 6 - 2 \times (65 - 64)$.

3. 2 cwt. 2 qrs. 14 lbs. of tallow at 56s. per cwt.

4. An oil tank is 4 feet high by 2·5 feet by 3·5 feet: what was the consumption per day in gallons, if in 14 days the oil in the tank is lowered 10½ inches in the glass? See Ex. 3, page 209.

5. What is the weight of gun metal in a screw-shaft liner, length 54 inches, diameter of shaft 18 inches, thickness of liner ¾ inch, if a cubic inch = ·3 of a lb.? See Ex. 4, p. 266.

6. 19′ 6″ × 3′ 4½″: find the product in square feet and decimals of square feet.

7. If 1 lb. of steam raises the temperature of 1000 lbs. of water 1°, and the sea water is 60°, and the discharge water 105°: how many lbs. of circulating water passes the condenser for each lb. of steam? See Ex. 6, page 396.

8. The piston is 52 inches diameter, and at the beginning of the stroke the pressure is 42 lbs. per square inch; if the crank-pin is 9 inches diameter, what length should it be in the bearing if the pressure on the oil is not to exceed 750 lbs. per square inch? See Ex. 12, page 405.

9. When the temperature in the funnel is 600° the consumption is 22 tons per day; after a few days it increases to 27 tons per day to raise the same amount of steam: what is now the temperature of the funnel, nothing else having altered; and what is the cause of the increased consumption?

Note.—Cost of evaporation is increased by (*n*) per cent. The alteration in the temperature of escaping gases alters to 22 (*n*) degrees. See Ex. 8, page 396.

10. Find the collapsing pressure on a furnace tube whose thickness of plate is $\frac{7}{16}$, diameter 2 ft. 11 in., and length 10 ft. 3 in.

Rule.—1°. *Square the thickness and multiply by* 806300.
2°. *Multiply the length in feet by the diameter in inches.*
3°. *Divide the first product by the second.* See Ex., page 233.

11. The formula for finding the N.H.P. of common engines is $\frac{D^2 \times V}{6000}$ What will be the N.H.P. of a cylinder whose diameter is 53 inches, length of stroke 2 ft. 9 in., and making 65 revolutions per minute?

12. Formula for rectangular cross-bars for combustion chamber crowns—

$$B = \frac{72000\, d^2\, t}{D\, L^2}$$

Where B = boiler pressure.
and D = distance apart of bars.
L = length of bars.
d = depth of bars.
t = thickness or width of bars.

The cross-bars are $4\frac{7}{8}'' \times \frac{5}{8}''$, 26" apart, and length of bars 36": what pressure will be allowed in the boiler if there are 3 stays? See Ex. 4, page 442.

13. What would be the pressure if there were 4 stays?

Increase $\frac{B}{N\,(N+2)}$ where N = No. of stays. See Ex. 5, page 442.

EXAMINATION PAPER—No. IV.

FOR SECOND-CLASS ENGINEERS.

1. Express in decimals:—4 times the 1-140 thousandth part of an inch.
2. (36 + 6·2) — 6 × 4.
3. Subtract 1' $2\frac{3}{8}''$ from 2' $4\frac{1}{4}''$, and multiply the remainder by 13.
4. What is the amount of wages—

From 21st June to 20th December, at £14 per month.
„ 15th March to 13th December, at £6 per month.
„ 28th October to 18th February, at £6 per month.

Note.—Allow 30 days to the month.

5. A compound engine has a steam receiver round the H.P. cylinder equal in diameter to the low pressure cylinder, steam chest for slides between the cylinders, distance between the centres of cylinders 50 inches, diameter of L.P. cylinder 40 inches, and depth 35 inches: how many square feet of felting will be required to cover it? See Ex. 22, page 446.

6. A tank 34 feet long by 20 feet 3 inches broad, and 6 feet 2 inches high: how long will it take to pump it out, the diameter of pump being 8 inches, stroke 10 inches, revolutions per minute 68, double-acting, if 10 per cent. be non-effective. See Ex. 42, page 450.

7. Formula $S = \frac{5 \cdot 1\, W\, a}{d^3}$

S = strain per sq. inch of section of the shaft.
W = weight at leverage (= a).
d = diameter of shaft.

Question.—The piston is 60 inches diameter, steam 32 lbs. per square inch, length of crank 18 inches, diameter of shaft 12 inches: is the shaft strong enough? and what is the strain per square inch? See No. 327, page 410.

8. The topmast of a ship is 84 feet above the level of the sea when upright, but when fore and aft sails are set she is lying over till the topmast is 74 feet above the sea level; the valve is 5 inches diameter, and loaded to 641 lbs.: what will she blow off at when lying at this angle? See Ex. 2, page 257.

9. On starting on a voyage the consumption is 10 tons per day, the temperature of the funnel is 560°, but at the end of the voyage it is 690°: what is the cause of this? and what was the consumption?

NOTE.—Increase $= \frac{F}{2200}$ See Ex. 19, page 398.

10. A coal bunker is 6 feet 9 inches at the top, and 5 feet 3 inches at the bottom, 20 feet 3 inches and 9 feet high: what was the weight in tons? State how many cubic feet you will allow per ton. See Ex. 4, page 203.

11. The I.H.P. of an engine is 750, and she uses 24 lbs. weight of steam per hour per I.H.P. If 1 lb. of steam on being condensed gives out sufficient heat to raise 1000 lbs. of water 1° Fahrenheit, and if the injection water has a temperature of 61°, and the discharge water 104°: how many tons of injection water are used per day of 10 hours? See Ex. 16, page 397.

12. An engine makes 45 revolutions per minute, when the steam pressure is 23 lbs., it is reduced to 41 revolutions: what is now the pressure? See Ex. 54, page 452.

EXAMINATION PAPER—No. VII.

FOR SECOND-CLASS ENGINEERS.

1. $488 \div 8 + 263 - 28 \times 5$.

2. Find the value of $\frac{10^4 - (12 - 8^2)}{6^3}$

3. A sheet of lead 16 ft. 3 in. long by 12 ft. 9 in. broad, and $\frac{6}{16}$ inch thick: what would it cost at 4d. per lb., if a cubic inch of lead weighs ·4126 of a lb.?

4. If a piston is 43 inches diameter, and assuming it to be $4\frac{1}{4}$ inches thick throughout: what should it weigh if 3·9 cubic inches of cast iron weigh 1 lb.? See Ex. 3, page 261.

5. The diameter of the barrel of the wheel and axle is $8\frac{1}{2}$ inches, and the diameter of the rope 1 inch, weight 200 lbs.: what force must be applied to the handle whose radius is 18 inches? See pp. 424-425.

6. If the bolts in a cylinder cover be $1\frac{1}{4}$ in. diameter, and the diameter of cylinder 40 inches, pressure 60 lbs. per square inch: how many bolts will be required if the strain per square inch of their section is not to exceed 1800 lbs.? See Ex. 6, page 403.

7. The sea water is 76°, the water in the hot-well is 120°: what will be the temperature of the hot-well if the sea water is raised to 80°, if you cannot use any of the injection water? See Ex. 3, page 395.

8. If the sea water be at 60° Fahr., and the water in the hot-well is to be at 102°: how many lbs. of sea water will be required to condense 1 lb. of steam? See Ex. 10, page 397.

Rule. $\frac{1149 \cdot 6^\circ - t'}{t' - t}$ Where t = temperature of sea water.
and t' = „ hot-well.

9. A waste steam pipe of a spring-loaded safety-valve is 23 feet high: what extra pressure per square inch is on the valve should the pipe become filled with water? What would be wrong? See Ex. 3, page 258.

10. An engineer has a bonus of £18 for coal economy; there were 84 tons left at 18s. per ton: what was his per centage of the value? See Ex. 5, page 164.

11. What boiler pressure should be allowed for a shaft 15 inches diameter: the diameter of cylinder is 58 inches, and length of stroke 36 inches?

Rule. $B = \frac{2830\,d^3}{D^2\,S}$

Where B = boiler pressure.
d = diameter of shaft.
D = „ cylinder.
S = length of stroke.

12. An air-pump is 16 in. in diameter and 18 in. stroke; the cylinder is 35 in. diameter and 36 in. stroke; the Board of Trade allows ⅛ the capacity of cylinders for the capacity of air-pump, single-acting; the air-pump is double-acting; is the capacity more or less than ⅛? Should air-pumps be larger or smaller for surface condensers, and why? See Ex. 2, page 242.

EXAMINATION PAPER—No. VIII.

FOR SECOND-CLASS ENGINEERS.

1. 92 − (58 − 24) + 14 × (21 − 9).

2. What is the cost of wrought iron forging, weighing 16 cwt. 3 qrs. 14 lbs., at 28s. 3d. per cwt.? See page 132.

3. The diameter of cylinder is 76 inches, length of stroke 42 inches, revolutions per minute 51, and pressure of steam per square inch 23·3 lbs.: what is the horse-power. See Rule CXII, page 243.

4. The travel of the slide 9¼ inches, cover 3¼ inches, lead $\frac{1}{16}$ inch, and stroke of the piston 42 inches: how far is the piston from the end of the stroke when the steam is cut off? See Rule CXXXI, page 291.

5. The area of the boiler level is 120 square feet; a single-acting donkey can pump up 17 inches in the glass per hour, diameter of pump 4 inches, stroke 5 inches, revolutions per minute 100: what is the efficient stroke of the pump? See Ex. 2, page 240.

6. Diameter of the piston-rod at the bottom of thread is 5½ inches; there are two bolts in the cross-head journal, each 3¼ inches diameter: what is the proportion of the area of the rod to that of the bolts? See Ex. 4, page 185.

7. What is the per centage of strength of a single-rivetted seam, diameter of rivets ⅞", pitch 1¾", and thickness of plate ⅝"?

8. There are 2 side bunkers, each 5 ft. 3 in. wide, and 14 ft. 6 in. high, and are to contain 70 tons of coal: what is the length of each? and how many cubic feet will you allow per ton?

9. What is the rubbing surface of a packing ring whose diameter is 65 inches, and width 5½ inches?

10. When the temperature of the hot-well was 110° Fahrenheit the evaporation was 7·1 lbs. of water for 1 lb. of coal: what will be the evaporation if the temperature of the hot-well is increased to 160°?

$$\text{Increase} = \frac{F}{1100}$$ See Ex. 15, page 397.

11. The speed of the ship is 8¼ knots per hour, with a consumption of 9 tons per day; but the speed is increased to 9·7 knots per hour, with a consumption of 10·1 tons per day: what is the rate of cost per day saved?

Rule. $\frac{C_{,}K - CK_{,}}{K_{,} - K}$ See Ex. 6, page 276.

12. Verify the above formula, working under the two conditions for a voyage of 1600 miles. See Ex. 7, page 276.

EXAMINATION PAPER—No. IX.

FOR SECOND-CLASS ENGINEERS.

1. Express in words: 900999009.

2. 48 ÷ 6 + 8 × 2 — 4.

3. Add together £29 19s. and £35 16s. 10d., and divide the sum by 25.

4. How many bolts 1¼ inch diameter are required for a cylinder cover 54 inches diameter, the pressure being 70 lbs., and the strain per square inch of section is not to exceed 2000 lbs.?

5. A coal bunker is 22 ft. 4 in. wide by 13 ft. 11 in. high, and in 12 days a section of the bunker 16 feet long was consumed: what was the consumption per day? See page 220.

6. In a wheel and axle the diameter of the barrel was 8¾ inches, and that of the rope ¾ inch; the crank handle was 15 inches long, and the weight was 250 lbs.: what was the force exerted, allowing 10 per cent. for friction? See pages 424-425.

7. Three engineers receive £120 between them as bonus; it is to be divided among them in proportion to the amount of wages they have received from the company. A's wages are £272 10s., B's are £224 15s., and C's are £102 15s.: what part of the £120 will each receive? See Ex. 4, page 160.

8. When the temperature of the hot-well was 114°, the evaporation was 8·3 lbs. of water: what should be the evaporation when the hot-well is at 156°? Increase $= \frac{F}{1100}$ See Ex. 15, page 397.

9. A slide-valve is worked by a hooped spindle, the hoop surrounding the body of the valve; the port of the valve is 20 inches wide, rectangular, and the recess is 4 inches deep; the exhaust port of the cylinder is 6¾ inches deep; the steam port 2¾ inches; the bars 1¾ inches; internal cover ⅛ inch; and lap 3 inches; the valve is 1½ inches on each side of the ports, face of the valve is 1 inch thick, and the rest of the valve is ¾ inch thick: find its weight if 3·9 cubic inches weigh 1 lb.? See page 268.

10. Add together 5′ 11½″, 17′ 3¾″, 14′ 1⅞″, 3′ $10\frac{15}{16}$″, and 7′ $0\frac{17}{32}$″.

11. The speed of a ship is 9·6 knots per hour, and her consumption 12 tons per day; at another time her speed is 10·1 knots, and her consumption 15 tons per day: what is the rate of cost per day saved? See Ex. 6, page 276.

$$\frac{C_1K - CK_1}{K_1 - K}$$

12. Verify the above formula by working at full length under the two conditions for a voyage of 2500 miles. See Ex. 7, page 276.

EXAMINATION PAPER—No. X.

FOR SECOND-CLASS ENGINEERS.

1. Add together £46 12s. 3d., £56 19s. 6d., £75 4s. 6d., £54 4s. 4½d., £96 12s. 6½d., £99 19s. 9½d., £99 3s. 7¼d., and £154 11s. 11½d.

The candidate is to leave out rows Nos. 4 and 7, and add together the remainder.

2. $84 + \frac{76 \times 24}{20 + 5}$

3. A steam cylinder 64 inches diameter, steam pipe 10 inches diameter: how many square inches area of piston is there for 1 square inch of sectional area of pipe?

4. If instead of using 12 gallons of oil per day at 2s. 11d. per gallon, you use 8 gallons per day of a different quality of oil which costs 3s. 4d. per gallon: what difference of expense will that amount to in 46 days?

5. What is the greatest weight that can be safely suspended by one round hemp rope 17 inches in girth?

Rule.—Girth squared ÷ $\frac{1}{32}$ = answer in tons.

6. There are 12 teeth in the ratchet of the turning gear, and 30 in the worm, which at each pull of the lever advances 3 teeth, at 12 pulls per minute: in what time will a crank be moved from the top centre to the bottom?

7. The steam ports are each 27 inches by 3½ inches, the steam lap is 2⅛ inches, the travel of the valve is 9 inches: what is the area of the greatest opening for steam? See Ex. 27, page 447.

8. The thickness of the shell plating of a cylindrical boiler of the best construction is 1$\frac{1}{16}$ inch thick, diameter of boiler 11 feet 8 inches: what is the safe working pressure?

9. The safety-valve is 5½ inches diameter, its bearing face is quite flat: what is the lift when the opening is ⅑ of the area of the valve? See Ex. 8, page 259.

10. The feed water contains 4½ ozs. of saline matter to the gallon: if ⅝ of the feed be blown off continuously, what will be the saltness of the water in the boiler?

11. A compound engine has one cylinder 32 inches, the other 58 inches diameter: what is the N.H.P. (nominal horse-power) at 30 circular inches of piston to 1 horse-power?

EXAMINATION PAPER—No. XI.

FOR FIRST-CLASS ENGINEERS.

1. The effective pressure per square inch on the surface of a piston is 35 lbs., the total pressure 89·3 tons: what is the diameter of the piston? See Ex. 44, page 450.

2. If the drag of friction be ·055 of the pressure, what H.P. is expended on the friction of a thrust bearing with 6 collars, the mean diameter of the collar being 14 inches, the shaft making 57 revolutions per minute, and the average pressure 7 tons? See Ex. 1 & 2, pages 413—414.

3. If the drag of friction be ·05 of the pressure, and if the pressure of the connecting-rod on the crank-pin be uniform: what per centage of that is lost by friction on the crank-pin, the stroke being 33 inches, and the diameter of the crank-pin 9 inches? See page 414.

4. If the maximum strain to be allowed on iron be taken as 51520 lbs. (23 tons) per square inch: what should be the depth at the fulcrum of an iron lever whose thickness is 1½ inches, to support 1½ tons at the extremity, the lever arm being 18 inches long? See Rule CXXXV, pages 406-407.

5. If the maximum strain to be allowed on iron be taken as 51520 lbs. per square inch: what pressure may be applied at the end of a crank 15 inches long, to be transmitted by a shaft 9½ inches diameter? Torsion only to be taken. See Rule CXXXVI, pages 408-409.

6. What is the maximum strain per square inch on a screw-shaft due to the over-hang of a propeller weighing 3 tons, the centre of weight being 24 inches from the point of support, and the shaft 9 inches diameter? See No. 328, page 410.

7. In a double-rivetted boiler the rivets are ⅞ inch diameter, and placed at 2½ pitch, the plates are ⅝ inch thick: what per centage of strength has the seam as compared with the strength of the solid plate? See Ex. 1, page 230.

8. A boiler measures 18 feet 11 inches long, 19 feet broad, rectangular; the water is 10 feet 4 inches high, and one-third that space is furnaces and tubes, the rest is water. The furnace consumes 12 cwt. of coal per hour, and each lb. of coal is equal to heating 12000 lbs. of water 1° Fahrenheit. The difference between the temperature of 65 lbs. of steam and of 70 lbs. of steam is 5°·1: in what time will the pressure rise from the former to the latter pressure if all the valves are shut and the fires in action? See page 223.

9. Work out the following indicator card, and show the point where the steam is cut off; whether the slide-valve is correct or requires alteration, and if the latter, what?

From the engine of the S.S. *Bremen*, by Caird & Co. Steam cut off $13\frac{1}{2}$ inches of stroke, cylinder 90 inches diameter, length of stroke 3 feet 6 inches.

Note.—This vessel steams fast with a low consumption of fuel.

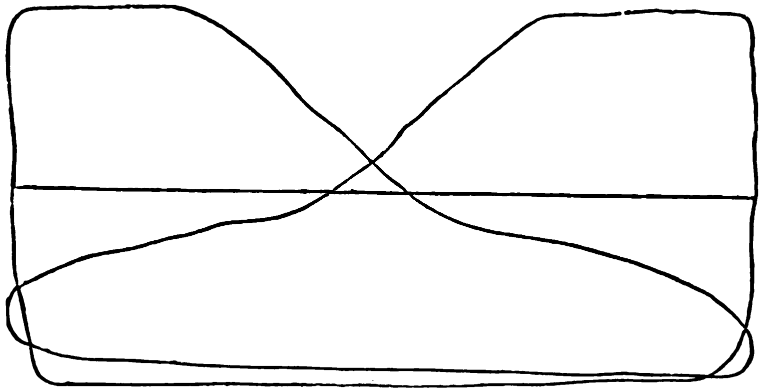


Fig. 135.
Scale of 10ths.

10. The stroke of an engine is 27 inches, the slide-valve travel is $6\frac{7}{8}$ inches, the cover on the steam port $1\frac{7}{16}$ inches, and the lead $\frac{3}{16}$ inch: at what distance from the end of the stroke will the piston be when the steam is cut off? See page 291.

EXAMINATION PAPER—No. XII.

FOR SECOND-CLASS ENGINEERS.

1. $\cdot 012 + \frac{17 \cdot 25}{6 \cdot 4}$.

2. Find the square of the third line and divide by the sixth line.

5·614
0·124
0·128
1·176
2·876
6·814

3. 3 firemen, 3 months 20 days, at £3 per month.
17 boilermakers, „ 6s. per day.
30 labourers, „ „ 3s. 3d. per day.

4. A cask of tallow weighs 600 lbs. and the price is 4d. per lb.; the cask alone is 60 lbs.: what is the cost, allowing 8 per cent. for discount?

5. From the 19th of May, at $7^h\ 15^m$ A.M., to the 2nd of July, at $10^h\ 30^m$ P.M., the consumption was 480 tons, and the distance gone 13891 knots: find the mean consumption and mean speed.

6. An engine of 170 N.H.P. is repaired three times in the year at the cost mentioned as No. 4 in the following list; the second time at the cost marked as No. 1 in the list, and the third time at the cost marked No. 6 in the list: what is the average cost per N.H.P. per year?

Cost 1st time as marked No. 4 in list, £120
„ 2nd „ 1 „ 176
„ 3rd „ 6 „ 85

7. A ship with a certain draught is put down 1" with a weight of 37 tons: find the area of the water-level of the ship in square feet. See Ex. 62, page 455.

8. The sectional area of a ship at the water level was 4821 square feet; her draught before taking in her bunker coals was 19·7 feet aft and 19·4 feet forward; after coaling the draught was 21 feet aft and 20·8 feet forward: how many tons of coal were taken in? See Ex. 63, page 455.

9. The scale of iron rust has been taken off which weighed 5 ozs., and scale consists of 112 parts of iron to 48 oxygen: how much iron was there in the scale?

10. The above scale was 7" by 4"; it was of uniform thickness: what was its thickness?

11. You can see the horizon 34 statute miles off: what is your height of eye?

Rule.—Square the miles and then divide by 1·5.

EXAMINATION PAPER—No. XIII.

FOR FIRST-CLASS ENGINEERS.

1. A propeller of 20 feet pitch is altered to 18 feet pitch, but an indicator diagram shows the same point of cut-off: the ship made 10½ knots with the first pitch. See Ex. 3, p. 275.

2. If the pitch was 18 feet and revolutions 60, the ship making 11 knots per hour, and the pitch was changed to 19·6 feet: how many revolutions and knots will she now make? See page 275.

3. A rectangular bar is 15 square inches in section and 60 inches long; it is weighted at the end by a weight of 2240 lbs.: what is the depth of the bar, the strain being 12000 lbs.?

Rule. $\frac{\text{Strain} \times B \times D \times D}{6} = W \times l$. See Ex. 1, page 405.

4. In the last question, if the breadth is 3 inches, depth 5 inches, length 60 inches, and weight 5000 lbs.: what will be the strain per square inch of section? Ex. 2, page 406.

5. A red pine beam is 16 feet long by 12 inches by 8 inches: what weight can be laid in the centre of the beam if the working weight is $\frac{1}{10}$ of the breaking weight? See Ex. 3, page 406.

Rule. $\frac{4 \times B \times D^2}{\text{length in feet}} =$ breaking weight in cwts.

6. If steam at 50 lbs. pressure gives 60 revolutions per minute: what number of revolutions should steam at 58 lbs. give? See Ex. 56, page 452.

7. Speed K knots, and P mean effective pressure: what is the speed of the vessel when the mean effective pressure is M? See Ex. 2, page 275.

$$K = 10;\ P = 29 \text{ lbs.};\ M = 32 \text{ lbs.}$$

8. If the shearing strength of rivets per square inch of section is taken as equal to the tensile strength of plates per square inch of section: what is the strength of a double seam of rivets of the following dimensions? See Ex. 4, page 403.

Diameter of rivets $= \frac{7}{8}''$; pitch of rivets $= 2\frac{1}{8}''$; breadth of plate $= 12\frac{1}{4}''$; thickness of plate $\frac{3}{4}''$; tensile strength $=$ 23 tons per square inch of section.

9. The diameter of crank-pin journal is $\frac{D + L}{9}$ and its length is the same size as diameter.

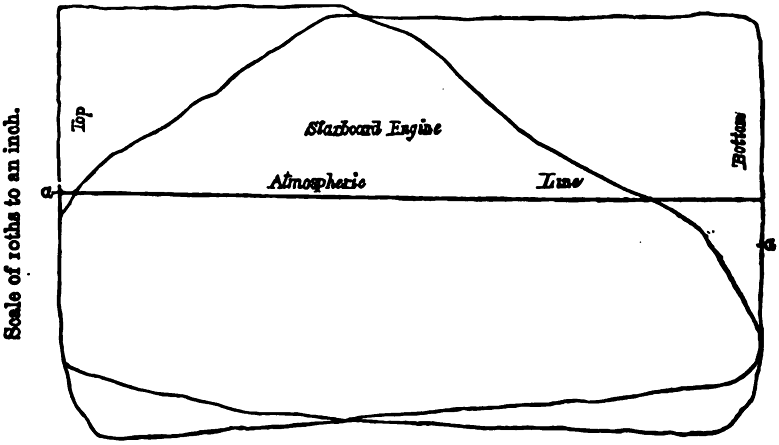
The diameter of the L.P. cylinder is 72", length of stroke 45", and the areas of the two cylinders are to each other as 1 to 4. The effective pressure per square inch at the beginning of the stroke is 18 lbs. on the L.P. piston, and 54 lbs. on the H.P. one: what is the pressure per square inch on the surface of the crank-pin, supposing the brasses to be a perfect fit? See Ex. 13, page 405.

10. Upon every square foot of fire-grate surface is burnt 12 lbs. of coal per hour; each lb. of coal evaporates 7·1 lbs. of water; the superheater is 3·1 cubic feet of capacity for every square foot of fire-grate; the volume is 512 times that of the water: what is the average time each portion of steam remains in the superheater? See Ex. 13, page 444.

11. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

From the S.S. *Penelope.*

Fig. 136.



EXAMINATION PAPER—No. XIV.

FOR FIRST-CLASS ENGINEERS.

1. If 28 lbs. of water at 112° Fahrenheit be added to 48 lbs. at 200° Fahrenheit: what will be the resulting temperature?

2. $\dfrac{(24+12)^{\frac{1}{2}}}{(4-2)^{2}}$

3. A bonus of £180 is to be divided between 3 engineers in the proportion of their wages. A's wages amounted to £198, B's £86, and C's to £76: what share of the £180 will each receive? See Ex. 4, page 160.

4. The steam pipe is 12 inches diameter, and goes inside the boiler; the part inside has slots 6 inches long and ½ inch wide: how many slots must there be to make an area equal to double the sectional area of the pipe? And what are the slots for? See page 442.

5. A T headed piston of an oscillating engine whose dimensions are 12 feet 6 inches over all, and 8 inches diameter; the T head is 30 inches long, 10 inches wide, and 5 inches deep: what is the weight of it? And make a sketch. See Ex. 37, page 449.

6. The piston is 48 inches diameter, and the pressure is 29 lbs. per square inch; the length of the crank is 24 inches, and the connecting-rod 8 feet 4 inches: what is the strain per square inch section of the face of the cross-head, the face being 12 inches by 10 inches? Sketch it, showing the position of the greatest strain.

NOTE.—The pressure on the guide has the same proportion to the pressure on the piston as the length of the crank has to the length of the connecting-rod.

7. A safety-valve, 6 inches diameter, has a spiral spring 5 inches in diameter outside, and ⅞ inch thickness of steel: what is the pressure per square inch?

Formula $\frac{8000\,s^3}{d}$; where s is the thickness of steel, and $d =$ diameter of spring from centre of steel to centre of steel.

8. What is the atmospheric pressure in lbs. per square inch in the following diagram, (the weather barometer showing 29 inches), if 2·04 inches of mercury were equal to 1 lb. per square inch?

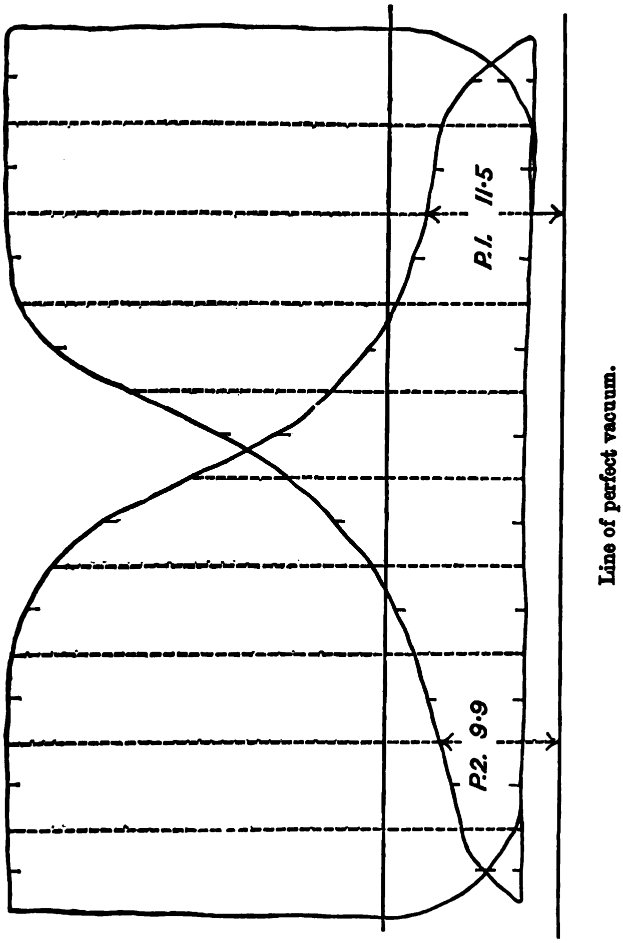


Fig. 137.
Scale of 16ths to an inch.

NOTE.—According to the density of steam at 18 lbs. gross pressure, that is, about 3 lbs. above the atmosphere by the steam gauge, it would take just 35 lbs. weight of steam per hour to produce one I.H.P., if worked without expansion, and also without back pressure or other loss. At higher pressures (less than 35 lbs.) steam would produce the same result; but there are several considerations which lead to the adoption of 35 lbs. as a convenient standard in estimating approximately the cost of evaporation and of H.P. from indicator diagrams.

9. Draw the line of perfect vacuum on the diagram, and measure off the pressure denoted by P_1 and P_2, viz., the gross pressure at $\frac{8}{10}$ of the stroke from the beginning for both strokes, P_1 being the down, and P_2 the up stroke, and determine the tons of steam for 24 hours as given by the card.

Rule. $\frac{(P_1 + P_2)\, d^2\, l\, r}{140000}$ = tons of steam for 24 hours. See pages 453-454.

10. What is the I.H.P of the engine, diameter of cylinder 66 inches, length of stroke 3 ft. 6 in., and revolutions per minute 60; the scale being 16 lbs. = 1 inch?

11. And what is the least consumption per I.H.P. per hour which would give 10 lbs. of evaporation for 1 lb. of coal? See Ex. 61, page 454.

EXAMINATION PAPER—No. XV.

FOR FIRST-CLASS ENGINEERS.

1. A ballast donkey discharges a tank in 3¼ hours alone, and the boiler donkey in 8½ hours alone: how long will they take to do it working together? See pages 236-237.

2. A sling chain when lifting a 5 ton weight forms a triangle with two sides of 7½ feet, the base is 12½ feet, and height of triangle 4·14 feet: what strain is there upon each half of the sling? See Ex. 1, page 411.

3. A packing ring of a cylinder is 73¼ inches diameter, before being cut was 75½ inches: how much must be cut out of the circumference to make it fit the cylinder? See Ex. 39, page 449.

4. The S.S. *Candidate* left port on the 1st instant on a voyage of 3120 miles with a stock of coals of 800 tons; distances run each day 250, 255, 270, 290, 280, 285, 270, and 265 miles; coals consumed each day 58, 60, 62, 65, 62, 65, 60, and 58 tons.

Make out a tabular statement, and say whether there is a sufficiency of coal to reach her port of destination. See Ex., page 217.

5. A lead wire ·072″ diameter after testing the brasses is flattened to ¾″ broad: how many thicknesses will make one inch? See Ex. 47, page 450.

6. For steam above 12 lbs. pressure above the atmosphere, the weight of steam that will escape through an orifice one square inch in area is, in 70 seconds, just equal to the pounds of the gross pressure of steam per square inch: to what breadth of opening must a 6 inch safety-valve rise to allow the escape of 10500 lbs. of steam per hour, the pressure carried being 70 lbs? See Exs. 1-2, page 255.

7. A pair of blocks and ropes are used to hoist a weight of 2 tons, the rope is fastened to the top block, and there are 3 sheaves in each block: what is the strain upon the single of the rope, neglecting friction? See pages 438-439.

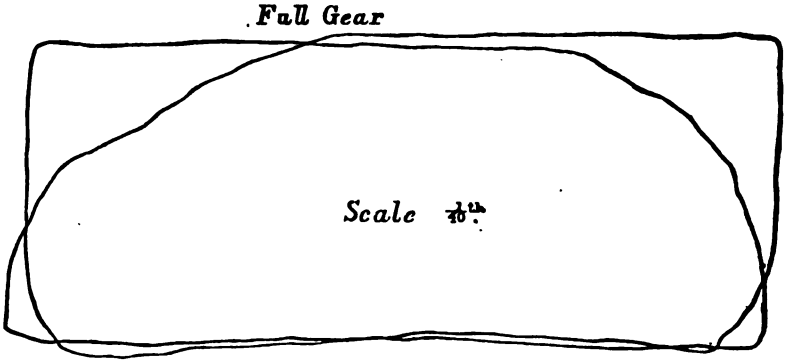
8. In the last question, supposing 10 per cent. to be lost at each sheave by friction: what will be the pull at the free end of the rope when raising the weight? See pages 438-439.

9. If the compression of a spring that loads a safety-valve be 2 inches, and the pressure to which the valve is loaded 65 lbs. per square inch: what will be the load per square inch on the valve when it is lifted to give an area of escape equal to ¼ of the area of the valve? The face of the valve is flat. The valve is 6 inches diameter. See Exs., page 256.

10. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

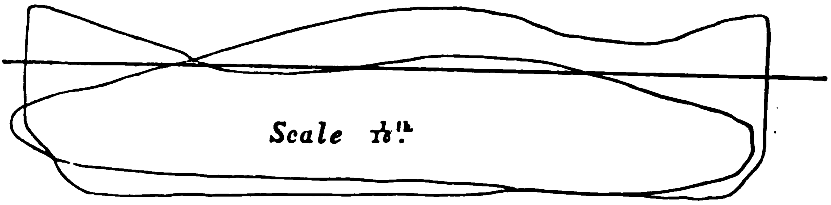
High pressure cylinder.—Fig. 138.

Mean pressure 56·2 lbs.
I.H.P. 443·6.



Low pressure cylinder.—Fig. 139.

Mean pressure 11·4 lbs.
I.H.P. 360.



EXAMINATION PAPER.—XVI.

FOR FIRST-CLASS ENGINEERS.

1. $(221 - 140)\frac{1}{3} + 7 \times 3$.

2. What is the sum of $\frac{8}{21}$ and $\frac{7}{18}$.

3. $\frac{7}{18}$ of 1 — 12th part of 72.

4. Some repairs took 9 men 9 hours per day for 9 days, and cost £20 3s. 9d.; if 10 men had been employed for 10 hours per day: how many days would they have taken? and what would have been the daily pay per man suppose the same total sum were paid for the job? See pages 152-157.

5. A tank 67 ft. long, 23 ft. 6 in. broad, and 4 ft. 5 in. high: how long will it take to pump it out by a double-acting donkey, the diameter of whose piston is 8 in., stroke 11 in., revolutions per minute 71, if 15 per cent. is non-efficient? See Ex. 42, page 450.

6. A condenser door is 5 ft. 10 in. by 4 ft. 9 in.; there are 21 bolts one way and 18 the other: find the number of bolts, and the pitch, if the bolts be 1¼ in. from the edge of the plate. See Ex. 34, page 448.

7. What is the pressure per square inch, when the temperature is 80° Fahrenheit, and the barometer shows 29½ in.?

Rule. $\frac{4907 \times h}{9057 \times t}$ See Ex. 7, page 396.

8. If the pressure gauge be 50 lbs., and the area of water level in the boiler be 120 square feet: how long would it take to lower it 6 in. through a ¾ in. rivet being blown out of the ash pit? See Ex. 17, page 444.

9. If it take 800 lbs. at a leverage of 12 in. to break a stud 1 in. diameter: what force will be necessary to break a ¾ in. stud at a leverage of 18 in.? See Ex. 1, page 412.

10. What is the weight of a cast iron ball of 8 inches diameter, if ·257 lbs. be equal to 1 cubic inch? See Rule LXXXVII, page 191.

11. Diameter of high pressure piston 32 inches, stroke 33 inches, and steam carried for ½ the stroke; there are 1200 cubic feet of steam capacity in boiler: how many strokes will be required to draw off that amount? See Ex. 43, page 450.

12. If 0·4536 kilograms be equal to 1 lb. English, and if you pay £9 10s. 4d. for 200 kilograms of tallow: how much will that be per lb.?

13. Work out the following indicator card, from a compound screw engine, and show whether the slide-valve is correct or requires alteration, and if the latter, what? Also determine the indicated horse-power for each cylinder.

Diagram taken from a compound engine.

Fig. 140.

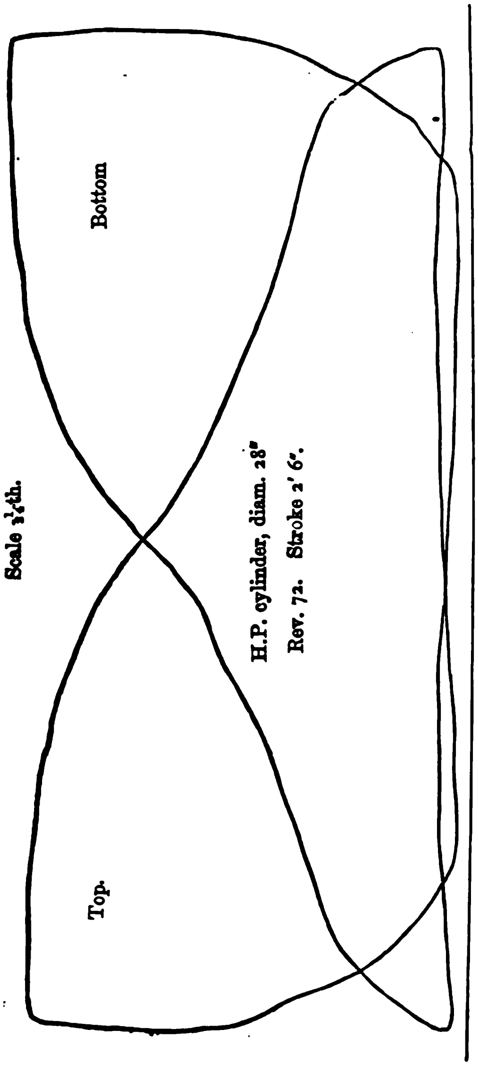
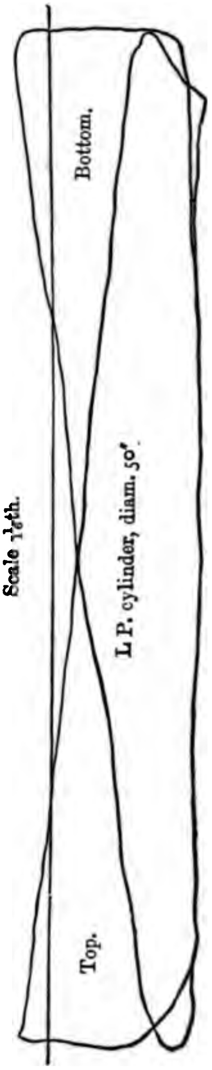


Fig. 141.



EXAMINATION PAPER.—No. XVII.

FOR FIRST-CLASS ENGINEERS.

1. $\sqrt{30+34}-(3+3)$.

2. What is the radius of a circle whose area is equal to that of a rectangle whose sides are 7963·7 and 674·3? Rule LXXVII, page 185.

3. Five boilers with 4 furnaces in each; there are 2 lengths of bars, and 17 bars in each length, each bar weighing 42 lbs.: what would a full set cost at 7s. 6d. per cwt.?

4. A screw-shaft 26 feet over all and 13 inches in diameter; the coupling flanges are 27 inches diameter and 4 inches thick, there are 6 holes in each coupling 3½ inches diameter: what is the weight of the shaft? See Ex. 1, page 265.

5. An engine counter has 6 counting wheels, and it stood at ·007436 at the commencement of the voyage, and you know that it has gone twice round, it now stands at ·000624: how many knots has the ship gone, the pitch being 21 feet?

6. A coal bunker is 32 feet at the top, 29 feet in the middle, and 24 feet at the bottom, it is 30 feet long and 20 feet high: how many tons will it hold at 45 cubic feet to the ton? Ex. 3, page 203.

7. In altering the pitch of a screw-propeller it is found that the mean effective pressure of the indicator diagram is not altered, the product of the pitch by the square of the knots per hour is not altered.

Question.—If the pitch is altered as under, what will be the speed of the vessel, the diagram remaining the same? Ex. 50, page 451.

Old pitch 14 feet, speed 8 knots, new pitch 13 feet.

8. The pressure forcing a slide-valve upon a cylinder face depends a great deal upon the state of the surface; but in a common slide-valve it is never far from equal to the difference between the pressure per square inch in the exhaust passages, calculated for an area equal to the sum of the area of the port in the valve added to the area of the steam port.

Question.—The port of the slide-valve is 9 inches by 28 inches, the port of the cylinder 1¾ inches by 28 inches, the steam gauge showed 36 lbs., and the back pressure is 4 lbs.: find the pressure forcing the slide upon the cylinder. Ex. 28, page 447.

9. There are 3 cross bunkers in a ship, the coals in the centre one do not alter the trim of the ship, but the coals in the other two will, unless properly proportioned between them; the forward bunker is 34 feet from its centre to the centre of the middle one, while the after bunker is 76 feet from the same centre; you have to put 400 tons of coal in these two bunkers so that the ship may keep her trim: how many tons must go in each?

10. A pair of common engines of 180 N.H.P. If 30 circular inches be equal to one N.H.P., what is the diameter of the cylinder?

11. A paddle-wheel which is 30½ feet from centre of float makes 20 revolutions per minute, the ship making 15 knots per hour: what is the amount of slip? See page 271.

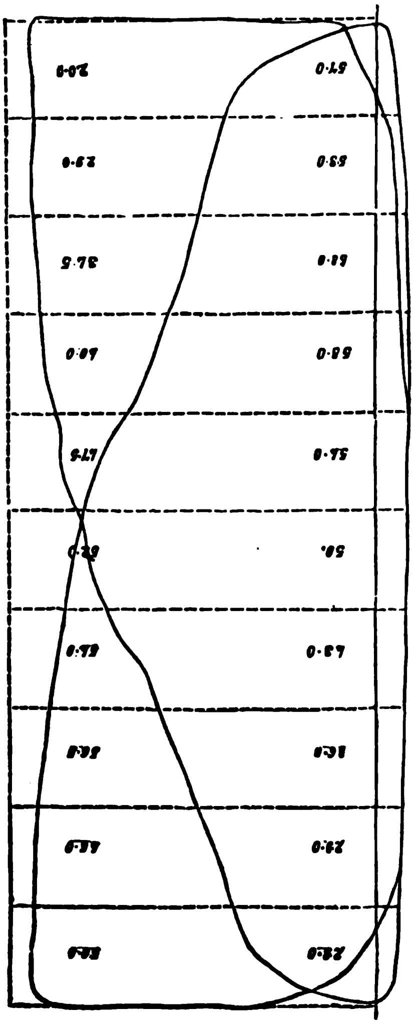
12. A steamer over the measured mile is 5 minutes 10 seconds with the tide, and 7 minutes 10 seconds against the tide: what was her correct rate of steaming?

13. Work out an indicator card, and show the point where the steam is cut off; whether the slide-valve is correct or requires alteration, and if the latter, what? Also determine the indicated horse-power for the high and low pressure cylinders respectively.

S.S. *Barrasford.* Oct. 15th, 1870.

Common Slide.

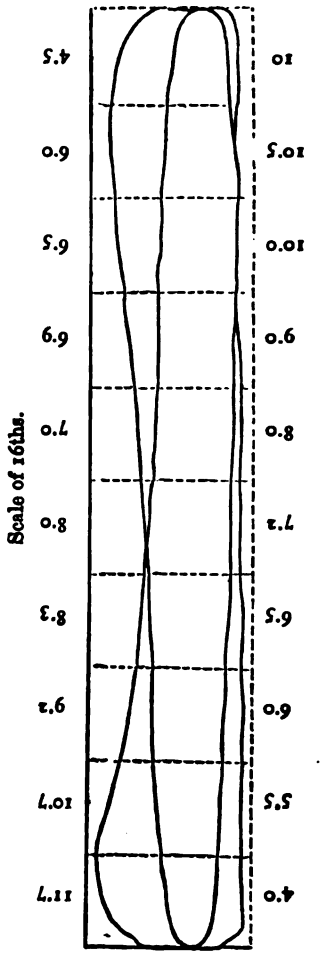
Fig. 142.



Steam 59 lbs.; vacuum 27½; revolutions 58; cylinders $\frac{28}{56}$; stroke 2 feet 6 inches.
Scale 30ths.

Common Slide.

Fig. 143.



EXAMINATION PAPER—No. XVIII.

FOR FIRST-CLASS ENGINEERS.

1. $\frac{(23 \cdot 7 - 7)^3}{40 - 36} + 36 \times 9.$

2. If 8000 lbs. is the strain allowed on plate iron, what would be the pressure on a boiler 12 feet in diameter, double rivetted, ⅞ inch rivets and 2⅜ inch pitch, thickness of plate ¾ inch, crossings not to be taken into account? See Ex. 7, page 443.

3. Upon every square foot of fire-grate is burnt 16 lbs. of coal per hour, each lb. of coal evaporates 8 lbs. of water, the superheater has 3·5 cubic feet of capacity for every square foot of fire-grate; the volume is 381 times that of water; what is the average time each portion of steam remains in the superheater? See Ex. 13, page 444.

4. The diameter of a piston-rod at the smallest part is $\frac{1}{11}$ of diameter of piston: what is the tensile strain on the piston-rod, the pressure being 40 lbs. per square inch, and the back pressure 3·4 lbs. per square inch, the diameter of the body of the rod being $\frac{1}{11}$ that of the cylinder? See Ex. 10, page 404.

5. The cable tank of a telegraph steamer is 21 feet 7 inches diameter and 21 feet 3 inches high; it is to be used as a coal bunker, but half the bottom for 3 feet is covered with gear: what quantity of coal will it contain, and how long will it last at 45 tons per day?

6. Slide-valve worked by a hooped spindle, the port of valve is 14 inches wide, the exhaust port 4 inches deep, steam port 2¼ inches, bars 1¼ inch, internal cover $\frac{1}{16}$ inch, the valve is 1⅛ inch on each side of the ports and has 2 inches lap, the face of valve 1 inch thick, and the rest ¾ inch thick; the port of the valve is rectangular, 3 inches deep: find the weight? 3·9 cubic inches equal 1 lb. See No. 273, page 268.

7. The diameter of a cylinder is 60 inches, stroke 40 inches, diameter of shaft 9 inches: find the effective pressure on the piston per square inch.

$$\text{Tunnel shaft } \frac{3200\,d^3}{S} = \text{pressure on piston.}$$

8. Sea water is 50°, and the water in the hot-well is 114° temperature: if the sea water rises to 83° and you cannot put on more injection: what is now the temperature of the hot-well? See Ex. 3, page 295.

9. What is the horse-power when the diameter of cylinder is 77 inches, stroke 36 inches, cut off at 10 inches, and revolutions 74 per minute, the steam gauge showing 68 lbs., and the back pressure 5·8 lbs.? See Ex. 3, page 290.

P is the gross pressure at the beginning of the stroke.

R is the number of times the steam is expanded.

$$\text{Then } P \times \left(\frac{18 - R}{40} + \frac{\cdot 85}{R}\right) = \text{mean pressure.}$$

10. Two side bunkers, each 6 feet 9 inches wide and 14 feet 6 inches high, and to contain 70 tons of coal: what should their length be? See page 203.

11. The piston is 75 inches diameter, and at the beginning of the stroke the pressure is 38 lbs. per square inch: if the crank-pin is 12 inches diameter, what length should it be in the bearing if the pressure on the oil is not to exceed 700 lbs. per square inch? See Ex. 12, page 405.

12. On starting on a voyage the consumption was 11 tons, the temperature of the funnel was 531°, but at the end of a few days the temperature was found to be 750°: what was the cause of this, and what was the increased cost of consumption? See Ex. 19, page 398.

$$\frac{F}{2200} = \text{increase; where } F = \text{diff. of temp.} \times \text{the original consumption.}$$

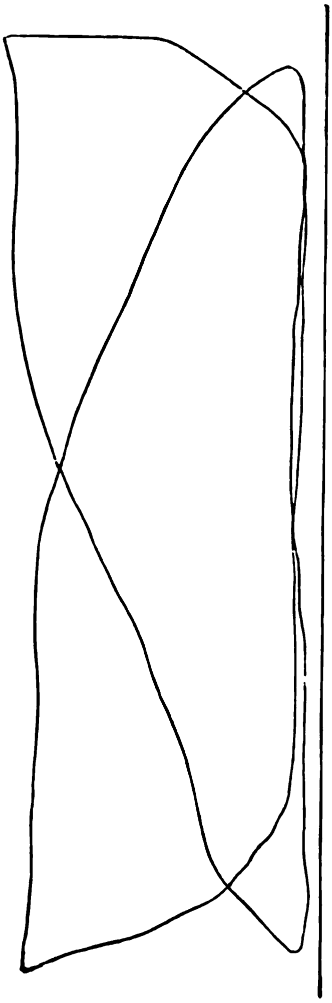
13. A shaft 29 feet 6 inches long over all, diameter of shaft 11 inches, there are 2 flanges each 3 inches thick and 20 inches diameter, there are 5 holes through each coupling 2½ inches diameter: find the weight and give a sketch of it.

14. The diameter of high pressure piston is 38 inches, stroke 36 inches, steam is carried for ½ stroke, there are 1400 cubic feet of steam capacity in the boilers: how many strokes will it require to draw that amount off? See Ex. 43, page 450.

15. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what? Also determine the total estimated horse-power.

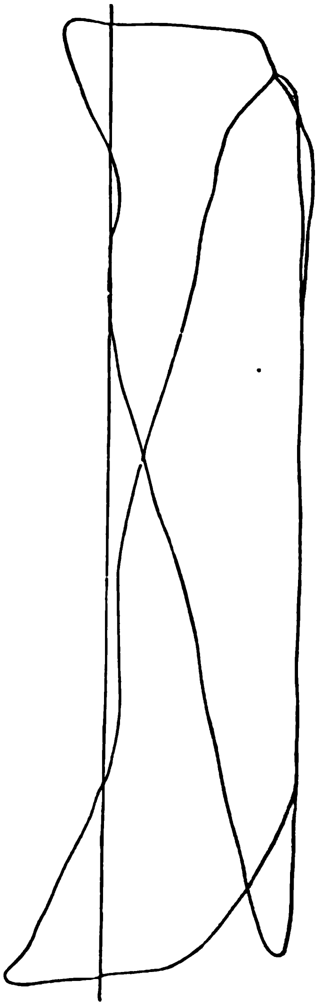
S.S. *J. C. Stevenson*. Voyage to Port Said. Jan. 5th, 1872.

Fig. 144.



Steam 65 lbs.; steam chest 8 lbs.; vacuum 27½; revolutions 66; wheel 3 turns back; scale $\frac{1}{16}$th.

Fig. 145.



Scale $\frac{1}{16}$.

EXAMINATION PAPER—No. XIX.

FOR FIRST-CLASS ENGINEERS.

1. ·675 is to what number as 4 to 9.

2. Find the value of £20·59.

3. Express the sum of ·525 and ·625 as a vulgar fraction.

4. $(340 - 16)^{\frac{1}{2}} + 2 \times 6$.

5. A compound engine has a steam receiver round the high pressure cylinder equal in diameter to that of the lower pressure cylinder, and the steam chest for slide-valves is between the cylinders; distance between the centres of cylinders 84 in., diameter of L.P. cylinder is 50 in., and depth 63 in.: how many square feet of felting will be required to cover it? See Ex. 22, page 446.

6. The topmast of a ship is 71 feet above the level of the sea when upright, but when fore and aft sails are set she heels over till the topmast is only 62 feet above the sea level; the diameter of the valve is 4 in. and is loaded to 480 lbs.: what will she blow off at when heeling over at this angle? See Ex. 2, page 257.

7. The piston is 93 in. diameter, effective pressure of the steam 45 lbs., length of the crank 21 in., and diameter of shaft 16 in.: is the shaft strong enough? and what is the strain per square inch of section?

Formula $S = \frac{5 \cdot 1\, W a}{d^3}$

S = strain per square inch section of shaft.
W = weight at leverage (= a).
d = diameter of shaft.

8. There are 3 boilers with 3 furnaces each; the diameter of each fire-grate is 3 ft. 3 in.: how many tons of coal will be consumed per day, and how many H.P. will be indicated?

9. An engine of 826 I.H.P. uses 21 lbs. of steam per I.H.P. per hour; one air-pump, single-acting, 20 in. diameter, 19 in. stroke, and making 50 revolutions per minute: how many inches of the stroke will be full?

10. The steam gauge shows 45 lbs., and is carried for 14 in. of the stroke, the length of the stroke being 30 in., and the mean effective pressure 40 lbs.: what is the co-efficient of efficiency?

$$e = \frac{\text{mean pressure} \times \text{length of stroke}}{\text{gross pressure} \times (\text{length of admission} + \text{clearance})}$$

11. If the loss by blowing off is ·12 of the fuel used, the temperature of the steam 320°, and of the feed 110°: what should the boiler water be at above the feed density?

$$n = \frac{T - t}{t\,(1113^\circ + 3\,t - T} + 1. \quad \text{See Ex. 20, page 398.}$$

12. If 10 lbs. of water at 80° Fahrenheit be added to 100 lbs. at 212° Fahrenheit: what will be the resulting temperature? See Ex. 5, page 396.

13. If 29 circular inches = 1 N.H.P., what is the mean effective pressure per square inch required to indicate 3·8 times the N.H.P., if the piston runs 400 feet per minute?

14. A rectangular bar is 6 inches deep, 3 inches thick, and is broken by a weight of 3200 lbs. placed 35 inches from the point of fracture: what was the strain per square inch at the broken part?

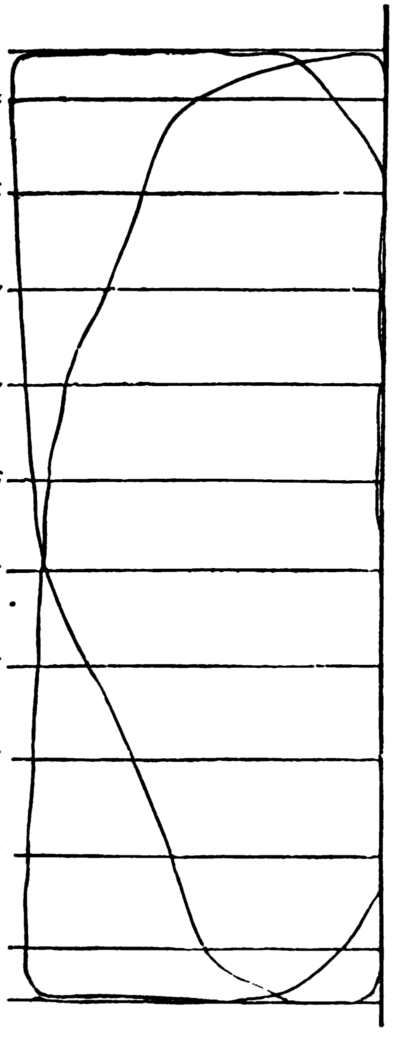
15. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

S.S. *Rothbury.*

Steam 60 lbs.; vacuum 25½; revolutions 56.

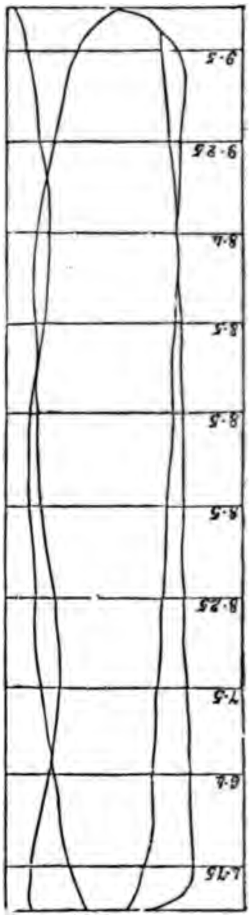
Scale $\frac{1}{80}$th.

Fig. 146.



Outridge's Patent Slide.—Full Gear.

Fig. 147.



Common Slide.

16. The boiler contains 60 tons of feed water when starting on a voyage. The water in the hot-well through some leak is ·15 of the saltness of the sea: how many tons of water must have been evaporated during the voyage, the water in the boiler at the end of the voyage being 2·2 times the saltness of the sea? Ex. 15, page 444.

EXAMINATION PAPER—No. XX.

FOR FIRST-CLASS ENGINEERS.

1. If 454 tons of Welsh coal in the bunkers is sufficient for a voyage of 2900 miles, and Lancashire coal is for space in the proportion of 46 : 40, and is inferior in quality as 8·7 : 10·5; what distance will the bunkers carry her if filled with Lancashire coal?

2. What pressure is allowed on a plate in the combustion chamber when the thickness of plate is $\frac{7}{16}''$, and the stays are 16″ apart?

Formula. $B = \frac{60\,(T+1)^2}{S'-6}$ Where B = boiler pressure.
T = thickness of plate in 16ths.
S = surface.

3. And what would be the smallest diameter of stay in this case?

4. What degree of Fahrenheit will correspond to 129° Centigrade, Centigrade being marked 0 at freezing and 100° at boiling point?

5. Three years ago wages were £14 per month, and were increased by 11 per cent.; it is now proposed to reduce them by 14 per cent.: what will be the new rate of wages?

6. The water in the hot-well is 112°, and the vacuum in the condenser is 11½ lbs., but by some accident the temperature of the water in the hot-well rises to 154°: what will the vacuum be?

Rule. $\frac{(T-t)\,(T-50)\,(t-50)}{100000}$ = loss of vacuum.

7. If, when the engines are making 41 revolutions, the mean effective pressure of the steam is 27 lbs.: how many revolutions will the engine make when the vacuum shows 1½ lbs. less with the same cut off of the steam?

8. The square of the diameter of the funnel in feet is equal to the whole width of the fire-grate; the fire-bars are 5 feet 9 inches long: how many square feet of fire-grate surface are there to 1 square foot of funnel section?

9. Make a sketch of half a thrust-block from the following, and find its weight, when 1 cubic inch = ·3 of a pound.

There are 9 collars each 1½″ thick, with 1″ space between them; diameter of shaft 9″, of collars 11″, of seat 13″, and of flanges 13″, and the flanges are 1½″ thick each. See No. 271, page 267.

10. What is the value of ⅜ of ⅝ of 63?

11. A beam of 11 cwt. rests on two supports A and B, 14 feet apart; a weight of 4 tons is placed 7 feet from B, and another of 7 tons 6 feet from A: what share of these weights will A and B each support?

12. Find the thermal units of heat in an engine of the following dimensions: diameter of cylinder 70″, stroke 36″, revolutions per minute 60, mean pressure 38 lbs.; and find the coals burnt per hour, and how much per I.H.P. per day?

NOTE.—1 lb. of good coal gives 8000 thermal units of heat in the furnace, but by the time it gets to the piston it equals about 800 thermal units.

13. The connecting-rod is adjusted by a screwed cutter, the screw has 13 threads to the inch, the taper of the cutter is ⅝″ to the foot, the nut is 6 canted: required the number of turns and cants of the nut to bring the brasses ·01″ closer.

14. When a safety-valve has the area required by the Board of Trade, and is spring loaded, the increase of pressure due to the increased compression of the spring in blowing off steam is, with bright fires and engines stopped, about equal to the quotient found by dividing the diameter of the valve by its original compression.

999

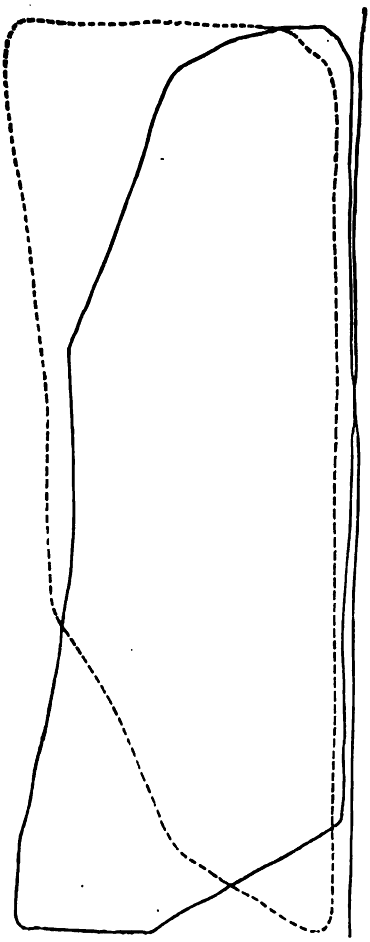
Question.—The spring has 6′ compression, there are 50 square feet of fire-grate to one valve, the loaded pressure is 50 lbs., the increase, irrespective of the spring, would be, say, 10 per cent.: to what pressure would the steam rise, including the effect of the spring?

15. The piston has 2827·44 square inches of area: find the length of a shoe 15 inches broad, allowing 10 square inches of piston to 1 square inch of valve.

16. There are two cylinders 40 inches and 76 inches diameter, 275 N.H.P.: how many circular inches is that per N.H.P., and what is the ratio of one to the other?

17. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

Fig. 148.

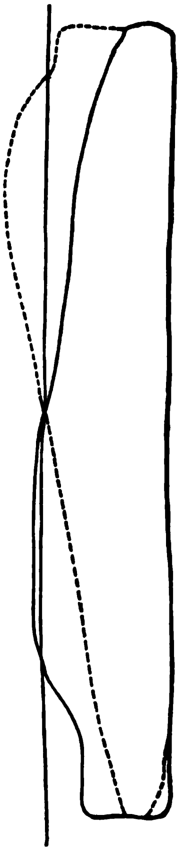


S.S. *H*———*n*. March, 1873.

Steam gauge 54 lbs.; vacuum 26 in.; revolutions 62; coals per measure 12 tons per day.

Scale $\frac{1}{16}$.

Fig. 149.



Scale $\frac{1}{11}$.

EXAMINATION PAPER—No. XXI.

FOR FIRST-CLASS ENGINEERS.

1. $\frac{(24 + 12)^{\frac{1}{2}}}{(4 - 2)^2}$ $(16\frac{1}{2} - 14\frac{1}{4}) + 1\frac{1}{8} \times 3 = 4\frac{7}{8}$.

2. A bonus of £150 is to be divided amongst 3 engineers in the proportion of their wages; A's wages amount to £120, B's to £90, C's to £30. See Ex. 4, page 150.

3. A steam pipe is 11½ in. diameter, and goes inside the boiler; the part inside the boiler has slots 5¼ in. long and ¼ in. wide: how many slots must there be to make an area equal to double the sectional area of pipe? See Ex. 3, page 442.

4. Find the weight of the rim of a fly-wheel whose outer diameter is 9 feet 6 inches, and inner diameter 7 feet 5 inches, and 7 inches thick, if ·257 of a circular inch = 1 lb.

5. The piston is 45 inches diameter, and the pressure is 30 lbs. per square inch; the length of crank is 21 in., and the connecting-rod is 7 ft. 6 in.: what is the strain per square inch of section of the face of the cross-head, the face being 11 in. by 9 in.? Sketch the position of the greatest strain.

NOTE.—The pressure on the guide is the same proportion to that on the piston as the length of the crank is to that of the connecting-rod.

6. A safety-valve 6 inches diameter has a spiral spring 5 inches diameter outside, and ⅞ inch thickness of steel: what is the pressure per square inch?

Rule. $\frac{8000 s^3}{d}$ Where s is the thickness of steel, and d is the diameter of the spring from centre of steel to centre of steel.

7. Find the H.P. exerted by a pump in pumping out a tank 21 feet by 15 feet by 4 feet; the top of the tank is 16 feet below the level of the sea, the loss through non-efficiency being 40 per cent.; a cubic foot of water weighs 64 lbs., and 31000 lbs. of water raised 1 foot high in 1 hour equals 1 H.P.; the tank was pumped out in 5 hours. See Ex. 41, page 450.

8. the stays of a boiler are 15 in. apart, and one of them breaks, thus throwing more strain on the 4 stays that surround the broken one, they are 1¼ in. diameter, and the pressure carried is 60 lbs. per square inch: what extra strain per square inch will be on the stays, allowing ¼ more than each had before? See Ex. 8, page 443.

9. Formula. $\frac{6\cdot55\, d^3}{L} = D^2$ Where d = diameter of crank shaft.
L = length of stroke in feet.
D = diameter of cylinder.

10. A crank-shaft is 13 in. diameter, the length of stroke is 5 ft. 9 in., the pressure about 30 lbs.: what diameter of cylinder will suit that size crank-shaft according to the above formula? See Ex. 23, page 446.

11. A rectangular seat for a brass 12 in. wide, shaft 10 in. diameter, flanges are 14 in. square, and 1⅛ in. thick, the depth of seat is 13 in., and the seat is 13 in. long: what is the weight of the two half brasses at ·3 lb. per cubic inch? See Ex. 36, page 449.

12. If $S = \frac{60\,(T+1)^2}{B} + 6.$ Where S = surface held up by one stay.
T = thickness of plate in sixteenths.
B = boiler pressure.

How far apart should the stays be pitched, when the pressure is 48 lbs. per square inch, and the plate $\frac{7}{16}$ in. thick? See Ex. 9, page 443.

13. And what is the least diameter of one of the stays? See Ex. 10, page 443.

14. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

S.S. ———. June 19th, 1871.

Diameter of H.P. cylinder 36".
" L.P. " 68".
Length of stroke 3' 0".

Revolutions 54; steam 57 lbs.; vacuum 25½.

Fig. 150.

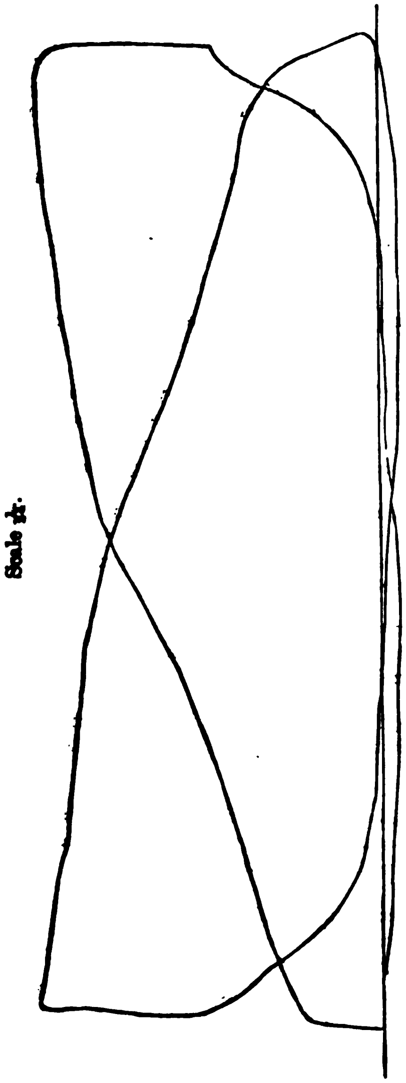
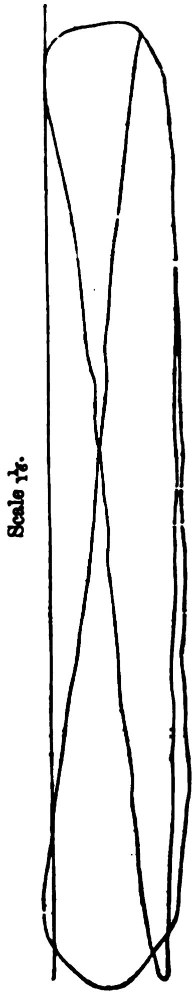


Fig. 151.



EXAMINATION PAPER—No. XXII.

FOR FIRST-CLASS ENGINEERS.

1. $\dfrac{(104{\cdot}4 - 4)^3 - 11 \times 6}{37 - 23}$

2. What is the length of a thrust block with 9 collars in the block, each 1⅛ inch thick, if the spaces between them be 1 inch?

3. What will a bill for the following amount to, if 17 per cent. be allowed for discount?

2 cwt. 3 qrs. 2 lbs. at 32s. per cwt.
2 tons 15 cwt. at £1 per ton.

4. A boiler has an area at the water level of 142 square feet, and a donkey can pump 10 inches into the glass per hour; the donkey is 3 inches diameter, 6 inches stroke, and making 120 revolutions per minute, single-acting: find the efficient stroke of the pump. See p. 240.

5. A piston-rod is 4½ inches diameter at the bottom of thread, there are two bolts at the cross-head each 2¾ inches diameter: what proportion does the section of the rod bear to the section of the bolt?

6. What is the corresponding temperature on the Fahrenheit scale to 146° on the Centigrade scale?

7. An air-pump lever has a weight of 5 tons at a distance of 1 foot 4 inches: what would be the weight on the centre bearing when the other arm is 3 feet 1 inch?

8. Sketch a crank for a paddle-shaft of the following dimensions, and give its weight in lbs.:

Pin 6 inches diameter, boss 11½ inches diameter, and length of boss 7¾ inches, shaft 9 inches diameter, boss 15 inches diameter, and length of boss 9¼ inches, bar at pin-boss 9¾ inches broad, at shaft boss 12 inches broad, and 4½ inches thick.

Add ⅛ of diameter of each boss to the length of bar for the fillets, the distance from centre of pin to centre of shaft being 26 inches.

9. Find the boiler pressure, the diameter of the two cylinders being 64 inches and 41 inches, length of stroke 32 inches, and diameter of crank 11 inches.

Rule. $B = \dfrac{f\,d^3 - 15\,S\,D^2}{S\,H^2}$ Where $f = 4936$ for a crank-shaft, and 5760 for a tunnel shaft, B = boiler pressure, S = length of stroke in inches, D = diameter of low pressure cylinder, and H = diameter of high pressure cylinder.

10. The speed of the ship is 12·7 knots per hour, and the consumption 29 tons per day; but the speed at another time is 13·5 knots per hour, and consumption 34 tons per day: what is the rate of cost per day saved?

Rule. $\dfrac{C_, K - C K_,}{K_, - K}$

11. Verify the above formula by working at full length under the two conditions the coal saved on a voyage of 3174 miles.

12. A Whitworth thread on a bolt of 1¼ inch diameter has 1200 lbs. strain per square inch of section: find the actual strain on the bolt.

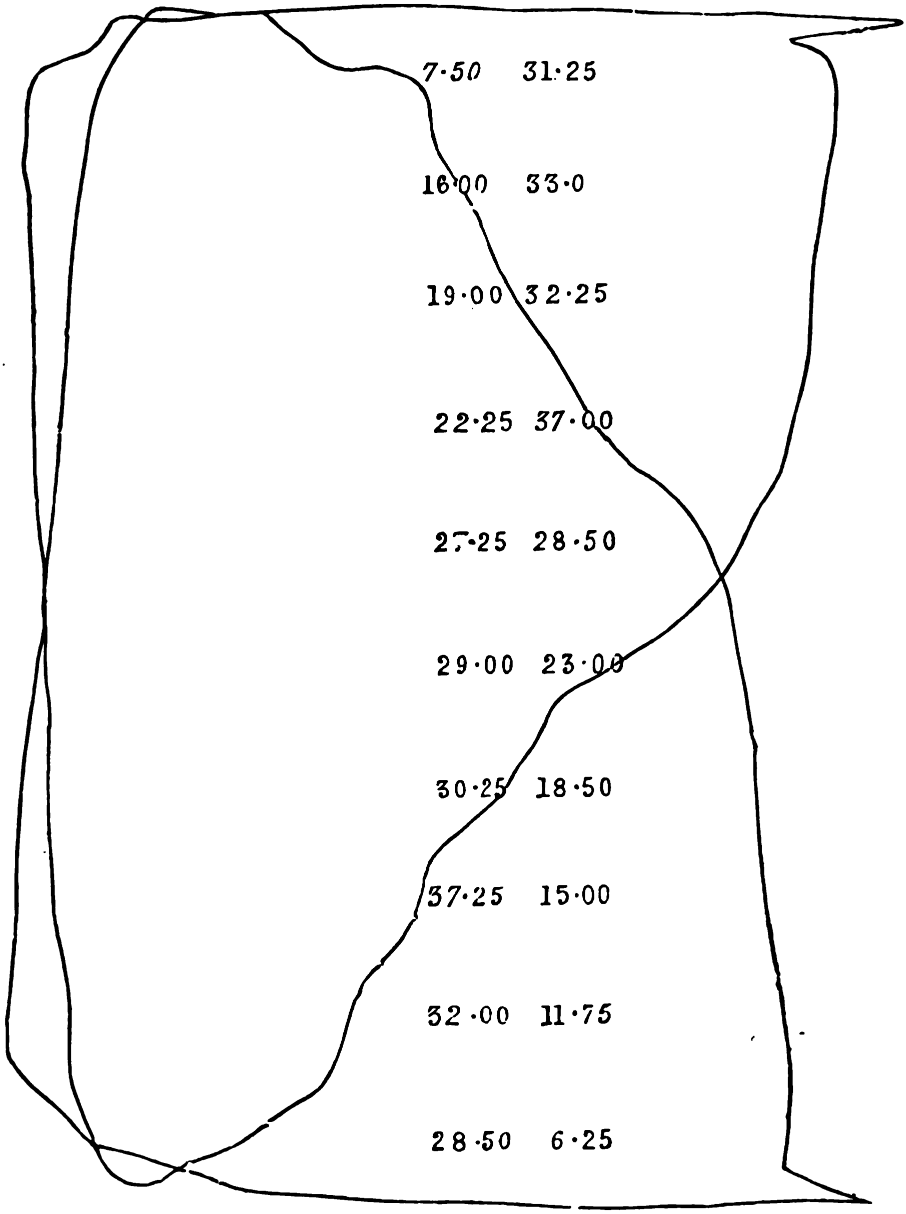
Rule.—Multiply the number of 8ths in the diameter by the number of 8ths less 1, and divide the product by 100; this quotient multiplied by the given strain per square inch is the answer.

13. The feed water is 4·2 oz. of salt to the gallon, and the density of the water in the boiler is not to exceed 9·5 oz.: what proportion of the feed water requires to be blown off?

14. Work out the following indicator card, and show the point where the steam is cut off, whether the slide-valve is correct or requires alteration, and if the latter, what?

Fig. 152.

Scale of 8ths.



Official trial of R.M.S.S. *China*, February 4th, 1862. Steam 21 lbs., vacuum 27 and 27½ inches, revolutions 30 per minute, I.H.P. 2389·5. (Average card).

EXAMINATION PAPER—No. XXIII.

FOR FIRST-CLASS ENGINEERS.

1. $(7+4)^3-(16-4)^2$.

2. $\frac{3}{8}$ of $\frac{1}{16}$ of 34.

3. £69 2s. 8d. ÷ £14 2s. 2d. Give the times it will go and the remainder in £ s. d.

4. The temperature of the hot-well was 114°, and the evaporation for 1 lb. of coal was 8·3 lbs. of water: what would the evaporation be if the temperature of the hot-well were increased to 156°. See Ex. 15, page 397.

$$\text{Increase} = \frac{F}{1100}$$

5. A stay is fastened by a double eye to an angle iron; the bolt is 1 in. diameter, the plate is $\frac{1}{2}$ in. thick: how much space must be left between the hole and the edge of the plate?

6. What is the radius of a circle whose area is 64?

7. Sketch a slide-valve to be worked by a hooped spindle, the hoop surrounding the body of the valve, the port in the valve is 14 in. by 7 in., the breadth of the face at each end is 3 in., and at each side $1\frac{1}{2}$ in., the recess is 4 in. deep, rectangular, the thickness of face is 1 in., and the thickness of the rest of the valves $\frac{3}{4}$ in.: what is the weight of the valve allowing 3·9 cubic inches of cast iron to weigh 1 lb.?

8. In a wheel and axle the barrel is 9 in. diameter, the rope is $1\frac{1}{4}$ in. diameter, the crank handle is 14 in. radius, and the weight to be lifted is 300 lbs.: what is the force to be applied to the handle, adding 10 per cent. for friction?

9. A bonus of £140 is to be divided amount 3 engineers in proportion to the amounts drawn as pay by each man; the first drew pay of £201, the second of £341, and tho third of £185: what amount of the £140 does each receive?

10. How many bolts $1\frac{1}{4}$ in. diameter are required for the cover of a cylinder 40 in. diameter, the boiler pressure being 60 lbs., and the strain per square inch on the section of the bolts is not to exceed 1800 lbs.?

11. When a safety-valve has the area required by the Board of Trade, and is spring loaded, the increase of pressure due to the increased compression of the spring in blowing off steam is, with bright fires and engines stopped, about equal to the quotient found by dividing the diameter of the valve by its original compression.

Question.—The spring has 5 in. compression, there are 70 square feet of fire-grate surface to one valve, the loaded pressure is 70 lbs., the increase, irrespective of the spring, would be, say 10 per cent.: to what pressure would the steam rise, including the effect of the spring?

12. The actual H.P. transmitted in the tunnel shaft is 400: what is the mean shearing strain per square inch on the bolts, 5 of them at each coupling, $2\frac{1}{2}$ inches diameter and $12\frac{1}{2}$ inches radius, 60 revolutions per minute?

13. Find the thermal units of heat in an engine of the following dimensions: diameter of cylinder 64 inches, stroke 3 feet, revolutions per minute 60, mean pressure 35 lbs., and find the coals burnt per hour.

14. The seat of a rectangular brass is 12 inches wide, depth 13 inches, and 13 inches long, the shaft is 10 inches diameter, and the flanges are 14 inches square and $1\frac{1}{2}$ inch thick: what is the weight of the brass at ·3 lb. per cubic inch?

15. The boiler contains 28 tons of fresh water on starting on a voyage; the water in the hot-well, through some leak, is ·2 of the saltness of the sea: how many tons of water have been evaporated during the voyage, the water in the boiler at the end of the voyage being 3·12 times the saltness of the sea?

EXAMINATION PAPER—No. XXIV.

FOR FIRST-CLASS ENGINEERS.

1. $24 - \sqrt{16} - (9 + 11)^2$.

2. If 1 ton stretch a bar of iron 1 inch in 19000, what will be the stretch of a bar 20 feet long with a weight of 8 tons?

3. A balance weight of a crank weighs 1400 lbs.; the effective radius is 1·5 feet, and making 90 revolutions per minute: what distance must the bolts be to hold it, there being 2 bolts, and the strain is not to exceed 5000 lbs. per square inch of their section?

Rule. $\frac{R^2 d}{5870}$ = number of times the weight is increased.

4. Molesworth, in the 17th edition of his "Pocket Book," page 258, says, "The feed water should be about ·07 cubic feet, or nearly a half gallon per N.H.P. per minute for condensing stationary engines."

If 1 lb. of coal evaporates 8 lbs. of water: how many lbs. of coal will be required to evaporate the above allowance per N.H.P.? Also, if the consumption of coal per N.H.P. be 10 lbs., what is the weight of feed water used as steam per minute?

5. An engine was compounded, and the same expenditure of fuel gives 20 per cent. more power than on the old principle; the engine works to the same H.P.: how many tons of coal will it now use, if before 12 tons per day were used?

6. The counter stood at 00734, since which time it has been three times at 0, and now stands at 21211: how many knots has the ship made?

7. According to the Board of Trade instructions, circular furnaces with longitudinal joints, welded or made with a butt and strap:—

$$\frac{90000 \times t^2}{(t+1) \times d} = \text{the working pressure per square inch.}$$

Without the Board's special approval of the plan the pressure is in no case to exceed

$$\frac{8000 \times t'}{d'}$$

Find the working pressure when the diameter is 36″, length of tubes 6′ 4″, and ⅝″ thick.

8. For arithmetical convenience, reckoning the total heat of steam of T° temperature to be equal to 1115° + ·3 T° (the temperature of the feed) is the number of units of heat to each lb. of water.

Example.—What is the total heat and what is the latent heat of steam of a temperature of 300°.

9. If a certain quantity of fuel evaporates 8 times its weight of water from and at 212°, how much will 1 lb. of coal evaporate if the steam is 300° and the feed water 110°?

10. The consumption of coal is 28 tons, and each watch of 4 hours throws overboard 36 baskets of ash, each weighing 48 lbs.: what per centage of coal is lost as ash?

11. A counter at starting stood at 918376 and has been round once and stands at 043721; the ship has been steaming from the 16th of January at noon to the 22nd at 5 A.M.: what has been the number of revolutions.

12. $88 - \frac{220 \times t}{np + 3t^2}$ = per centage of strength of longitudinal seams of a boiler.

Where t = thickness of plate, p = pitch of rivets, and n = number of rows of rivets.

Question.—The thickness of plate = 1″, pitch 2″, the number of rows 2, single shear: what is the per centage of strength?

13. The thrust block takes only 32 per cent. of I.H.P.; the I.H.P. is 500, revolutions per minute 80, pitch 20 feet, and the slip 20 per cent.: what weight is against the thrust block,

EXAMINATION PAPER—No. XXV.

FOR FIRST-CLASS ENGINEERS.

1. Find the N.H.P. of the engines you were last with and the maximum strain on the crank-pin.

Example.—Say the diameter of the cylinders are 32″ and 63″, steam 60 lbs., and thence the diameter of crank-pin will be, say 12″, and length of same 18″.

2. A ship carries 1800 tons of coal as cargo with a speed of 8·4 knots per hour on a consumption of 11 tons per day; the ship is lengthened and fitted with compound engines and now carries 2200 tons at the same speed and with a consumption of 10·8 tons: what quantity of coal is saved on a voyage of 7000 miles?

3. A portion of air is heated up to 39° and 14·7 lbs. pressure; if it be afterwards heated up to 79°: what is then its pressure?

Note.—461° below zero of Fahrenheit is absolute zero.

4. The stays of a boiler bottom are to be fixed by collars, the depth of collars being 3 times the thickness, the diameter of stays 1⅞″: what size must the end be swelled out to to have uniform strength?

Rule. $D = 1 + \left(\frac{\cdot 08}{n} + \frac{\cdot 4}{\sqrt{n}}\right) d.$ Where $n = 4.$
$d = 1\tfrac{7}{8}.$

5. An engine is constructed so as to have 28 circular inches of cylinder per N.H.P. and is to indicate 4 times the N.H.P., to have 42″ stroke, and to have 20 lbs. mean effective pressure of steam: how many revolutions must the engine make to complete the contract?

6. If the counter advance from 081376 to 418397 in 84 hours: what will it stand at after 360 hours?

7. If a continuous blow off at ·125 of the feed, and if the boiler water retains 11 ounces of salt to the gallon: what saltness has the feed been at?

Rule.—Multiply the ounces of salt by the blow off.

8. The area of water-level in the boiler is 2·1 times the area of fire grate; there is consumed 13 lbs. of coal on each square foot of fire grate per hour; each pound of coal evaporates 8 lbs. of water. The feed is shut off when there are 4 inches of water in the glass, in how many minutes will the feed be out of the glass?

9. A shaft 12″ diameter is put under torsion strain by a weight of 500 lbs. acting with a leverage of 68″: find the maximum stress per square inch of section.

10. The stroke of the piston is 46″, the length of the connecting-rod 8 feet; the steam was cut off at 22″ from the bottom: required the distance from the cross-head to the centre of the shaft. And the lap and lead being the same top and bottom, where does she cut off from the top?

11. How far can the visible horizon be seen from a height of 26 feet?

Rule.—Multiply the height of the eye by 1·5, and extract the square root of the product.

12. The temperature of the gases in the combustion chamber was 1200° Fahrenheit, and the front plate 600° Fah., the velocity at the back was 1200 feet per minute: what was the velocity at the front?

Note.—Absolute zero is (taken as) 461° below zero of Fahrenheit.

13. The ship's draught of water before taking in her bunker coal was 17 feet forward and 18½ feet aft, and it was found that every 16 tons put her down one inch; after taking in her bunker coal her trim was and now is 18½ feet forward and 19¾ feet aft: find how many tons were put in the bunkers, and the area of section of ship at the water line in square feet?

EXAMINATION PAPER—No. XXVI.

FOR FIRST-CLASS ENGINEERS.

1. 1256·64 square inches are added to $1314\frac{1}{16}$ circular inches: find the diameter of a circle which would have the same area as their sum.

2. If the heat required to melt 1 lb. of ice is sufficient to raise the temperature of 144 lbs. of water 1°, and if 42 lbs. of ice be mixed with 120 lbs. of water at 212°: what will be the resulting temperature?

3. A circular safety-valve weighs 84 lbs., and a cubic inch of the metal weighs ·257 of a lb.: what is the diameter of the weight when its thickness is $1\frac{1}{4}$ inches?

4. Supposing there are 16 of these weights on a valve of 5 inches diameter: what is the pressure per square inch? And if the bottom of the weights be 2″ above the valve, what extra pressure will there be, supposing the box to be full of water to the top of the weight?

5. If 9200 lbs. be expended on a thrust, the mean radius of the rubbing surface of which is 5″, the co-efficient of friction being ·09, and the engines working 43 revolutions per minute: what is the loss with this friction in I.H.P.?

6. The surface of a screw propeller being oblique to the direction of the thrust, the area of screw disc is less effective for thrust than an equal area of feathering float immersion with the same ship.

$A = \left(·7854 - \frac{P}{5D}\right)(D^2 - B^2)$; where A is effective area of screw disc in square feet, P = pitch, D = diameter, and B = diameter of boss.

Question.—The diameter of the screw propeller is 16 feet, and that of the boss 3·2 feet; the pitch is 22 feet: what is the equivalent perpendicular area of feathering paddle float?

7. What is the thrust in lbs. when the ship is making 14 knots per hour with 64 revolutions per minute?

$T = 5·67 \times K \times k \times$ equivalent equal area in square feet; where T is the thrust in lbs.; K = knots per hour; k = slip in knots per hour.

8. According to the above statement, what is the loss of work by slip in actual H.P.?

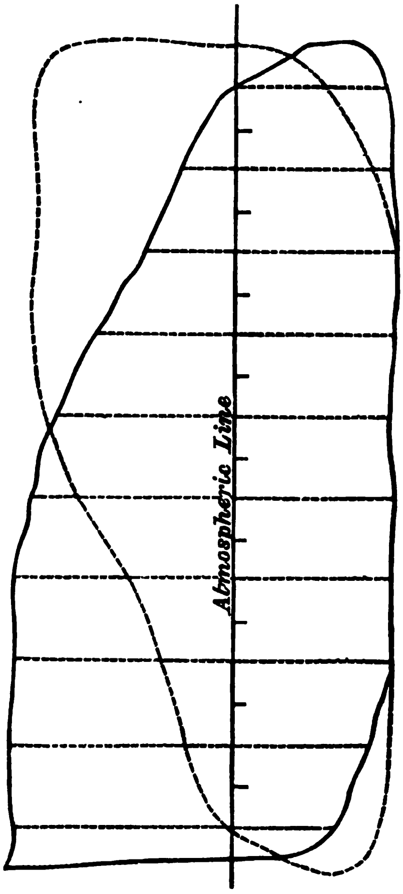
9. Practically, we find that $38\frac{1}{2}$ per cent. of the I.H.P is lost by slip: what was the I.H.P. in the above engine?

10. If a boiler uses 1800 lbs. of coal per hour, and each lb. of coal evaporates 8 lbs. of water (using fresh water): what is the smallest diameter in inches of the nozzle of an injector to put in double feed at 60 lbs. pressure?

Rule. *Smallest area in square inches* $= \dfrac{\text{cubic feet per hour of feed}}{800\sqrt{P}}$

Or, smallest area in circular inches $= \dfrac{\text{cubic feet per hour of feed}}{630\sqrt{P}}$

11. Work out the following indicator card, and show the point where the steam is cut off; whether the slide-valve is correct or requires alteration, and if the latter, what?



U.S.S. *Shenandoah.* May 14th, 1872.

Engines designed by Isherwood.

Steam 20 lbs.; Revolutions 45; Vacuum 27. Scale 16 lbs.' = 1 inch.

In this diagram the point of cut off is not sharply defined, the line being *convex upwards* for a considerable distance after the pressure begins to lessen. This peculiarity is chiefly due to the wire-drawing action of the slide-valve, worked by the eccentric and link, as it gradually closes the port of the cylinder and stops the admission of steam. The cylinders are not jacketed, but the steam is superheated, with the object of preventing liquifaction.

EXAMINATION PAPER—No. XXVII.

FOR FIRST-CLASS ENGINEERS.

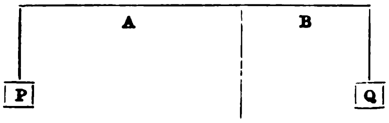
1. Find the value of $(14 - {\cdot}07)^3 - (10 - {\cdot}003)^2$.

2. A paddle float is 6′ 4½″ by 2′ 1″ broad: how many square feet and decimals of a foot?

3. Babbitt's metal is composed of 10 parts of tin to 1 of antimony and 1 of copper: how many lbs. of the metal can be made out of 82 lbs. of tin, 16 lbs. of antimony, and 24 lbs. of copper?

4. Suppose antimony to be ¾d. per lb., tin to be 1s. 3d. per lb., and copper to be 7d. per lb.: what would be the value of 28 lbs. of Babbitt's metal?

5. A beam as in the sketch is exactly balanced.



Let A = long arm, and B = the short arm.

P = weight when W is on A, and Q when weight is on B, and W = correct weight (all in lbs.)

$W \times A = P \times B$, and also $W \times B = QA$.
Hence $W^2 = AP = QP \times AB$.

$$W^2 = QP$$
$$\text{and } W = \sqrt{QP}$$

Question.—The weight P = 296 lbs., and Q = 336 lbs.: what is the correct weight?

6. Steam is carried for 22″ of the stroke, and the clearance is 1½″; the gauge is 60 lbs.: if the steam of one stroke were condensed, what would be the thickness of the layer of water in 16ths on the top of the piston?

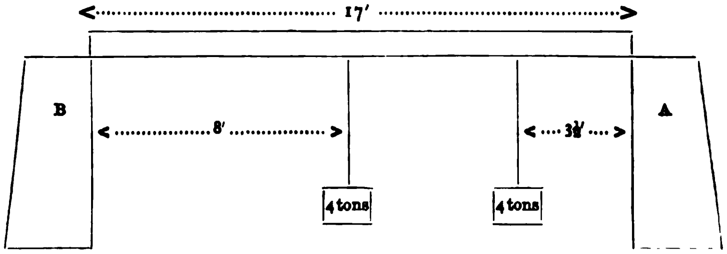
$$\frac{P + 1600}{P + 1} = \text{the proportion of steam to water.}$$

7. The inside diameter of the condenser tubes is ⅝″, there are 2½ square feet of surface per I.H.P., the tubes being 6 feet long; there are 800 lbs. of water per I.H.P., and it circulates three times through the tubes: find the velocity of its passage.

Rule. $V = \dfrac{L\,T\,P}{80\,P^2\,S}$

8. Sea water is 4½ oz. to the gallon, the hot-well ·18 oz. to the gallon: how much water is leaking through condenser tubes and supplementary feed, if any?

9. A beam, 17 feet long and weighing 17 cwt., is supported at both ends; the weight 3½ feet from A is 4 tons, and 8 feet from B is 4 tons: what share of these weights has A and B?



10. The safe working strain of a rope is $\frac{1}{24}$ of the girth squared, and 75 per cent. is allowed off this girth squared for the breaking strain in tons: what should be the least girth of a rope to lift $\frac{4}{10}$ of a ton, and also what would be the breaking strain?

EXAMINATION PAPER—No. XXVIII.

FOR FIRST-CLASS ENGINEERS.

1. The thermometer scale, according to the Centigrade division, reads 0° at freezing point, and 100° at boiling point: what is the temperature, Fahrenheit, corresponding to 172° Centigrade?

2. What is the following equal to?

$$(6 + 2)^3 - (16 - 12)^2$$

3. What is the $\frac{1}{32}$ of $\frac{1}{12}$ part of 186?

4. Three years ago the pay in a certain company's service was increased 18 per cent., this year the same company made a reduction 14 per cent. on all wages; if the pay of one of the engineers was £17 before the first alteration, what is it now?

5. If the height of the eye is 40 feet, what is the distance of the visible horizon?

NOTE.—The square of the distance of the visible horizon in statute miles is equal to $1\frac{1}{2}$ times the height of the eye in feet.

6. At the mean draughts referred to in this question, 14 tons puts the vessel down 1 inch; before coaling the draught of water was 18·2 feet forward and 19·1 feet aft., after coaling the draught was 19·8 feet forward and 19·4 feet aft.: how many tons have been taken in? also, what is the area of the water-level section of the hull?

7. A round shaft 9 inches in diameter is subjected to torsion; a weight, 18000 lbs., acts at the end of an arm 42 inches in length: what is the maximum intensity of stress produced?

8. A pair of common trunk engines, stroke 42 inches, diameter of cylinder 66 inches, diameter of trunks at each end of cylinder 32 inches, making 50 revolutions per minute, shows on the indicator diagrams the mean effective pressure to be 22 lbs. per square inch, and uses 23 lbs. of steam per horse-power per hour. Each pound of coal produces $8\frac{1}{2}$ lbs. of that steam when the surface condenser is in perfect order, and without scumming. How many tons of coal are burnt in 24 hours?

9. Referring to Question No. 8, the surface condenser gets out of order, and jet condensation and scumming have to be resorted to. The temperature of the feed is 110° Fahr., and the temperature of the steam is 312° Fahr., the density of the water in the boiler is maintained at 1·8 times the density of the feed water: what will be now the rate of consumption of fuel if the power be maintained the same as before?

NOTE.—In comparing differences in quantities of heat in steam or in water in questions about the steam engine, it is generally sufficient to write the heat units in water as numerically equal to the temperature, Fahr., and the heat units in steam as numerically equal to 1115° + ·3 times the temperature, Fahr.

10. Referring to the preceding questions on this leaf, what per centage more feed must be supplied, working with the jet condenser, and at that density.

11. Referring to the preceding question on this leaf, work out the consumption of coal per indicated horse-power per hour under each of the two sets of conditions, and then find what per centage less of fuel is required with perfect surface condensation as compared with the jet condensation, as above.

Exn. 17a.

EXAMINATION IN ROUGH WORKING DRAWING

FOR A

FIRST-CLASS ENGINEER'S CERTIFICATE OF COMPETENCY.

The regulations of the Board of Trade in regard to the qualifications of a Candidate for a First-class Engineer's Certificate of Competency specify that,—

"He must be able to make rough working drawings of the different parts of the engines and boilers.

"He must be able to state the general proportions borne by the principal parts of the machinery to each other."

In accordance with these clauses, a Candidate for a First-class Certificate is required to make a rough working drawing of the parts specified on the other side of this leaf. A mechanic who has been some years in charge of marine engines and boilers ought by this time to have familiarly in his mind the general construction of at least one set of engines and boilers, say that set he was last with. Fine drawing is not expected, and in the proportions of the parts a wide margin will be allowed; absurd dimensions will be failure in practical knowledge.

The drawing must, however, be practically a working drawing, giving a sufficient number of views to show the parts fully—sections, plans, or elevations, just as the Candidate would require to be supplied to him if he had to make the parts to the design of another person.

A clear hand sketch showing the construction completely, and fully dimensioned, will be accepted if the Candidate prefers this alternative.

A portion only of the parts specified may be accepted in place of the whole, if that portion is sufficient to show that the Candidate has a good practical idea of the construction of the parts, and a fair notion of their general proportions or dimensions.

Candidates are hereby cautioned not to put on paper what they have not fully condsidered, and deliberately intend to be understood, as their statement of what they know about the construction of any part required.

The statements given in by a Candidate may be in themselves, apparently, of little importance, but, as sample material from which the state of the Candidate's knowledge of engines and boilers is to be inferred, every detail which is glaringly inconsistent with a sound knowledge of the use of the part, or in which an essential consideration has evidently been overlooked, is an important element in the description which the Candidate is giving of his own qualifications.

The Candidate is advised not to begin more than he can clearly finish in the time allowed. An important object in this part of the examination is to ascertain whether the Candidate can be trusted to mark all necessary dimensions upon a sketch or a drawing. The test of this is, practically, the making of the part from the sketch without having to supply additional dimensions, and without measuring the drawing. To prove this ability the Candidate must fully dimension the parts shown in his sketch or drawing, notwithstanding that the parts may be correctly drawn to scale. A drawing is fully dimensioned when no part of it is left to the option of the party who is to work to the drawing.

To prevent misunderstanding, however, when the Candidate has been led into showing more of the details than he has time fully to finish, he should name, in the statement on the other side, the particular parts which he has fully dimensioned.

All dimensions should have lines and darts, to indicate distinctly the points between which the dimensions are given.

Beware of writing cross dimensions upon centre lines, or upon longitudinal dimension lines. This is not an order, but a recommendation.

The Candidate is not expected to design anything; he has merely to sketch or draw a something with which he is expected to be already familiar.

Pencil in nothing after half-past three; all the dimensions, the figures, and the darts must be inked in; employ the remaining time in examining the drawing and in inking in any figures that may have been before overlooked, and in checking the dimensions.

Make sure that you will have sufficient room on the drawing sheet to show all the necessary views. You can have another sheet of drawing paper if necessary. All the paper used must be forwarded with the drawing.

Fill in and sign the statement on the other side of this leaf.

Exn. 17a.

SUBJECT FOR EXAMINATION IN ROUGH WORKING DRAWING.

☞ *Read the general instructions on the other side of this leaf.*

A common slide-valve with its spindle. Show also an outline section of the ports at the cylinder face. Show the provision for connecting the slide-valve to the spindle.

The Candidate is requested to fill up the following, and to attach this paper to his drawing.

STATEMENT BY THE CANDIDATE.

The accompanying drawing, made by me this day without referring to any document, and without the assistance of any person, is intended by me to be sufficient for the new construction of the parts above described to fit the places of similar parts which are to be removed. The construction is similar to what I have been with in the steamer , but the dimensions may be different.

The diameter of the cylinder is

The stroke of the piston is

The stroke of the valve is

The cover at top end on steam side is

The cover at bottom end on steam side is

The lead at top is intended to be

The lead at bottom is intended to be

The inside cover is + or —

The thickness of the face of valve is

The thickness of the body of valve is

The greatest opening for steam will be

That gives an area equal to th of piston

The opening for exhaust when the crank is on the top centre is

That gives an area equal to th of piston

The Candidate may omit this part if he chooses.

The length of the connecting-rod is

The valve will cut off steam on the down stroke at

The valve will cut off steam on the up stroke at

The parts fully dimensioned, in ink, are

Dated at this day of 18 .

Applicant.

Exn. 17a.

SUBJECT FOR EXAMINATION IN ROUGH WORKING DRAWING.

☞ *Read the general instructions on the other side of this leaf.*

An air-pump cover, with its packing gland, and the rod, and its nuts or cotters, complete.

The candidate is requested to fill up the following, and to attach this paper to his drawing.

STATEMENT BY THE CANDIDATE.

The accompanying drawing, made by me this day without referring to any document, and without the assistance of any person, is intended by me to be sufficient for the new construction of the parts above described to fit the places of similar parts which are to be removed. The construction is similar to what I have been with in the
steamer , but the dimensions may be different.

The diameter of the bucket is

The top of air-pump above the barrel is made of

The diameter of that top part, at mouth, is

The cover is held truly central by the

The rod is made of

The diameter of the body of the rod is the one th of the diameter of the pump.

The diameter of the screwed part at bottom of thread the one th of the diameter of the pump.

The pull on this section will, therefore, be on every square inch times what the effective load is per square inch of the bucket.

There are two kinds of air-pump rods in general use; their respective advantages or disadvantages are the following:—

The parts fully dimensioned, in ink, are

Dated at this day of 18 .

Applicant.

ELEMENTARY QUESTIONS.

1.—The parts of an engine generally made of **wrought iron** are all those parts which are subjected to great strains or sudden shocks, such as the crank-shaft and crank-pin, cross-head, propeller and paddle-shafts, connecting-rods, piston-rods, slide-rods, eccentric-rods, links, levers, draw-bars, weigh-shaft, bolts, studs, and nuts.

2.—The parts of an engine generally made of **cast iron** are the parts requiring great rigidness, such as the bed-plate, the cylinder and its covers, pistons, packing-rings, junk-rings (when not of wrought iron), pillars, condenser, guide-bars, steam-chest and steam covers, slides, and pumps.

3.—The parts of an engine for which **steel** is sometimes used are the working parts liable to much wear, such as piston-rods, slide-rods, pins on single cranks, piston springs, rod, and eye (sometimes).

4.—The parts of an engine generally made of **brass or gun metal** are all rubbing surfaces liable to great friction or corrosion, such as main bearings, top and bottom ends of connecting-rods, feed and bilge-pump plungers, air and circulating pump rods with buckets, barrels, and head and foot valve guards, feed and bilge-pump valves and seats, tubes and tube plates of surface condensers, bushes in piston and slide-rod glands, all the steam valves and cocks.

5.—**White metal** is principally used for the stern tube and outer paddle bearings, sometimes for the main bearing brasses and connecting-rod bottom end brasses. It is adopted on account of its anti-frictional properties (due probably to the zinc in its composition). It would be more generally used but it is very brittle, melts at a low temperature, and is apt to come out (if it gets hot it will run out).

6.—**Muntz Metal** is sometimes used for those parts where bolts, studs, and nuts are brought under the action of salt water or steam, such as air-pump rods, for holding bolts, and bolts inside of condensers, for fixing cast iron faces to cylinders, and it is valued for its ductility and freedom from corrosion. It is also malleable, and can be readily forged into any shape.

7.*—**Cast iron, wrought iron, and steel** have only two essential ingredients, viz., pure iron and carbon. **Cast or pig iron** is composed of about 91 per cent. of iron, 5 of carbon, 2 of silicon, and the remaining 2 parts of sulphur, phosphorus, and other impurities in different proportions. The best **malleable**

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* This question is very clumsily put. There is no actual difference in the component parts of cast iron, wrought iron, and steel; in fact there are no component parts, as they are all homogeneous, the only difference being in the difference of treatment.

iron is composed of 99·5 per cent. of iron, 0·035 of carbon, 0·076 of silicon, and the rest of sulphur and phosphorus. Steel is composed of 98·6 of iron, and 1·4 of carbon. The hardness of steel varies with its proportion of carbon. The steel poorest in carbon is least hard, and the steel richest approaches white pig iron; but its other properties—strength, fusibility, weldability, &c., are then modified.[*]

8.—**Cast iron, wrought iron, and steel** can be distinguished from each other, in the first place, by the difference in the fineness of the grain; wrought iron being finer in the grain than cast, and steel finer than wrought. Again: cast iron is short and brittle, wrought iron is fibrous, whilst steel is void of the fibre so characteristic of wrought iron, and, like cast iron, is a granulated or crystallized metal.

9.—**Steel and cast iron** possess the property of fusibility; **wrought iron** is malleable, ductile, tough, fibrous, and possesses the quality of **welding** in the highest degree; **steel** also is both malleable and weldable.[†] From this it may be seen that steel possesses properties in common with both wrought and cast iron. Further: steel is characterized by its softness at a glowing heat, and its becoming hard upon sudden cooling. Cast iron resists a greater crushing strain than malleable iron; wrought iron resists great torsion and tensile strains, while steel can resist a great tensile strain, nearly twice that of wrought iron, and seven times that of cast iron.

10.—By the term "**breaking strain**" is meant that strain which will break or produce fracture in a girder, beam, or bar, or whatever the subject may be; by "**proof strain**" is meant the greatest strain of a specific kind consistent with safety, that is, with the retention of the strength of the material unimpaired; and by "**safe working strain,**" the strain which may with perfect safety be imposed upon it.*

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[*] Cast iron, wrought iron, and steel have only two essential ingredients, viz., pure iron and carbon; indeed, upon the degree of carburization of iron, modified by the presence of other elements, as sulphur, phosphorus, silicon, and manganese, depends their classification into cast iron, steel, and malleable iron, with the widely different qualities possessed by these bodies.

In pig or cast iron, iron attains its maximum degree of carburization, the amount of carbon varying from 2 per cent. to 5 per cent., the lower figure being just beyond the limit (about 1·8 per cent.) of hard cast steel. Carbon may exist in pig iron, either in a state of chemical combination, or as flakes or crystals of graphite, distributed in an uncombined state through the mass of iron, or partly combined and partly graphitic; under these conditions we have various grades of *white*, *grey*, and *mottled* pig iron respectively.

Iron in its minimum state of carburization is *designated* as *malleable*, *bar*, or *wrought iron;* it then possesses a colour varying with its quality, and the mechanical treatment it has undergone, from bluish to blackish shades of grey.

Malleable iron usually contains from 0·02 per cent. to 0·25 or 0·3 per cent. of carbon, but iron containing the latter amount of carbon is essentially steely in its character, 0·2 or 0·25 per cent. being the more usual limit of carburization of malleable iron, which is, however, influenced by its freedom from other ingredients, especially silicon, sulphur, and phosphorus; the freer the iron is from these elements, the larger is the proportion of carbon that can be combined with it without producing the steely quality.

The material called steel is a nearly pure alloy of iron with a small portion of carbon, or it is a combination of iron with from 0·3 per cent. to 0·8 per cent. of carbon, which is cast in a fluid state into a malleable ingot, and has a *bluish* colour, with a tinge of brown. In cast steel it is usually considered that the *bluer* the shade in colour the better the metal; when hardened it is whiter than in its soft condition.

[†] When red hot, steel is nearly as malleable as bar iron, and may be welded, but very careful management is required to prevent its becoming decarbonized.

11.—**The cohesive strength,** or breaking strain of wrought iron is, on the average, about 47,000 lbs., or 21 tons per square inch.†

12.—**Steel is tempered** by heating it until it has attained a cherry redness, then plunge the parts to be tempered suddenly into the liquid (water or oil, as the case may be) and withdraw it almost immediately, and watch the point until it has attained the required colour, and then immediately cool it out. **The colours come in the following order,** commencing with the colour for the hardest description ("dead hard") of steel—very pale yellow, pale straw, full yellow, brown, brown dappled with purple spots, purple, light blue, full blue, dark blue. On slowly cooling, these temper-colours appear in reverse order, and the steel is plunged into the liquid as soon as the proper colour becomes apparent.

13.—**Case-hardening** is a process by which the surface of articles made of wrought iron are superficially coated with a skin or layer of steel. Case-hardening, in its chemical nature, is simply the addition of carbon to the surface of wrought iron, thus converting it into the character of steel. The necessary carburization is effected after the article has received its finished form and dimensions, by heating it to a bright redness, and if a small article, sprinkling powdered yellow prussiate of potass over its surface, when the article is returned to the fire for a few minutes, and then cooled by immersion in water; for giving a thicker coating, or for case-hardening larger articles—such bodies are placed in wrought iron boxes, embedded in substances rich in carbon, as fragments of horn, bones, and leather cuttings; the box is then luted up so as to become air-tight, and is placed in a furnace, where it is slowly heated and retained at a cherry red heat for about twelve hours, when the fire is allowed to burn out, the box removed, and its contents thrown into water. **The parts of an engine generally case-hardened** are those small in size, but subject to great friction, such as the working surfaces of the slide link, the slide block, bolts, and eccentric-rod eyes.

14.—**The common metals** that can be **forged** are wrought iron, steel, copper, and Muntz metal. Cast iron, cast steel, brass, white metal, gun metal, zinc, tin, bell metal, &c., are short or brittle.

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* The "*working load*" on each piece of machinery is made less than the proof strain in a certain ratio determined by practical experience, in order to provide for unforseen circumstances.

† "The ultimate tensile strength of wrought iron is usually set down as 25 tons per square inch, but this is above the present general average, 23 tons being nearer as an approximate round number; at the same time, hard, steely wrought iron frequently has a tenacity of 30 tons, and inferior kinds only 19 tons. As a rule, square or round bars are stronger than plate, by at least 3 tons to the square inch, but weaker iron is not inferior for some purposes; such iron frequently has the welding property in a marked degree."—*The Strength of Materials and Structures*, by JOHN ANDERSON, C.E., LL.D.

14a.—In speaking of metals, as gun metal, brass, bronze, &c., why are they designated alloys?

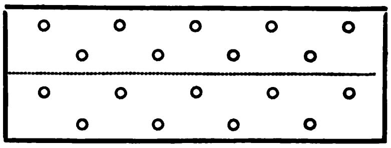
A.—Because these metals are composed of two or more having a chemical affinity for each other, and hence said to be an alloy, or combined mixture.

15.—By **welding** is meant the joining of two pieces of metal by heating the two ends to be joined to a good white heat, placing them together so that one end slightly overlaps the other, and then hammering them till they are completely and solidly one. The common metals that can be welded are wrought iron and steel.

16.—All metals **expand** more or less by the application of heat; for example, if a slide-valve be set up on a spindle when cold, it will, when hot, expand, and there is great danger of the valve spindle breaking; similarly, any valve or cock screwed up moderately tight when cold, will be found immovable when hot. The upright standards of vertical engines are inclined inwards at the top, so that they become parallel to each other when heated; and in the boiler the furnace bars are kept short to allow them to expand without buckling up. In consequence of this property of expansion, any connection between two boilers, such as the steam-pipe, must be fitted so as to allow for expansion and contraction, similarly, between the boilers and the engines; the steam-pipe is fitted with a stuffing-box or other expansion joint.

17.—The **parts** of a boiler usually made of the **Staffordshire** quality of plate are the shell plates, furnace bottoms, and gusset stays; while those parts of the boiler which are exposed to more severe or more irregular strains than the rest, or to a more intense heat, such as flanging, the furnace tops, tube plates, and back tube plates, are made of the **Yorkshire** quality of plate.

18.—**Double rivetting** is rivetting with a double row of rivets on each side of the joint, the rivet holes being generally arranged in a zig-zag fashion as shown in the sketch. All longitudinal seams in both common flat and circular boilers are double rivetted.



19.—**Caulking** is the thorough closing of the seams of a boiler by means of hammering the inner edges of the plates with a caulking tool; the edges being first planed quite smooth and bevelled.

20.—There are various ways of **fastening** the ends of boiler **main stays**, but the method, perhaps, most in vogue is by screwing both ends of the stays (the ends being swelled out so that no strength is lost in screwing), and securing them to the plates by means of nuts and washers inside and out. Another way is by rivetting the flattened out ends of stays to double angle-irons rivetted to the boiler plates. Yet another method is by putting eye-bolts through the plates and securing them on the outside with nuts, the end of the stays being a double eye, which is secured to the single eye of the bolt by means of a pin passed through and fastened with a cotter. Each method has its own particular **advantage** or **disadvantage**. For instance, those that are movable are not all likely to have an equal strain, and from the fact that the pins may not fit exactly, the stays may not all be of the same length; but on the other hand they are easily removed for cleaning or repairing. Screwed stays can be put in so that all of them have about an equal strain, but they cannot be removed for cleaning.

21.—The **greatest strain** to which a boiler stay should be exposed is 5000 lbs. per square inch of section at the smallest part.

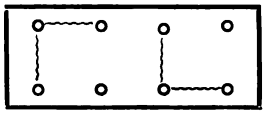
22.—**A rivetted stay** is a stay screwed into the plates, and the ends rivetted over. They are generally used in the narrow water spaces of boilers, between the flame-box sides and back, and the shell of the boiler, also between the furnaces.

23.—**Thin plates** are generally looked for in a boiler as it wears on the furnace tops, flame-box backs, and in those plates generally upon which the flame acts with greatest force. The thinness may be detected by the peculiar sound emitted whilst going over each plate carefully with a hammer.

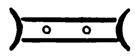
24.—**Boiler tubes** are fixed by driving a tapered mandril into the ends, and thus causing those parts of the tube to expand so as to fill the holes previously prepared for their reception; they are then clinked over after that has been done. **Stay tubes** are ordinary tubes with the ends screwed, one end being screwed into the *back* tube plate, which is tapped to suit, and the other being screwed to the front tube plate by two nuts, one inside and the other outside.

25.—**Boiler tubes** generally leak at the flame-box end. This defect is repaired by re-mandrilling the end of the tube. The leakage is caused by the tube ends burning off or corroding.

26.—The **spaces** between the tubes becoming salted up, the water cannot reach the plate, which consequently gets burnt and cracks. The cracks always take place in a line between the centres of the tubes as shown in the sketch.



These cracks are repaired by what are known as "spectacle eyes," which are pieces of flat iron shaped as shown in the sketch, which are placed between the tubes over the cracks and secured by two tap bolts to the tube plates.



27.—**A dry uptake** is external, going up outside of the boiler, and is generally made of thin plate, say $\frac{1}{4}''$ thick, it being under no pressure. A **wet uptake**, on the other hand, goes up through the boiler, and is internal, and consequently under a collapsing pressure. The wet uptake requires the **more repairing** for the reason just stated, viz., that it has to be both steam and water tight at the working pressure.

28.—**The Superheater** is a vessel attached to the boiler into which the steam is received and superheated. They are sometimes made as a dome rising up within the uptake, and sometimes as a long cylinder, with rounded ends, passing through the uptake. The stop-valve and safety-valves are placed upon it, and sometimes a pressure gauge is attached to it in order that the engineer may be able to compare the pressure in the superheater with that in the boiler. There is **sometimes a glass gauge** to show if priming is going on.

29.—The **parts** of a **marine tubular boiler** first **injured** by shortness of water are the top of the flame-box, or the top row of tubes.

30.—**Angle irons** are generally used in the construction of a boiler for joining the shell and end plates together, and to stay the front and back plates of shell. They are also used as a means of strengthening flat parts. **Flanged plates** are generally used for the end plates, tube plates, and the top of steam chest.

31.—**Priming** is a violent agitation of the water within the boiler, in consequence of which a large quantity of water passes off with the steam in the shape of froth or spray. It **may arise** from a variety of causes; one frequent cause is a smallness of steam space in the boilers, another is the too crowded arrangement of tubes, which hinders the steam from rising freely as it is generated, and so causes the water to rise with it. The too sudden opening of the safety-valve will also cause priming; or, in some cases, the too rapid opening of the throttle-valve to the full. Priming often takes place when the ship is passing from fresh into salt water, and *vice versâ.* Increasing the steam space by adding a receiver, enlarging the boiler, or improving the arrangement of the tubes. At the time of its taking place it may be checked by opening the furnace doors, partially closing the throttle-valve, or injecting melted tallow. The water passing through the steam-pipe into the engines is apt to cause a break down.

32.—**The draught** in the funnel is caused by the rush of heated air away from the furnaces, and the cold air rushing to supply its place, or in other words, draught is caused by the difference of pressure between the heated air within and the atmosphere without. It can be checked by closing the damper placed in the funnel, opening the furnace and smoke-box doors, or placing dampers in front of the ashpits.

33.—**Flame** seen at the **top of the funnel** is the burning of carbonic, oxide, and other gases, which ought to have been consumed in the furnace, but which ignites when it comes in contact with the oxygen of the atmosphere. It is **caused** by the bridges of the furnaces being too low. It is **detrimental,** inasmuch as there is a great waste of heat, and, consequently, great waste of fuel.

34.—A Blast Pipe is a small pipe generally tapered towards the end, fitted with a stop-cock, and leading from the superheater into the funnel. Steam can be blown through it from the superheater. It is used for increasing the draught in the furnaces.

35.—Two **blow-off cocks** are generally fitted to each boiler, one on the boiler, the other in the ship's bottom. They are so fitted to prevent the serious consequences that would ensue if the pipe between the boiler and the ship's bottom, fitted with only *one* cock, was getting damaged; if the single cock were on the boiler, then there would be a rush of sea water into the ship, and if it were on the ship's bottom, then there would be an explosion of steam and scalding water into the stoke-hole.

36.—The Spanner Guard on blow-off cocks is so arranged that the key can only be removed when the cock is shut.

37.—The Test Cocks are three in number. They are placed in a diagonal row on the boiler front. The middle one is placed just on the water-line, or in other words, 9 inches above the top row of tubes; the top one is generally placed about 4 inches above that, and the bottom one 4 inches below. The **cocks themselves** must be at that height. For the **purpose of cleaning** these cocks there is a small brass plug screwed into the cock just at the bend; this plug being unscrewed, a wire can be passed straight through the cock.

38.—A Dead-weight Safety-valve is a valve which has the load placed directly upon it. The **rubbing surfaces** are generally composed of brass. The top of the spindle of a **lock-up** safety-valve is fitted with a cast iron cap with two handles cast on; a flat key passes through both spindle and cap, and the key-way is so arranged that, although the valve can be turned round or lifted up, no extra weight above that which is already in the valve box can be placed upon the valve.

39.—The area of safety-valve required by the Board of Trade is about ½ square inch of valve area per square foot of fire-grate area. The effect of **suddenly opening a safety-valve** when the steam is up is to cause the boilers to prime. A common 6-inch safety-valve, when blowing off, rises without being eased by the hand about one-fifth of an inch, but this varies with the pressure of steam. The amount of lift, as calculated by the Board of Trade, is found by dividing twice the diameter of the valve by the pressure of steam on the gauge.

40.—The Spring-loaded Safety-valve has one important advantage which the dead-weight valve has not, viz., it is constant and equal in its pressure in all positions of the ship at sea. In the case of boilers fitted with the dead-weight safety-valves, when the ship rolls there is so much of the weight upon the valve neutralized, and this increases with the angle of inclination, until, supposing the ship to be completely on her beam ends, there is really no weight on the valve at all.* On the other hand, the spring has its disadvantages, it does not indicate quite so true as the dead-weight valve, because the resistance of the spring tends to increase as the valve lifts.

41.—A Glass water gauge consists of three pieces, viz., two brass sockets fitted with cocks, and secured to the boiler front, and a strong glass tube fitted between the sockets. The two cocks being open to the boiler, the water passes through and shows itself in the glass, rising, of course, to the water-level in the boiler. It is **placed** on the boiler front, and in such a position that it can readily be seen by the engineer. It is also placed at such a **height** that, when there is the proper depth of water over the tubes, the water will show about half-way up the glass. It is **liable to derangement** from the cocks becoming salted up. Its **working** can be **tested** by trying the test-cocks on the boiler front.

42.—Glass water gauges have sometimes pipe connections to the top and bottom of the boiler; the object of this arrangement is to obtain the steam and water for the gauge as free as possible from those impurities which collect on the water surface in the boiler, especially when priming is going on. It is desirable to have cocks at the extremities of these pipes, so that if anything were to happen to the pipes, their connection with the boiler could be immediately cut off.

43.—The principle of construction of dial steam gauges is, that a thin elliptical metal tube, if bent into a ring, will seek to coil or uncoil itself if subjected to external pressure, and to an extent proportionate to the pressure applied. A **Bourdon's steam gauge** is of the following construction:—A hollow brass hoop of an oval cross section (thus ⬭), into which the steam passes, is secured at one end to a strong brass plate, which is fitted over the steam entrance to the gauge, the joint being made steam tight; the other end, which is closed, is attached by means of a small arm to a toothed sector, which is geared into a small pinion upon the spindle to which the index pointer is attached. The steam entering the hoop causes the end which is attached to the sector to move outwards from the centre; this causes a corresponding movement in the pointer, transmitted by means of the arm, sector, and pinion; the movement continues as the pressure of the steam increases. The hoop, arm, sector, pinion, and frame to which they are hung, are all of brass. The **use of the inverted syphon pipe** is to collect a small quantity of water, so that it may act as a cushion between the steam and the working parts of the gauge; also to prevent injury, as the steam permeating the gauge would, in time, spoil its action, loosen the enamel from the dial, and also dim the glass so that it would become unreadable.

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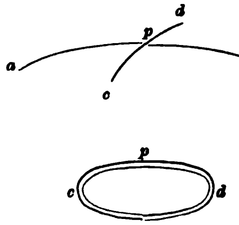
* With the spring-loaded safety-valve there is thus a considerable saving of steam, there being no blowing off when in a sea-way as in the case of a dead weight or weighted lever valve.

NOTE.—It remains to examine the reason for the shortening of the tube under pressure. One way of looking at the question is based a theorem in geometry, by which it is proved that if *p* be a point in a surface, and *a p b*, *c p d* be two sections of the surface made by planes at right-angles to each other, and passing through the line perpendicular to the surface at the point *p*, the sum of the curvatures of the two sections *a b*, *c d* will be a constant quantity. The word "curvature" is a technical term presenting the idea which is implied in its popular signification, and its meaning will be understood when we say that the curvature of a circle is inversely proportional to its radius, and is measured by the fraction $\frac{1}{\text{radius.}}$



The form of the tube is given in cross-section by the curve *c p d*, and the line *a p b* may be taken to represent the curving of the tube in the direction of its length. If the sum of the two curvatures, viz., those in *a b* and *c d*, respectively, be constant, it will follow that when one increases the other must decrease, and conversely, that is to say, if the tube *e e* becomes more curved in the direction *c p d*, it will become less curved in the direction *a p b*. But the effect of air under pressure is to bulge *c p d* outwards, and to make it more convex, or to give it a larger curvature, and the direct result is that the tube *e e* straightens and its end moves outwards.

It is easy to show that such an action will certainly occur, for, if we bend a vulcanised india rubber tube it will become more and more flat, and will finally straighten in the direction across that in which it is being bent.

44.—The **small cock** on the pipe is used to draw off any water which may have collected in the pipe. This cock should be placed at the lowest part of the bend in the syphon. If the cock was not opened, and the water properly blown off, it would rise into the gauge, and the steam pressure would in consequence be incorrectly indicated.

45.—**Steam Gauges** do not indicate the total pressure of the steam, only the pressure above the atmosphere. The steam pressure is measured from the pressure of the atmosphere, that is 0, or *zero*, on the steam gauge equals the pressure of the atmosphere, or 15 lbs. per square inch.

46.—By the **Salting of the Boiler** is meant the accumulation of salt on and between the tubes, on the flame-boxes, the furnaces, and tube plates. This can be prevented by not allowing the water in the boiler to attain beyond a certain fixed limit of density, which is accomplished by regularly and frequently scumming the boilers.

The **density** or **specific gravity** of ordinary sea water is 1·027, pure water being 1, and contains 4 oz. of salt. The density of the water may be ascertained by means of the Hydrometer. The specific gravity, or density, of the water in the boilers should not be allowed to exceed 8 oz. to the gallon, or 1½-32 to 2-32 by Hydrometer.

47.—The **Scum Cocks** and pipes are so arranged that the surface water of the boiler can be conveniently blown off through the ship's side or bottom into the sea. One cock is placed on the boiler, and the other on the ship's bottom or side, as the case may be. The cock on the boiler is placed a little below the working water level. Generally speaking, these cocks are used when the water in the boiler has attained that limit of density beyond which it ought not to be worked; but they ought to be systematically opened, and the boilers well scummed at certain fixed intervals. They must be shut before the water has been blown too far down in the boilers, otherwise there is a danger of burning the tubes if this be neglected.

48.—The principle ingredient of the substance called scale, which is formed in boilers using sea water, is sulphate of lime. Where spring water, or what is commonly known as hard water, is used, carbonate of lime is the principal ingredient of the scale. It is most objectionable when it forms upon the furnace tops and tubes. It can be removed by chipping with hammers and slices. Its formation can be most effectually prevented by a regular system of blowing off the water before it becomes too highly charged with impurities, and, to a great extent, by the suspension of zinc plates in the water space in the boiler; these plates absorb the sulphate of lime with great avidity. Iron having about thirty times the conducting power of scale, when the latter is allowed to form on the boiler by its resisting the conduction of heat, it impairs at once the evaporative power of the boiler, its durability, and its safety.

49.—The Salinometer, or Hydrometer as it is sometimes called, is an instrument by which the density of the water can be measured. It consists of two hollow bulbs of glass or metal—the lower bulb very much less than the upper one—from the upper bulb there rises a tall stem, graduated with degrees, and the instrument is ballasted so as to swim upright. The graduated stem is more or less immersed in proportion to the density or specific gravity of the water; in other words, the more salt there is in the water, the *less* the instrument will sink in it. It is graduated to one-thirty-twos ($\frac{1}{32}$) for a certain fixed temperature, generally 200° Fah., and a thermometer is used in connection with it so that the water tested may be of the proper temperature. Salinometers being graduated for certain fixed temperatures, cannot be used at any temperature indiscriminately; but if water varies a few degrees from the standard we can allow for it by making a correction of $\frac{1}{8}$ of $\frac{1}{32}$ for every 10°, adding if the temperature is greater than 200°, but subtracting if less.

50.—The reason of some boilers being fitted with division plates (usually called dash plates or wash plates) is to prevent the scend of the water to one side of the boiler when the vessel rolls or lurches heavily, thus leaving the tubes on the opposite side bare, whence there is a risk of injuring the crowns of the fire places or upper tubes. Such division plates are neither steam nor water tight. They stand fore-and-aft in the ship, but they are also sometimes fitted along the sides of the boilers, being set at a certain angle—not lying horizontal; they are also occasionally placed across the boiler above the flame-box.

51.—A Split Tube is plugged up by means of either wooden or iron plugs. The best method is to take two cast iron discs, one part of which just fits the inside of the tube, and the other comes against the tube plate, and place one in each end of the damaged tubes, the discs being provided with holes, a long bolt is passed through the whole length of the tube, and secured on the outside with nuts, the whole being tightly screwed up, a steam-tight joint being first made by placing a little spunyarn and white lead under the flanges of the disc. In the case of wooden plugs being used, the method is as follows:—Make the plug a driving fit in the tube, and drive through till it has passed the split, take then another plug and drive it well in from the same end.

52.—Dampers are used to reduce the draught of air to the fires. They are sometimes fitted in the funnel, and sometimes before the ashpit doors. They should be used when the engines are slowed or stopped.

53.—When there are **no dampers fitted**, the furnaces or smoke boxes are opened instead. The cold air coming into immediate contact with the hot plates or tubes is apt to have an injurious effect.

54.—The **piston** of a steam cylinder is a hollow disc of cast iron, strongly ribbed, with a tapered hole in the centre, which fits into a corresponding taper on the end of the piston-rod, and which is secured to the piston by means of a strong brass nut and a check plate. Around the outside of the piston, a little back from its greatest diameter, is arranged a series of recesses containing alternately a steel spring and a square brass nut; immediately outside these is fitted a cast iron ring, called the packing ring, and which is kept in its place by a flat circular cast iron hoop, which is placed on the top, and secured by bolts passing down through it, and screwing into the brass nuts above mentioned—this is called the junk ring. The packing ring being turned and finished a little greater in diameter than the cylinder which it has to fit, is cut through at one point and sprung in, so that it fits the cylinder accurately, but not too tightly. The steel springs above mentioned are employed to exercise an uniform pressure all round the packing ring, and keep it against the barrel of the cylinder, so that no steam can pass. The **round pieces let in flush**, generally on the top side of the piston, are tapered cast iron plugs, filling up the core holes which are necessary in the moulding of the piston.

55.—Cylinder Drain Cocks are used to drain off any water which may have collected in the cylinder, as the cylinder escape-valve only comes into action when violent priming goes on; they are always opened when starting the engine, and shut when fairly under weigh. **The valve** upon the drain cock serves the purpose of preventing any water or air being sucked into the cylinder by the vacuum when the drain cocks are open.

56.—Cylinder Escape-valves are generally strong valves (resembling an ordinary safety-valve in form) fitted into the top and bottom of the cylinder, and kept in their places by helical rings, compressed to the required pressure. **They are protected** by means of a cylindrical cast iron casing, which fits down over the top of them. The escape-valves **are regulated** by means of a brass nut screwed into the top of the metal casing already mentioned, and against the bottom of which the spring presses; by means of this nut the spring can be compressed or relaxed as circumstances may require. **These valves are most needed** when the engines are being started, as from various causes (such as condensation, priming, &c.) water is apt to collect in the cylinder, both above and below the piston, and since it is incompressible, if it accumulate to any considerable extent, it must find an exit, or damage will be done to the engine. The engineer is **exposed to the danger of scalding** from steam or hot water being suddenly, and with great force, ejected from these valves. It is customary, as a **precaution against** this, to turn the hole which is in the metal casing to allow of the free passage of the hot water or steam away from the front of the engines, and in such a direction as will make it impossible that the ejected steam or hot water, in the event of the valves blowing through, should come in contact with any of the engine-room hands; but **sometimes**, as a precaution against scalding, a pipe is fixed to the outlet, and led away to the bilge.

57.—A Compound Engine is a high and low pressure condensing engine, in which the mechanical action of the steam is *begun in one cylinder and ended in a larger cylinder*. The steam at a high pressure first enters the small cylinder until the piston has been moved through a certain distance, where the valve is so regulated that the communication with the boiler is entirely shut off, and the remainder of the space to be passed through by the piston is performed by the expansion of the steam now shut up in the cylinder, and which, having done its work there, rushes to the condensing cylinder, where it does an equal or proportionate amount of work, and out of which it is exhausted into the condenser. With respect to the number and arrangement of the cranks and cylinders,* there may be said to be about five different kinds of compound engines used for screw steamers. The first, and most generally adopted kind is that of the inverted direct-acting engine, with the cylinders, high and low pressure, placed alongside of each other in the fore-and-aft direction of the ship, with the steam-chests between them, and connected direct with the two cranks on the shaft beneath. Another kind occasionally adopted is that also of an inverted direct-acting engine, but with one cylinder placed immediately above the other—the high pressure being on the top. In this case there is only one piston-rod, which is continued through both cylinders and pistons, one connecting-rod, and consequently only one crank; a fly-wheel being fitted on to the shaft to help the engine over the dead centre. Another type, similar to the last as far as the number of cranks goes, is that in which the high pressure cylinder is placed within the low pressure. A fourth arrangement, and one adopted by a well-known firm of marine engineers, is to have four cylinders, two high pressure, and two low; the two high pressure cylinders being placed immediately on the top of the low, that is, each low pressure cylinder has a high pressure cylinder on the top of it; the piston-rods (two in number), as in the case mentioned above, passing through both cylinders, and connected direct with the two cranks on the shaft lying immediately beneath. And lastly, the arrangement called the horizontal direct-acting engine, used chiefly for war vessels, and in which we have two cylinders, high and low pressure, placed on their sides alongside of each other with their cranks to the side of the ship, and their centres on a level with the centre of the crank shaft, to the two cranks of which they are directly connected.

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* Compound engines usually belong to one or other of two classes. (1) Those in which the motion of the first piston is always in the opposite direction to that of the second. (2) Those in which the motion of the first piston is sometimes in the opposite direction to that of the second, and sometimes in the same direction; there being an intermediate *casing* filled with steam.

58.—The **Link Motion** is an arrangement of slide-valve gear for reversing engines, and varying the rate of expansion at will. It consists of two eccentrics, with their appropriate hoops and rods, and a connecting curved link of iron for joining the ends of the eccentric-rods together. Usually, the link is made open, and a metal block, which is in connection with the valve, may be made to traverse it from end to end. The eccentrics are so placed that when one end is in the right position for the engine to move forward, the other is in the position for moving backward, and by raising up or lowering the link, therefore, so as to bring the metal block in connection with the valves, opposite to the one or other eccentric-rod, the motion proper to that rod will be communicated to the valve, and the engine will move backward or forward accordingly. The result of this combination is, that the link receives a reciprocating motion in its centre, since, when one eccentric is moving the one end of the link in one direction, the other eccentric will be moving the other end of the link in the other direction, so that the link will have nearly the same species of motion communicated to it as if it were suspended by a pivot at its centre. Thus, the amount of travel which will be communicated to the valve will depend upon the distance at which the movable metal block may be from the central point of the link; and by moving the block up or down in the link, the amount of travel of the valve may be modified as desired; and since the travel of the valve is the measure of the lap, to reduce the travel is tantamount to increasing the lap, and hence the link motion becomes an expedient for regulating the amount of expansion with which the engine works. **Its advantages** are, its simplicity of construction, and the ease and readiness with which it can be worked. **When there is no link motion** the loose eccentric is used instead.

59.—A separate expansion-valve is one which works separately and independently of the main slide-valve. It is generally of the description called the "gridiron" valve. It is worked by an independent eccentric and rod, and is so arranged that the cut-off can be regulated to any required portion of the stroke of the piston. It is not fitted to all engines, as it may be deemed, in some cases, superfluous, since sufficient expansion may be got by lap on the ordinary valve; it may also be objected to as increasing in complication the working parts of the engine; any saving of fuel it may effect not making up for this and the additional trouble it may give. The effect of an expansion-valve is often to hinder and retard the expeditious starting or reversing of the engine.

60.—The friction of a slide-valve may be reduced by so arranging the passages in the slide, that the steam pressure is equalized on both sides, and the valve placed *in equilibrio.* The arrangement to effect this purpose is generally a ring cast on the back of the slide-valve, and working steam-tight against a planed surface inside of the casing cover, so as to reduce the area of the valve exposed to the pressure of the steam in the valve casing. The friction of a slide-valve is due to the pressure of the steam acting upon the unbalanced area of the slide-valve.

61.—A Loose Eccentric consists of the eccentric pulley, of cast iron, which is loose on the intermediate shaft, and the hoop of wrought iron lined with brass, which encircles the pulley and gives motion to the eccentric-rod. The eccentric stops are two little projections fixed on the intermediate shaft for the purpose of carrying round the eccentric pulley, according as it is wanted to go ahead or astern, the pulley itself being free to revolve backwards or forwards between the two stops. The weight of the pulley is balanced on the shaft by the back balance cast on it. A link is generally used instead.

62.—The travel of the Eccentric-rod is the distance travelled by the rod in half a revolution of the eccentric, and is equal to twice the throw of the eccentric. It can be measured on the eccentric by taking the distance from the outside of the shaft to the outside of the eccentric at the *smallest* part, and subtracting it from the distance measured from the outside of the shaft to the outside of the eccentric at the *largest* part.

63.—Double-beat Valves consist of two circular discs, or valves, threaded on the same spindle, with a space between into which the steam enters, and by pressing on the top of the lower valve, and on the bottom of the top one, places them in equilibrium, so that they are easily opened or closed. They are not generally used for safety-valves as it is difficult to keep them tight, consequent upon the unequal expansion of the brass valves and the iron chest. They are sometimes used instead of slide-valves, and worked by means of a cam shaft; but are objectionable for the reason that they are not easily kept tight, and sometimes require a weight or spring to cause them to shut. Their action is also limited to the difference in the area of the two valves.

NOTE.—As a mechanical device for effecting the object in view, viz., the opening of a close pipe against fluid pressure with a small expenditure of force—the double-beat valve is a perfect apparatus. The principle relied upon, as just stated, is the opposition of fluid pressure on two surfaces, but the mode of application is different. Steam is supposed to be on its way to the cylinder and to have passed a regulator, viz., the throttle-valve, so as to be entering the space between the discs. It is manifest that the tendency of the steam pressure is to lift one disc upwards and to press the other disc downwards, and if the areas of the two discs be equal, these opposing forces will balance and the valve may be lifted with a very small exertion of force.

An important advantage attaches to a double-beat valve in respect of the area of opening for the passage of steam under a given amount of lift. The question is one of geometry.

Let $2r$ be the internal diameter of a pipe covered at one end by a disc of the same diameter. When the valve is raised, let x be the linear motion of the disc along the axis of the pipe, and we have

$$\text{Area of opening} = 2\pi r \times x.$$

But if the pipe be fully open, area of opening = area of pipe,

$$\text{Or, } 2\pi r x = \pi r^2,$$

$$\therefore x = \frac{r}{2} = \tfrac{1}{4}(2r),$$

which prove a well-known rule, viz., that a pipe closed by an ordinary plate valve is fully open when the lift of the valve is one-fourth the diameter of the pipe. (See Rule CXXII, page 255).

64.—A Circulating Pump is a pump used to pump water from the sea and force it through the condenser. It is **not always** worked by the main engine, but sometimes by an independent auxiliary engine.

65.—The Air-valve, sometimes fitted to a circulating, reciprocating, or double-acting pump, is for the purpose of admitting air to the water on the up-stroke of the pump, but the valve is closed against the down-stroke; the air admitted serves to soften the dead shock of the water.

66.—The difference between a **bucket air-pump**, a **piston air-pump**, and a **plunger air-pump** is, a **bucket air-pump** is a single-acting pump, the bucket being, in fact, a piston with a valve fitted to it, which closes on the up-stroke and opens on the down, lifting, therefore, a quantity of water equal to only once the capacity of the air-pump in one revolution of the engine. It is fitted with foot and head valves. The **piston air-pump** is a double-acting pump, the piston being solid. It is fitted with suction and delivery valves, and discharges with each stroke. The **plunger air-pump** is also a double-acting pump, but with an arrangement resembling the bucket air-pump in every thing, except that there are no head valves, and that the bucket rod is fitted with a large plunger. The effect of this is, that, whereas the ordinary bucket air-pump discharges only with the up-stroke, the plunger air-pump, owing to the displacement caused by the bulk of the plunger, discharges on the down-stroke as well.

67.—Double-acting air-pumps are generally made with pistons, sometimes, however, also with plungers, as shown above.

68.—An air-pump trunk is a hollow cylinder of brass attached to the bucket or piston, and worked through the stuffing box in the same manner as an air-pump rod. Inside, at the end next the bucket or piston, is fixed a single eye, which takes the connecting-rod. It is rendered necessary when the pump is worked direct off the crank-shaft, or where the pump is arranged so close to the levers through which the motion is transmitted from the engine, as to render the application of an intervening cross-head and links impossible. The difference in the discharge is equal to the relative difference between the displacement caused by an ordinary air-pump rod, and that caused by a large trunk.

69.—The double-acting air-pump must have both suction and delivery valves; but it is possible with the single-acting pump in some cases to do without either one or the other.

70.—An air-pump with a foot-valve and no discharge-valve would be most affected with a leaking stuffing box. If the foot-valve be allowed to remain, the pump would still, however, draw water, but if it were removed, the pump would probably miss altogether.

71.—In the case of the **delivery-valves** being at the **sides of the air-pump,** the gland will be between it and the bucket-valve, and if leaky will spoil the vacuum above the bucket, and consequently the quantity of water in the up-stroke will be diminished. In the other case the leaking is of little or no moment. In the first case, if there is a foot-valve there will always be a reserve of water left at each stroke, and the bucket dipping into this must necessarily take up what water is above it at the bottom of its stroke, whereas were there no foot-valve the bucket would only draw a quantity of water equal to the amount of vacuum above it, and, as before stated, if the gland was leaking this vacuum would be diminished.

72.—The air-pump pet-cock or valve is placed immediately below the head-valve, and above the bucket when at the top of the pump. It opens with the down-stroke of the pump, and air is drawn in to act as a cushion on the top of the water when opening the delivery-valve. By it the engineer can tell whether the pump is drawing properly or not. It will necessarily reduce the effectiveness of the pump. It would not be applicable to double-acting pumps, as the air reduces their effective capacity.

73.—The water in the hot-well of a surface condensing engine is generally **worked at a temperature** of about 100° to 110° Fah., and getting warmer as the ship approaches the equator. The **effect of a higher temperature** would be to injure the india rubber air-pump valves. The **effect of a lower temperature** would be to cool down the cylinder too much, and to cause a waste of fuel by the condensation of the steam. The lowness of the temperature is limited by the temperature of the sea water after it has passed through the condenser, finding its way into the hot-well either through the auxiliary feed-pipe (used generally when the boilers are leaking) or condenser tubes. **A very low temperature has two disadvantages**, viz., the increased consumption of fuel caused by it, and also the increased risk of salting the boilers—the lowness of the temperature of the feed water indicating the presence of pure sea water in greater or smaller quantities.

74.—The necessary fittings for a bilge injection with a common condenser would be, a cock on the side of the condenser with a leaden pipe leading from it to the bilge, the end of the pipe being fitted with a strum. It is used when the ship is making more water than the bilge-pumps can get rid of. The strum must be kept free from small coal or other matter in the bilge which is apt to choke it, and care must also be taken that the condenser does not get hot.

75.—When surface condensers are used, the bilge water is used as circulating water instead of injection water, as in the case of a bilge injection. A stop-valve is fitted to the circulating suction pipe, or on to the bed-plate immediately beneath the circulating pump. A bilge injection-valve should always be a clack-valve, whose lift can be regulated by means of a screw spindle. This is evidently necessary, as a clack-valve will allow of water being pumped from the bilge, although it will, from its construction, prevent any flow of water to the bilge.

76.—The injection cocks are regulated by means of rods with loose ends, which fit on to the tops of the cocks, and with handles attached. These rods and handles are so arranged that they can be worked from the front of the engine. If the injection cocks are too much open the delivery valves of the air-pump will lift too soon, thereby causing a defective vacuum in the condenser; if, on the other hand, they are too little open, the condenser will get hot.

77.—The feed-pump pet-cock or valve is placed on the barrel of the pump just above the suction valve. It is used to let air in to mix with the water, so as to soften the shock on the pipes when discharging, and to show whether the pump is working properly or not. If an air-vessel is fitted on the discharge pipe, the pet-cock is not so necessary.

78.—The method mostly used of fastening the ends of condenser tubes is to drive a wooden ferrule, which fits the outside of the tube, tightly in between it and the hole in the tube plate. Another way is to have the tube ends projecting a little through the tube plates, an india rubber washer is placed around these ends, and a second plate is bolted down over all, the holes in which are so arranged that the tube ends go so far through, then by decreasing the diameter of the hole, a shoulder is formed which comes against the end of the tubes. A third way is to make a stuffing-box of the hole in the tube plate, into which an india rubber washer is inserted as packing, and a small gland screwed down on to it. Condenser tubes are generally ¾ inch diameter, and No. 18 wire gauge, or not quite $\frac{1}{16}$ of an inch thick.

79.—The Blow-through Valve or Cock is an ordinary disc valve with a spindle and lever, and the casing of general form. It is used to blow out the air and gases which may have collected there. It is attached to the steam casing and connected to the condenser by means of a pipe. In the case of compound engines it is fitted to the high pressure slide casing, and so arranged, that in the event of the high pressure engine breaking down, by opening it steam can be admitted to the exhaust port, and so direct to the low pressure steam chest.

80.—A Snifting-valve is an ordinary mitre valve fitted to the condenser, opening upwards, and loaded with lead to withstand a certain pressure. Through it the air and gases are blown when the engines are blown through before starting; by it, also, should the tubes be leaking, the condenser is relieved of the water after it has attained a certain height. If it be placed too high it will not act efficiently; on the other hand, it cannot well be placed too low, in fact it should be placed as low down on the condenser as possible, and so weighted with lead, that when the water which may collect in the condenser (the tubes being leaky, the engine stationary, with the discharge valve open and the sea washing in) has reached the top, it will open the valve and flow out. It is evident that under these circumstances, if the condenser were not fitted with a valve of this description, there is nothing to prevent the water from rising into the cylinder and thus choking the engines.

81.—The connections generally fitted to the **hand pump** are pipe connections from the bilge and sea to the boilers, deck, and overboard. It can be used for pumping up and feeding the boilers, clearing the bilges, washing decks, and as a fire engine in case of fire.

82.—When the **engines are stopped** with the **steam up**, shut the stop-valve and ease the safety-valve (if too much steam), close the damper, shut off the feeds, and if going to stop for some time, shut the injection and service pipes, and open the drain cocks on high pressure cylinders.

83.—The engine is heated up before starting by opening the main stop-valve a little. If the cylinder is jacketed, open the jacket inlet cock, the blow through valve, and also the cylinder drain cocks. See, first of all, that the engine is properly warmed through, and the cylinders free from water; then see that the discharge, kingston, and stop-valves are open, and that all the gear of the engine is clear and in working order; and if the ship is in dock or harbour, a man should be sent on deck to see that there is nothing foul of the propeller.

84.—An interceptor, or separator as it is sometimes called, is generally just a square or oblong box of cast iron or brass, with a diaphragm or partition plate extending from the top of it to better than half-way down; the steam enters at the top of one side and strikes full against the partition plate, passing under it and out to the engine at the top of the other side; if any water comes with the steam it strikes against the plate and drops down to the bottom of the interceptor, where it collects. A cock is fitted on the bottom for the purpose of drawing off the water. Care must be taken that the water does not rise too high and fill up the steam passage under the plate. The interceptor is generally fitted on to the superheater or steam receiver, as the case may be.

85.—An air-pump bucket is a hollow piston made of brass with a grating cast in the top, and a boss which receives the rod in its centre, from which run strengthening ribs to the rim of the bucket. The outside of the bucket is grooved to receive packing, which generally consists of spunyarn lapped tightly round. On the top of the grating comes the india rubber valve, whose lift is regulated by a brass guard secured by means of a brass nut, and against which the valve is pressed when the bucket is on the down stroke, and the water rushing through. The valves are generally made of india rubber.

86.—Air-pump rods are made sometimes of Muntz metal, at other times of brass or wrought iron cased with brass. They are made so on account of the brass being proof against the oxidization to which wrought or cast iron rods and plungers are subjected.*

87.—The racing of the engine is a suddenly accelerated motion of the engines caused by the propeller being relieved from the resistance of the water when the ship is rolling and pitching. Racing takes place in a heavy sea-way. When it occurs there is a danger of some part of the machinery giving way from the strain caused by the engine flying off at a high rate of velocity; also, of the propeller blades being snapped off by coming violently in contact with a sea as the ship settles down again. The means used to prevent racing is the application of the Marine Governor, by means of which the steam is shut off from the engine; or, if no governor is fitted, regulating the throttle-valve by hand.

88.—The Marine Governor is an apparatus which is used to regulate the speed of the engines and to prevent racing when in a sea-way. They are of various constructions, the commonest, perhaps, being that of the friction clutch and spring. It is driven direct from the shaft by means of a rope and pulleys. On the governor spindle is a fly-wheel to ensure steadiness of motion, and a friction clutch, the sliding half of which is connected by an arrangement of rods and levers with the throttle-valve on the steam-pipe. When any suddenly increased speed of the engine takes place it is immediately communicated to the governor, and the sliding half of the clutch is forced away from the fixed half, this motion is communicated by the rods and levers to the throttle-valve, which is partially closed; as soon as the speed of the engine is thus reduced to its normal condition, the spring attached to the sliding half fetches it back into close contact with the fixed half.

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* Voltaic electricity is produced whenever two dissimilar metals are in the same liquid touching one another. The most oxidisable metal is acted upon very rapidly; the least oxidisable metal is not acted upon at all.

88a.—Define the terms, (1) the thread of a screw, (2) angle of pitch, (3) length, (4) area of blade, (5) and diameter.

A.—(1) The thread is the distance along the edge of the blade; (2) the angle is the inclination of the thread to the horizon; (3) the length is the fraction of the pitch actually used; (4) the area is the surface of the blade; (5) is the diameter of the cylinder from which the screw is taken, or its perpendicular distance between the extreme outside points of the blades.

89.—**The pitch of a screw propeller** is the distance in the line of the shaft from one convolution to the next, or the distance between the threads measured in a line with the shaft, supposing the screw to consist of two or more convolutions. The operation of **measuring the pitch** proceeds upon the fact, that if a line representing the circumference of the disc of the screw at its extreme diameter, and another representing the pitch of the screw, be drawn at right-angles to one another to a scale of parts, the hypothenuse of this right-angled triangle will represent the length of the winding thread.* After measuring the diameter, that is, length of the blade or fan to the centre of screw shaft, in order to find the circumference from it, the length of the blade along its penphery is to be measured, this being a portion of the hypothenuse of the triangle whose sides we wish to determine: then measure the distance that one corner of the blade is before the other in the line of the axis, and this being a portion of the pitch, is a part of another side of the triangle. Now, two sides of a right-angled triangle being known, the third can be found, in this case, by subtracting the square of the side from the square of the hypothenuse, and

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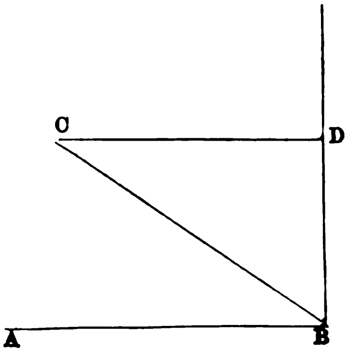
* To measure the pitch;

Let AB = line of shafting.

Angle A B C = the angle that the blade makes with a shaft at x feet from centre of shaft. At B raise a perpendicular on A B and let B D = $2\,x \times 3{\cdot}1416$

From the point D draw the line C D parallel to A B.

C D = the pitch.



finding the square root of the remainder, and the dimension so found will represent a portion of the circumference. Next make the following proportion:—*The measured fraction of the circumference* : *the whole circumference* :: *the measured fraction of the pitch* : *the whole pitch.*

90.—A right-handed propeller is a propeller whose blades form part of a right-handed screw of a certain pitch. It revolves from port to starboard of the ship. **A left-handed propeller's** blades form part of a left-handed screw, and it revolves from starboard to port of the ship.

90a.—What is the meaning of the term screw of increasing pitch?

A.—A screw of which the pitch increases from the leading edge to the following edge of the blade, or from the centre to the circumference, or both.

90b.—What is a screw of uniform pitch?

A.—A screw of uniform pitch is one in which the distance between the threads is a constant.

90c.—Give illustrations of the difference between a screw of an even and one of an increasing pitch, and of one in which no two parts of the pitch is equal.

A.—(1) A right-angle triangle wound round a cylinder.
(2) A right-angle wound round a cone.
(3) A right-angle triangle wound round a parabola.

90d.—Why do engine makers, as a general rule, fit a right and left-handed propeller to twin screw vessels?

A.—It is advisable that the crank should revolve in the same direction with regard to the cylinders, and it is more convenient to have propellers, one right-handed, the other left, than to place the cylinders on the same side of the ship.

91.—The slip of a screw propeller is the difference between the actual speed of the ship and the speed of the screw; or, in other words, it is the amount of work lost from the screw revolving in water, a yielding substance. **Its amount** is expressed in figures as so much per cent. of the work which would be accomplished if the screw were revolving in an unyielding medium, that is, as so much per cent. of the speed due to the pitch multiplied by the number of revolutions.

Thus: suppose the propeller to have a pitch of 25 feet, and to make 50 revolutions per minute, then the distance in knots it should move in an hour is—

$$(50 \times 60 \times 25) \div 6080 = 12{\cdot}3.$$

But the ship, we suppose, has an actual speed of 11 knots, consequently,

$$12{\cdot}3 - 11 = 1{\cdot}3 \text{ knots} = \text{slip, or,}$$

$$1{\cdot}3 \times 100 \div 12{\cdot}3 = 10{\cdot}6 \text{ per cent. slip.}$$

92.—The valves about engines and boilers which have to be worked by hand are the stop valve, the throttle-valve, the starting valve, the blow-through valve or cock, the safety-valves, the kingston-valves, and the delivery valves. **Those which are self-acting** are, the circulating pump head bucket and foot valves, air-pump ditto, all the feed and bilge-pump valves, the feed stop-valves on boilers, the escape-valves, the snifting-valves, and locked safety-valves. The slide-valves are the only ones which are, strictly speaking, worked by the motion of the engine, although all the others, with the exception of the lock safety-valve, may be said to be worked indirectly by the engine.

93.—The tallow is forced into a boiler by means of an injecting syringe, and is sometimes done to allay any priming that may be going on in the boiler. It is necessarily sent into the water space; if sent into the steam space it would probably not mix with the water of the boiler at all, but be carried off by the steam to the engine.

94.—Tallow cups with two small cups and two cocks, and those with one small cock and a valve, are suitable for high pressure cylinders, and those with only one small cock, or with one large hollow plug cock, for the cylinder of a condensing engine. The cup with only one small cock is used in the following manner: fill the cup up with tallow, then by opening the cock when the engine is exhausting on that side of the cylinder to which the tallow cup is attached, the tallow will be immediately forced in by the air, as soon as it has disappeared, shut the cock again immediately.

95.—A cylinder escape-valve, self-acting, does not allow all the water to escape. The depth of water left in the cylinder, it is possible, will be equal to the amount of clearance between the piston and cylinder cover or bottom, as the case may be.

96.—The steam lubricator, or **Impermeator,** as it is sometimes called, is a contrivance by which the cylinders and slides of an engine can be regularly and unfailingly oiled without any further attention on the part of the engineer than merely, at long intervals, filling up the Impermeator with oil. The Impermeator is so arranged that the steam entering it at a high temperature expands and rushes back towards the cylinder after the cut-off has taken place, carrying some of the oil with it. It is placed either on the high pressure cylinder or in the steam-pipe close to the cylinder. Throwing cold water on the Impermeator causes a more rapid flow of the oil to the engine, by condensing the steam, which then, as water, sinks beneath the oil and causes it to float higher, and so overflow down the pipe.

97.—A common or **radial paddle-wheel** is one in which the floats are fixed rigidly to the arms; the centres are made of cast iron; the arms are formed of wrought iron. **The bolts for the floats** have one end turned back so as to grip the back of the arm, the other is screwed and fitted with a nut and plate or washer. **The arms** are fitted into recesses in the centres, and secured by wrought iron bolts and nuts.

98.—Some paddle-wheels are fitted with one or more cast iron floats for the purpose of counterbalancing the weight of the moving parts of the engine.

Another method often adopted is the bolting of a metal weight to the outer rim of the wheel. Side lever engines derive the most benefit from counterbalancing. The metal float or weight is placed so that when the centre of the weight is at the bottom of the wheel, the crank has a slight lead for the downward stroke.

99.—**Paddle-wheel floats are sometimes made of different breadths** to prevent racing and to render the working of the engines smooth and regular. Single engines with common wheels most require the adoption of this plan. The broad floats take the water when the engine is at half stroke, and the narrow ones when the engine is on top or bottom centre.

100.—The difference between a **radial paddle-wheel** and one with feathering floats is, that in a radial wheel the floats are fixed to the arms with their faces parallel to lines drawn from the centre to the circumference of the wheel. **In the feathering wheel** the floats oscillate or move on their centres, and are so regulated by an eccentric that their faces, while immersed, are nearly at right-angles to the surface of the water. In a feathering float there is but little power wasted in friction, the force applied to the float being nearly all expended in a direct fore-and-aft thrust on the water. One eccentric-rod, called the driving rod, is rigidly fastened to the eccentric strap (the sheave of the eccentric being fixed on the side of the ship); the other eccentric-rods work freely on pins attached to the eccentric strap.

101.—**The centre of the wheel eccentric** is placed forward of and in a horizontal line with the centre of the shafting. **The eccentric sheave** is cast in two pieces; it has a recess turned in it to receive the strap, is bolted to the ship's side, and sometimes has a seating cast on to carry a plummer block for the shaft bearing.

102.—**The working surfaces** of a feathering wheel are made of brass and are lubricated with the water thrown up by the wheel.

103.—**A disconnecting paddle-engine** is a double engine by which the wheels can be worked either separately or together; this is effected at the inner port bearing by a wheel on the intermediate shaft; this wheel slides on keys let into the shaft, and can be moved by a lever so that the end of the crank-pin either enters a bearing on the circumference of the wheel, or else works perfectly clear. **The crank-pins** in a disconnecting engine are fixed in the outer crank.

104.—**Disconnecting paddle-engines** are generally fitted **with loose eccentrics**, and are frequently used for tugs or ferries in which great handiness is required.

105.—**Expansion joints** are stuffing-box joints used when a straight metal pipe, which is exposed to considerable variations of temperature, has no elbow or curve in its length to allow it to expand without injury. In such a case the pipe is divided into two lengths, which are united by a steam-tight joint, accurately bored out and turned, so as to allow one pipe to slip within the other when they lengthen by expansion. **They are necessary** on pipes connecting the engines and boilers, one boiler with another, and sometimes the engine with the ship's side. **They require** that the packing be kept in order, and the surfaces not allowed to corrode or stick together. **The working surfaces** should be made of brass.

106.—The neglect of fitting chains or stops to prevent the pipe being blown out of the stuffing-box may lead to a serious accident. **By soldering a collar** on to the pipe, outside of the stuffing-box, and carrying a stop from the bed-plate to the back of the collar.

107.—**An oil cup** is a hollow box or cylinder made of brass, sometimes of cast iron, with a pipe standing upright in the centre to the height, perhaps, of three-fourths or more of the height of the oil cup inside; this pipe leads direct to the working surfaces. A hinged lid is fitted over this cup to prevent dirt getting into the pipe. The **worsted consists** of a number of threads of worsted of equal length attached to small pieces of lead at one end and to a piece of bent wire at the other; the lead serves to sink the worsted in the oil, and by means of the wire it is thrust down the pipe towards the bearing, the bend in the wire catching the top of the pipe prevents the syphon from going down the pipe altogether. **The worsted may be cleaned** by blowing some of the water out of the boiler upon it. **The syphon ought to extend** so far down the tube that its extreme end is below the bottom of the oil cup.

108.—**A thrust bearing** generally consists of a number of rings or corrugations upon the shaft, which work in a number of corresponding recesses in the thrust block. This bearing receives the full thrust of the propeller, either going ahead or astern, and so prevents any lateral strain upon the gear of the engine; it is placed as near to the after engine as convenient, and must, at all events, be easily accessible from the engine room. The **lateral faces fore-and-aft of the rings** or corrugations will wear, and consequently **also of the corresponding recesses in the blocks**, but the horizontal surfaces of the rings or recesses **will not**, because the block being bored a little larger than the bearing, it does not take any of the weight of the shaft, that being borne by the ordinary shaft carriages. The construction of the bearing, with its numerous rings and recesses, renders it absolutely necessary to have a **number of oil tubes** instead of one, as in the ordinary bearings, it being evidently impossible that the oil from one tube could be disseminated equally over the whole bearing.

109.—**The bearing parts** of the tail and shaft within the stern tube **are covered with brass.** These parts being subject to the action of the water would rapidly oxidize, and the bearing be injured, were they not lined with brass. The brass is generally made about ½ inch thick.

110.—The stern tube is a cast iron tube or pipe fitted between the stern frame and the aftermost bulkhead of the ship, its connections with both being made water-tight. It carries the tail end shaft through that part of the ship which is so narrowed by the run of the ship's lines aft. **Its length is fixed** by the distance between the stern post and the after bulkhead. At the outer end, or the end nearest to the propeller, it is generally secured to the stern post by means of a large nut screwed on to the end projecting through the stern post, a collar on the tube coming against the inside of the post, forming the resistance to the nut. The inner end, or the end in the after bulkhead, is secured by means of a flange of sufficient diameter, cast on the tube, being bolted to the bulkhead.

111.—A lignum vitæ bearing is a bearing made of that description of hard wood. The wood is fitted in long narrow strips, dovetailed together, and which are prevented from working out by means of a check plate. Such bearings are **generally used at the outer end** of the stern tube, or outer end bearing in a paddle engine.

112.—A screw propeller is generally **fixed on the shaft** by means of a large flat key, driven through the boss of the propeller and the shaft, a large nut coming on behind the propeller, and behind the nut a tapered check pin being driven through the shaft to prevent its working loose. **The end of the large flat key** is fitted with a check plate secured to it by means of small bolts, to prevent its getting loose at sea.

113.—Sluice valves are fitted on the water-tight bulkheads. On almost all screw steamers a **large sluice valve** is fitted on the **after engine room** bulkhead; through this valve admittance is gained by the engineers to the screw shaft tunnel. This valve **ought to be worked** from the **upper platform of the engine room. This is necessary,** because in the **case of any sudden and serious leakage** into the tunnel, the engineer would, in all probability, not be able to get at the valve from below. These **valves** ought to be **frequently looked to,** so that they may not be allowed to set fast, and so be unworkable if any emergency arose requiring their use.

114.—With a condensing engine the following **valves or cocks** are on the skin of the ship in the engine room:—The main kingston-valve for injection water, the main discharge-valve from condenser, and the bilge discharge-valves; and in the stoke-hole there are the blow-off and scum kingston-valves, and cooling cock.

115.—The necessary **fittings of a marine boiler** are as follows:—The furnace bars, dumb plates, bridge and dampers, safety-valves, the stop-valves, the blow-off and scum-cocks, the feed and check-valves, for both main engines and feed donkey, the water gauge glass and fittings, and the test cocks; a small cock is, also, generally fitted, by which water may be taken from the boiler, and its density tested by the salinometer.

116.—With a **surface condensing engine** before preparing to start, it is necessary to open the stop-valves on the boiler so as to get all parts nicely heated; the injection and discharge valves; the drain cocks in the cylinders and slide jackets are also opened so as to prevent the accumulation of any water, and the throttle-valve, if perfectly tight, to be left open a little (but there is generally sufficient leakage from this valve to allow of its being kept shut); at the same time the engineer must not forget to slack back the check-valves on the boiler so as to allow the feed water to pass when the engines are in motion.

117.—**A steam jacket** is an outer cast iron casing enclosing the cylinder, the intermediate space being filled with the hot steam from the boiler; its object being to prevent condensation or loss of heat in the steam. **A drain cock** is placed on the **bottom of it,** to drain off any water that may collect there; and **on its side a cock** for the admission of steam from the steam-pipe preparatory to starting the engines. Steam jackets are **most generally used** in compound engines: they require to be **felted** to prevent, as far as possible, loss of heat by radiation.

118.—**The parts and fittings** of an engine requiring to be felted or otherwise **protected from radiation** are the sides of the cylinders and the steam-pipes; the boilers ought also to be felted and the uptake protected from radiation by means of these plates secured to the uptake plates by means of bolts, and kept at a distance of two or three inches from them by means of small hollow distance pieces, through which the bolts pass.

119.—**The small cylinders** sometimes fitted on the **slide-valve casing cover** of vertical engines, are balance cylinders which serve to take the weight of the slide off the slide-rods and gear. The top end of the slide-rod is fitted with a small piston, which works steam tight in the balance cylinder. They are connected on the top side by a pipe to the condenser, so that the vacuum in the condenser acting on the upper side of the piston in the small cylinder, will carry up any weight proportionate to the area of the piston in square inches multiplied by the vacuum in lbs. per square inch. It also carries away any steam arising from leakage.

120.—**The principal pipes** in connection with the engines and boilers of a steamer, and to what the ends are connected, are as follows, viz.: the **waste steam-pipe** leading from the safety-valve to the open air; the **main steam-pipe,** connecting the stop-valves on the boiler with the steam-chest on the engine; the **circulating suction-pipe,** connecting the main kingston-valve with the bottom of the circulating-pump; the **circulating discharge-pipe,** connecting the discharge branch of the condenser with the main discharge-valve in the ship's side; the **air-pump suction,** connecting the exhaust steam compartment of the condenser with the bottom of the air-pump; the **main exhaust-pipe,** connecting the exhaust port of the low pressure cylinder with the condenser; the **feed suction-pipe**, connecting the hot-well with the feed-pump; the **feed discharge-pipe**, connecting the feed-pump with the feed check-valves on the boiler; the **bilge suction**, connecting a strum box in the bilge with the bilge-pump; the **bilge discharge-pipe**, connecting the bilge-pumps with the bilge discharge-valves in the ship's side; the **scum-pipe**, connecting the scum-cocks on the boiler with the blow-off kingston on the skin of the ship; the **blow-off pipe**, connecting the blow-off cocks on the boilers, likewise with the blow-off kingston on the skin of the ship.

120a.—Name any connections between the donkey and other parts of the engines of the ship.

A.—Steam-pipe from, and feed-pipe to, the main boiler; pipe from the sea for the feed; one from the bilge, so that the donkey can pump the ship out in case of leak; one on the water ballast tanks for pumping them out; and one to the surface condenser to keep the water circulating when the main engines are stopped for a short time.

120b.—Name any pipes that lead from or go to the sea.

A.—Blow-off pipe, donkey feed suction-pipe, sea injection-pipe, and discharge pipe.

121.—**The injection water**, on its way from the sea injection rose-plate, passes to the water space of the boilers, first through the main kingston-valve on the ship's side, then along the injection-pipe through the injection-cock into the condenser, from which it is taken by the feed-pump, passing through the pump-valve and pump chamber, through the bucket-valve and discharge-valve into the hot-well, then through the feed suction-valves, pump barrel, feed discharge-valve, along the feed-pipes, and through the feed-valve chest on the boiler, thence down a pipe inside of the boiler leading to the water space.

122.—**The circulating water of a surface condenser** passes first of all through the main kingston-valve on the bottom of the ship, along the circulating suction-pipe to the bottom of the circulating pump, through the circulating pump and circulating discharge pipe to the condenser, passing amongst or through the tubes of the condenser, circulating twice or thrice as the case may be, then along the main discharge-pipe, through the discharge-valve, and so again into the sea.

123.—The **steam**, from the time it leaves the boiler until it is in the form of water in the hot-well, passes first through the superheater, then the main steam stop-valve steam-pipe, the regulating and throttle-valves, steam-chest, steam-port, into cylinder, return through steam-port, through inside of slide-valve, exhaust port and exhaust passage, condenser, air-pump, foot-valve, bucket, chamber and head-valve into the hot-well. If a jet condenser, part goes into the sea through the discharge pipe, past the stop or sluice valve.

124.—The pieces of the engine through which the **pressure** of the **steam** is **transmitted** from the piston to the screw propeller, and the order in which they take the strain, are as follows:—The piston, the piston-rod, piston-rod gudgeon, connecting-rod, crank-pin, crank-shaft, crank-shaft couplings, intermediate shafting and couplings to screw-shaft and propeller.

125.—**An air vessel** is a pipe or chamber containing air, fitted to the upper side of pump-valve boxes or discharge pipe. The discharge water from the pump compresses the air contained in the vessel during the pump's delivery, and fills part of the vessel. During the suction stroke in a single-acting pump, or that part of the stroke when the motion of the bucket is slow in a double-acting pump, the compressed air in the vessel reacts on the water inside of it and discharges it; it thus keeps the discharge more constant than it would be without it. By the use of an air vessel in a single-acting pump the size of the discharge pipe may be reduced, for since it extends the duration of the discharge over double the time, its greatest velocity would be reduced by about one half. The parts of an engine or its fittings where air vessels are generally applied are the discharge pipes of air, feed, and circulating pumps.

126.—**A mud box** is a cast iron box of oblong form, fitted with a hinged lid that can be readily opened, but which will be quite air-tight when closed; the box is divided into two compartments by a loose division plate, which is perforated with small holes, to allow of the water passing through, but no obstruction in the shape of pieces of wood or small coal can get through and probably choke the valves of the pump, as these collect in that half of the box in direct communication with the bilge, and the mud box being generally placed upon the engine room floor, they can be readily removed by the engineer on duty.

127.—**A trunk engine** has a hollow cylindrical casing or "trunk" fastened to the upper face of the piston; this casing passes steam tight through and works in stuffing boxes at both ends of the cylinder, in the same manner as the piston-rod of an ordinary engine. The lower end of the connecting-rod descends through the hollow trunk to the upper side of the piston, to which it is attached by a pin or gudgeon at its centre in the direction of its length, the top of the connecting-rod being connected, as usual, with the crank-pin. Its internal diameter is such as must allow of the necessary vibration of the connecting-rod. Its object is to reduce the distance between the centres of the crank-shaft and the cylinder, still retaining a good length of connecting-rod. They should be placed on the starboard side. When they are so placed, the oblique strain of the connecting-rod will tend to lift the piston from the bottom of the cylinder, and thus neutralize, in some measure, the effect of the weight of the piston and trunk, which would wear the cylinder oval.

128.—An oscillating engine is one in which the cylinder is mounted on **gudgeons** or **trunnions**, generally near the middle of its length, on which it is capable of swaying to and fro through a small arc so as to enable the piston to follow the movement of the crank, to which it is directly attached without the intervention of a connecting-rod. The piston-rod thus fulfils both the functions of piston and connecting-rod. This construction is very advantageous in point of economy of space and weight. The trunnions are hollow, and are connected by steam-tight joints, one with a steam pipe leading from the boiler, the other with an exhaust pipe. The valve chest oscillates with the cylinder. Paddle steamers generally. Since they have no separate connecting-rod, a low engine may be constructed with a good length of stroke. **Through** the trunnions, which are made hollow for that purpose.

129.—The valve motion gear of an oscillating engine consists of the eccentric or link motion attached in the usual way, while various arrangements are used to adapt the valve gearing to the oscillating motion of the cylinder and valve chest; one of the simplest being to communicate motion from the eccentric to a sliding-rod, on which is a cross-head of the form of an arc of a circle described about the axis of the trunnions when the valve is in its middle position, and having in it a slot of the same figure; in that slot is a slider attached to the end of a lever and projecting from a rocking shaft carried by the cylinder; another arm projecting from that shaft moves the slide-valve rod.

129a.—Why is the stuffing-box deeper in these engines than in those fitted to ordinary cylinders?

A.—There is much tendency in the stuffing-box to wear oval, and hence it is made of great depth.

130.—Geared engines are sometimes used for screw propulsion—the gearing is introduced to increase the number of revolutions of the shaft, and thus enable the screw to make revolutions enough without giving undue velocity to the piston. The cogs of the large wheel are made of hard wood, such as crab tree, horn beam, live oak, or beech.

131.—The part of a screw steamer that the **pressure which propels it is applied to the hull** is at the thrust block, or the engine keelson, generally amidships.

132.—The part of a paddle steamer that the **pressure which propels it is applied to the hull** is at the paddle beams and bed-plates, the engines being firmly attached to both.

133.—The fuel per indicated horse-power per hour required by modern compound engines is from 1¾ lb. to 3 lbs., and by common condensing engines from 3 lbs. to 5 lbs.

134.—In a surface condensing engine the air-pump has merely to pump the condensed steam and uncondensed vapour from the condenser; in a jet condensing engine it has, besides this, to withdraw the condensing water; thus with a surface condenser the air-pump may be made much smaller than in a jet condenser. Now since the air-pump works against the pressure of the atmosphere, the smaller it can be made the less energy will it absorb to drive it, and since, then, it can be made smaller in the case of a surface condensing than in a jet condensing engine, that engine must be the more economical of the two. It also allows the boiler to be supplied with water comparatively free from salt, and at a high temperature.

135.—A surface condenser consists of a box or chest through which are carried tubes. At each end there is a compartment or chamber with doors which are open to the tubes. In some surface condensers the water flows through the tubes while the steam exhausts in among them; in other cases the steam exhausts into the tubes while the water flows through amongst them in the central part of the condenser. The tubes are made of brass. **They are fixed** in various ways. At the end of the condensers, and forming the division between the centre in which the steam is condensed and the chambers through which the water flows, are placed plates made variously of cast iron or brass through which the tubes pass. In some instances the tubes are fastened into this by means of wooden ferrules being driven in between the tube and the plate—the holes in the plate being left clear of the tubes for that purpose. They may be fastened by means of india rubber ferrules which are put over the tube ends and into recesses in the tube plates. They may also be fastened by allowing each tube end to pass through a stuffing-box and gland in the tube plate. The packing round the tubes which fastens them to the tube plate serves to keep them tight. It is plugged up at both ends until it can be taken out and renewed.

136.—They foul from grease on the steam side and from deposits of saline matter on the water side. The latter occur to a considerable extent if the condenser is worked at a high temperature. By injecting soda into the condenser with the exhaust steam, and by having a door on a convenient part of the condenser which may be removed and a hose allowed to play upon the tubes.

Note.—The manner of cleansing the tubes from grease depends upon the circulation of the steam. Care must be observed whether the detergent is steam, caustic potash, or simply sea water through the medium of the jet injection. With the first, if the wood ferrules are fitted, and the condenser is allowod to get too hot, they become charred and drop out, or this may happen if the ferrules are allowed to get dry. Potash attacks the packing of glands. India rubber is destroyed if too high a temperature is maintained in the condenser. It is an essential question for the engineer to decide what temperature of condenser gives the most economical results. A higher vacuum in many engines can be obtained by working the condenser at a lower temperature, and so lowering the temperature of feed water entering the boilers.

137.—The non-conducting substances employed to prevent radiation are [illegible]lt, sawdust, and a cement specially prepared, made up of jute, flax or wool choppings, or fire-clay. The general mode of applying it is to put on about two or three inches of the cement, then a layer of hair felt, and cover the whole with wood, canvas, orpaint.

138.—In the construction of **smoke-box doors** and dry uptakes provision is made to lessen the amount of radiation by having an extra thickness of plate fastened at about three inches from the uptake all round, which allows the air to circulate freely around them.

139.—The formation of **black smoke** can be prevented by careful firing, and introducing more air into the furnace, or by a jet of dry steam let in horizontally above the dead plates. Smoke preventing apparatus, as applied to furnaces of marine boilers, generally consists of some method whereby a jet of air may be introduced, so as to unite with the heated products of the furnace behind the bridge.

140.—By **circulation** in a boiler is meant the motion imparted to the water in contact with the furnaces and other hot surfaces. The water expands on being heated, it becomes lighter and rises to the top; in turn it gets heavy and comes down again with other cool particles, which stream in from the sides to supply the place of the axial upward currents, and this going on continually, causes the whole water to be constantly circulating, or in motion. The results of defective circulation are unequal expansion of the boiler, salting up, exposed plates getting burnt through, besides decreasing the evaporation.

141.—The means sometimes adopted to improve the **circulation** in a boiler is by circulating tubes put through the furnaces, either vertical or inclined, with a view to separate the descending from the ascending currents.

142.—**The arrangements for promoting circulation** in a haystack boiler is by cutting cross tubes in the flame-box.

143.—**A ship's side air-pump discharge-valve** is an ordinary mitred valve in the usual case. There is generally a spindle connected with the valve and carried through a gland in the cover and a weight placed upon the valve, so that its tendency is to keep shut. When discharging, the air or water is forced through the valve, which, as soon as the discharge is completed, shuts of its own accord. Sometimes the spindle has attached to it the means of either keeping the valve permanently open or closed. It differs from a stop-valve in not having a screw lift and being weighted. It requires attention to see that it does not stick fast open and allow a return of salt water down the feed-pipe and into the hot-well.

144.—The construction of a **feed escape-valve** is that of an ordinary mitre valve, loaded directly with a helical spring of a square or round suction. The discharge may be connected to the hot-well or suction pipe, or allowed to go to the bilge. By compressing the spring. To effect this there is at each end of the spring a screw, between them is a cross-head which receives the thrust of the spring and is transferred by it to the screw. By adjusting the nut on the screw the pressure on the valves may be increased or diminished.

145.—When there is no **feed escape-valve fitted** to the feed-pump, there is usually a suction cock or valve on the hot-well, and a check-valve on the boiler, which is not fitted with an adjusting screw. There must not be a cock placed between the delivery of the pump and the boiler.

146.—The measure of a **horse-power** is a working force that will raise 33,000 lbs. one foot high in one minute, or it may be defined as 33,000 *foot pounds in one minute.* Indicated horse-power (I.H.P.) is ascertained from diagrams taken with the indicator from the top and bottom of the cylinders of the engines when at work. The mean or average pressure during the stroke having been ascertained from the "Indicator Diagram," the calculation of the indicated horse-power is as follows:—

$$\text{I.H.P.} = \frac{\text{area of piston in inches} \times \text{mean pressure per square inch} \times \text{speed of piston in feet per minute.}}{33{,}000}$$

147.—The **nominal horse-power** is generally fixed by the area of the piston, without any reference to the speed of the piston, or the pressure, and has really no definite meaning, because there is no fixed area per horse in general use—nearly every manufacturer has his own particular measure. Its use is only as a commercial expression for the sizes and capabilities of engines.

148.—**Back pressure** in the cylinder is the pressure on the piston at the side opposite to the steam. In a low pressure cylinder it is the difference between the indication of the vacuum gauge and a perfect vacuum. Its amount is generally about 3 lbs. per square inch of piston area.

149.—By **speed of piston** is meant the number of feet passed through by the piston in one minute, and is the measure of the revolutions per minute × twice the length of stroke in feet. In modern marine engines the speed of piston is about $250 \sqrt[3]{\text{stroke in feet}}$.

150.—**Atmospheric pressure** is the pressure exerted by the weight of the atmosphere at the sea level. Its average amount is about 14·7 lbs. per square inch. It is measured by the barometer.

151.—**Gross, or absolute pressure** is the pressure per square inch above a perfect vacuum; that is to say, it is the pressure above the atmosphere with 14·7 lbs. added. The pressure that is shown by the steam gauge is only the pressure above the atmospheric pressure.

152.—By **cutting off steam** is meant shutting off the supply of steam to the cylinder before the piston has completed its stroke. It is done by arranging the valve so that when at half-stroke it overlaps the ports; or by an independent expansion valve. The part of the valve that regulated the cut-off is the outer edge or lap.

153.—The time of exhausting is fixed by the inside edge of the slide-valve and the angle of the eccentric on the shaft. This angle depends upon the lap or cut-off, and the lead of the valve. The exhausting depends on the inside edge.

154.—The time of closing the exhaust is fixed by the amount of lap there may be on the exhaust side of the valve. After the exhaust is shut, and before the port opens for steam, the steam that is in the cylinder is compressed and cushions the piston at the end of the stroke.

155.—The lead of the valve is the amount which the port of the valve is open for steam when the piston is at the beginning of its stroke. Its object is that the piston may begin its stroke with a pressure. The amount of lead varies from $\frac{1}{16}$ to $\frac{3}{8}$ of an inch.

156.—The cover or lap of the valve is the amount which one edge of the valve overlaps the steam port when the valve is at half-stroke. Its object is to cut off the supply of steam to the cylinder before the piston has completed its stroke. Its proportion to the travel of the valve depends upon the cut-off required. To cut off at the parts $\frac{1}{2}$ stroke, $\frac{5}{8}$ stroke, and $\frac{3}{4}$ stroke, the lap must be ·354, ·305, and ·25 of the valve stroke respectively—neglecting lead. If lead is taken into account $\frac{1}{2}$ the lead must be subtracted from the lap found by the above.

157.—By exhaust cover or exhaust is the amount which one inside edge of the slide-valve covers the steam port when the valve is at half-stroke. It delays the period of exhaust and increases the amount of cushioning.

158.—It is the amount which one inside edge of the slide-valve is *clear* of the steam port edge, or the amount of opening which one steam port has to the exhaust when the valve is at half-stroke. It hastens the period of exhaust and decreases the amount of cushioning.

159.—Cushioning or compression is the "shutting up" of the steam in the clearance space and port on the return stroke of the piston, the elasticity of such compressed steam acting as a kind of cushion (hence the name) for the piston at each end of the stroke, and thus taking off the heavy knock that otherwise would take place in the cylinder. Cushioning (as explained in 157) is increased by cover on the exhaust port of the valve, or diminished by minus cover on the steam side of the valve (see No. 158).

160.—Mean effective pressure is a constant pressure, whose effect during the stroke would be the same as that of the variable pressure of the cylinder. Its amount is found by ascertaining the mean width of an indicator diagram.

161.—A dial vacuum gauge on Bourdon's principle, is an instrument which indicates the difference of pressure in the condenser and atmospheric pressure.

It consists of a case about 7 inches diameter, with a dial plate in front. Inside the case, and communicating with the condenser by means of pipes, is a bent hollow tube, oval in section, the other end of which is closed air-tight, pointed off and attached to a sector which works into a small pinion, the spindle of which carries a pointer working against the dial in front. Its construction, as just described, is the same as the steam gauge. In the steam gauge the pressure of the steam within the tube tends to straighten it; in the vacuum gauge the pressure of the atmosphere outside of it tends to compress or bend it (the arrangement of gearing being accordingly reversed), and thus move the pointer, which indicates the amount of exhaust or vacuum on the dial plate. When the engine is working all right the vacuum will be from 25 to 28 inches (usual working vacuum 25 to 26 inches). Its variations indicate an increase or decrease of the back pressure on the piston, which decreases or increases respectively the I.H.P.

162.—The vacuum gauge does not of itself tell what pressure there is in the condenser; recourse also must be had to the barometer. To ascertain the actual amount of back pressure there is in the condenser, take the difference between the indications of the vacuum gauge and that of the barometer; thus:—Vacuum by vacuum gauge, 26½ in.; vacuum, by barometer, 30 in.; therefore, 30 in. — 26½ in. = 3½ in., or 1¾ lbs. per square inch.

163.—A Barometer is an instrument by means of which the pressure of the atmosphere is ascertained. It consists of a graduated tube of glass, about 30 inches long, hermetically sealed at one end, which has been filled with mercury and inverted with its open end in a cup of the same metal. A species of barometer is sometimes used as a vacuum gauge. The upper end of the tube in the vacuum gauge is connected by a pipe to the condenser. Instead of a cup the glass tube may be bent up again, and this serves as a receptacle for the mercury when it falls; in such an one an inch rise or fall must be read as about double the gain or loss when compared with an inch of the other.

164.—The graduations in the common **steam-gauge** and the common vacuum gauge are both marked from atmospheric pressure; the one upwards, the other downwards. The steam-gauge indicates the pressure above the atmosphere, and requires the pressure of the atmosphere, as shown by the barometer, to be added in order to tell us what is the actual pressure in the boiler. The vacuum gauge indicates the pressure below the atmosphere, *i.e.*, so many lbs. below the atmospheric pressure.

NOTE.—The indications of the vacuum-gauge gives us directly the amount of the atmospheric pressure which is allowed to act per square inch on the piston.

165.—If the **steam-gauge** shows 60 lbs. per square inch, the vacuum gauge 10 lbs. on the square inch, and the weather barometer equals 15 lbs. on the square inch, the difference between the pressure in the condenser and the pressure in the boiler will be 70 lbs.; thus:—Total pressure of steam 60 + 15 = 75 lbs., difference between the barometer and condenser 15 — 10 = 5 lbs.; therefore, 75 lbs. — 5 lbs. = 70 lbs., the difference between the pressure in the condenser and that in the boiler.

166.—Vacuum is generally stated as so many inches, because the gauges are graduated to correspond with a column of mercury 30 inches high, which balances the weight of the atmosphere, and is equal to 14·7 lbs., or nearly 15 lbs. per square inch at the level of the sea; so that by 20 inches vacuum is meant that the pressure of the atmosphere above the pressure in the condenser will support a column of mercury of 20 inches vertical height. It tells us that the pressure of the vapour in the condenser is equal to the difference between the indications of the barometer and the vacuum gauge; thus, if the former stands at 30 inches = 15 lbs., and there is 20 inches vacuum = 10 lbs., then the pressure of the vapour in the condenser is 15 lbs. — 10 lbs. = 5 lbs.

167.—There is a limit from which a common **pump** will draw water; that limit is about 32 feet. The reason of it is that the atmospheric pressure will not support a column of water of a weight greater than its own. This weight is attained, about 15 lbs. per square inch, by a column of water 32 feet high. But from various causes, such as imperfect exhaustion of air above the water in the pump, the friction of water on the sides of the pipes, and the opening of the valves, 26 feet is about the greatest practical height it is possible to draw water by a common pump.

168.—Vacuum means, properly speaking, vacancy. Commonly amongst engineers vacuum is spoken of as an absence of pressure; the one definition includes the other. A vacuum cannot work an engine: atmospheric pressure, however, can do so when it is allowed to exert itself upon the piston through a vacuum being produced on one side. When the temperature of the water in the condenser is 212°, there can be no vacuum in the condenser, as that temperature is the boiling point of water at the atmospheric pressure, and ebullition would follow the slightest attempt to create a vacuum.

169.—A thermometer is an instrument by means of which the temperature or hotness of bodies is measured. The mercurial thermometer consists of a glass tube of very fine bore, having a bulb of thin glass at one extremity, and closed at the other. The bulb and part of the tube contains mercury; the rest of the tube is a vacuum, and affords space for the expansion of the liquid. A graduated scale is attached to the tube to indicate the expansion of the mercury—the graduation of the instrument being effected in the following manner:—Two invariable or fixed points are selected, viz., the *freezing point* and the *boiling point* of water at the sea level, and under the mean pressure of the atmosphere. The former point is obtained by plunging the tube into melting ice, when the column gradually sinks, and eventually comes to rest; the lowest level of the mercury is then marked off on the scale attached to the tube. The latter point is determined by placing the instrument in the steam escaping from boiling water; this being done, the mercury rises and finally settles at a certain height—the highest level attained is marked off as before. The space between these two fixed points thus determined is divided into a number of equal and arbitrary parts, and the instrument receives different names according to the number of equal parts into which it is divided. Thus, if the freezing point be called 32°, and the boiling point 212°, giving 180 equal divisions between these points, it is called Fahrenheit's, and is the one commonly used in England; if the space is divided into 100 equal parts, and the freezing point be called 0°, it is called a Centigrade thermometer; and if, again, the freezing point be called 0° and boiling point 80°, it is called Reaumer's thermometer. The graduations in every case are carried up and down the scale as far as the tube of the thermometer extends. The principle upon which its construction depends is, that within certain limits bodies expand or contract with the increase or decrease of the degree of temperature to which they are subject. The thermometer is used by marine engineers to ascertain the temperature, and from thence the density or saturation of the water in the boiler.

170.—The temperature of melting ice is 32° on Fahrenheit's scale for thermometers, which is the one in general use in this country. The temperature of boiling water at atmospheric pressure = 212° Fahr. Steam at 60 lbs. pressure by the steam gauge = 311° Fahr. Steam of the same pressure, but superheated, may probably be = say 318° Fahr. The smoke generally passes up the funnel at a temperature of about 500° Fahr. A temperature of 600° in the heated gases passing up the chimney give the most efficient draught. The temperature of water in the hot-well from 100° to 120° Fahr.

Heat travels from point to point in space in three ways: (1) by conduction; (2) by convection; (3) by emanation or radiation.

171.—By conduction of heat is meant the flow of heat through an unequally heated body from places of higher to places of lower temperature; or conduction may be defined to be the transfer of heat between two bodies, or parts of a body, which touch each other. It is distinguished into **internal and external** conduction, according as it takes place between the parts of one continuous body, or through the surface of contact of a pair of distinct bodies. When the end of a metal bar is heated, a certain increase of temperature is presently observed along the bar; when the heat is transmitted in the mass of the body itself, as in this case, it is said to be **conducted**. Another example of conduction is the heating of the water in the boiler, from the heat being conducted to the water through the particles in the plate between, and in the engine by the steam jacket that often surrounds the cylinder to replace the heat lost by condensation. Bodies conduct heat with different degrees of facility. **Good conductors** are those which readily transmit the heat, such as the metals, while **bad conductors**, to which class belong the resins, glass, wood, and more especially liquids and gases, offer a greater or less resistance to the transmission of heat. A knowledge of the difference in the conducting powers of substances will teach an engineer on which to bed or surround his boilers, so that the least possible heat may be conducted out of it; also steam cylinders and steam pipes are frequently coated with felt or wood, because they are bad conductors. As an instance of the bad conducting property of water, it may be mentioned, that when steam is first raised in a boiler, the hand may be applied to the lower part, although the temperature of the water be 240° or 250°. The non-conducting property of water must be borne in mind in the construction of steam boilers.

172.—Convection or carrying of heat means the transfer and diffusion of the state of heat by means of the motion of the particles of that mass. Liquids and gases being bad conductors, transmit heat by convection. The water in a boiler is heated by convection, which may be explained thus:—When the particles of water in immediate contact with the hot plates or tubes become heated, which causes them to expand, and therefore their specific gravity becomes less than that of the upper strata; consequently, a displacement of the particles ensues, the lighter rise to the surface and are replaced by the heavier ones, which in their turn are heated and are replaced by the others, and so by this process, the heat from the surface having passed through the plate is conveyed throughout the general mass. Again, this principle explains why fires in boilers should be applied as low down as possible; the surfaces exposed to the fires should be large, and there should be a facility for the heated particles to rise to the surface. The heat from the walls of the steam cylinder is conveyed to the condenser by convection. The draught in a chimney is also an illustration of the principle of convection, as the column of air and gases in the chimney being at a higher temperature than an equal column outside the chimney, the hot air column is pressed upwards by the cold air column.

173.—By radiation of heat* is meant the heating effects produced by direct rays from a hot body, for as a bright body sends out rays of light in all directions, in straight lines, so does a hot body send out rays of heat in all directions, in straight lines. Bodies are continually radiating heat. A body in the neighbourhood of other bodies hotter than itself, receives more heat than it gives out, and in consequence rises in temperature. From this it will be seen that, unless the radiation from a hot boiler or steam-pipe is prevented as far as possible, by the application of some non-conducting substance, a great loss of heat will take place—all the surrounding objects being at so much lower a temperature.

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* "The phrases radiation of heat and radiant heat are not quite scientifically correct, and must be used with caution. Heat is certainly communicated from one body to another by a process which we call radiation, which takes place in the region between the two bodies; we have no right, however, to speak of this process of radiation as heat."—*Theory of Heat*, by J. Clerk Maxwell, M.A.

174.—(1) **Heat** from the glowing fuel to the furnace crowns is the heat of radiation. (2) Heat passing from one side of the furnace crown-plate to the other, is the heat of conduction. (3) Heat passing from the steam-pipe in the engine room is heat by radiation. (4) Heat of evaporation is the heat of convection.

175.—**The effective heating surfaces** in a marine boiler are as follows:—(1) The furnace crown-plates (taking about ⅝ of the circumference of the furnaces, (2) the flame-box back and sides, (3) the outside surfaces of the tubes, (4) the tube-plates back and front, with the areas of the tubes deducted. The objection to vertical heating surfaces is that the flame, heated air, and gases having a strong tendency to rise, do not impinge upon a vertical heating surface with as much effect as a horizontal one.

176.—The **parts** of a marine engine **exposed to danger** when the temperature is below freezing point are—all pipes, air-pump, circulating, feed, and bilge-pumps: in other words all the parts through which the sea water passes.

177.—The **precautions necessary** in cold climates when the temperature is below freezing point is to keep all the pipes and pumps covered to keep their temperature as high as possible, and the water within them to be kept constantly in motion, or if obliged to stand, it is necessary to draw all the water out of them.

178.—If a boiler or one of a set of **boilers gets short of water**, although the feed-valve is open its proper amount, it may be due to more than one cause. The first and most probable reason would be that the boiler was leaking badly; another reason might be that the water in the hot-well is too hot to be drawn into the suction-valve; or, the spring of the escape-valve may have been accidently relieved of its load, thus causing all the water to pass through it, or the packing at the feed-pump may be too slack; or again, in a set of boilers, the one boiler may be priming into another and so get short of water.

179.—**Furnace bars** are usually made of cast iron, about 1¼ inch thick at the top to ½ an inch at bottom. The space between them generally runs about ½ to ⅝ of an inch. The bars are put farther apart for Welsh coal, as it contains more carbon, and consequently requires a supply of more oxygen or air for combustion than Newcastle coal.

180.—Newcastle coal burns faster than Welsh. Newcastle coal is the flaming coal. Newcastle makes the most smoke.

181.—The quantity of **steam** coal that can be burnt per day in four furnaces, each 3 feet wide and of about the usual length of 5 feet, having about 60 square feet of surface, about 10 tons at the lowest computation, because in marine boilers the rate of combustion in the furnaces is from 16 to 20 lbs. of steam coal per square foot of fire grate per hour.

A simpler rule is, that 1 foot in width of fire bar equals 1 ton of coal per day; thus, in the example above, the total width of four furnaces is $3 \times 4 = 12$ feet; hence, allowing 1 ton for each foot of width, we have 12 tons per day.

$$\text{Thus } \frac{60 \text{ ft.} \times 16 \text{ lbs.}}{112} = 8{\cdot}3 \text{ cwt. per hour} = \frac{8{\cdot}3 \times 24}{20} = 10 \text{ tons per day.}$$

182.—The number of tons of **steam** coal that will be burnt per day, with common average engines, to drive an ordinary steamer of 40 feet beam 10 knots an hour, by steam alone, will be about 32 tons. Taking the immersed midship section for a ship of 40 feet beam at 800 square feet, she will require about 1200 I.H.P. to drive her at the rate of 10 knots per hour, which at 2½ lbs. of coal per I.H.P. will equal 32 tons. An approximate rule sometimes used is that the square of the beam of a steamer with average engines is about equal to the quantity of coal that would be consumed in 40 days, and as $\frac{40^2}{40} = 40$, then the consumption will be 40 tons per day for ordinary condensing engines.

$$40^2 = 40 \times 40 = 1600\text{; then } 1600 \div 40 = \text{tons per day.}$$

183.—The number of tons of **steam** coal that will be burnt per day with a pair of common average engines, surface condensers, the cylinders 40 inches diameter, doing average work, will be about 15 tons; for, assuming that the engines in question are of 100 nominal horse-power (that being in a measure settled by the diameter of the cylinders) and with 40 lbs. pressure of steam in the boilers, and a piston speed of 350 per minute, they would indicate four times the nominal horse-power (at least), and that the consumption is 3½ lbs. per indicated horse-power per hour, then the quantity of coal consumed in a day of 24 hours will be 15 tons. An approximate rule sometimes used is that "if the diameter of one of the cylinders of a pair of surface condensing engines be squared and divided by 100, the result will be equal to the consumption per day": thus—

$$40^2 = 40 \times 40 = 1600\text{; then, } \frac{1600}{100} = 16 \text{ tons per day.}$$

184.—If in a pair of **inverted cylinder direct-acting engines** there is a liner half-an-inch thick between the ahead eccentric-rod and the eccentric-strap, and if, on overhauling the engine, this piece should be lost or forgotten, it would cause the valve to work half-an-inch lower down in the casing than formerly, and on the upstroke the admission of steam would take place later, the upper exhaust will open later, the cut-off be earlier, and the exhaust would close earlier; on the down stroke the admission would take place earlier, opening of the exhaust from the lower end earlier, and the cut-off and the closing of exhaust proportionately later. The probabilities are that if the slides of such an engine had less than half-an-inch lead at the bottom, the engine would not turn the bottom centre.

185.—In a pair of inverted cylinder direct-acting engines driving a right hand screw, the pressure on both the up and down strokes will be greatest on the port cross-head guide bars.

186.—In the case of a screw propeller getting loose and having a little play on the shaft sideways on the key or feather, the noise and vibration would be unmistakeably felt in the engine room.

187.—To prove whether the centre line of the trunnions of an oscillating engine be fair with the centre line of the main shaft, disconnect the piston-rod from the crank-pin and then putting the crank alternately on top and bottom centres, and at half stroke, see whether the piston-rod takes the bearing fairly in each position.

188.—The fairness of a line of screw shafting can be tested without lifting the shafts, by taking out the coupling bolts and noticing of the shaft falls out of the line, which will show the amount the shaft is down, or by stretching a line alongside the shaft and measuring the shafting at different points along the line.

PROPERTIES OF STEAM.

384. **Steam, or Vapour of Water.**—Water gives off vapour at all temperatures. The vapour given off has a certain density and elastic force corresponding to each temperature.

The density and elastic force of the water given off are the greatest possible for the temperature, that is to say, if an attempt be made to increase them by compressing the vapour into a smaller bulk while the temperature remains the same, some of the vapour will be reduced to the liquid state, the density and elastic force of the remainder suffering no change.

If a given weight of this vapour, while the temperature remains constant, expand into a larger bulk, the density and elastic force will follow BOYLE and MARRIOTTE's law. If its temperature be increased, the density and elastic force will follow DALTON and GAY-LUSSAC's law.

If the elastic force be less than the external pressure on the surface of the water, vapour is given off only at the surface. This process is called *Evaporation*. When the temperature rises so high that the elastic force becomes equal to the external pressure, vapour is given off in bubbles throughout the whole body of the water; this process is called *Ebullition*. The vapour given off when the water boils is called *Steam*.

The boiling point of water depends upon pressure.—The temperature at which water boils under the standard atmospheric pressure of 30 inches of mercury at the sea level, or 14·7 pounds on the square inch, is called its *Boiling Point*. The boiling point of water on Fahrenheit's scale is 212°.[*]

Water may be made to boil at any lower temperature by reducing the external pressure, while an increase of pressure will raise the temperature of ebullition to any required degree.[†]

The weight and density of steam.—The specific gravity or absolute density of steam, that is to say, the ratio of its weight, bulk for bulk, to that of atmospheric air of the same temperature and elastic force, has been shown by REGNAULT to be 622 : 1000, which is nearly identical with the earlier determination of GAY-LUSSAC, namely 10 : 16.

Relative volume.—The ratio of the volume of steam to the volume of the water from which it is produced is called its *Relative Volume*. DE PAMBOUR's definition is, "The relative volume of steam is the quotient of the absolute volume of the steam by the corresponding volume of water."

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[*] In measuring temperatures we employ Fahrenheit's scale as the one in common use, although the Centigrade scale is far more convenient, and it is to be hoped may ultimately be adopted.

[†] "The boiling point of water is that temperature at which the tension of its vapour exactly balances the pressure of the atmosphere."—TYNDALL's *Heat as a Mode of Motion*.

Measure of the pressure of steam.—The pressure of steam is measured by atmospheres. Steam of 15 lbs. pressure is termed steam of one atmosphere, of 30 lbs. pressure, of two atmospheres, &c. It is frequently used as high as six or seven atmospheres; but even ten, or 150 lbs. pressure is employed. Steam below two atmospheres is termed *low pressure steam*, and all pressures above, *high pressure steam.*

The latent heat of steam.—When a cubic inch of water is converted into steam at 212°, it nearly fills one cubic foot; from this it follows that the particles of the water in passing from the state of liquid to that of steam, recede from each other to twelve times their original distance.

The heat which is required to effect this change is insensible to the thermometer, and for this reason, was called by its inventor, Dr. Black, *latent heat.** Its whole energy is expended in so modifying the molecular forces that the particles of the water assume a new arrangement, at a much greater distance, constituting that state which is commonly called vapour or steam.

In order to calculate the amount of this heat, after the manner first adopted by Dr. Black, let the following definition of a unit of heat be adopted:—

Definition of a Unit of Heat.—*The amount of heat which is required to raise one pound of water through one degree of temperature in the Fahrenheit scale, is a Unit of Heat.*

Let a pound of water at 32° Fahr., having a thermometer immersed in it, be placed over a constant source of heat. As the heat enters the water the temperature will increase until it reaches 212°, the boiling point under the standard pressure. After this the heat which enters the water produces no impression on the thermometer, which continues to stand at 212°, but seems to be altogether expended in converting the water into steam. If the time required to raise the water from 32° to 212°, *i.e.*, through 180°, be observed, it will be found to be five-and-a-half times as long as will be required to convert it into steam. During the first period the water had absorbed 180 units of heat, and, therefore, during the second period $183 \times 5\frac{1}{2}$ or 990 units of heat. According to this method of proceeding the latent, or potential, heat of steam would appear to be 990 units.

More accurate experiments have since been derived for determining this important element; on the authority of Regnault it is now generally assumed that the latent heat of steam at 212° = 966·6 units.

The total heat of steam.—Although the whole amount of heat in a given mass of matter cannot be estimated, yet it is usual to call the amount over and above that which is contained at a certain temperature, say, the zero of the Fahrenheit scale the total amount; according to this view, the total heat of a pound of steam at 212°, is the sum of 212° and 966·6 units; or, total heat of steam at 212° = 1178·6 units.

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* "*Definition.—Latent heat is the quantity of heat which must be communicated to a body in a given state in order to convert it into another state without changing its temperature.*

We here recognize the fact that heat when applied to a body may act in two ways—by changing its state, or by raising its temperature—and that in certain cases it may act by changing the state without increasing the temperatures."—*Theory of Heat*, by J. Clerk Maxwell, M.A., LL.D.

It was for a long time a disputed point whether the total amount of steam raised at different temperatures was a constant quantity or not. WATT maintained the doctrine that as the sensible heat increased the latent diminished by equal degrees so that the total amount remains constant.

SOUTHERN and CREIGHTON, on the other hand, maintained that the latent heat of steam at all temperatures is constant, and that the total heat is the sum of this constant quantity and the sensible heat.

REGNAULT, as the result of an elaborate series of experiments in steam varying from 3 lbs. to 200 lbs. pressure, arrived at the conclusion that the total amount of heat in a quantity of steam increases slowly with every increase of temperature. He constructed the following formula, which gives pretty nearly the total amount of heat in steam at all temperatures,

Total heat of steam = 1081·94 + 0·305 T (Fahr.)

The following Table, taken from his results, shows that there is a difference of about 117° between the total heat of the vapour of the water at pressures corresponding to 32° and 466° respectively.

SENSIBLE AND LATENT HEATS OF STEAM,* BY M. REGNAULT.

Temperature in Degrees Fahrenheit.	Latent Heat.	Sum of Sensible and Latent Heats.	Temperature in Degrees Fahrenheit.	Latent Heat.	Sum of Sensible and Latent Heats.
32	1092·6	1124·6	248	936·6	1187·6
53	1080·0	1130·0	266	927·0	1193·0
68	1067·4	1135·4	284	914·4	1198·4
86	1054·8	1140·8	302	901·8	1203·8
104	1042·2	1146·2	320	889·2	1209·2
122	1029·6	1151·6	338	874·8	1212·8
140	1017·0	1157·0	356	862·2	1218·2
158	1004·4	1162·4	374	849·6	1223·6
176	991·8	1167·8	392	835·2	1227·2
194	979·2	1173·2	410	822·6	1232·6
212	966·6	1178·6	428	808·2	1236·2
230	952·2	1182·2	446	795·6	1241·6

Condensation of Steam.—When steam parts with its (potential) latent heat, it is reduced to the state of water having the same temperature.

If cold water be injected into hot steam, the steam parts with its heat to the water, and the result is a quantity of water of intermediate temperature.

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* The student is recommended to study carefully *An Introduction to the Science of Heat*, by TEMPLE AUGUSTUS ORME. London: Groombridge and Sons. The reader, after perusing this, is recommended to study Dr. TYNDALL'S *Heat as a Mode of Motion*, several portions of which are master-pieces of lucid explanation. Dr. BALFOUR STEWART'S *Elementary Treatise on Heat*, Prof. TAIT'S *Thermodynamics*, and *The Theory of Heat*, by J. CLERK MAXWELL, M.A. (Longmans' Text Books of Science), will be most valuable aids to him when he has a connected view of the main facts of the science.

Thermal Unit.—A certain quantity of heat, with which all other quantities are compared, is called a *Thermal Unit.*

A thermal unit of heat is the heat necessary to raise a pound of water one degree in temperature.*

"The quantity of heat necessary to raise one pound of water 1° Fahrenheit in temperature is equal to that generated by a pound weight falling from a height of 772 feet against the earth, or it would raise 772 pounds 1 foot high."

Foot-Pound.—The term "foot-pound" has been introduced to express in a convenient way the lifting of a pound to the height of a foot. Thus the quantity of heat necessary to raise the temperature of a pound of water 1° Fahrenheit being taken as a standard, 772 foot-pounds constitute what is called the *mechanical equivalent of heat.*† If the degrees be centigrade, 1,390 foot-pounds constitute the equivalent.‡

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* GARNOT's *Physics*, page 401.

† Dr. MAYER, of Heilbroun, in the year 1842, determined the exact equivalance which exists between a quantity of heat and work done expressed in foot-pounds. Founding his calculations on previous experiments on the specific heat of gases, he arrived at the following results, viz., that a unit of heat converted into work was expressed by 669 foot-pounds.

In the year 1843, Dr. JOULE, of Manchester, independently of Dr. MAYER's labours, and by the method of direct experiment, determined the mechanical equivalent of heat. As a final result of an elaborate and varied series of experiments, he arrived at numbers:—

$$J = 772 \text{ foot-pounds.}$$

This number expresses the whole work due to the quantity of heat which is able to raise one pound of water through one degree Fahrenheit.

‡ TYNDALL's *Heat as a Mode of Motion*, 4th edition, page 40.

PROPORTION OF PARTS.

Modern engines work to about 4 to 5 times N.H.P.

Grate surface required per N.H.P.—about ¾ of a square foot; **heating surface**—about 22 square feet; **coal burnt on grate**—about 16 lbs. per square foot per hour, which is about 12 lbs. per hour per N.H.P.; and if engines work up to 5 times the N.H.P., this would be 2½ lbs. per hour per I.H.P.

Weight of water evaporated by each pound of coal—from 8 to 14 lbs. In practice 10 lbs. may be considered good.

Boiler space—One cubic yard per N.H.P., of which not more than one-half should be water space, and not less than one-half steam space.

Water required to turn into steam per N.H.P.—from 1½ to 2 cubic feet.

Sectional area of tubes—10 square inches.

Area above the bridges—14 square inches.

Area of safety-valves—½ square inch for every square foot of grate surface. ⅜ square inch of valve surface equals a N.H.P.

Area of chimney—7 square inches.

Area of piston for low-pressure condensing engine—about 22 square inches.

High-pressure (non-condensing engine)—11 square inches.

Size of shaft—diameter a little less than ⅙ diameter of low-pressure cylinder.

Main steam pipe—the same diameter as shaft.

Crank pin—about ¼ diameter less than the shaft.

Eduction pipe—about ⅓ the diameter greater than the steam pipe.

The piston-rod—about $\frac{1}{10}$ the diameter of low-pressure cylinder.

The air-pump—capacity between ⅙ and ⅛ of that of low-pressure cylinder.

ANSWERS.

NOTATION, *Pages* 11 *and* 12.

1. 100. 2. 101. 3. 110. 4. 9009. 5. 9090. 6. 9909. 7. 5074. 8. 10700. 9. 90090. 10. 305000. 11. 900900. 12. 505550. 13. 1003008. 14. 5030049. 15. 9900006. 16. 58000009. 17 70302441. 18. 222000035. 19. 604060005. 20. 800003033. 21. 900900900. 22. 700000007. 23. 180000000. 24. 500000000. 25. 580245192. 26. 707007077.

NUMERATION, *Page* 13.

1. Forty-three. 2. Sixty. 3. Twelve. 4. Twenty-one. 5. One hundred. 6. One hundred and one. 7. One hundred and ten. 8. Five hundred. 9. Five hundred and five. 10. Five hundred and fifty. 11. One thousand. 12. Two thousand and twenty. 13. Three thousand three hundred and three. 14. Four thousand and four. 15. Seven thousand seven hundred and seven. 16. Eight thousand eight hundred and eighty. 17. Eighty-seven thousand and fifty-four. 18. Seventy thousand seven hundred and seven. 19. Sixty thousand eight hundred and eighty. 20. Ninety-nine thousand four hundred and four 21. Nine hundred and three thousand seven hundred and fifty-six. 22. Two hundred and two thousand two hundred and two. 23. Four hundred thousand and four hundred. 24. Five hundred and fifty thousand five hundred and fifty. 25. One million and one. 26. Eight million and forty-seven thousand three hundred and twenty-eight. 27. Four million and ninety thousand and three hundred. 28. Five million two hundred and ten thousand and seven. 29. Six million and thirty thousand four hundred and five. 30. Nine million nine thousand and nine hundred. 31. Forty-one million forty-one thousand and fourteen. 32. Three million and six. 33. Twenty million eighty-four thousand two hundred and sixteen. 34. Five million one thousand eight hundred and sixty. 35. Eight million eighty thousand eight hundred and eight. 36. Fifty-five million seven hundred thousand and five. 37. Seventy-six million and fourteen thousand and fifty-nine. 38. Six million six thousand six hundred and six. 39. Fifty-six million seven hundred thousand five hundred and five. 40. One hundred and twenty million fifteen thousand and fifteen. 41. Two hundred and two million two hundred and two thousand and two hundred. 42. One hundred million one hundred thousand one hundred and one. 43. Two hundred and seventy-five million eight thousand and five. 44. One hundred million ten thousand and one. 45. Seventy-nine million thirty thousand one hundred and eighty-four. 46. Four hundred and eight million seventy-six thousand and thirty-two. 47. Four hundred and one million four hundred thousand and fifty-six. 48. Nine hundred and eight million five hundred thousand and sixty.

ADDITION, *Page* 15.

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Page 26.

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(9) 16-5 (11) 4-10 (12) 2-11
5. (2) 31261 (3) 2744-1 (4) 488-1 (5) 128 (6) 42-5 (7) 17 (8) 7-5
(9) 3-6 (11) 1-1 (12) 0-8
6. (2) 146406 (3) 12853 (4) 2287-2 (5) 599-3 (6) 200-4 (7) 79-4
(8) 35-5 (9) 17-5 (11) 5-3 (12) 3-1
7. (2) 154907 (3) 13599-1 (4) 2420-1 (5) 634-2 (6) 212-2 (7) 84-1
(8) 37-6 (9) 18-5 (11) 5-6 (12) 3-3
8. (2) 73531 (3) 6455-1 (4) 1148-3 (5) 301 (6) 100-5 (7) 40 (8) 17-7
(9) 8-7 (11) 2-7 (12) 1-6

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1. 75638-2
2. 542370
3. 34785-7
4. 542370
5. 220950-14
6. 150898-27
7. 127474-28
8. 100045-25
9. 461002-18
10. 725983
11. 706321-515
12. 95248-834
13. 8607936214-143
14. 576845
15. 245728
16. 567977-3852
17. 107356-2031
18. 100010-10
19. 999000-1000
20. 9000090000-10000
21. 8485852-43614
22. 1068392-117002
23. 544023-195858
24. 70030401
25. 730956788-980154321
26. 63245553
27. 9007609600
28. 304529878174100474
29. 4342944819-49016833

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1. 46351967376353 3-5
2. 27201490438560034-10
3. 1582874324701-32
4. 95022741046776-8
5. 149644594409277-63
6. 4489339863279-30
7. 27206980239559-123
8. 34045491087172-1

1. 18715729675979 29-46
2. 962869563912-25
3. 329218107-670
4. 6897234900
5. 5719069838-108
6. 1035-640
7. 40316322081 10056-69
8. 1039682584-834
9. 3396-5094687
10. 3854-26167

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1. 33; 136; 26
2. 24; 30; 132
3. 26; 24
4. 185635-1
5. 147802420
6. 8903·17
7. 11769431
8. 41097703
9. 3; 84

REDUCTION, *Page* 43.

1. 87854 farthings; 68810 farthings; 65998 halfpence; 5753 threepences; 1563 threepenny pieces; 1516746 halfcrowns.
2. £299 6s. 2d.; £9 17s. 1¾d.; 23764 halfcrowns; 441 gs. 8s.; £5909 18s. 9½d.
3. 458589 ounces; 71701738 drams; 162 tons 17 cwt. 3 qrs. 25 lbs. 9 oz.; 21411 drams; 617661 ounces; 23597760 drams; 269 tons 0 cwt. 1 qr. 26 lbs. 3 oz; 43 lbs. 3 oz. 10 dwt. 22 grains (troy); 35 lbs. 10 oz. 14 grains (avoirdupois); 8508 grains (troy); 27901 tons 15 cwt. 2 qrs. 24 lbs.; 6 st. 13 lbs. 14 oz. 315 grs.
4. 13360 poles; 40534565 inches; 1056816 feet, or 12681792 inches; 157 mls. 6 fur. 24 pls. 5 yds. 2 ft. 4 in.; 1080303 fur. 1 pl. 1 yd. 0 ft. 6 in.; 4516300900 sq. in.; 428 yds. 18 ft. 128 in.; 1730 acres 1 rd. 14 pls. 25½ yds.
5. 4504 pints; 11432 gals. 2 qts.; 180 pints; 160 pecks; 1129 pints; 9259 hhds. 14 gals.; 15331056 qts.; 2560230 gals. 1 qt.
6. 22266000 secs.; 2674859 secs; 19 yrs. 0 hrs. 40 mins. 16 secs; 31556928 secs.; 3 mos. 25 dys. 17 hrs. 46 mins. 40 secs.
7. 17280 lbs., avoirdupois. 8. 253·2 grains, troy. 9. 35 ft. 1568 in.

COMPOUND ADDITION, *Pages* 44 *and* 45.

1. £327537 13s. 3¼d. 2. £332311 18s. 5d. 3. £412105 6s. 11d.
(1 & 2) £659849 11s. 8¼d. (2 & 3) £744417 5s. 4d.
4. £1050668258 15s. 6½d. 5. £1500543139 10s. 5½d.
6. £904933435 0s. 9½d. 7. £1080997983 7s. 5¼d.
8. 22913 tons 6 cwt. 1 qr. 9 lbs. 1 oz. 14 drs. 9. 3659 lbs. 6 oz. 3 dwts.
10. 405 lbs. 1 oz. 5 drs. 0 sc. 8 grs.
11. 15875 mls. 7 fur. 23 pls. 0½ yd. 2 ft. 8 in, or 15875 mls. 7 fur. 23 pls. 1 yd. 1 ft. 2 in.
12. 23361 acres 3 rds. 7 pls. 7¼ yds. 4 ft. 61 in., or 23361 acres 3 rds. 7 pls. 7 yds. 6 ft. 97 in.
13. 2973 yds. 17 ft. 24 in. 14. 15 ft. 2 in.

COMPOUND SUBTRACTION, *Page* 46.

1. £33 7s. 2¼d.
2. £120 10s. 8½d.
3. £117 14s. 9½d.
4. £2077 17s. 7¾d.
5. £2607 15s. 8d.
6. 200 tons 18 cwt. 3 qrs. 26 lbs. 15 ozs.
7. 2 lbs. 7 ozs. 0 dwts. 22 grs.
8. 3 mls. 3 fur. 109 yds. 0 ft. 9 in.
9. 1308 acres 2 rds. 29 pls.
10. 8 sq. yds. 1 sq. ft. 125 sq. in.
11. 2^h 35^m 34^s.
12. 1 c. yd. 18 c. ft. 1493 c. in.
13. 2 gals. 2 qts. 1 pt.
14. 3 mos. 2 wks. 6 dys.
15. 8 yrs. 39 wks. 5 dys.
16. 6 yrs. 346 dys. 14 hrs.

COMPOUND MULTIPLICATION, *Pages* 50 *and* 51.

1. £639 12s. 7½d.; £1066 1s. 0½d.; £1492 9s. 5½d.; £1918 17s. 10½d.
2. £24097 13s. 9d.; £36146 10s. 7½d.; £48195 7s. 6d.; £60244 4s. 4½d.
3. £6829 0s. 4d.; £7682 12s. 10½d.; £8536 5s. 5d.; £9389 17s. 11½d.; £10243 10s. 6d.
4. £9082 10s. 11¼d.; £24220 2s. 6d.; £15137 11s. 6¾d.; £21192 12s. 2¼d.; £36330 3s. 9d.
5. 6 cwt. 1 qr. 26 lbs. 15 ozs. 8 drs.
6. 41 tons 18 cwt. 1 qr. 18 lbs. 10 ozs.
7. 11 cu. yds. 1 cu. ft. 956 cu. in.
8. 70 gals. 3 qts. 0 pts. 0 gills; 88 gals. 1 qt. 1 pt. 2 gills.; 106 gals. 0 qts. 1 pt.
9. £67565 6s. 3d.; £57913 2s. 6d.; £130304 10s. 7½d.; £135130 12s. 6d.; £101347 19s. 4½d.; £56304 8s. 6½d.; £43434 16s. 10½d.
10. £223121 0s. 2¾d.; £397737 9s. 1¼d.; £252223 15s. 0½d.; £329831 1s. 2½d.; £1037997 14s. 11¾d.
11. 115 tons 0 cwt. 2 qrs.; 244 tons 8 cwt. 2 qrs. 7 lbs.; 603 tons 17 cwt. 2 qrs. 14 lbs.; 8698 tons 15 cwt. 1 qr. 7 lbs.; 273 tons 3 cwt. 2 qrs. 21 lbs.; 2746 tons 4 cwt. 1 qr. 21 lbs.
12. 302 gals.; 320 gals. 3 qts. 1 pt.; 358 gals. 2 qts. 1 pt.; 7568 gals. 3 qts. 1 pt.; 28312 gals. 2 qts.
13. 57 tons 5 cwts. 1 qr. 21 lbs.
14. 1841 tons 10 cwt. 1 qr. 12 lbs.; £1383 6s. 8d.
15. 30 mls. 6 fur. 32 perches.
16. 382 mls. 7 fur. 91 ft. 10½ in.
17. 375 feet per hour.

COMPOUND DIVISION, *Page* 51.

1. £42 3s. 9d.; £28 2s. 6d.; £25 10s. 8d.
2. £27 9s. 3¼d.; £47 8s. 5½d.; £27 9s. 3¼d.
3. £275 10s. 2½d.; £38 17s. 6¼d.; £632 17s. 4⅝d.
4. £554 3s. 8d.; £58 13s. 4¼d.; £259 13s. 7¼d.
5. £962 4s. 1½d.; £427 19s. 5½d.; £843 9s. 6¼d.
6. 3 lbs. 4 oz. 13 dwt. 1 gr.; 6 lbs. 11 oz. 17 dwt. 9 grs.; 14 tons 5 cwt. 2 qrs. 17 lbs.
7. .8 cwt. 3 qrs. 25 lbs. 13 oz.; 27 yds. 2 ft. 6 in.; 3 mls. 4 fur. 16 per. 4 yds.
8. 4 mls. 7 fur. 23 per. 3⅛ yds.; 27 acres 3 roods 27 poles; 57 gals. 3 qts.

Page 53.

1. £79 5s. 8¼d.; £17 0s. 0½d.; £19 16s. 6¾d.
2. £4 19s. 8¼d.; £7261 1s. 11¼d.; £22 2s. 2d.
3. £12 15s. 11½d.; £17 7s. 9¼d.
4. £8690 8s. 0¼d.; £41 13s. 5¼d.; £100 17s. 4d.
5. £26 8s. 8¾d.; £8820 16s. 3¾d.; £845 17s. 4½d.

1. £13 8s. 2½d.
2. £15 6 4¾
3. £13 16 5
4. £5 14s. 9⅝d.
5. £7 8 2
6. £4 9 10¼
7. £2 8s. 4¼d.
8. £4 0 0¼
9. £1 2 10¾
10. £0 9s. 5¾d.
11. £0 8 7¾
12. £0 1 2½

Page 54.

1. £17 13s. 8d.
2. £ 3 5s. 9¾d.
3. £ 1 10s. 2¾d.
4. £0 1s. 3½d.
5. £0 2s. 2¼d.
6. £1 8s. 11½d.
7. £ 1 1s. 1¼d.
8. £11 2s. 6d.
9. 236–6.

1. 2435 2. 13. 3. 117 4. 126 5. 1111 6. 25 7. 42 8. 67 9. 248 10. 349.

GREATEST COMMON MEASURE, *Page* 58.

1. 12; 9; 11; 29; 1.
2. 9; 11; 27; 1; 313.
3. 1; 939; 23; 1.
4. 87; 111; 11; 29.
5. 19; 2; 37.
6. 1; 9187; 1.
7. 3; 1.

LEAST COMMON MULTIPLE, *Page* 61.

1. 12; 60; 840; 900.
2. 11803; 18648; 50337; 72.
3. 1890; 3000; 1728.
4. 1008; 2260.
5. 900.
6. 1512.
7. 1008; 79560.
8. 68904; 49098049.
9. 3956121; 1336783.
10. 5754840; 18522000.
11. 1116864; 2549004064039.

MISCELLANEOUS EXAMPLES IN G.C.M. and L.C.M., *Page* 61.

1. $(314725 \times 9 \div 5) = 566505$.
2. $(148995 \div 9 \times 5) = 82775$.
3. 3 the greatest No. of times, 7905 least.
4. 371280.
5. 4.
6. 1391.
7. 2, 3, 4, 6, 9, 12, 18, 36, and 228096.
8. 24 dollars.
9. 160 ounces.
10. 14 minutes.
11. 1 day.
12. 540; 1080; 1620.
13. 300.
14. 1 hour, 15; 12; 10 times.
15. 252. A has 20 heaps, & B has 21 heaps.
16. 23.
17. 8

VULGAR FRACTIONS, *Page* 64.

1. $\frac{35}{7}, \frac{42}{7}, \frac{49}{7}, \frac{56}{7}, \frac{63}{7}, \frac{70}{7}, \frac{77}{7}, \frac{84}{7}$. $\frac{75}{15}, \frac{90}{15}, \frac{105}{15}, \frac{120}{15}, \frac{135}{15}, \frac{150}{15}, \frac{165}{15}, \frac{180}{15}$.
2. $\frac{17}{7}, \frac{31}{7}, \frac{1001}{7}, \frac{9999}{11}, \frac{21809}{101}$.

Page 65.

1. $\frac{43}{9}, \frac{119}{84}, \frac{362}{63}, \frac{113}{21}, \frac{313}{39}, \frac{6699}{840}, \frac{810}{836}, \frac{18013}{17}, \frac{38703}{10001}, \frac{67398738}{2119274}, \frac{16404378843}{169227767}, \frac{60088003}{10001}$, $\frac{28960731468888}{1003001}$.

Page 66.

1. $\frac{1}{8}, \frac{1}{8}, \frac{4}{25}, \frac{3}{5}, \frac{3}{10}, \frac{5}{108}$.
2. $\frac{4}{15}, \frac{16}{147}, \frac{5}{131}, \frac{247}{5691}, \frac{1}{1260}, \frac{1}{1809}$.
3. $\frac{107}{1303040}, \frac{1}{2}, \frac{77}{130013}, 1, \frac{1}{147}, \frac{1}{28}$.
4. $\frac{9}{70}, \frac{1}{120}, \frac{1}{510}, \frac{488}{687}, \frac{13}{21033}$.
5. $\frac{389}{11011}, \frac{23}{20817}$.

Page 67.

1. $2\frac{1}{3}, 3\frac{4}{5}, 4\frac{2}{5}, 13\frac{9}{11}, 16\frac{7}{13}, 143\frac{1}{7}, 4\frac{1}{17}, 3\frac{11}{19}, 2\frac{29}{133}$.
2. $33\frac{11}{13}, 13, 13, 130\frac{27}{50}, 823\frac{29}{37}, 7, 511\frac{128}{161}$.
3. $125\frac{70}{5058}, 3\frac{429602}{3745037}, 59168\frac{487}{1483}, 2\frac{2847751}{7543268}, 12\frac{2884}{8371}, 2329\frac{888}{1847}$.

Page 68.

1. $\frac{19}{7}, \frac{40}{9}, \frac{78}{31}, \frac{871}{48}, \frac{497}{13}, \frac{4909}{71}, \frac{1859}{16}, \frac{1007}{7}$.
2. $\frac{784840}{729}, \frac{10099}{100}, \frac{1000}{3}, \frac{186409}{1001}, \frac{129025}{1441}, \frac{991188}{10011}$.
3. $\frac{1000}{9}, \frac{1008000}{1001}, \frac{1000000}{99}, \frac{1000000}{9}, \frac{185889878}{94898}$
4. $\frac{2417}{1409}, \frac{1808109}{6000}, \frac{881052}{90725}, \frac{5841}{2346}, \frac{183171}{6285}$

Page 69.

1. $\frac{2}{3}, \frac{2}{3}, \frac{3}{4}, \frac{3}{4}, \frac{9}{10}, \frac{1}{2}, \frac{2}{3}, \frac{9}{10}, \frac{1}{3}, \frac{7}{16}$.
2. $\frac{2}{3}, \frac{11}{13}, \frac{7}{9}, \frac{5}{11}, \frac{5}{9}, \frac{17}{20}, \frac{1}{30}, \frac{13}{17}, \frac{1}{3}, \frac{2}{3}$.
3. $\frac{4}{51}, \frac{2}{3}, \frac{5}{51}, \frac{85}{96}, \frac{3}{35}, \frac{11}{30}$.
4. $\frac{60}{161}, \frac{4}{7}, \frac{1}{540}, \frac{11}{15}, \frac{20845}{887}, \frac{2472}{981}, \frac{7048}{11781}, \frac{9003}{38807}$.
5. $\frac{3331}{5360}, \frac{231598}{3497000}, \frac{860149}{3000018}, \frac{138549}{832211}, \frac{859233}{3310600}, \frac{17}{16}, \frac{789211801}{4963109383}$.
6. $\frac{18807}{30020}, \frac{21}{25}$.

Page 71.

1. $\frac{12,\ 8,\ 15,\ 14}{36}$, $\frac{36,\ 54,\ 68,\ 75}{84}$, $\frac{1155,\ 792,\ 616,\ 252}{1386}$, $\frac{675,\ 680,\ 684}{720}$.
2. $\frac{10368,\ 2016,\ 5103,\ 3724}{15876}$, $\frac{1560,\ 1530,\ 1782,\ 1700}{1800}$.
3. $\frac{63,\ 88,\ 102,\ 76,\ 75}{144}$, $\frac{720,\ 378,\ 525,\ 612,\ 80,\ 675}{1260}$.
4. $\frac{162,\ 108,\ 144,\ 24,\ 16}{243}$, $\frac{540,\ 630,\ 216,\ 330,\ 260,\ 345}{900}$, $\frac{1572,\ 225,\ 1110}{2760}$.
5. $\frac{4320,\ 6930,\ 1008,\ 560,\ 1728,\ 3213}{7560}$, $\frac{162,\ 108,\ 252,\ 36,\ 36,\ 32}{486}$.
6. $\frac{1440,\ 2025,\ 3600,\ 3240,\ 3150,\ 3456,\ 4000,\ 3960,\ 2880,\ 2925,\ 2800,\ 1920}{10800}$.
7. $\frac{6930,\ 7546,\ 24745,\ 24206,\ 16016,\ 16874,\ 22295}{35035}$, $\frac{1995,\ 600,\ 2278,\ 2023,\ 1810,\ 3383}{3570}$.
8. $\frac{154,\ 99,\ 84}{231}$, $\frac{195,\ 260,\ 234,\ 270}{390}$, $\frac{140,\ 231,\ 330}{105}$, $\frac{4312,\ 385,\ 6720,\ 16104}{616}$.

Page 72.

1. $\frac{1}{3}, \frac{1}{3}, \frac{7}{40}, \frac{5}{12}, \frac{1}{4}, \frac{77}{2}$. 2. $\frac{13}{16}, \frac{1}{6}, 1, 9\frac{1}{3}$. 3. $1\frac{2}{3}, 3, 6$. 4. $\frac{40}{1263}, 8\frac{202}{363}, 251\frac{92}{185}$.

Page 73.

1. $\frac{95}{116}, \frac{84}{143}, \frac{16}{25}, \frac{133}{166}, 2\frac{6}{13}, \frac{11}{14}, \frac{28}{33}, \frac{119}{600}, 1\frac{23}{27}$.

Page 75.

1. (1) $\frac{8}{11}$. (2) $\frac{13}{16}$. (3) $\frac{29}{32}$. (4) $\frac{63}{86}$.
2. **Greatest** $\frac{5}{8}$, **least** $\frac{4}{9}$, **greatest** $\frac{4}{5}$.
3. **Greatest** $\frac{9}{10}$, **least** $\frac{2}{3}$.
4. **Greatest** $\frac{7}{46}$, **least** $\frac{3}{20}$.
5. $\frac{3}{4}, \frac{28}{37}, \frac{25}{33}, \frac{22}{29}, \frac{19}{25}, \frac{16}{21}, \frac{13}{17}, \frac{10}{13}, \frac{7}{9}, \frac{4}{5}$.

ADDITION OF VULGAR FRACTIONS, *Page* 78.

1. $\frac{2}{3}, 1, \frac{5}{6}, 1\frac{5}{12}, 2\frac{6}{7}$.
2. $\frac{23}{30}, \frac{1145}{1820324}, 5\frac{109}{1170}, 1\frac{11}{105}$.
3. $2\frac{1961}{2100}, 3$.
4. $3\frac{2226}{360360}, 1\frac{3643337143}{3326633100}$.
5. $56712\frac{77}{118}, 10\frac{9}{8000}$.
6. $1\frac{2}{3}, \frac{2707}{2800}, 4$.
7. $1\frac{64}{75}, 1\frac{81877}{362880}$.
8. $319\frac{8299}{8487}$.

SUBTRACTION OF VULGAR FRACTIONS, *Page* 80.

1. $\frac{1}{3}, \frac{7}{30}, 2\frac{1}{2}, 3\frac{1}{3}, \frac{1}{6}, \frac{1}{15}$.
2. $\frac{2}{7}, \frac{19}{40}, \frac{179}{290}, \frac{66}{121}, \frac{5}{12}, \frac{2647}{15625}$.
3. $\frac{1888}{4025}, \frac{1}{33}, \frac{87466}{469455}, \frac{90482041}{118169253}$.
4. $\frac{697687}{1620000}, \frac{376043}{740883}, 1\frac{1}{12}, \frac{37}{35}, 1\frac{9}{35}$.
5. $\frac{91}{100}, \frac{9}{20}, 11\frac{9}{44}, 1\frac{13}{35}, 2\frac{277}{3120}, 1\frac{3867}{7001}$.
6. $\frac{1}{742}, \frac{9}{560}, 1, \frac{1}{164}$.
7. $2\frac{14}{17}, 1\frac{13}{660}$.
8. $12, 11\frac{65}{78}, 11\frac{29}{62}, 11\frac{26}{39}, 11\frac{65}{104}, 11\frac{11}{18}, \frac{109}{609805}$.
9. $\frac{314618}{362880}, \frac{60872}{7036796861}$.
10. $\frac{266914884966891}{1013331361769600}$.

MULTIPLICATION OF VULGAR FRACTIONS, *Page* 82.

1. $\frac{9}{20}, \frac{14}{27}, 2, 8\frac{1}{6}, 15, 35$.
2. $99990, 999900, \frac{15}{48}, \frac{11}{64}, \frac{3}{4}, 1$.
3. $\frac{24072}{41580}, \frac{183664}{189783}, \frac{222}{1627747}, \frac{11626}{30482}, \frac{17}{30}$.
4. $3\frac{7}{12}, \frac{2}{3}$.
5. $1, \frac{466660000000}{616208374}$.
6. $\frac{827}{828}, \frac{48}{55}, 19\frac{371}{703}$.
7. $414\frac{16}{81}, 5\frac{2}{3}, \frac{22}{216}$.
8. $26, 1\frac{7}{18}$.
9. 2.

DIVISION OF VULGAR FRACTIONS, *Page* 84.

1. $\frac{1}{3}, 1\frac{1}{3}, \frac{45}{68}, \frac{10}{21}, 1\frac{7}{20}, \frac{33}{38}, \frac{22}{83}$.
2. $\frac{4}{21}, 7\frac{1}{12}, 14, 169, 26645, 1, 3753\frac{3}{8}$.
3. $\frac{10}{7}, \frac{4}{5}, 1\frac{1}{8}, 1\frac{1}{15}, \frac{14}{15}, \frac{9}{10}$.
4. $\frac{1}{2}, \frac{99}{145}, \frac{7}{9}, 2\frac{76}{333}, 37\frac{8}{9}$.
5. $9\frac{3337}{9333}, \frac{1}{341}$.
6. $14\frac{7}{15}, 8, 19\frac{37}{55}$.
7. $7\frac{19}{33}, \frac{27}{35}, 2\frac{3}{35}$.

REDUCTION OF FRACTIONS, *Page* 89.

1. $\frac{1}{3}, \frac{4}{5}, \frac{11}{12}, \frac{421}{480}, \frac{191}{192}, 1\frac{44}{63}$.
2. $\frac{1}{6}, \frac{5}{16}, \frac{23}{44}, \frac{3}{320}, \frac{1}{8}, 46, \frac{462}{661}, \frac{20}{63}, \frac{1}{280}$.
3. $\frac{67}{560}, \frac{19}{112}, 5\frac{5}{63}, \frac{1}{256}, \frac{2}{3}, \frac{281499}{302782}, \frac{877}{887}, 19\frac{149}{480}$.
4. $\frac{761}{10560}, \frac{3617}{11664}, \frac{3035}{6396}, \frac{36}{1}, \frac{1169}{10368}$.
5. $\frac{79}{88}, \frac{131}{421}, \frac{2005}{28807}$.
6. $\frac{16801}{630}, \frac{11}{13}$.
7. $\frac{1}{4}$.

Page 91.

1. 7s. 6d., 6s. 3d., 1s. 1d., £20 15s. 6d., £22 10s., £99 16s. $9\frac{1}{2}$d. & $\frac{3}{4}$, £1 7s. $0\frac{3}{4}$d. $\frac{83}{868}$.
2. £14 18s. $10\frac{1}{2}$d., £9 10s. 4d., £124 12s.
3. £136 5s. $7\frac{1}{2}$d , £4 1s. $8\frac{1}{2}$d., 3 mls. 25 fur. 2 yds. 1 ft. 6 in., 7^h 12^m, 2 ft.
4. 15 cwt., 1 ton 6 cwt. 1 qr., 7 cwt. 2 qrs., 4 cwt. 0 qrs. 11 lbs. 6 ozs.
5. 4 cwt. 3 qrs. 18 lbs. $8\frac{3}{8}$ ozs., 1 ton 14 cwt. 0 qrs. 23 lbs. 3 ozs., 8263366 miles 66 acres, $\frac{180527624}{1008008001}$. 6. 111 cwt., £11 1s. $1\frac{1}{2}$d.
7. £329, £51562 10s., 16 cwt. 8. £304722, 31296 tons 6 cwt. 2 qrs.
9. 3 cwt. 1 qr. 6 lbs., £566 2s. 11d., 103 lbs., 3^d 11^h 13^m.

Page 93.

1. $\frac{5}{12}$, $\frac{7}{80}$, $\frac{10}{1}$. 2. $\frac{7}{192}$, $\frac{3}{10}$, $\frac{200}{9}$. 3. $\frac{20}{29}$, $\frac{1}{736}$, $\frac{192}{1}$.
4. $\frac{16}{33}$, $\frac{2}{245}$, $5\frac{166}{187}$. 5. $\frac{1}{12}$ troy and $\frac{3}{56}$, avoir. $\frac{3}{896}$. 6. $\frac{14}{3535}$, $\frac{128}{3375}$.

Page 94.

1. 12 cwt. 2 qrs. 14 lbs. 10 ozs. $10\frac{2}{3}$ drs., 6 ozs. 12 dwt. $5\frac{1}{3}$ grs., 4 fur. 39 pls. 2 yds.
2. 5 cub. ft. $110\frac{34}{37}$ cub. in., 2725 days 18 hrs. 34 min.
3. $\frac{2680775}{120862}$ cwt., $\frac{6509767}{86317440}$ tons. 4. $21\frac{88109}{88488}$.
5. The relative values are as 62, 61, 54 (the absolute values being 15s. 6d., 15s. 3d., and 13s. 6d.)
6. $\frac{7}{1860}$. 7. 31 sq. inches, $\frac{1}{80}$ furlong.

NOTATION OF DECIMALS, *Pages* 98 *and* 99.

1. ·1, ·3, ·7, ·33, ·07, ·003, ·0011, ·9178, ·09178, 91·78.
2. ·000028, ·001041, ·00000006, ·0000009, 520·3, ·90, 3·0142.
3. 6·72819, ·000672819, 6728·19, ·0000003, 400·01, 53·00415, 50·0000101, ·00000000501.
4. 7·060, 43·2143, 9·07823457, 1·000001, 35·721341.
5. 53·9, 6·039, 8·000004, 3·7000, 9·000400537, 902·030401.
6. $\frac{7}{10}$, $\frac{7}{100}$, $\frac{7}{1000}$, $\frac{7}{10000000}$, $\frac{827}{1000}$, $3\frac{27}{100}$, $\frac{46897}{100000}$, $456\frac{97}{100}$, $\frac{898}{1000}$, $\frac{893}{100000000}$, $47\frac{82}{125}$, $\frac{10001}{8000000}$.
7. $\frac{1}{9765625}$, $\frac{9}{1562500000000}$, $\frac{2241}{10000}$, $6\frac{9}{100}$, $\frac{2064919}{10000000}$, $\frac{1}{1525878906250000}$.
8. ·073, ·0197, ·000001, ·00261, ·0001001.
9. 1·54, 24·079, 315·008, ·000050, ·001100, ·009.
10. ·1, ·03, ·005, ·105, ·000002, ·000060, 41·08, 1000·001, 30·000006, ·100, ·00002375.
11. 4004, ·0000345, 3·00004, 524000624·0008034, 100000·000001.

12. Six-tenths. Ninety-two hundreds. Five thousand four hundred and ninety-eight ten thousandths. Seven, and seven hundredths. Twenty-six, and four hundred and five thousandths. One millionth. Thirty-seven hundred thousandths. Eleven, and one hundred and one thousand one hundred and one millionth. Four hundred and forty thousand three hundred and eight ten millionths. Eighty-two thousand three hundred and forty-four hundred thousandths. Thirteen thousand two hundred and thirty-six hundred thousandths.

13. Nine, and four hundred and fifty-seven ten thousandths. Four thousand and four, and three hundred and forty-five ten millionths. Three, and four thousand and one ten thousandths. Five hundred and twenty-four million six hundred and thirty-four, and eight thousand and thirty-four ten millionths. Three thousand seven hundred and five thousand millionths. Twenty-four thousand and fifty-six thousand millionths. Seven thousand and five, and six hundred and seventy-four thousand millionths. One hundred thousand, and one ten millionths. Ten, and one thousandth. Nine, and twenty-eight millionths. One, and six thousand and three ten millionths.

14. One, and one millionth. One million and one ten millionth. One hundred millionths. One, and thirteen thousand and four hundred thousandths. Nine, and two hundred and three thousand one hundred and sixty-seven millionths. Four, and three million eight thousand and four ten millionths. Twenty-seven and four million six hundred and twenty-seven thousand three hundred and fifty ten millionths.

15. ·7453, 7·453, 74·53, 48956·21, 489562·1, 4895621, 8764·30071, 87643·0071.
876430·071.
·000531674, ·0000531674, ·00000531674.
·000000000317, ·0000000000317, ·00000000000317
902·030401, 90·2030401, 9·02030401.
16. 478·3, 4783, 47830, 4·783, ·4783, ·04783, 23915, 3587·25.

ADDITION OF DECIMALS, *Page* 100.

1. 745·796, 2935·5073
2. 886·9326, 1680·679
3. 1436·4179
4. 4009
5. 538·6422021
6. 140·1996, 5082·3192995
7. 53·6769
8. 9181·6074975
9. 1·1111, 1·2345
10. 230143·035741
11. 2·9291474

SUBTRACTION OF DECIMALS, *Page* 101.

1. 91·7423, 1427·633782, ·066721522, 24680·12377. 2. 3·431, ·01, ·0011, 9·999999, 189585
3. 39·8479194, ·000099, 7·336606, ·000396, 64·0317753, 8·20001, 699·93, 537·15997
4. 8·000001, 98·99999901, 72·5193401, 2·291. 5. ·378, ·0051, 6·3, 6·999993
6. 272·732875, 41·7011. 7. 0. 8. 5555·6049095.

MULTIPLICATION OF DECIMALS, *Page* 103.

1. 10·0, 10·0, 1190·0, 11·9, ·0119, ·00119.
2. ·000000202, 3·06034, ·000000112, ·00210175.
3. ·0360963, 26·5344, ·000604476, 2·02100.
4. 11505·3503, 417962·601, 257·049, 408·04, ·00043.
5. ·0108575, ·032016, ·000000072, 264396·22160671.
6. ·0306002448. 470116914·436, 536·672000592.
7. ·00164389993, 160·86701632806, ·062884059091 56.
8. 2·5067823, ·000011826009, ·00000000667654 2672.
9. ·013014, 1·5, ·20736. 10. 5·31441, 4·096.

DIVISION OF DECIMALS, *Page* 105.

1. 19·82421, 14·16015, 11·01345, 2·7533625, 1·12637557, 1·66704, 0·83352, ·2667264, ·01058438.
2. 311·487360333, 66·6188183, 2·2258942, 5387·4.
3. ·0000186, ·00186, 45, 450, 4·5.
4. ·03, 74·84, 43206·7, ·00007375, 83671000, ·000000000003, ·061096, ·8, ·4.
5. 14·789983, 255·121, 1210·234426, ·02.
6. 2681·081081, ·0000360074, ·0001, ·006578947, ·00862.
7. 15000, 5060000, ·008, ·0375313.
8. 73939·3$\dot{9}$, 114000, ·125, ·266$\dot{6}$, 3·75.
9. ·0000000125, 125, 125, 24·3. 10. ·00000125, 8·004, ·0002, ·00000584795.
11. ·099, ·0000112, ·009375, 2000. 12. ·10, 10·0, ·001, ·001, ·1000, 10000, ·00001.
13. (1) ·0882352941 (2) ·017256637168 (3) ·0000009699321047526673132
(4) 0090009000900090009000900 (5) ·000000123456791 (6) ·001163
(7) 126·325277 (8) ·007141.

REDUCTION OF DECIMALS, *Page* 112.

1. ·1875, ·225, ·208, ·136, ·848, ·0176, ·4375, ·5.
2. ·$\dot{3}$, ·$\dot{1}4285\dot{7}$, ·$\dot{0}\dot{9}$, ·0$\dot{5}88235294117 64\dot{7}$, ·$\dot{9}\dot{0}$, ·$\dot{4}$, ·$\dot{5}$, ·$\dot{1}\dot{3}$, 2·$\dot{3}4\dot{5}$, ·00$\dot{8}$, ·$\dot{1}\dot{7}$.
3. ·0$\dot{3}448275862068965517241379\dot{3}1$, ·$\dot{8}4615\dot{3}$, ·$\dot{0}4273\dot{5}$, ·08$\dot{3}$, ·$\dot{0}00109998\dot{9}$, ·$\dot{0}13121\dot{1}$, ·$\dot{0}0171699828\dot{3}$, ·$\dot{1}50943396226\dot{4}$, 2·$\dot{0}014285\dot{7}$.

Page 115.

1. $\frac{1}{3}, \frac{7}{9}, \frac{8}{11}, \frac{7}{11}$.
2. $\frac{8}{11}, 1\frac{1}{11}, \frac{10}{11}, \frac{17}{30}$.
3. $\frac{1}{180}, \frac{1}{12}, \frac{3}{22}$.
4. $7\frac{16}{33}, 106\frac{1}{10}, 10\frac{2}{3}$.
5. $\frac{43481}{99900}, 3\frac{233}{1998}$.
6. $357\frac{1043}{333300}, 27\frac{2693}{4950}$.
7. $21\frac{47}{225000}, 52\frac{814160}{999999}$.
8. $3\frac{1867}{180000}, \frac{6606}{124875}$.
9. $\frac{11}{104}, \frac{84}{85}$.
10. $\frac{17}{28}, \frac{41}{176}$.
11. $2000\frac{1863}{16000}, 7030\frac{8}{9}$.
12. $106\frac{69719}{99900}, \frac{26681}{49950}$.
13. $\frac{28}{187}$.
14. $\frac{1}{9901}$.
15. $2\frac{1}{700}$.
16. $10\frac{1}{7000}$.
17. $1\frac{1}{478}$.
18. $\frac{1}{17}$.
19. $\frac{3}{65}$.
20. $\frac{1}{39}$.

Page 119.

1. $\cdot 31\dot{6}$, $\cdot 43541\dot{6}$, $\cdot 23125$, $\cdot 796875$, $\cdot 99479\dot{1}\dot{6}$, $1\cdot\dot{6}9841\dot{2}$.
2. $\cdot\dot{2}$, $\cdot 3125$, $\cdot 52\dot{2}\dot{7}$, $\cdot 009375$.
3. $\cdot 9375$, $\cdot 6196\dot{4}2857\dot{1}$, $\cdot 7125$, $\cdot 51875$, $\cdot 7452287946\dot{2}857\dot{1}$, $13\cdot 303125$, $1\cdot 65$, $17\cdot 9015625$, $\cdot 20208\dot{3}$.
4. $\cdot 82215\dot{9}\dot{0}$, (1) $\cdot 0224185\dot{6}\dot{0}$, (2) $\cdot 5380\dot{4}\dot{5}$, (3) $\cdot 1793\dot{4}\dot{8}$.
5. $1\cdot 3$, $1\cdot 1630208\dot{3}$ lbs., $13\cdot 95625$ oz. 6. $12\cdot 6102430\dot{5}$, $23\cdot 986\dot{1}$.
7. $\cdot 788260\dot{4}4973\dot{5}$, $29\cdot 53058819\dot{4}$.
8. $\cdot 0002\dot{4}\dot{3}5064\dot{9}$, $\cdot 0000\dot{8}034894398530762167 12\dot{5}$, $\cdot 00030420564301467779$, $12\cdot 2724$, $\cdot 12\dot{7}$, $\cdot 0002051\dot{7}\dot{6}$.
9. $\cdot 9518072$, $\cdot 4299287$, $\cdot 0695317$, $\cdot 5789474$, $\cdot 25$.

Page 123.

1. £9, £2 8s., £17 7s. 6d., £10 12s. 6d., £2 16s. 8d.
2. 1 qr., 10 lbs. 8 oz., 10 oz., 19 cwt. 2 qrs., 15 cwt. 2 qrs. 4 lbs. 0 oz. 8·192 drs., 4 cwt. 3 qrs. 27 lbs. 0 oz. 3·6864 drs., 7 lbs. 8 oz. 15·36 drs.
3. 1 rd. 14 pls., 10 oz. 9 dwt. 18·24 grs., 1 dwt. 10$\frac{1}{8}$ grs., 6 oz. 7 drs. 0 sc. 12 grs.
4. 3 drs. 2 scr. 6$\frac{14}{15}$ grs., 18$\frac{9}{10}$ grs., 11^h 48^m $28\frac{4}{5}^s$, 2^d 9^h 37^m $26\frac{2}{5}^s$, 195^d 1^h 2^m $38\frac{2}{5}^s$.
5. 3^d 8^h, 6 fur. 26 pls. 3 yds. 2 ft., 12 pls. 3 yds. 1 ft. 6$\frac{18}{25}$ in., 5 ft. 47·468 in., 185^d 12^h 21^m $44\cdot 16048^s$.
6. 15s. 0·23604d., 3 cwt. 1 qr. 8 lbs. 8 oz. 15·314944 dwt.
7. 62 cwt. 3 qrs. 9 lbs. 2 oz. 10·889216 dwts., 112594·680576 oz., 49260172·752 oz., troy.
8. 57° 17′ 44·806128″, 206264·806128″
9. 369·4164159168 acres, 16091779·077335808 sq. ft.
10. 17 cu. ft. 29511·59847552 cu. in. 11. 106·434785 gals. 12. £2 7s. 1$\frac{7}{20}$d.
13. 9s., 3s. 4$\frac{1}{2}$d., £2 7s. 4$\frac{1}{2}$d., 10s. 1$\frac{1}{2}$d., 85 mls. 0 fur. 7$\frac{3}{11}$ pls., 73 ac. 0 rd. 2 pls. 20$\frac{1}{4}$ sq. yds. 1 qr. 4 lbs.

Page 124 *and* 125.

1. 65·510217, &c.
2. ·03493
3. ·000066
4. $\frac{333}{108}$ by ·0011865, &c.
5. 2$\frac{71}{81}$
6. 1·25 lbs.
7. $\frac{219}{728}$
8. £2 7s. 1$\frac{7}{20}$d.
9. 532$\frac{4}{15}$
10. $\cdot 345\dot{6}$, $\cdot 34\dot{5}\dot{6}$, $\cdot 3\dot{4}5\dot{6}$, $\cdot\dot{3}45\dot{6}$, $\cdot 3456$.
11. 19
12. ·000038
13. 7590 feet
14. $\cdot 000662\dot{8}\dot{7}$
15. 7 ft. 5$\frac{3}{4}$ in.
16. 43·039783 ft., 6·1575164 yds., 89·03894378 mls.
17. 277·2738 cubic inches
18. 3·883 inches
19. 62·321 lbs., 35·943 cubic feet
20. ·246857
21. £2 6s. 7$\frac{807}{1007}$d.
22. 22·7205 tons, 8639·6805 tons
23. £500
24. ·0338235, 71·0030, ·49867
25. $\cdot 043402\dot{7}$ seconds.
26. $\frac{4}{9}$
27. 16000000 cubic inches.
28. 1 N.M. = 1·15202 I.M., 1 I.M. = ·86804 N.M., 1·1515, ·868421
29. $\frac{4}{9}$ or $\cdot\dot{4}$.

PRACTICE, *Page* 135.

1. £392 10s., £196 5s., £157, £261 13s. 4d., £65 8s. 4d.
2. £49 1s. 3d., £273, £22 7s. 6d., £14 18s. 4d., £2 15s. 11$\frac{1}{4}$d.
3. £1912 10s., £1593 15s., £1753 2s. 6d., £5462 12s.
4. £34765 18s. 3d., £36761 12s. 6d., £1785 11s. 3d., £69 7s. 2d.
5. £4628, £965, £7281 1s. 3d., £1352 10s. 7$\frac{1}{2}$d.
6. 9s. 10$\frac{3}{4}$d., £836 16s. 4$\frac{7}{8}$d., £18 17s. 2$\frac{1}{4}$d.
7. £117 0s. 3d., £1657 5s. 6$\frac{1}{2}$ $\frac{1}{4}$d., £4612 14s. 0$\frac{3}{4}$d.
8. 1373 tons 17 cwt. 2 qrs., 2229 lbs., 2 ozs., 2 mls. 76 yds., £59 3s. 8d.
9. £6581 5s., £2515 10s., £661 10s.
10. £89 0s. 0$\frac{86}{112}$d., £696 6s. 4$\frac{1}{20}$d., £867 12s.
11. £354 3s. 4d., £48958 6s. 8d., £4582 19s. 2d., £16835 13s.

12. £344 13s. 4d., £966, £42.
13. 37 tons 15 cwt. 0 qr. 4 lbs., 7352 tons, 1 cwt. 2 qrs. 8 lbs., £562500.
14. £609 9s. $5\frac{129}{160}$d.
15. 1 ton 1 cwt. 3 qrs. 21 lbs. $10\frac{1}{3}$ ozs.; cost £5 7s. $2\frac{1}{4}$d.
16. 1336 tons 10 cwt.; cost £11805 15s.
17. £1211 16s. $6\frac{3}{7}$d.; £297 5s. $7\frac{1}{2}$d. 18. 5 tons 17 cwt. 3 qrs. 12 lbs.; £24 7s. 6d.
19. £4533 1s. 1d. 20. £1119 18s. 9d.
21. £1153 19s.; £444 16s. 8d.; £719 11s. 8d.; £1327 4s.; £302 16s. 8d.; £691 5s.
22. £513 6s. $6\frac{1}{4}$d. 23. £255 13s. $3\frac{3}{8}$d. 24. £23 14s. $0\frac{3}{4}$d.
25. £98 5s. $4\frac{4}{5}$d. 26. £4114 11s. 5d.

PROPORTION, *Page* 140.

1. 27
2. 64
3. 14
4. 9
5. 8
6. 3249
7. 18
8. 25
9. $7\frac{7}{8}$
10. $6\frac{2}{9}$
11. $\frac{1}{2}$
12. $2\frac{1}{2}$
13. 10·0693̇.
14. $1\frac{1}{3}$
15. 1 yard
16. $26\frac{2}{3}$ yards
17. $4\frac{4}{9}$ tons.
18. £5 13s. 8d.

Pages 149 *and* 151.

1. £55 16s.
2. £7697.
3. 16s. $6\frac{3}{4}$d.
4. 12 cwt. 3 qrs.
5. £39 6s. 11d.
6. £492 11s. $7\frac{101}{121}$d.
7. 2 tons 5 cwt.
8. 24d 7h 40m $24\frac{24}{35}$s.
9. 12 days.
10. 13580.
11. £4 15s. $10\frac{1}{3}$d.
12. £16 16s. $10\frac{149}{187}$d.
13. 21 lbs. 5 oz. 16 dwt. 6 gr.
14. 2500.
15. $350\frac{14}{15}$.
16. 13 yds. 0 qr. 1 nl.
17. £552 10s. 6d.
18. £306 19s. 8d.
19. £6 10s. $8\frac{4}{25}$d.
20. 108 yds.
21. $168\frac{8}{9}$ lbs.
22. 13 ft. 2·382 in.
23. 7 years 261 days.
24. 18 acres.
25. $233333333\frac{1}{3}$ miles.
26. $947\frac{1}{2}$ gallons.
27. $5\frac{5}{8}$ feet.
28. 28 cwt. 3 qrs. 8 lbs.
29. 108 days.
30. $1\frac{1}{3}$ day.
31. 4 cwt. 2 qrs. $17\frac{97}{105}$ lbs.
32. 5 dwt. $3\frac{111}{323}$ grs.
33. £3168000.
34. 64 days.
35. £5 8s.
36. £1 1s. 3d.
37. £2.
38. 10m $20\frac{20}{23}$s past 5 o'clock.
39. 120 days.
40. £15 10s.
41. 3000.
42. 3000.
43. 45 feet.
44. $73\frac{88}{89}$ feet.
45. £13 9s. $0\frac{3}{4}$d.
46. $26\frac{1}{24}$ lbs.
47. $206\frac{2}{15}$ lbs.
48. 9 seconds.
49. 11·9396 lbs., &c.
50. 216000.

COMPOUND PROPORTION, *Pages* 157 *and* 158.

1. 5 hours.
2. £2 14s. $1\frac{3}{5}$d.
3. 10 cwt.
4. 44 ozs.
5. 125 reams.
6. £273 18s.
7. £79 6s. 8d.
8. 1 ton 0 cwt. 3 qrs. 16 lbs.
9. 280000 words.
10. $\frac{1}{51}$ inch.
11. $61\frac{1}{9}$ feet.
12. $678857\frac{1}{7}$ gallons.
13. 11 hours.
14. 48519·2187 tons.
15. 13 tons 19 cwt.
16. 32 days.
17. 9 months.
18. 2530 yards.
19. 24 days.
20. 8 men.
21. 7 tons 4 cwt.
22. 15 pumps.

PROPORTIONAL PARTS, *Page* 160.

1. 40 : 60. 2. 21, 15, 9. 3. 1140, 855, 684.
4. 48. (16 + 12 + 6 + 4 + 10). 5. 1st, £78, 2nd, £63, 3rd, £9.
6. 1st, £56 16s., 2nd, £46 8s., 3rd, £16 16s.

INTEREST, &c., *Page* 164.

1. £61 5s., £74 18s. $2\frac{4}{25}$d. 2. £167. 3. £75 12s. $9\frac{3}{5}$d.
4. 8s. $5\frac{1}{5}$d., £13 7s. $9\frac{88}{100}$d., £32 19s. $7\frac{1}{5}$d.
5. $15\frac{5}{8}$, $23\frac{11}{31}$, $8\frac{88}{82}$, $2\frac{16888}{20007}$. 6. £16 17s. $0\frac{12}{125}$d.

SQUARE ROOT, *Page* 171.

1. 3163, 490·07, 5764801
2. 2·23606, ·707106, ·02236, ·07071
3. 9859, 3117·6895451, 985·9, 311·7689545
4. 98·59, 31·1768954, 9·859, 3·1176895
5. ·9859, ·3117689, ·09859, ·0311768
6. 9509900499, 1·7724538
7. 43046721, 6561, 81, 9, 3
8. $\frac{110}{121}$, $\frac{13}{16}$, $\frac{29}{37}$, $\frac{48}{65}$, $2\frac{1}{6}$, $2\frac{3}{7}$, $18\frac{2}{3}$, $122\frac{8}{11}$.
9. 3005, ·12, ·1, ·08, ·073, ·0231
10. ·2005, ·003, ·$\dot{3}\dot{3}\dot{3}$, ·1$\dot{6}$6, ·22222

CUBE ROOT, *Page* 176.

1. 15, 22, 42, 157, 179, 6872
2. 36·4, 30·02, 2·35, 3002, 279
3. ·404, ·287, ·0313, ·101, ·195
4. 1·587, 1·91, 2·06
5. ·5714, ·172, 3·667, 19·625, 2·$\dot{3}\dot{3}\dot{3}\dot{3}$, 3·142
6. 1·442249.

MENSURATION, *Page* 180.

1. 1278·0625 2. 35 ft. 9 in. 3. 385 4. 296·484375
5. 104·125 6. 19 in. 7. 144 in. 8. 25 ft. 9. 20 ft 3 in.

Page 181.

1. 1326·775 2. 97·5208

Page 184.

1. 29·8452 3. 23·562 5. 95·557
2. 15·3153 4. 56·5488 6. 263·8944

1. 15·9155 2. 15 3. 56 4. 33 5. 105 6. 8·992

Page 185.

1. 3216·9984. 2. 78·54. 3. 2237·518696. 4. 37·122421. 5. 74·6620.
6. 8143·0272. 7. 8535·7762875. 8. Result nearly the same.

Page 186.

1. 62. 2. 58. 3. 67·75. 4. 94·499. 5. 9·499. 6. 4·5.

Page 187.

1. 255·92. 2. 19·098. 3. 12·73. 4. 5 lbs. per sq. in., 3·927 cir. inch.

Page 188.

1. 19·0896 sq. ft. 2. 5·399625 sq. ft. 3. 84·8232 sq. in. 4. 14·1372 sq. ft.

Page 189.

1. 78·54. 2. 157. 3. 15·708. 4. 7·854.

Page 190.

1. 375 ft. 2. 30 ft. $5\frac{3}{4}$ in. 3. 3402 ft. 4. 24 ft. 5. 50 ft. 6. 2524·09375 ft.

SIMPSON'S RULE FOR AREAS, *Pages* 195 *and* 196.

1. Length × breadth = area; 3 breadth × breadth = 5808 sq. yards; $(\text{breadth})^2 = \frac{5808 \times 9}{3}$; breadth $= \sqrt{(5808 \times 3)}$ sq. ft.; breadth $= \sqrt{17424} = 132$; whence length 396 ft.

2. $10\frac{2}{3}$ sq. inches. 3. 3·927 4. 12·5664 + 7·0686 = 19·635.

5. Each square mile contains 1760 × 1760 square yards, and 60 feet = 20 yards; therefore the contents of coalfield is 1000 × 1760 × 1760 × 20 cubic yards, and this weighs 1000 × 1760 × 1760 × 20 tons; hence number of years $= \frac{1000 \times 1760 \times 1760 \times 20}{70000000} = \frac{176 \times 176 \times 2}{70} = 885\frac{1}{35}$ years. Also volume of required pyramid $= \frac{1}{3} \times$ height $\times 110 \times 90 =$ height $\times 110 \times 30$; therefore height $= \frac{70000000}{110 \times 30}$ yards $= \frac{700000}{33} = 21212\frac{4}{33}$ yards = 12 miles $92\frac{4}{33}$ yards.

6. 457 sq. ft. 18 in., and 35·7804 sq. in. 7. $21\frac{1}{3}$ in.

8. $\frac{5}{8}$. 9. $1856\frac{1}{3}$. 10. 5114·5248. 11. 43·9824 sq. in.

12. 50·2656. 13. $27\frac{1}{3}$ sq. in.

14. 264 cu. in. 15. Surface 572·5566; volume 1288·25235 cu. ft.
16. 19·712 cu. ft. 17. $\frac{187}{800}$ part, nearly. 18. 9149976000 cu. miles.
19. $\frac{1}{384375}$. 20. 5 lbs. per sq. in.; 3·927 circular inches.
21. 15 ft. 11½ in. 22. 64.
23. The surfaces are as 13½ to 1, nearly, and volume as 49½ to 1, nearly.
24. Internal diameter 49·2477; area 157·85817432 in.
25. Surface 907·9224 in.; solidity 2572·4468.
26. ·0000042921 in. 27. 3 ft. 10 in. 28. 2 ft., 4 ft., 8 ft.
29. 6. 30. Upper 13·867, middle 3·605, lower 2·525.
31. ·2073 thick. 32. 78⅜ lbs. 33. Surface 87·9648 sq. in., solidity 62·832 sq. in.
34. 27·143424 cu. in. 35. 5·18364 cu. in. 36. 231·928716 + cu. in.
37. Radius 6·77 in. 38. 6221·1534 area. 3884·1534 waste.
39. 59·446926 area. 45·146926 waste. 40. 29·338, nearly, diameter.
41. 7·54 diameter. 42. 376·992 area. 43. 21·27 in., nearly.
44. ·000000116 part of the earth.

SPECIFIC GRAVITY, *Page* 200.

1. 16523·59375 lbs., 7·3764 tons. 2. 1139·328 lbs. 3. 15·04321785 lbs.
4. 24·338809 lbs. 5. 88·8994 lbs. 6. 8 dwt. 12·86 grs. 7. 20½ gr. 8. 2·823 lbs.

BUNKERS, TANKS, &c., *Page* 204.

1. 15 tons 4 cwt. 3 qrs. 7$\frac{1}{11}$ lbs.
2. 15 tons 12 cwt. 2 qrs. 6½ lbs. nearly.
3. 52 tons 4 cwt. 1 qr. 4 lbs.
4. 117 tons 4 cwt. 3 qrs. 27 lbs.
5. 72 tons 3 cwt.
6. 60 tons 19 cwt. 1 qr. 27 lbs.

Page 205.

1. 3 ft. 6⅝ in. 2. 4 ft. 9 in. nearly. 3. 3 ft. 3 in. nearly. 4. 6 ft. 6 in.
5. 5 ft. 5½ in. nly. 6. 7 ft. 1¾ in. 7. 6 ft. 10·08 in. 8. 18 ft. 2⅝ in.

CONSUMPTION OF FUEL, *Page* 212.

1. 160 tons.
2. 172 tons 12 cwt. 1 qr. 14·672 lbs.
3. 76 tons 13 cwt. 1 qr. 9½ lbs.
4. 93 tons 16 cwt. 3 qrs. 6·9 lbs.
5. 207 tons 15 cwt. 0 qr. 11·2 lbs.
6. 70 tons 11 cwt. 3 qrs. 1¼ lbs.
7. 84 tons 16 cwt. 0 qr. 17$\frac{3}{13}$ lbs.
8. 320 tons 3 cwt. 3 qrs. 10·8 lbs. nearly.
9. 373 tons 18 cwt. 1 qr. 16¾ lbs.
10. 624 tons 17 cwt. 1 qr. 3·65 lbs.

Page 214.

1. 181·26 tons. 2. 126 tons. 3. 194·4 tons. 4. 98·9 tons.

Page 215.

1. 2·568 lbs.
2. 2·196 „
3. 2·666 „
4. 2·022 lbs.
5. 1·97 „
6. 2·388 „
7. 2·528 lbs.
8. 2·644 „
9. 4·236 „
10. 2·082 lbs.
11. 4·32 „
12. 3·43 „

Page 216.

1. 97·92 basketfuls.
2. 67·89 „
3. 81·66 basketfuls.
4. 57·44 „
5. 105·77 basketfuls.
6. 70 „

1. 45·14 tons.
2. 49·39 „
3. 27·87 tons.
4. 41·37 „
5. 39·37 tons.
6. 42·05 „

Page 219.

1. 48½ tons. 11·615 tons short.
1586 miles to run.
2. 32½ tons. 4·43 tons over.
694 miles to run.
3. 316 tons. 156·2 tons over.
3605 miles to run.
4. 335½ tons. 5·79 tons over.
4867 miles to run.
5. 35¼ tons. 3·76 tons short.
438 miles to run.
6. 363 tons. 77·71 tons over.
6437 miles to run.

BOILERS, &c., *Page* 221.

1. 8·026 lbs.
2. 5·423 „
3. 6·652 lbs.
4. 7·122 „
5. 6·038 lbs.
6. 7·230 „

Page 223.

1. Heating surface of tubes = 2·5 × 3·1416 (= 7·854 circumference) × 76 (= 596·904) × 130 = 77597·52; area of plate 77 (= 6 ft. 5 in.) × 69 (= 5 ft. 9 in.) = 5313; area of tube mouths 2½^2 (= 6·25) × ·7854 × 130 = 638·1375; heating surface in 2 plates 9349·725; total heating surface = 86947·245 square inches = 603·8 square feet.

2. Heating surface of tubes = 162556·5942; area of plate = 11070; area of tube mouths = 1518·12957; heating surface in 2 plates = 19103·74; total heating surface = 181660·334 square inches = 1261·53 square feet.

3. Heating surface of tubes = 71062·992; area of plate = 4860; area of tube mouths = 455·532; heating surface of 2 plates = 8808·936; total heating surface = 79871·928 square inches = 554·666 square feet.

4. Heating surface of tubes = 119148·3216; area of plate = 4408; area of tube mouths = 1212·2649; heating surface of 2 plates = 6391·47; total heating surface = 125539·79 square inches = 871·80 square feet.

5. 795·688 square feet.
6. 1286·028 square feet.

Page 224.

1. Pressure of water per square inch = $\frac{6}{2\cdot305}$ = 2·603 lbs.; 2·603 + 20 = 22·603 = total pressure per square inch; 114 (= 9 ft. 6 in.) × 90 (= 7 ft. 6 in.) = 10260 = area of bottom of boiler; 10260 × 22·603 = 231906·78 lbs., the total pressure on the bottom of the boiler; each stay bears 1·25^2 × ·7854 (= 1·22719) × 11200 = 13744·528; number of stays = 231906·78 ÷ 13744·528 = 16·8 or 17 stays, nearly.

2. Pressure of water = 1·735 lbs.; total pressure per square inch = 1·735 + 20 = 21·735 lbs.; total pressure on bottom of boiler = 262906·56; each stay bears 1·5^2 × ·7854 × 6000 = 10602·9; number of stays = 262906·56 ÷ 10602·9 = 24·8 or 25 stays, nearly.

3. Number of stays = 53·2 or 54 stays.
4. Number of stays = 25·9 or 26 stays.
5. Number of stays = 223·4 or 224 stays.
6. Number of stays = 178·8 or 179 stays.

PUMPS, &c., *Page* 235.

1. Capacity of pump = 3·25^2 × ·7854 × 12 = 99·54945 cubic inches; quantity delivered = 99·54945 × ⅔ = 66·3663 cubic inches; quantity delivered per hour = 66·3663 × 15 × 60 = 59729·67 cubic inches, which being divided by 1728 gives 34·5658 cubic feet.

2. 3·5^2 × ·7854 × 14 = 134·6961; then 134·6961 × ¾ = 101·022075; again, 101·022075 × 12 × 60 = 72735·894 cubic inches, which being divided by 1728 gives 42·0925 cubic feet.

3. 31^2 × ·7854 × 30 × ⅓ = 7547·694; 7547·694 × 36 × 60 = 16303019·04 cubic inches = 9434·6175 cubic feet.

4. 276117·18 cu. in. = 159·79 cu. ft.
5. 890643·6 cu. in. = 515·4187 cu. ft.
6. 77404·50 cu. in. = 44·794 cu. ft.
7. 603187·2 cu. in. = 349·07 cu. ft.
8. 743114·8 cu. in. = 424·2562 cu. ft.
9. 941171·7201 cu. in. = 544·6596 cu. ft.
10. 446753·19168 cu. in. = 258·5377 cu. ft.
11. 580000·68 cu. in. = 335·6485 cu. ft.
12. 3016359·36 cu. in. = 1745·5783 cu. ft.

HORSE-POWER, &c., *Page* 245.

1. Area of cylinder = 62^2 × ·7854 = 3019·0776; as there are 15 lbs. on each square inch, there will be a total pressure of 45286·164 lbs., and as the length of the double stroke is 9 feet, the power exerted will be equal to 9 times 45286·164 lbs., or 407575·476 foot-pounds per stroke, and as there are 26 strokes per minute, there will be 26 times this, or 10596962·376 foot-pounds exerted per minute. As an actual horse-power is assumed to be 33000 foot-pounds per minute, we shall, by dividing by 33000, get the actual power exerted by the engines, viz., 321·1.

2. Area of cylinder = 103^2 × ·7854 = 8332·3086; pressure in lbs. = 8332·3086 × 9 = 74990·7774; units of work = 74990·7774 × 252 = 18897675·9048. Horse-power = 18897676 ÷ 33000 = 572·6.

3. Area of cylinder = 2970·57915; units of work = 6149098·8405. Horse-power = 6149098 ÷ 33000 = 186·3.

4. Area of cylinder = 4778·3736; pressure = 57340·4832 lbs.; units of work = 12672246·7872 ÷ 33000 = 384 horse-power.

5. Area of cylinder = 3848·46; pressure = 51954·21 lbs.; units of work = 14547178·8 ÷ 33000 = 440·8 horse-power.

6. Area of cylinder = 1963·5; pressure = 23562 lbs.; units of work = 6832980; horse-power = 207·06.

7. Area of cylinder = 3019·0776; pressure = 30190·776 lbs.; units of work = 7849601·76; horse-power 237·9.

8. Area of cylinder = 706·86; pressure = 11309·76 lbs.; units of work = 2290226·4; horse-power 69·4.

9. Horse-power 262·06. 10. Horse-power 584·4.

PRESSURE ON PISTON, *Page* 250.

1. Total pressure 34·3 lbs. 3. Total pressure 37·5 lbs.
2. „ 33·5 „ 4. „ 30 „

SAFETY-VALVES, &c., *Page* 252.

1. Area of valve $4{\cdot}6^2$ × ·7854 = 16·619064; upward pressure 299·143, deducting the weight of lever 63 lbs. + weight of valve 10 lbs. = 73 = effective pressure 226·143; multiplying by distance from F to V, 6 = 1356·858, and dividing by distance from fulcrum to weight 12 + 6 = 18 gives 75·381 lbs., the weight to be placed on the end of the valve.

2. Area of valve = $3{\cdot}625^2$ × ·7854 = 10·3206468; upward pressure 258·01617 lbs.; weight of lever and valve 27 lbs.; effective pressure 231·01617 lbs.; this multiplied by distance from F to V and divided by distance from F to W gives 23·8982 lbs.

3. Area of valve = 4^2 × ·7854 = 12·5664; upward pressure 289·0272 lbs.; this being multiplied by 2½, and the product being divided by 19, gives 32·8058 lbs.

4. Area of valve = 113·0976; upward pressure 2035·7568; weight of lever and valve 10·5; effective pressure 2025·2568 lbs. × 3½ ÷ 21 = 337·5428 lbs.

5. Area of valve = 143·13915; upward pressure 5296·14855; weight of lever and valve 23·75; effective pressure 5272·39855 × 2·5 ÷ 28 = 470·7499 lbs.

6. 93·396 lbs. 7. 50·1421 lbs. 8. 98·126 lbs. 9. 747·09 lbs. 10. 79·68 lbs.

Page 253.

1. By Rule CXVIII. $10^2 - 9^2 = 100 - 81 = 19$; and 19 × ·7854 × 20 = 298·452 lbs.
By Rule CXIX. 10 + 9 = 19; 10 − 9 = 1; and 19 × 1 = 19; then 19 × ·7854 × 20 = 298·452 lbs.

2. $13{\cdot}5^2 - 12^2 = 182{\cdot}25 - 144 = 38{\cdot}25$; and 38·25 × ·7854 × 25 = 751·03875 lbs.

3. $9^2 - 6^2 = 81 - 36 = 45$; and 45 × ·7854 × 20 = 706·86 lbs.

4. 8 + 7 = 15; 8 − 7 = 1; and 15 × 1 = 15; then 15 × 26 × ·7854 = 306·306 lbs.

5. 6 + 4 = 10; 6 − 4 = 2; and 10 × 2 = 20; then 20 × 24 × ·7854 = 376·992 lbs.

6. 12 + 9·5 = 21·5; 12 − 9·5 = 2·5; and 21·5 × 2·5 = 53·75; then 53·75 × 35 = 1881·25 × ·7854 = 1477·53 lbs.

7. 1249·375 lbs. 8. 108·385 lbs.

Page 254.

1. 40 : 32 :: 450 : x, or dividing first and second terms by 8 we have 5 : 4 :: 450 : x. Again, we can divide the first and third terms by 5, whence 1 : 4 :: 90 : 360 lbs. the answer.
2. 36 : 24 :: 980 : $653\frac{1}{3}$ lbs. Divide 36 and 24 by 12; then 3 : 2 :: 980 : $653\frac{1}{3}$ lbs.

3. 476·3 lbs. 4. 799·9 lbs. 5. 500 lbs. 6. 329·3 lbs.

7. 27 — 24 = 3; then 27 : 3 :: 775 : 86·1, or 9 : 1 :: 775 : 86·1.

8. 111. 9. 170. 10. 51·4 lbs. to be taken off.

Page 255.

1. 1·625 4. 1·7 7. 1·296875
2. 1·1875 5. 2·1 8. 1·15625
3. 1·71875 6. 1·9 9. 2·1875

RATE AND SLIP OF SCREW, *Page* 273.

1. 12 × 6080 = 72960 feet; 72960 ÷ 19 = 3840 revolutions.
2. 14·8 × 6080 = 89984 ÷ 17 = 5293·18 revolutions.
3. 10 × 6080 = 60800 ÷ 15 = 4053·3 revolutions.
4. 11·8 × 6080 = 71744 ÷ 11·5 = 6238·6 revolutions.
5. 12 × 6080 = 72960 ÷ $6\frac{2}{3}$ = 10944 revolutions.
6. 12 × 6080 = 72960 ÷ $21\frac{5}{7}$ = 3360 revolutions.

Page 275.

1. Progression of propeller = 19 × 69 × 60 ÷ 6080 = 12·94 knots; then 100 : (100 — 25 =) 75 :: 12·94 : 9·70 knots, rate of ship.
2. Progression of propeller = 17·1 × 74 × 60 ÷ 6080 = 12·49 knots; then 100 : (100 — 20 =) 80 :: 12·49 : 9·99 knots, rate of ship.
3. Progression of propeller = 14·5 × 126 × 60 = 109620 ÷ 6080 = 18·029; then 100 : (100 — 15·5 =) 84·5 :: 18·029 : 15·23, rate of ship.
4. Progression of propeller = $6\frac{2}{3}$ × 84 × 60 = 33600 ÷ 6080 = 5·526; then 100 : (100 — 13 =) 87 :: 5·526 : 4·81, rate of ship.

5. 10·65, rate of ship. 6. 9·39, rate of ship.

MEAN PRESSURE, &c., *Page* 289.

1. (*a*) BY METHOD I.—The effective pressure per square inch = 20 lbs. + 15 = 35. The steam is cut off at $\frac{3}{8}$ of the stroke, therefore the cylinder is divided into 16ths, $\frac{3}{8} = \frac{6}{16}$. There are 6 parts before expansion, and 10 under expansion; then according to Marriotte's law 7 : 6 :: 35 : 30 even; 210 ÷ 8 = 26·25 odd; 210 ÷ 9 = 23·333 even; 210 ÷ 10 = 21 odd; 210 ÷ 11 = 19·091 even; 210 ÷ 12 = 17·5 odd; 210 ÷ 13 = 16·153 even; 210 ÷ 14 = 15 odd; 210 ÷ 15 = 14 even; 210 ÷ 16 = 13·125 last. *Evens* 30 + 23·333 + 19·091 + 16·153 + 14 (= 102·577) × 4 = 410·308; *odds* 26·25 + 21 + 17·5 + 15 (= 79·75) × 2 = 159·5; next (first) 35 + (last) 13·125 + 410·308 + 159·5 = 617·933; dividing by 3 = 205·978 = units of work done upon each square inch since expansion began; and units of work done *before* expansion = 35 × 6 = 210; whence units of work done during a single stroke = (205·978 + 210) = 415·978 ÷ 16 = 25·998 lbs.

(*b*) BY METHOD II.—Ratio of expansion 8 ÷ 3 = 2·667; hyperbolic log. of 2·66 is ·97833; which being increased by 1 gives 1·97833; then 35 ÷ 2·667 = 13·123, and 1·97833 × 13·123 = 25·962 lbs., the mean pressure.

(*c*) BY METHOD III.—Ratio of expansion 2·667; 2·6 gives in the table of multipliers ·75212 and 2·7 gives ·73824, the difference is 1388, which being multiplied by 67, and then divided by 100 gives 930 to subtract from ·75212, whence the multiplier is ·74282, and multiplying by 35 gives the mean pressure 25·999 lbs.

(*d*) By Method IV.—⅜ = 3 ÷ 8 = ·375. Taking 11 ordinates, according to rule, we have 0 (½ ordinate) ·05; 1st, ·1; 2nd, ·1; 3rd, ·1; 4th, ·09375; 5th, ·075; 6th, ·0625; 7th, ·05357; 8th, ·04687; 9th, ·04167; 10th, (½ ordinate) ·01875; sum ·74211, multiplying by 35 = 25·974 lbs., mean pressure.

2. Method I.—Effective pressure 43; ⅓ = $\frac{4}{12}$. 12 parts. 4 before expansion, 8 during expansion. *Evens* 93·718 × 4 = 374·872; *odds* 67·367 × 2 = 134·734; ⅓ sum of first, last, evens, and odds = 566·939 ÷ 3 = 188·980 lbs.; 43 × 4 = 172 + 188·980 = 360·980 ÷ 12 = 30·082 lbs., mean pressure.

Method II.—Ratio of expansion 3; its hyperbolic log. = 1·09861 + 1 = 2·09861 × 43 ÷ 3 = 30·08 lbs., mean pressure.

Method III.—Ratio of expansion 3; multiplier in expansion table ·69954 × 43 = 30·08 lbs., mean pressure.

Method IV.—⅓ = ·33333; 0 (½ ordinate) gives ·05; 1st, ·1; 2nd, ·1; 3rd, ·1; 4th, ·08333; 5th, ·06667; 6th, ·05555; 7th, ·04762; 8th, ·04167; 9th, ·03704; 10th, (½ ordinate) ·01667 = sum ·69855 × 43 = 30·038 lbs., mean pressure.

3. Method I.—4 times *sum of evens* = 64·147 × 4 = 256·588; twice *sum of odds* = 41·85 × 2 = 83·7; ⅓ sum of first, last, evens, and odds = 386·788 ÷ 3 = 128·929; 128·929 + 186 = 314·929; dividing by 12 = 26·244 lbs., mean pressure.

Method II.—Ratio of expansion 2; its hyperbolic log. ·69315 + 1 = 1·69315 × 31 ÷ 2 = 26·244 lbs., mean pressure.

Method III.—26·244 lbs.

Method IV.—Ratio of expansion ·5; 0 (½ ordinate) ·05; 1st, ·1; 2nd, ·1; 3rd, ·1; 4th, ·1; 5th, ·1; 6th, ·08333; 7th, ·07143; 8th, ·0625; 9th, ·05555; 10th, (½ ordinate) ·025; sum ·84781 × 31 = 26·282 lbs., mean pressure.

4.—Method I.—4 times *sum of evens* = 72·424 × 4 = 289·696; twice *sum of odds* 94·5; ⅓ sum of first, last, evens, and odds = 436·696 ÷ 3 = 145·565; 145·565 + 210 = 355·565 ÷ 12 = 29·63.

Method II.—Ratio of expansion 2; its hyperbolic log. ·69315 + 1 = 1·69315 × 35 ÷ 2 = 29·630 lbs., mean pressure.

Method III.—29·631 lbs.

Method IV.—Ratio of expansion ·5; 0 (½ ordinate) ·05; 1st, ·1; 2nd, ·1; 3rd, ·1; 4th, ·1; 5th, ·1; 6th, ·08333; 7th, ·07143; 8th, ·0625; 9th, ·05555; 10th, (½ ordinate) ·025; sum ·84781 × 35 = 29·673 lbs., mean pressure.

5. Method I.—4 times *sum of evens* = 69·003 × 4 = 276·012; twice *sum of odds* = 88·6666; ⅓ sum of first, last evens, and odds = 139·2928; 139·2928 + 152 = 291·2928 ÷ 10 = 29·1292, mean pressure.

Method II.—Ratio of expansion 2·5; its hyperbolic log. + 1 = 1·91629 × 15·2 = 29·1276, mean pressure.

Method III.—Ratio of expansion 2·5; its tabular multiplier ·76652 × 38 = 29·12776, mean pressure.

Method IV.—Ratio of expansion ·4; 0 (½ ordinate) ·05; 1st, ·1; 2nd, ·1; 3rd, ·1; 4th, ·1; 5th, ·08; 6th, ·06666; 7th, ·05714; 8th, ·05; 9th, ·04444; 10th, (½ ordinate) ·02; sum ·768247 × 38 = 29·2033, mean pressure.

6. Method I.—4 times *sum of evens* = 226·264; twice *sum of odds* = 56; ⅓ sum of first, last, evens, and odds = 113·532; 113·532 + 280 = 393·532; dividing by 12 = 32·724 lbs.

Method II.—Ratio of expansion 1·5; its hyperbolic log. + 1 = 1·40547 × 35 × 2 = 98·383 ÷ 3 = 32·794 lbs.

Method III.—32·794 lbs., mean pressure.

Method IV.—Ratio of expansion ·66667; 0 (½ ordinate) ·05; 1st, ·1; 2nd, ·1; 3rd, ·1; 4th, ·1; 5th, ·1; 6th, ·1; 7th, ·09524; 8th, ·08333; 9th, ·07407; 10th (½ ordinate) ·03333; sum ·93579 × 35 = 32·795 lbs., mean pressure.

EXAMINATION PAPER—No. I, *Page* 458.

FOR SECOND-CLASS.

1. 6,006,606.

2. Sum 86 tons 19 cwt. 1 qr. 18 lbs.; product 695 tons 15 cwt. 1 qr. 4 lbs.

3. Product 27·266624. 4. Quotient 5·5386.

5. Quantity remaining on board 19 tons 5 cwt. 1 qr. 11 lbs.

6. By common decimals. 9·6667 × 7·3333 = 70·8888 × 3·5 = 248·11080 cubic feet, which divided by 36 gives 6·89197 tons = 6 tons 17 cwt. 3 qrs. 10 lbs.

By circulating decimals. 9·$\dot{6}$ × 7·$\dot{3}$ = 70·$\dot{8}$ × 3·5 = 248·$\dot{1}$ cubic feet, which divided by 36 gives 6·891975 tons = 6 tons 17 cwt. 3 qrs. 10 lbs.

7. 10·25 × 24 = 246 knots; then 246 : 3704 :: 48 : 722 tons 14 cwt. 2 qrs. 15 lbs.

8. Effective pressure 25 lbs.; otherwise 30 — 26 = 4, and 4 ÷ 2 = 2; then 15 + 2 = 17 lbs., effective pressure above the atmosphere.

9. Pressure necessary to blow out 5·423 lbs.

10. Circumference 94·248; feet in knots sailed 82080; revolutions 870·9, nearly.

11. Revolutions 5211·4. 12. 839·69 lbs. to be taken off. 13. 1·875 inch, the height.

14. Capacity of pump 5·875^2 (= 34·515625) × ·7854 = 27·10857 × 17·5 = 474·399975 cubic inches; then 474·4000 × $\frac{2}{3}$ × 11 × 60 = 208736·022 cubic inches: quantity delivered per hour = 120·796 cubic feet.

15. 94^2 (= 8836) × ·7854 (= 6939·7944) × 21·5 (= 149205·5796) × 561·6 = 83793·8535 ÷ 33000 = 2539·208 horse-power.

16. 5^2 = 25 × 4500 = 112500 lbs.

FOR FIRST-CLASS.

1. 30·003. 2. £736 3s. 4d.

3. 5·5^2 × ·7854 × 28 lbs. = 665·2338 — 101 = 564·2338 = effective pressure on valve, which being multiplied by 3 and divided by 35 gives 48·36 lbs., the required weight.

4. 14·5 + 12·5 = 27 × 2 (or 14·5 — 12·5) = 54 diff. squares diameters; and 54 × ·7854 × 26 = 1102·70 lbs., the required weight.

5. Pressure of water 2·386; total pressure of steam water 32·386; total pressure = 280 × 152 × 32·386 = 1378348·16; pressure each stay bears 2^2 × ·7854 × 6500 = 20420·4 lbs.; number of stays 67·5 or 68 nearly.

6. Heating surface in tubes 2·75 × 3·1416 × 78 × 134 = 90299·0088; area of tube mouths 2·75^2 × ·7854 × 134 = 795·9047; area of plate 81 × 48 = 3888 — 795·9047 = 3092·0953 × 2 = 6184·1906 heating surface of 2 plates; total heating surface 96483·1994 square inches, or 670·022 square feet.

7. 11·5 × 108 × 60 ÷ 6080 = 12·257 knots, advance of screw per hour; then 100 : (100 — slip) 82·5 :: 12·257 : 10·11 knots, rate of ship per hour.

8. Ratio 1 : 2 or $\frac{1}{2}$.

9. Method I.—$\frac{3}{8} = \frac{6}{16}$; divide the cylinder into 16 equal parts, then $\frac{3}{8}$ of the cylinder will equal 6 of these parts; 25 lbs. pressure above the atmosphere + 15 lbs. atmospheric pressure = 40 lbs., effective pressure. There are 6 parts without expansion, and 16 — 6 = 10 parts under expansion. 4 times the *sum* of the *even* pressures is 117·232 × 4 = 468·928; twice the *sum* of the *odd* pressures = 91·143 × 2 = 182·286; then $\frac{1}{3}$ sum of first, last, evens, and odds — 706·214 ÷ 3 = 235·405, to which *add* (40 × 6 =) 240, and the sum 475·405 divided by 16 gives 29·713 lbs., the mean pressure.

Method II.—Ratio of expansion 2·66; its hyperbolic log. + 1 = 1·97833, which multiplied by 40 × $\frac{3}{8}$ or 15, gives 29·675 lbs.

Method III.—The multiplier is ·74287, which multiplied by the effective pressure 40 gives 29·715 lbs., mean pressure.

Method IV.—Ratio of expansion ·375; take 11 ordinates, according to rule, we have 0 ($\frac{1}{2}$ ordinate) ·05; 1st, ·1; 2nd, ·1; 3rd, ·1; 4th, ·09375; 5th, ·075; 6th, ·0625; 7th, ·05357; 8th, ·04687; 9th, ·04167; 10th ($\frac{1}{2}$ ordinate) ·01875; sum ·74211, multiplying by 40 = 29·684 lbs. 10. Mean pressure 25·22 lbs.

EXAMINATION PAPER—No. II, *Page* 460.

FOR SECOND-CLASS.

1. 506,055,084.

2. Sum 1282 tons 1 cwt. 0 qr. 19 lbs.; product 20512 tons 18 cwt. 2 qrs. 24 lbs.

3. Product 3320161·0691092.   4. Quotient ·471036768.

5. Quantity remaining on board 122 tons 4 cwt. 0 qr. 17 lbs.

6. 28·75 × 10·5 × 8·25 = 2490·46875; dividing by 42 = 59·296875 tons = 59 tons 5 cwt. 3 qrs. 21 lbs.

7. $9\frac{1}{2}$ × 24 = 228 knots per day; then 228 : 1507 :: 20 : 132 tons 3 cwt. 3 qrs. 12 lbs.

8. Effective pressure 40·75 lbs.   9. Pressure to blow off 5·748 lbs.

10. Circumference of wheel 89·5356; feet in knots 59280; number of revolutions 662·1.

11. 13·3 × 6080 ÷ 25 = 3234·56 number of revolutions.

12. 24 : (24 — $10\frac{1}{2}$) $13\frac{1}{2}$ :: 240 : 135 lbs.   13. Height of valve to be raised 2·25 in.

14. Capacity of pump ·7854 × $7{\cdot}3^2$ × 15 = 627·80949 cubic inches; this × $\frac{2}{3}$ × 13 × 60 = 326460·9348 cubic inches, quantity delivered in an hour; this divided by 1728 = (12 × 12 × 12) gives 188·924 cubic feet.

15. 64^2 × ·7854 × 22·3 = 71739·06432 × 6 × 66 ÷ 33000 = 860·87 horse-power.

16. $4{\cdot}6^2$ × 4500 = 95220 lbs.

FOR FIRST-CLASS.

1. Side of square 31·1496; remainder ·141984.   2. Wages to be paid £179 14s. 6d.

3. 31·7677 lbs., the required weight.

4. 12 + 10·5 = 22·5 × (12 — 10·5) 1·5 = 33·75 diff. of squares of diameters; then ·7854 × 33·75 × 20 = 530·145 lbs.

5. 9365·895 lbs.; number of stays 92·6 or 93 stays.

6. Total heating surface 112790·958 square inches, or 783·27 square feet.

7. 11·75 × 106 × 60 ÷ 6080 = 12·29 knots, advance of screw per hour; then 100 : (100 — 17·5) 82·5 :: 12·29 : 10·14 knots.*

8. Ratio 1 : 4·2, or 5 : 21, or $\frac{5}{21}$.

9. Method I.—4 times *sum of evens* = 98·078 × 4 = 392·312; twice *sum of odds* = 70·5 × 2 = 141; $\frac{1}{3}$ sum of first, last, evens, and odds = 593·312 ÷ 3 = 197·771; to this *add* 45 × 4 = 180 before expansion; and the sum 377·771, divided by 12, the number of parts into which the cylinder is divided, gives 31·481 lbs., mean pressure.

Method II.—Ratio 3; its hyperbolic log. + 1 = 2·09861 × $\frac{45}{3}$ or 15 = 31·479 lbs.

Method III.—Ratio 3; its tabular multiplier ·69954, multiplied by the effective pressure 45 gives 31·478 lbs., mean pressure.

Method IV.—Ratio ·33333; ordinates ·05; ·1; ·1; ·1; ·08333; ·06667; ·05555; ·04762; ·04167; ·03704; ·01667; sum ·69855 × 45 = 31·435 lbs.

10. Top 17·9 lbs.; bottom 19·53 lbs.; mean 18·71 lbs.

EXAMINATION PAPER—No. III, *Page* 461.

1. Forty-six thousand seven hundred and twelve million three hundred and eighty-nine thousand four hundred and sixty-seven.

2. 64 — 5 = 59; 59 × 3·1416 ÷ 6 = 185·3544 ÷ 6 = 30·8924, or 31 bolts.

3. No. of rivets in circular seam (36 × 6) — 6 × (7 + 1) = 1680 rivets. No. of rivets in vertical seam 14 × 6 — 12 × 7 = 504 rivets. Total number 2184 rivets.

4. 801001 — 6001 = 795000. 10^{d} 10^{h} = 15000 min. 795000 ÷ 15000 = 53 rev.

5. ·16 × 4800 = 268; 18·5 × 5·75 = 106·375, ∴ 268 ÷ 106·375 = 7 ft. $2\frac{5}{8}$ in.

6. £436 1s., £478 8s., and £347 4s.

7. 33 × 2240 = 73920 lbs.; 73920 ÷ (1480 × 24) = 2·081 lbs.

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* The pitch of a screw, or the circumference of a paddle-wheel, multiplied by the revolutions, and two figures cut off for decimals, give the speed in knots, nearly. It is too much in the exact proportion of 304 to 300.

8. 38^2 × ·7854 (= 1134·1176) × 61 = 69181·1736 × 66 = 4565957·4576 × 60 × $\frac{7}{8}$ × 2 = 479425533·0480 ÷ 1728 = 277445·331 lbs.

9. $63·75^2$ (= 4064·0625) × ·7854 (= 3191·9146875) × 6 = (19151·488125) × 58 × 2 = 2221572·6225 = 1166 × 33000 = 38478000 ÷ 2221572·6225 = 17·32 lbs.

10. 120″ × 96″ × 5 = 57600, 1728 × 35 = 60480, ∴ 57600 ÷ 60480 = ·9524 tons = 19 cwt. 0 qr. 5 lbs.

11. 21 tons × 2240 = 47040 lbs.; 66 × 6 = 396, ∴ 47040 ÷ 396 = 118·79 basketfuls.

12. Length of shaft 19 ft. 3 in. — 6 in. = 18 ft. 9 in. or 225 in. $8·5^2$ × ·7854 = 56·74515 × 225 = 12767·65875. Flanges = 18^2 × ·7854 × 6 = 1526·8176; 12767·65875 + 1526·8176 = 14294·47635, ∴ 14294·47635 ÷ 3·6 = 3971·6879 lbs., or 1 ton 15 cwt. 1 qr. 22·699 lbs.

EXAMINATION PAPER—No. IV, *Page* 462.

1. 6 tons 15 cwt. 1 qr. 22 lbs.

2. 4500 — 2450 = 2050; 250 tons — 120 tons = 130 tons; 2450 : 4500 :: 120 : 220·4 ∴ 250 — 220·4 = 29·6 tons left.

3. 94 × ·16 = 15·04; 2·5 × 4·25 = 10·625; 15·04 ÷ 10·625 = 1 ft. 4·95 in.; ∴ 2 ft. 3 in. — 1 ft. 4·95 in. = 0 ft. 10·05 in.

4. 1 cwt. 3 qrs. 16·5 lbs. 5. $13\frac{3}{4}$ gills per day.

6. 6 tons 9 cwt. 2 qrs. 12 lbs. 7. 13·42 lbs.

8. 15^2 × 25 = 5625; $1·125^2$ × ·7854 = ·9940218750; 5625 ÷ ·99402 = 5658·8, &c., lbs.

9. $10\frac{1}{2}$d. per lb. 10. £17 15s. 3d.

11. 4^d 15^h 45^m. Tuesday 11^h 45^m P.M.

12. Cubic inches discharged each stroke = 10^2 × ·7854 × 14 × $\frac{11}{16}$ = 755·9475; 755·9475 × (95 × 2) × 60 = 8617801·500 cubic inches discharged in an hour ÷ 1728 = 4987·15364 cubic feet discharged in an hour. 350 tons × 35 = 12250 cubic feet contained in tank, ∴ $\frac{12250}{4987·15364}$ = 2^h·456 = 2^h 27^m 21^s·6.

NOTE.—$\frac{5}{16}$ empty is $\frac{11}{16}$ full.

EXAMINATION PAPER—No. V, *Page* 462.

1. 15 ft. $7\frac{3}{4}$ in. 2. 10. 3. £7 7s. 4. 3·41797, &c., gals. per day.

5. Dia. of shaft 18 in. + $1\frac{1}{2}$ in. = $19\frac{1}{2}$ in., ∴ 18 in. + $19\frac{1}{2}$ in. = 37·5 in.; $19\frac{1}{2}$ in. — 18 in. = 1·5 in.; 37·5 × 1·5 × ·7854 × 54 × ·3 = 715·69575 lbs.

6. $3·3541\dot{6}$ × 19·5 = 65·406120, &c., sq. ft.

7. 105° — 60° = 45°; 1000° ÷ 45 = $22\frac{2}{9}$ lbs.

8. 52^2 × ·7854 × 42 ÷ (750 × 9) = 13·214 lbs.

9. 27° — 22° = 5°; 22 : 100 :: 5 : 22·727 per cent. 22·727 × 22 = 1099·9984.

10. $\frac{7}{16}^2$ × 806300 ÷ (10·25 × 35) = 430·19 lbs. 11. 167·37 N.H.P.

12. $4·875^2$ × ·625 × 72000 ÷ (36^2 × 26) = 31·738 lbs.

13. 31·738 ÷ 24 = 1·322 + 31·738 = 33·06 lbs. pressure.

EXAMINATION PAPER—No. VI, *Page* 463.

1. $·0000\dot{2}8571\dot{4}$. 2. 18·2. 3. 14 ft. $5\frac{7}{8}$ in.

4. £85 8s., £54 16s., and £22 16s.; total £163.

5. 3·1416 × 40 × 35 = 4398·24; 35 × 50 × 2 = 3500; 4398·24 + 3500 = 7898·24 ÷ 144 = 54·84889 sq. ft.

6. 6·1667 × 20·25 × 34 = 4245·77295; 8^2 × ·7854 × 10 × (68 × 2) × 90 (= 61525·0944) ÷ 1728 gives 35·6048; ∴ 4245·77295 ÷ 354·6048 gives 119^m·2472, or 1^h $59\frac{1}{4}^m$.

7. 60^2 × ·7854 × 32 × 18 × 5·1 ÷ 1728 = 4806·6458; being less than 5000 is not strong enough.

8. 5^2 × ·7854 = 19·635; 641 ÷ 19·635 = 32·646, ∴ 84 : 74 :: 32·646 : 28·76 lbs.

9. 690° — 560°, 130° ÷ 2240 = ·05909 × 10 = ·59 + 10·00 = 10·59 tons.

10. Mean width 6 feet.; 20·25 × 9 × 6 ÷ 45 = 24·3 tons, or 24 tons 6 cwt.

11. 750 × 24 × 23·256 × 10 = 4186080 ÷ 2240 = 1868·785 tons of coal.

12. 45^2 : 41^2 :: 23 lbs. : 19·09 lbs.

EXAMINATION PAPER—No. VII, *Page* 464.

1. 22. 2. $\frac{10000-16}{216} = \frac{9984}{216} = 46·2$.

3. 195″ × 153″ × ·3125 (= $\frac{5}{16}$″) × ·4126 = 3846·8503125 lbs., ∴ £64 2s. 3·4d.

4. 43^2 × ·7854 × 4·25 (= 6171·869550) ÷ 3·9 = 1582·53 lbs.

5. $8\frac{1}{2}$″ + 1″ = $9\frac{1}{2}$″; 200 × $9\frac{1}{2}$ ÷ 36 = 1900 ÷ 36 = $52\frac{7}{9}$ lbs.

6. 40^2 × ·7854 × 60 = 75398·4 ÷ ($1\frac{1}{2}^2$ × ·7854 × 1800 =) 2208·9375 = 34·13 bolts.

7. 80° — 76° = 4° + 120° = 124°. 8. 1149·6 — 102 ÷ 42 = 24·7.

9. 23 ÷ 2·305 = 9·978 lbs. 10. 23·8 per cent.

11. 2880 × 15^2 = 9720000; 58^2 × 36 = 121104; 80·26 lbs.

12. 16^2 × 18 × ·7854 = 3619·1252 capacity of air-pump; 35^2 × ·7854 × 36 = 34636·14 capacity of cylinders. If air-pump had been single-acting it would not have met the requirements of the Board of Trade, but being double-acting it is of greater capacity in the proportion of 4329·5175 : (3619·1252 × 2), or as 1 : 1·67.

EXAMINATION PAPER—No. VIII, *Page* 465.

1. 226. 2. £23 16s. $8\frac{4}{5}$d.

3. 76^2 × ·7854 × 7 × 51 × 23·3 = 37734814·43424 ÷ 33000 = 1143·47 H.P.

4. $\frac{(3·25 \times 2 + \frac{1}{16})^2}{9·25}$ = $·7094^2$ × 42 = 21·1364, &c.

5. 4^2 × ·7854 × 5 × 60 ÷ 1728 = 218·166; 120 × $\frac{17}{12}$ = 170; 3″·9 or ·78 stroke.

6. $5·5^2$ × ·7854 = 23·75835; $3·25^2$ × ·7854 × 2 = 16·591575; 1 : ·69.

7. 1·75 — ·875 ÷ 1·75 × 100 = 50. 8. At 45 cubic feet per ton 20′ $8\frac{1}{4}$″ long.

9. 3·1416 × 65 × 5·5 = 1123·122.

10. 160° — 110° (= 50°) × 7·1 = 355·0 ÷ 1100 = ·32 + 7·1 = 7·42.

11. 9 × 9·7 — 10·1 × 8·25 = 3·975 ÷ (9·7 — 8·25) 1·45 = 2·74.

12. 8·25 × 24 = 198; 1600 ÷ 198 = 8·08; 9·7 × 24 = 232·8; 1600 ÷ 232·8 = 6·87. 1st consumption 72·72 — 2nd consumption 69·3872 = 3·333 ÷ 1·21 = 2·75.

EXAMINATION PAPER—No. IX, *Page* 466.

1. Nine hundred millions nine hundred and ninety-nine thousand and nine.

2. 20. 3. Sum £65 15s. 10d.; quotient £2 12s. $7\frac{3}{8}$d.

4. $\frac{54^2 \times ·7854 \times 70}{1·5^2 \times ·7854 \times 2000} = \frac{160315·848}{3536·55}$ = 45·3 bolts.

5. 22′ 4″ × 13′ 11″ × 16′ = 22·333 × 13·916 (= 310·786028) × 16′ = 4972·576448 ÷ 45 (= 110·501699) ÷ 12 = 9·208, &c., tons. (Note.—45 cubic feet is allowed for 1 ton).

6. Leverage of weight = 4″·375 + ·4375 = 4·8125, ∴ P = $\frac{250 \times 4·8125}{15}$ = 802·083; with friction 802·083 + $\frac{1}{10}$ of 802·083 = 882·291 lbs.

7. A's share, £54 10s.; B's share, £44 19s.; and C's share, £20 11s.

8. F = 156° — 114° × 8·3 = 42° × 8·3 = 348·6, and $\frac{F}{1200}$ = 348·6 ÷ 1200 = ·29, ∴ 8·3 + ·29 = 8·59.

9. Whole depth of face of valve = $5\frac{7}{8}$″; whole width of face of valve = 23″; depth of valve port = whole depth of valve — 2 faces = 10″; height of walls above the face = depth of port — thickness of back and face of valve = $3\frac{3}{4}$″; total length of 4 walls = 63″; cubic inch of face = 270·25; sides = 30; back 150; 4 walls = 177·1875 = total 627·4375 cubic inches ÷ 3·9 = 160·882 lbs., nearly.

10. 48′ $3\frac{21}{32}$″.

11. $O_{,}K$ = 15 × 9·6 = 144·0; $OK_{,}$ = 12 × 10·1 = 121·2; $K_{,}$ — K = 10·1 — 9·6 = 0·5, ∴ 144·0 — 121·2 = 22·8, and 22·8 ÷ 0·5 = 45·6.

12. Days on voyage at 1st rate, 10·851; days on voyage at 2nd rate, 10·313. Tons burnt per day at 1st rate, 130·212; tons burnt per day at 2nd rate, 154·695. Difference of consumption, 154·695 — 130·212 = 24·483; difference of days, 10·851 — 10·313 = 0·538 ∴ 24·483 ÷ 0·538 = 45·5, &c. *Answers* as before.

EXAMINATION PAPER—No. X, *Page* 466.

1. £530 0s. 6½d. 2. $156\frac{34}{35}$.

3. 64^2 × ·7854 = 3216·9984; 10^2 × ·7854 = 78·54; 40·96 sq. in. to 1 sq. in. of pipe.

4. £19 3s. 4d. 5. 12 tons.

6. 30 ÷ 2 = 15 teeth; 1 pull = 3 teeth × 12 = 36; 15 × 12 = 180; 36 : 180 :: 1 : 5^m.

7. 9 ÷ 2 = 4½; 4½ − 2⅛ = 2⅜ open; 27 × 2⅜ = 64⅛ area open for steam.

8. 23 × 2240 × $1\frac{1}{16}$ × 2 (= 109480) ÷ 140 = 782; 782 × $\frac{70}{100}$ ÷ 6 = 91·233 lbs.

9. 5·5 ÷ 4 = 1·375 area; 1·375 ÷ 9 = ·1528 lift of valve.

10. ⅝ of 4½, or ⅝ of $\frac{9}{2}$ = $\frac{45}{16}$ = $2\frac{13}{16}$; $4\frac{6}{16}$ + $2\frac{13}{16}$ = $7\frac{3}{16}$ oz. per gallon.

11. 32^2 + 58^2 ÷ 30 = 1024 + 3364 = 4388 ÷ 30 = 146·26, &c., N.H.P.

EXAMINATION PAPER—No. XI, *Page* 467.

1. 89·3 tons × 2240 = 200032 lbs.; 35 × ·7854 = 27·489; 200032 ÷ 27·489 = 7276·8002 ∴ $\sqrt[2]{\ }$ 7276·8002 = 85·3, diameter of piston.

2. 7 × 2240 (= 15680) × ·055 = 862·4; 3·1416 × $\frac{14}{12}$ (= 3·6652) × 862·4 (= 3160·86848) × 57 = 180169·50336 ÷ 33000 = 5·4897 H.P.

3. 3·1416 × 9 (= 28·2744) ÷ 66 (= 33″ × 2) = ·4284 × ·05 × 100 = 2·142.

4. 1½ × 2240 × 18 × 6 = 362880 ÷ (51520 × 1¼ =) 64400 = 5·6348 ∴ $\sqrt[2]{\ }$ 5·6348 = 2·37.

5. $9·5^2$ (= 857·375) × 51520 = 44171960; 5·1 × 15 = 76·5; 44171960 ÷ 76·5 = 577411·24 ÷ 2240 = 257·77 tons.

6. 3 × 2240 × 24 × 10·2 = 1645056 ÷ (9^3 =) 729 = 2256·59.

7. 2·5 − ·875 ÷ 2·5 = 1·625 ÷ 2·5 = ·65; $·875^2$ (= ·765625) × ·7854 × 200 = 120·264375 ÷ (2½ × ⅝ =) 1·5625 = 76·973; 1·625 ÷ 2·5 × 100 = 65·0.

8. 18·9166 × 19 (= 359·4154) × 10·333 (= 3713·8393282) × ⅔ = 2475·8928855 × 62·5 = 154743·3 ÷ 12000 × 5·1 = 65·7659025; 12 cwt. × 112 = 1344 lbs., ∴ 1344 : 65·766 :: 60^m : $2^m·936$, or 2^m $56^s·16$.

9. Top 12·22 lbs.; bottom 12·56 lbs.; mean 12·39 lbs.

10. 1·437 × 2 + ·1875 = 3·0625 ÷ 6·875 = ·4454; $·4454^2$ × 27 = 5·356287.

EXAMINATION PAPER—No. XII, *Page* 468.

1. ·012 + 2·715 = 2·727.

2. $0·128^2$ = ·0163840; ·0163840 ÷ 6·814 = ·00240.

3. £11 0s. 0d. + £5 2s. 0d. + £4 17s. 6d. = £20 19s. 6d.

4. 600 lbs. − 60 lbs. = 540 lbs. 540 lbs. at 4d. per lb. = £9; discount at 8 per cent. = 14s. 4·8d., whence cost = £8 15s. 7·2d.

5. From May 19th to July 2nd is 44 days; from 7^h 15^m A.M. to 7^h 15^m P.M. is 12 hours; and from 7^h 15^m P.M. to 10^h 30^m P.M. is 3^h 15^m. Total 44^d $15\frac{1}{4}^h$ = $44^d·6354$. Mean consumption per day = 480 ÷ 44·6354 = 10·757 tons. Mean speed per day = 13891 ÷ 44·6354 = 311·21 knots.

6. £120 + 170 + 85 = £375 per year, ∴ 170 N.H.P. : 1 N.H.P. :: £375 : £2·2057 = £2 4s. 1·3680d.

7. 37 tons × 35 = 1295 cubic feet. 1″ = $\frac{1}{12}$ of foot = ·08333; 1295 ÷ ·08333 = 15540·621 sq. feet.

8. 34^2 = 34 × 34 = 11560, ∴ 11560 ÷ 1·5 = 770·66 feet above the sea.

9. Even keel before taking in bunkers = 19·7 ft. + 19·4 ft. ÷ 2 = 19·55 feet; even keel after coaling = 21·0 ft. + 20·8 ft. ÷ 2 = 20·9 ft., ∴ 20·9 − 17·55 = 1·35 ft.; 4821 × 1·35 = 6508·35; 6508·35 ÷ 35 = 185·95 tons taken in.

10. 112 + 48 = 160; 160 : 112 :: 5 oz. : 3·5 oz.

11. 3·6 inches = 1 lb. = 16 oz.; 5 oz. = $\frac{5}{16}$ lb. = ·3125; 7″ × 4″ = 28 sq. inch; ·3125 × 3·6 = 11·2500; 11·2500 ÷ 28 = ·04017 of an inch; ·04017 × 32 = 1·28544.

EXAMINATION PAPER—No. XIII, *Page* 469.

1. $10{\cdot}5^2 \times 18$ (= 1984·5) ÷ 20 = 99·225; $\sqrt{}^{2}$ 99·225 = 9·96 knots.

2. $60^2 \times 18$ (= 64800) ÷ 19·6 = 3306·122; $\sqrt{}^{2}$ 3306·122 = 57·5 rev.; $60^2 : 57{\cdot}5^2 ::$ 11 : 10·1 knots.

3. 12000 × 15 ÷ 6 = 30000; 2240 × 60 ÷ 30000 = 4·48 inches.

4. $5^2 \times 3 \div 6 = 12{\cdot}5$; 5000 × 60 = 300000 ÷ 12·5 = 24000 lbs.

5. $4 \times 8 \times 12^2 \div 16 = 288$ cwt., the breaking strain, and $\frac{1}{10}$ of 288 = 28·8 cwt., the working strain.

6. $50 : 58 :: 60^2 \div 4176$: $\sqrt{}^{2}$ 4176 = 64·6 revolutions.

7. $29. : 32 :: 10^2$: 110·3348, ∴ $\sqrt{}^{2}$ 110·3348 = 10·5 knots.

8. 12·25 × ·75 (= 9·1875, section of plate) × 23 = 211·3125 tons; $\cdot 875^2 \times \cdot 7854$ = ·601321875 × 2 = 1·20264375 ÷ (·75 × 2·5) = 1·875 = ·64141; 211·3125 × ·64141 = 135·5379506, &c., tons.

9. Length of crank-pin = 13^2 = 169 sq. inches; area of L.P. piston = $72^2 \times \cdot 7854$ = 4071·5136; area of H.P. piston = ¼ of 4071·5136 = 1017·8784; pressure on low pressure piston = 4071·5136 × 18 = 73287·2448; pressure on high pressure piston = 1017·8784 × 54 = 54965·4336; pressure on low pressure pin = 73287·2448 ÷ 169 = 433·652 lbs.; pressure on high pressure pin = 54965·4336 ÷ 169 = 325·23, &c., lbs.

10. 7·1 × 12 ÷ 62·5 = 1·3632 evaporation in cubic feet of water; 1·3632 × 512 = 697·9584 = cubic feet of steam; 697·9584 : 3·1 :: 3600 : 15·99 seconds.

11. Top 16·92; bottom 18·60; mean 17·76 lbs.

EXAMINATION PAPER—No. XIV, *Page* 470.

1. 48 + 28 = 76; $112^{\circ} \times 28 = 3136$; 48 × 200 = 9600; $167^{\circ}{\cdot}59$ Fahrenheit.

2. 1½. 3. A's share, £99; B's share, £43; C's share, £38.

4. $12^2 \times \cdot 7854 \times 2$ = 226·1952 ÷ (·25 × 6 =) 1·5 = 150·797, number of slots. The slots are placed there to prevent the water getting into the cylinder when priming.

5. $8^2 \times \cdot 7854 \times 145''$ = 7288·512; 30 × 10 × 5 = 1500; 7288·512 + 1500 = 8788·512 ÷ 3·6 = 2441·25 lbs.

6. $48^2 \times \cdot 7854$ (= 1809·5616) × 29 = 52477·2864; 100 : 24 :: 52477·2864 : 12594·548736 ÷ (12 × 10 =) 120 = 104·9545, &c., lbs. per square inch on the guide.

7. 5 — ·875 = 4·125; $8000 \times \cdot 875^2$ = 5359·375 ÷ 4·125 = 1299·242; $6^2 \times \cdot 7854$ = 28·2744; 1299·2424 ÷ 28·2744 = 45·95 lbs.

8. 29·00 ÷ 2·04 = 14·21 lbs. pressure.

9. P_1 = 13·25; P_2 = 17·75; ∴ (13·25 + 17·75) × 66^2 × 3·5 × 60 = 28357560 ÷ 140000 = 202·554 tons.

10. $66^2 \times \cdot 7854$ × 7 × 60 × 18·36 = 26381575·94688 ÷ 33000 = 799·44169, &c., H.P.

11. 202·554 ÷ 10 = 20·2554 tons of coal; 20·2554 × 2240 = 45372·0960 lbs. ÷ 24 = 1890·504, which divided by 79944 = 2·365 lbs. of coal per I.H.P.

EXAMINATION PAPER—No. XV, *Page* 472.

1. 3¼ + 8½ = $11\frac{3}{4}^{h}$ = $11^{h}{\cdot}75$; 3·25 × 8·5 = 27·625; 27·625 ÷ 11·75 = $2{\cdot}35106^{h}$ = $2^{h}\ 21^{m}\ 3^{s}{\cdot}8$.

2. 5 × 7·5 = 37·5; 4·14 × 2 = 8·28; ∴ 37·5 ÷ 8·28 = 4·529 tons.

3. 75·5 — 73·25 = 2·25 × 3·1416 = 7·0686 inches.

4. Total distance run 2165 miles; 3120 — 2165 = 955 miles yet to go; coals burnt 490 tons; 800 — 490 = 310, coals left for remainder of voyage; 2165 : 955 :: 490 : 216·14 tons, ∴ 310 — 216·14 = 93·86 tons remaining at the end of the voyage.

5. ·072 × ·072 × ·7854 (= ·004071536) ÷ ·75 = ·0054286; $1''{\cdot}00$ ÷ ·0054286 = 184·1 thickness.

EXAMINATION PAPER—No. X, *Page* 466.

1. £530 0s. $6\frac{1}{2}$d. 2. $156\frac{11}{16}$.

3. $64^2 \times {}^{\cdot}7854 = 3216{\cdot}9984$; $10^2 \times {}^{\cdot}7854 = 78{\cdot}54$; 40·96 sq. in. to 1 sq. in. of pipe.

4. £19 3s. 4d. 5. 12 tons.

6. $30 \div 2 = 15$ teeth; 1 pull $= 3$ teeth $\times 12 = 36$; $15 \times 12 = 180$; $36 : 180 :: 1 : 5^m$.

7. $9 \div 2 = 4\frac{1}{2}$; $4\frac{1}{2} - 2\frac{1}{8} = 2\frac{3}{8}$ open; $27 \times 2\frac{3}{8} = 64\frac{1}{8}$ area open for steam.

8. $23 \times 2240 \times 1\frac{1}{16} \times 2\ (= 109480) \div 140 = 782$; $782 \times \frac{70}{100} \div 6 = 91{\cdot}233$ lbs.

9. $5{\cdot}5 \div 4 = 1{\cdot}375$ area; $1{\cdot}375 \div 9 = {}^{\cdot}1528$ lift of valve.

10. $\frac{5}{8}$ of $4\frac{1}{2}$, or $\frac{5}{8}$ of $\frac{9}{2} = \frac{45}{16} = 2\frac{13}{16}$; $4\frac{9}{16} + 2\frac{13}{16} = 7\frac{6}{16}$ oz. per gallon.

11. $32^2 + 58^2 \div 30 = 1024 + 3364 = 4388 \div 30 = 146{\cdot}26$, &c., N.H.P.

EXAMINATION PAPER—No. XI, *Page* 467.

1. 89·3 tons $\times 2240 = 200032$ lbs.; $35 \times {}^{\cdot}7854 = 27{\cdot}489$; $200032 \div 27{\cdot}489 = 7276{\cdot}8002$ $\therefore \sqrt[2]{7276{\cdot}8002} = 85{\cdot}3$, diameter of piston.

2. $7 \times 2240\ (= 15680) \times {}^{\cdot}055 = 862{\cdot}4$; $3{\cdot}1416 \times \frac{14}{12}\ (= 3{\cdot}6652) \times 862{\cdot}4\ (= 3160{\cdot}86848)$ $\times 57 = 180169{\cdot}50336 \div 33000 = 5{\cdot}4897$ H.P.

3. $3{\cdot}1416 \times 9\ (= 28{\cdot}2744) \div 66\ (= 33'' \times 2) = {}^{\cdot}4284 \times {}^{\cdot}05 \times 100 = 2{\cdot}142$.

4. $1\frac{1}{2} \times 2240 \times 18 \times 6 = 362880 \div (51520 \times 1\frac{1}{4} =)\ 64400 = 5{\cdot}6348$ $\therefore \sqrt[2]{5{\cdot}6348} = 2{\cdot}37$.

5. $9{\cdot}5^3\ (= 857{\cdot}375) \times 51520 = 44171960$; $5{\cdot}1 \times 15 = 76{\cdot}5$; $44171960 \div 76{\cdot}5 = 577411{\cdot}24 \div 2240 = 257{\cdot}77$ tons.

6. $3 \times 2240 \times 24 \times 10{\cdot}2 = 1645056 \div (9^3 =)\ 729 = 2256{\cdot}59$.

7. $2{\cdot}5 - {}^{\cdot}875 \div 2{\cdot}5 = 1{\cdot}625 \div 2{\cdot}5 = {}^{\cdot}65$; $875^2\ (= {}^{\cdot}765625) \times {}^{\cdot}7854 \times 200 = 120{\cdot}264375 \div (2\frac{1}{2} \times \frac{5}{8} =)\ 1{\cdot}5625 = 76{\cdot}973$; $1{\cdot}625 \div 2{\cdot}5 \times 100 = 65{\cdot}0$.

8. $18{\cdot}9166 \times 19\ (= 359{\cdot}4154) \times 10{\cdot}333\ (= 3713{\cdot}8393282) \times \frac{2}{3} = 2475{\cdot}8928855 \times 62{\cdot}5 = 154743{\cdot}3 \div 12000 \times 5{\cdot}1 = 65{\cdot}7659025$; 12 cwt. $\times 112 = 1344$ lbs., $\therefore 1344 : 65{\cdot}766 :: 60^m : 2^m{\cdot}936$, or $2^m\ 56^s{\cdot}16$.

9. Top 12·22 lbs.; bottom 12·56 lbs.; mean 12·39 lbs.

10. $1{\cdot}437 \times 2 + {}^{\cdot}1875 = 3{\cdot}0625 \div 6{\cdot}875 = {}^{\cdot}4454$; ${}^{\cdot}4454^2 \times 27 = 5{\cdot}356287$.

EXAMINATION PAPER—No. XII, *Page* 468.

1. ${}^{\cdot}012 + 2{\cdot}715 = 2{\cdot}727$.

2. $0{\cdot}128^2 = {}^{\cdot}0163840$; ${}^{\cdot}0163840 \div 6{\cdot}814 = {}^{\cdot}00240$.

3. £11 0s. 0d. + £5 2s. 0d. + £4 17s. 6d. = £20 19s. 6d.

4. 600 lbs. — 60 lbs. = 540 lbs. 540 lbs. at 4d. per lb. = £9; discount at 8 per cent. = 14s. 4·8d., whence cost = £8 15s. 7·2d.

5. From May 19th to July 2nd is 44 days; from $7^h\ 15^m$ A.M. to $7^h\ 15^m$ P.M. is 12 hours; and from $7^h\ 15^m$ P.M. to $10^h\ 30^m$ P.M. is $3^h\ 15^m$. Total $44^d\ 15\frac{1}{4}^h = 44^d{\cdot}6354$. Mean consumption per day $= 480 \div 44{\cdot}6354 = 10{\cdot}757$ tons. Mean speed per day $= 13891 \div 44{\cdot}6354 = 311{\cdot}21$ knots.

6. £120 + 170 + 85 = £375 per year, ∴ 170 N.H.P. : 1 N.H.P. :: £375 : £2·2057 = £2 4s. 1·3680d.

7. 37 tons $\times 35 = 1295$ cubic feet. $1'' = \frac{1}{12}$ of foot $= {}^{\cdot}08333$; $1295 \div {}^{\cdot}08333 = 15540{\cdot}621$ sq. feet.

8. $34^2 = 34 \times 34 = 11560$, $\therefore 11560 \div 1{\cdot}5 = 770{\cdot}66$ feet above the sea.

9. Even keel before taking in bunkers $= 19{\cdot}7$ ft. $+ 19{\cdot}4$ ft. $\div 2 = 19{\cdot}55$ feet; even keel after coaling $= 21{\cdot}0$ ft. $+ 20{\cdot}8$ ft. $\div 2 = 20{\cdot}9$ ft., $\therefore 20{\cdot}9 - 17{\cdot}55 = 1{\cdot}35$ ft.; $4821 \times 1{\cdot}35 = 6508{\cdot}35$; $6508{\cdot}35 \div 35 = 185{\cdot}95$ tons taken in.

10. $112 + 48 = 160$; $160 : 112 :: 5$ oz. : 3·5 oz.

11. 3·6 inches = 1 lb. = 16 oz.; 5 oz. $= \frac{5}{16}$ lb. $= {}^{\cdot}3125$; $7'' \times 4'' = 28$ sq. inch; ${}^{\cdot}3125 \times 3{\cdot}6 = 11{\cdot}2500$; $11{\cdot}2500 \div 28 = {}^{\cdot}04017$ of an inch; ${}^{\cdot}04017 \times 32 = 1{\cdot}28544$.

EXAMINATION PAPER—No. XIII, *Page* 469.

1. $10{\cdot}5^2 \times 18\ (= 1984{\cdot}5) \div 20 = 99{\cdot}225$; $\sqrt[2]{99{\cdot}225} = 9{\cdot}96$ knots.

2. $60^2 \times 18\ (= 64800) \div 19{\cdot}6 = 3306{\cdot}122$; $\sqrt[2]{3306{\cdot}122} = 57{\cdot}5$ rev.; $60^2 : 57{\cdot}5^2 :: 11 : 10{\cdot}1$ knots.

3. $12000 \times 15 \div 6 = 30000$; $2240 \times 60 \div 30000 = 4{\cdot}48$ inches.

4. $5^2 \times 3 \div 6 = 12{\cdot}5$; $5000 \times 60 = 300000 \div 12{\cdot}5 = 24000$ lbs.

5. $4 \times 8 \times 12^2 \div 16 = 288$ cwt., the breaking strain, and $\frac{1}{10}$ of $288 = 28{\cdot}8$ cwt., the working strain.

6. $50 : 58 :: 60^2 \div 4176 : \sqrt[2]{4176} = 64{\cdot}6$ revolutions.

7. $29 : 32 :: 10^2 : 110{\cdot}3348$, $\therefore \sqrt[2]{110{\cdot}3348} = 10{\cdot}5$ knots.

8. $12{\cdot}25 \times {\cdot}75$ ($= 9{\cdot}1875$, section of plate) $\times 23 = 211{\cdot}3125$ tons; $875^2 \times {\cdot}7854 = {\cdot}601321875 \times 2 = 1{\cdot}20264375 \div ({\cdot}75 \times 2{\cdot}5) = 1{\cdot}875 = {\cdot}64141$; $211{\cdot}3125 \times {\cdot}64141 = 135{\cdot}5379506$, &c., tons.

9. Length of crank-pin $= 13^2 = 169$ sq. inches; area of L.P. piston $= 72^2 \times {\cdot}7854 = 4071{\cdot}5136$; area of H.P. piston $= \frac{1}{4}$ of $4071{\cdot}5136 = 1017{\cdot}8784$; pressure on low pressure piston $= 4071{\cdot}5136 \times 18 = 73287{\cdot}2448$; pressure on high pressure piston $= 1017{\cdot}8784 \times 54 = 54965{\cdot}4336$; pressure on low pressure pin $= 73287{\cdot}2448 \div 169 = 433{\cdot}652$ lbs.; pressure on high pressure pin $= 54965{\cdot}4336 \div 169 = 325{\cdot}23$, &c., lbs.

10. $7{\cdot}1 \times 12 \div 62{\cdot}5 = 1{\cdot}3632$ evaporation in cubic feet of water; $1{\cdot}3632 \times 512 = 697{\cdot}9584$ = cubic feet of steam; $697{\cdot}9584 : 3{\cdot}1 :: 3600 : 15{\cdot}99$ seconds.

11. Top $16{\cdot}92$; bottom $18{\cdot}60$; mean $17{\cdot}76$ lbs.

EXAMINATION PAPER—No. XIV, *Page* 470.

1. $48 + 28 = 76$; $112^\circ \times 28 = 3136$; $48 \times 200 = 9600$; $167^\circ{\cdot}59$ Fahrenheit.

2. $1\frac{1}{3}$. 3. A's share, £99; B's share, £43; C's share, £38.

4. $12^2 \times {\cdot}7854 \times 2 = 226{\cdot}1952 \div ({\cdot}25 \times 6 =)\ 1{\cdot}5 = 150{\cdot}797$, number of slots. The slots are placed there to prevent the water getting into the cylinder when priming.

5. $8^2 \times {\cdot}7854 \times 145'' = 7288{\cdot}512$; $30 \times 10 \times 5 = 1500$; $7288{\cdot}512 + 1500 = 8788{\cdot}512 \div 3{\cdot}6 = 2441{\cdot}25$ lbs.

6. $48^2 \times {\cdot}7854\ (= 1809{\cdot}5616) \times 29 = 52477{\cdot}2864$; $100 : 24 :: 52477{\cdot}2864 : 12594{\cdot}548736 \div (12 \times 10 =)\ 120 = 104{\cdot}9545$, &c., lbs. per square inch on the guide.

7. $5 - {\cdot}875 = 4{\cdot}125$; $8000 \times {\cdot}875^2 = 5359{\cdot}375 \div 4{\cdot}125 = 1299{\cdot}242$; $6^2 \times {\cdot}7854 = 28{\cdot}2744$; $1299{\cdot}2424 \div 28{\cdot}2744 = 45{\cdot}95$ lbs.

8. $29{\cdot}00 \div 2{\cdot}04 = 14{\cdot}21$ lbs. pressure.

9. $P_1 = 13{\cdot}25$; $P_2 = 17{\cdot}75$; $\therefore (13{\cdot}25 + 17{\cdot}75) \times 66^2 \times 3{\cdot}5 \times 60 = 28357560 \div 140000 = 202{\cdot}554$ tons.

10. $66^2 \times {\cdot}7854 \times 7 \times 60 \times 18{\cdot}36 = 26381575{\cdot}94688 \div 33000 = 799{\cdot}44169$, &c., H.P.

11. $202{\cdot}554 \div 10 = 20{\cdot}2554$ tons of coal; $20{\cdot}2554 \times 2240 = 45372{\cdot}0960$ lbs. $\div 24 = 1890{\cdot}504$, which divided by $79944 = 2{\cdot}365$ lbs. of coal per I.H.P.

EXAMINATION PAPER—No. XV, *Page* 472.

1. $3\frac{1}{4} + 8\frac{1}{2} = 11\frac{3}{4}^{\text{h}} = 11^{\text{h}}{\cdot}75$; $3{\cdot}25 \times 8{\cdot}5 = 27{\cdot}625$; $27{\cdot}625 \div 11{\cdot}75 = 2{\cdot}35106^{\text{h}} = 2^{\text{h}}\ 21^{\text{m}}\ 3^{\text{s}}{\cdot}8$.

2. $5 \times 7{\cdot}5 = 37{\cdot}5$; $4{\cdot}14 \times 2 = 8{\cdot}28$; $\therefore 37{\cdot}5 \div 8{\cdot}28 = 4{\cdot}529$ tons.

3. $75{\cdot}5 - 73{\cdot}25 = 2{\cdot}25 \times 3{\cdot}1416 = 7{\cdot}0686$ inches.

4. Total distance run 2165 miles; $3120 - 2165 = 955$ miles yet to go; coals burnt 490 tons; $800 - 490 = 310$, coals left for remainder of voyage; $2165 : 955 :: 490 : 216{\cdot}14$ tons, $\therefore 310 - 216{\cdot}14 = 93{\cdot}86$ tons remaining at the end of the voyage.

5. ${\cdot}072 \times {\cdot}072 \times {\cdot}7854\ (= {\cdot}004071536) \div {\cdot}75 = {\cdot}0054286$; $1''{\cdot}00 \div {\cdot}0054286 = 184{\cdot}1$ thickness.

6. 70^s : 3600^s :: 85 lbs. : 4371·43; 4371 lbs. : 10500 lbs. :: 1″ : 2·4019; 3·1416 × 6 = 18·8496, ∴ 2·4019 ÷ 18·8496 = ·127 or $\frac{1}{8}$″ lift of valve.

7. 2 × 2240 = 4480 ÷ 6 = 746·66 lbs.

8. 100 — 10 = 90; ·9 + ·81 + ·729 + ·6561 + ·59049 + ·531441 = 4·217031; 4 tons × 2240 = 8960 lbs.; 8960 = 4·217 = 1062·365.

9. Diameter of valve 6 ÷ 4 = 1·5; 1·5 ÷ 4 = ·375; 65 + 15 = 80 lbs.; 2″ : ·375″ :: 65 lbs. : 12·1875 extra, ∴ 65 + 12·1875 = 77·1875 lbs.

EXAMINATION PAPER—No. XVI, *Page* 473.

1. 30. 2. $\frac{87}{128}$. 3. $2\frac{4}{15}$. 4. 7·29 days and 5s. $6\frac{1}{2}$d., nearly.

5. 100 — 15 = 85 efficient; work to be done = 4·4167 × 23·5 × 67 = 6954·09415; work per minute done by pump 8^2 × ·7854 × 11 (= 552·9216) × 142 × ·85 = 66737·637120 ÷ 1728 = 38·621318, ∴ time required to pump out 6954·094150 ÷ 38·62132 = 180^m·057 = 3^h 0^m 3^s·4.

6. 5′ 10″ — 3″ = 67″ ÷ 20 = 3·35, pitch one way; 4′ 9″ — 3″ = 4′ 6″ = 54 ÷ 17 = 3·1765, pitch the other way; number of bolts 74.

7. 4907 × 29·5 = 144756·5 ÷ (9057 + 80) 15·842.

8. $·875^2$ = 2·5 × √50 = ·875 × ·875 × 2·5 (= 1·9140625) × 7·071 = 13·5343359375, cubic feet blown out per minute; cubic feet to be blown out 120 × $\frac{6}{12}$ = 60 cubic feet, ∴ 60 ÷ 13·5343 = 4^m·4332, or 4^m 25^s·99.

9. 18 : 12 :: 800 : 533·333, &c.; 1^3 : $·75^3$ :: 533·333 : 224·999, &c., or 225 lbs.

10. 8^3 × ·5236 × 257 = 68·8973, &c., lbs.

11. Cubic feet each stroke 32^2 × ·7854 × 16·5 ÷ 1728 = 7·67946; number of strokes 200 ÷ 7·67946 = 156·26 strokes.

12. 200 ÷ ·4536 = 440·917; £9 10s. 4d. = 2284d., ∴ 2284 ÷ 440·917 = 5·18d., nly.

EXAMINATION PAPER—No. XVII, *Page* 475.

1. 2.

2. 7963·7 × 674·3 (= 5369922·91) ÷ ·7854 = 6837182·21; extracting $^2\surd$ given diameter of cube 2614·8; radius = 1307·4.

3. 17 × 2 × 4 × 42 ÷ 112 = 51 cwt.; 51 cwt. at 7s. 6d. gives £95 12s. 6d.

4. Length of shaft without flanges 304″, contents of shaft 13^2 × ·7854 × 304 = 40350·7104; contents of 2 flanges 27^2 × ·7854 × 8 = 4580·4528; 4580·4528 + 40350·7104 = 44931·1632 — 461·8152 = 44469·3480 ÷ 3·6 = 12352·6 lbs., or 5 tons 10 cwt. 1 qr. 4·6 lbs.

5. One revolution of counting wheel is 1000000, therefore 2 revolutions = 2000000; then from 007436 to zero = 1000000 — 007436 = 992564, and it now stands at 000624; hence 2000000 + 992564 + 624 = 2993188 revolutions, which multiplied by pitch 21 gives 62856948 and divided by 6080 gives 10338·3 knots.

6. 32 + 29 × 4 + 24 = 32 + 116 + 24 = 172 ÷ 6 = 28·6667 × (30 × 20 =) 600 = 17200 ÷ 45 = 382·2 tons.

7. 8^2 × 14 ÷ 13 = 68·92306; √68·92306 = 8·3 knots.

8. 28″ × 9″ = 252″; 28″ × 1″·5 = 42″; 252 + 42 = 294 × (36 + 15 — 4 =) 47 = 13818 lbs., or 6 tons 3 cwt. 2 qrs. 14 lbs. pressure.

9. 110 : 34 :: 400 : 123·64, ∴ 400 — 123·64 = 276·36; the greatest weight must be placed in forward one, and the less weight in the aft one.

10. 3·1416 × 30·5 × 20 × 60 = 114982·560 ÷ 608 = 18·9116 — 15 = 3·9116 slip; 18·9116 : 100 :: 3·9116 : 20·6 per cent.

EXAMINATION PAPER—No. XVIII, *Page* 477.

1. $(23·7 — 7)^3$ = 4657·463 ÷ (40 — 36) = 1164·36575 + 324 = 1488·36575.

2. $·875^2$ × ·7854 (= ·60120224) × 2 = 1·20240448 ÷ (·75 × 2·5) = 6418.

EXAMINATION PAPER—No. XXII, *Page* 485.

1. 72284·43. 2. $22\frac{1}{8}$ inches. 3. £5 19s. 2d. 4. About 4″, or $\frac{3}{8}$ stroke.
5. As 1 : 746, &c. 6. 294°·8 Cent. 7. 7·16 tons. 8. 686·6965, &c., lbs.
9. 85·584 lbs. 10. 50·37. 11. Working to 4 places of decimals, 50·37.
12. 1080 lbs. 13. ·442 of a gallon.

EXAMINATION PAPER—No. XXIII, *Page* 487.

1. 1187. 2. $1\frac{11}{20}$. 3. 4 times and remainder £12 14s. 4. 8·617.
5. 1·5708. 6. 4·518, &c. 7. 80·336, &c. 8. 120·802 lbs.
9. 1st, £38 14s. $1\frac{487}{727}$d.; 2nd, £65 13s. $4\frac{89}{727}$d. 10. 17·4 bolts.
11. 78·335. 12. 4388·29 lbs. 13. 52505·414 thermal units.
14. 381·3795 lbs. 15. 436·8 tons.

EXAMINATION PAPER—No. XXIV, *Page* 488.

1. 20 — 400 = 380.

2. 20′ × 12″ × 8 = 1920 ÷ 19000 = ·101″ stretch.

3. 90 × 90 (= 8100) × 3 (= 24300) ÷ 5870 = 4·13, ∴ 4·13 × 14000 (= 5782·00) ÷ 5000 (= 11·564) ÷ 2 = 5·782, area of 1 bolt; 5·782 ÷ ·7854 = 7·3745, and $\sqrt{7{\cdot}3745}$ = 2″·71 diameter of bolt.

4. ·07 × 60 = 4·2; coals per hour = 62·5 × 4·2 (= 262·50) ÷ 8 = 32·81 lbs.; feed used as steam per minute = 8 × 10 (= 80) ÷ 60 = 1·33 lbs. NOTE.—1 cubic foot fresh water = 62·5 lbs.

5. New pressure = old pressure + 20 per cent. = 100 + 20 = 120; 120 : 100 :: 12 : 10 tons.

6. 100000 — 00734 = 99266; 99266 + 200000 + 21211 = 320477 × 20 = 6409540 ÷ 6080 = 1054·2 knots.

7. 6·33 + 1 (= 7·33) × 36 = 263·88; ·375 × ·375 (= ·140625) × 90000 (= 12656·25) ÷ 263·88 = 47·9 lbs., safe working pressure. Again, ·375 × 8000 (= 3000) ÷ 36 = 83·3 lbs.

8. 300 × ·3 (= 90) + 1115 = 1205, total heat, ∴ total heat 1205 — 300 sensible heat = 905, latent heat.

9. 1115 + (·3 × 212) — 212 = 1115 + 63·6 — 212 = 966·6, the latent heat of steam; 300 × ·3 = 90·0 + 1115 — 110 = 1095, No. of units of heat to each lb. of water evaporated, ∴ 1095 : 966·6 :: 8 lbs. : 7·06 lbs. of water.

10. 28 tons × 2240 = 62720 lbs. of coal; 36 × 6 (= 216) × 48 = 10368 lbs. of ashes, ∴ 62720 : 10368 :: 100 : 16·5, the rate per cent. of coal lost.

11. Time, $5^{d}\ 17^{h} = 8220^{m}$; 1000000 — 918376 = 81624; then 81624 + 43721 + 1000000 = 1125345; 1125345 ÷ 8220 = 136·9 revolutions per minute.

12. $88 - \frac{220 \times 1}{2 \times 2 + 3 \times 1} = 88 - \frac{220}{7} = 88 - 31{\cdot}4 = 56{\cdot}6$ per cent. of strength.

13. 20 × 80 (= 1600) ÷ 100 (= 16) × 80 = 1280; 32 × 500 ÷ 100 = 160; 160 × 33000 = 5280000 ÷ 1280 = 4125 lbs.

EXAMINATION PAPER—No. XXV.

1. NOTE.—Allow 32 circular inches per N.H.P. $32^2 + 63^2 \div 32$ = 1024 + 3969 (= 4993) ÷ 32 = 156 N.H.P. 12 × 18 = 216 sq. inches. Diameter H.P. cylinder squared = 32^2 = 1024; 1024 × ·7854 (= 804·2496) × 60 (= 48254·9760) ÷ 216 = 223·4 lbs., the strain per sq. inch on crank-pin.

2. Diff. of consumption = 11 — 10·8 = 0·2 tons. Knots per day = 8·4 × 24 = 201·6; time of voyage = 7000 ÷ 201·6 = 34·7 days. Tons saved in consumption on a run of 7000 miles = 34·7 × ·2 = 6·94 tons. Coals carried = 2200 tons — 1800 tons = 400 tons, ∴ 400 tons + 6·944 tons = 406·94 tons, the total quantity saved.

10. 1800 × 8 ÷ 62·5 = 230·4 cubic feet of feed; double feed = 230·4 × 2 = 460·8; 60 lbs. (gauge) + 15 lbs. (atmosphere) = 75 lbs.; $\sqrt{75} = 8\cdot64$; 8·64 × 800 = 6912; 460·81 ÷ 6912 = ·0666; ·0666 ÷ ·7854 = ·0848; $\sqrt{\cdot0848} = \cdot29$ of an inch diameter.

EXAMINATION PAPER—No. XXVII.

1. $(14 - \cdot07)^2 = 13\cdot93^2 = 194\cdot0449$; $10 - \cdot003 = 9\cdot997^2 = 99\cdot970002$; 194·0449 — 99·970002 = 94·074891.

2. 6 ft. 4½ in. = 76·5; 2 ft. 1 in. = 25 in., ∴ 76·5 × 25 = 1912·5 sq. in. ÷ 144 = 13·28125 sq. feet.

3. 82 of tin, 8·28 antimony, 8·28 copper = 98·56 lbs. of metal.

4. 28 × 10 ÷ 12 = 23·3 lbs.; 23·3 lbs. of tin at 1s. 3d. = £1 9s. 2d.; 2⅓ lbs. of antimony at 3s. 4d. = 7s. 9⅓d.; 2⅓ lbs. of copper at 7d. per lb. = 1s. 4⅓d.; total £1 18s. 3⅔d.

5. 296 × 336 = 99456; $\sqrt{99456} = 315\cdot3$ lbs.

6. Gauge 60 lbs. + (atmosphere) 15 lbs. = 75 lbs.; $\frac{75 + 1600}{75 + 1} = \frac{1675}{76} = 22\cdot0394$ of steam to $\frac{1}{76}$th of water; cut off = 22″ + (clearance) = 1″·5 = 23·5 lbs., ∴ 23·5000 ÷ 22·0394 = 1·06.

7. $\frac{6 \times 3 \times 800}{80 \times \cdot875 \times 2\cdot5} = \frac{14400}{153\cdot125} = 94$ feet per second.

9. 17 ft. : 8 ft. :: 4 tons : 1·882 tons = 1 ton 17 cwt. 2 qrs. 15·68 lbs.; 17 ft. : 3·5 ft. :: 4 tons : 0·823 tons = 16 cwt. 1 qr. 23·52 lbs.; this weight on B = 4 tons — 1 ton 17 cwt. 2 qrs. 15·6 lbs. = 2 tons 2 cwt. 1 qr. 12·4 lbs. + 16 cwt. 1 qr. 23·5 lbs. + 8 cwt. 2 qrs. (½ weight of beam) = 3 tons 7 cwt. 1 qr. 7·9 lbs.

10. $\frac{C^2}{24} = \cdot4$ of a ton; $C^2 = 24 \times \cdot4 = 9\cdot6$; $\sqrt{9\cdot6} = 3\cdot098$, girth; 100 — 75 = 25; 100 : 25 :: 9·6 : 2·4 lbs.; then 9·6 — 2·4 = 7·2 tons, the breaking strain.

EXAMINATION PAPER—No. XXVIII.

1. $341^\circ\cdot6$. 2. 496. 3. 3·19. 4. £17 4s.

6. 159·6 tons; area 5880 sq. feet. 7. 5288 lbs. per sq. inch.

8. $(66^2 - 32^2) \times \cdot7854$ (= 2616·9528) × 22 (= 57572·9616) × 7 × 50 × 2 = 40301073·1200 ÷ 3300 = 1221·245 I.H.P.; consumption in 24 hours = 1221·24 × 23 × 24 ÷ 8¼ = 81712·37 lbs. ÷ 2240 = 36·478 tons.

TABLE I.

Showing the weight of the atmosphere, in lbs. avoirdupois, on 1 square inch, corresponding with different heights of the barometer, from 28 inches to 31 inches, varying by tenths of an inch.

Barometer in inches.	Atmosphere in lbs.	Barometer in inches.	Atmosphere in lbs.	Barometer in inches.	Atmosphere in lbs.
28·0	13·72	29·0	14·21	30·0	14·70
28·1	13·77	29·1	14·26	30·1	14·75
28·2	13·82	29·2	14·31	30·2	14·80
28·3	13·87	29·3	14·36	30·3	14·85
28·4	13·92	29·4	14·41	30·4	14·90
28·5	13·97	29·5	14·46	30·5	14·95
28·6	14·02	29·6	14·51	30·6	15·00
28·7	14·07	29·7	14·56	30·7	15·05
28·8	14·12	29·8	14·61	30·8	15·10
28·9	14·17	29·9	14·66	30·9	15·15
				31·0	15·19

TABLE II.

Showing the elastic force and temperature of steam, at temperatures from 32° to 387°·3 Fahrenheit, varying by 5° of temperature up to the boiling point, then by ½ lbs. of pressure on the square inch up to 25 lbs., then by lbs. of pressure up to 85 lbs., and then by 5 lbs. of pressure up to 200 lbs.

Temperature.	Pounds per sq. inch	Temperature.	Pounds per sq. inch	Temperature.	Pounds per sq. inch	Temperature.	Pounds per sq. inch
32·	·098	205·	12·8	264·3	37·	309·3	73·
35·	·108	210·	14·13	265·9	38·	310·3	74·
40·	·129	211·	14·41	267·5	39·	311·2	75·
45·	·155	212·	14·7	269·1	40·	312·2	76·
50·	·184	212·8	15·	270·6	41·	313·1	77·
55·	·217	214·5	15·5	272·1	42·	314·	78·
60·	257	216·3	16·	273·6	43·	314·9	79·
65·	302	218·	16·5	275·	44·	315·8	80·
70·	·353	219·6	17·	276·4	45·	316·7	81·
75·	·417	221·2	17·5	277·8	46·	317·6	82·
80·	·49	222·7	18·	279·2	47·	318·4	83·
85·	·573	224·2	18·5	280·5	48·	319·3	84·
90·	·666	225·6	19·	281·9	49·	320·1	85·
95·	·774	227·1	19·5	283·2	50·	324·3	90·
100·	·911	228·5	20·	284·4	51·	328·2	95·
103·	1·	229·9	20·5	285·7	52·	332·	100·
105·	1·068	231·2	21·	286·9	53·	335·8	105·
110·	1·240	232·5	21·5	288·1	54·	339·2	110·
115·	1·431	233·8	22·	289·3	55·	342·7	115·
120·	1·632	235·1	22·5	290·5	56·	345·8	120·
125·	1·857	236·3	23·	291·7	57·	349·1	125·
130·	2·129	237·5	23·5	292·9	58·	352·1	130·
135·	2·450	238·7	24·	294·2	59·	355·	135·
140·	2·813	239·9	24·5	295·6	60·	357·9	140·
145·	3·100	241·	25·	296·9	61·	360·6	145·
150·	3·636	243·3	26·	298·1	62·	363·4	150·
155·	4·116	245·5	27·	299·2	63·	366·	155·
160·	4·635	247·6	28·	300·3	64·	368·7	160·
165·	5·23	249·6	39·	301·3	65·	371·1	165·
170·	5·94	251·6	30·	302·4	66·	373·6	170·
175·	6·67	253·6	31·	303·4	67·	376·	175·
180·	7·42	255·5	32·	304·4	68·	378·4	180·
185·	8·33	257·3	33·	305·4	69·	380·6	185·
190·	9·31	259·1	34·	306·4	70·	382·9	190·
195·	10·4	260·9	35·	307·4	71·	384·7	195·
200·	11·58	262·6	36·	308·4	72·	387·3	200·